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Design considerations for a cylindrical pressure vessel with a spherical launching lens

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Abstract

This paper presents analytical calculations for a switch system surrounded by a uniform spherical launching lens. The height of the pressure vessel and the relative dielectric constant of the launching lens are determined as a function of the pressure vessel radius. A simple transmission line model is used to calculate the transmission coefficient for a wave propagating through the switch system and the launching lens.

1 Introduction

The switch system consists of the switch cones, hydrogen chamber and pressure vessel [1, 2]. Various switch and guiding structure configurations were investigated in [3–6]. This paper explores the integration of the hydrogen chamber and pressure vessel with the switch cones. The design of a launching lens, surrounding the pressure vessel, is also explored. One of the more important features of the switch designs in [3–6] is that the geometric center of the switch cones is the first focal point. This allows for the use of a uniform spherical launching lens; compared to the more complex designs in [7–10]. Two parameters are analytically investigated in this paper,

1. The radius (and height) of the pressure vessel.
2. The relative permittivity of the spherical launching lens.

2 Design of the pressure vessel and launching lens

Figure 2.1 shows the setup of a cylindrical pressure vessel with a spherical launching lens. The objective is to determine the optimum dimensions of the pressure vessel which leads to a practically reasonable relative dielectric constant for the launching lens. The following notations are used,

$\epsilon_{r_{\text{hc}}} = 1.0$	= relative permittivity of hydrogen chamber
$\epsilon_{r_{\text{pv}}} = 3.7$	= relative permittivity of pressure vessel
$\epsilon_{r_{ll}}$	= relative permittivity of launching lens; to be determined
θ	= switch cone half-angle
θ_i	= incidence angle for ray OA travelling from hydrogen chamber into pressure vessel
θ_t	= transmitted angle for ray AB travelling from hydrogen chamber into pressure vessel
h_{sw}	= height of switch cone
r_{sw}	= radius of switch cone
h_{hc}	= height of hydrogen chamber
r_{hc}	= radius of hydrogen chamber
h_{pv}	= (half-)height of pressure vessel
r_{pv}	= radius of pressure vessel
r_{ll}	= radius of spherical launching lens
h_{swgp}	= height of switch gap

For a 200 Ω bicone switch immersed in the pressure vessel medium, $Z_c = (200\ \Omega/\sqrt{\epsilon_{r_{\text{pv}}}})$. Therefore, the half-angle of the switch cones is $\theta = \frac{\pi}{2} - \theta_i = 45.58°$.

We require the ray travelling along the edge of the switch cone, ray OA, to be refracted such that it takes path AB where B is the edge of the pressure vessel. Therefore, we have from Snell's law,

$$\sqrt{\epsilon_{r_{\text{hc}}}}\sin(\theta_i) = \sqrt{\epsilon_{r_{\text{pv}}}}\sin(\theta_t) \Rightarrow \theta_t = \arcsin\left(\sqrt{\frac{\epsilon_{r_{\text{hc}}}}{\epsilon_{r_{\text{pv}}}}}\cos(\theta)\right); \tag{2.1}$$

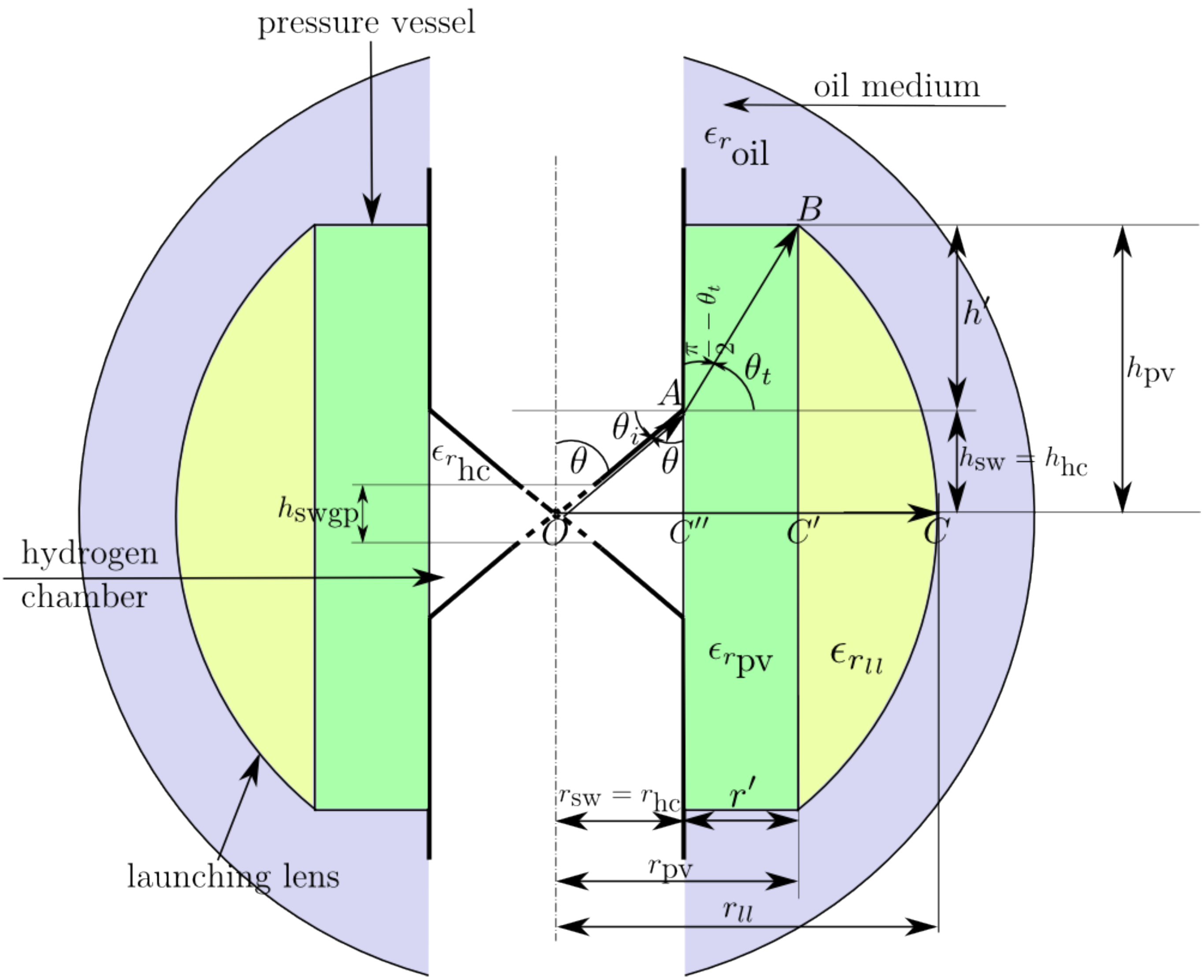

Figure 2.1: Diagram for cylindrical pressure vessel and launching lens calculations.

Also,

$$\tan(\theta) = \frac{r_{\text{hc}}}{h_{\text{hc}}} = \frac{r_{\text{sw}}}{h_{\text{sw}}} \Rightarrow h_{\text{hc}} = r_{\text{hc}} \cot(\theta) \tag{2.2}$$

$$\tan(\theta_t) = \frac{h'}{r'} \Rightarrow \theta_t = \arctan\left(\frac{h'}{r'}\right) \tag{2.3}$$

Further,

$$r' = r_{\text{pv}} - r_{\text{hc}} = \text{ “thickness” of the pressure vessel} \tag{2.4}$$

$$h' = h_{\text{pv}} - h_{\text{hc}} = h_{\text{pv}} - r_{\text{hc}} \cot(\theta) \tag{2.5}$$

Substituting (2.4) and (2.5) in (2.3),

$$\theta_t = \arctan\left(\frac{h_{\text{pv}} - h_{\text{hc}}}{r_{\text{pv}} - r_{\text{hc}}}\right) \tag{2.6}$$

From (2.1) and (2.6)

$$\arctan\left(\frac{h_{\text{pv}} - h_{\text{hc}}}{r_{\text{pv}} - r_{\text{hc}}}\right) = \arcsin\left(\sqrt{\frac{\epsilon_{r_{\text{hc}}}}{\epsilon_{r_{\text{pv}}}}}\cos(\theta)\right) \tag{2.7}$$

$$\Rightarrow \frac{h_{\text{pv}} - r_{\text{hc}}\cot(\theta)}{r_{\text{pv}} - r_{\text{hc}}} = \tan\left(\arcsin\left(\sqrt{\frac{\epsilon_{r_{\text{hc}}}}{\epsilon_{r_{\text{pv}}}}}\cos(\theta)\right)\right) \tag{2.8}$$

Therefore, the height of the pressure vessel can be determined as a function of its radius, i.e.,

$$h_{\text{pv}} = [r_{\text{pv}} - r_{\text{hc}}]\tan\left(\arcsin\left(\sqrt{\frac{\epsilon_{r_{\text{hc}}}}{\epsilon_{r_{\text{pv}}}}}\cos(\theta)\right)\right) + r_{\text{hc}}\cot(\theta) \tag{2.9}$$

To determine the relative permittivity of the spherical launching lens, the equal time condition must be satisfied, i.e.,

$$OA\sqrt{\epsilon_{r_{\text{hc}}}} + AB\sqrt{\epsilon_{r_{\text{pv}}}} = OC''\sqrt{\epsilon_{r_{\text{hc}}}} + C''C'\sqrt{\epsilon_{r_{\text{pv}}}} + C'C\sqrt{\epsilon_{r_{ll}}} \tag{2.10}$$

From Fig. 2.1,

$$\begin{aligned}
OC'' &= r_{\text{sw}} = r_{\text{hc}} \\
\sin(\theta) &= \frac{OC''}{OA} \Rightarrow OA = \frac{r_{\text{hc}}}{\sin(\theta)} \\
C''C' &= r' = r_{\text{pv}} - r_{\text{hc}} \\
\sin(\theta_t) &= \frac{h'}{AB} \Rightarrow AB = \frac{h_{\text{pv}} - r_{\text{hc}}\cot(\theta)}{\sin(\theta_t)} \\
OB^2 &= OC'^2 + C'B^2 \Rightarrow r_{ll} = \sqrt{r_{\text{pv}}^2 + h_{\text{pv}}^2} \\
C'C &= r_{ll} - r_{\text{pv}} = \sqrt{r_{\text{pv}}^2 + h_{\text{pv}}^2} - r_{\text{pv}}
\end{aligned} \tag{2.11}$$

Substituting (2.11) in (2.10),

$$\left[\frac{r_{\text{hc}}}{\sin(\theta)}\right]\sqrt{\epsilon_{r_{\text{hc}}}} + \left[\frac{h_{\text{pv}} - r_{\text{hc}}\cot(\theta)}{\sin(\theta_t)}\right]\sqrt{\epsilon_{r_{\text{pv}}}} = r_{\text{hc}}\sqrt{\epsilon_{r_{\text{hc}}}} + [r_{\text{pv}} - r_{\text{hc}}]\sqrt{\epsilon_{r_{\text{pv}}}} + \left[\sqrt{r_{\text{pv}}^2 + h_{\text{pv}}^2} - r_{\text{pv}}\right]\sqrt{\epsilon_{r_{ll}}} \tag{2.12}$$

$\therefore \epsilon_{r_{ll}}$ is determined as

$$\epsilon_{r_{ll}} = \left[\frac{[\csc(\theta) - 1]\, r_{\text{hc}}\sqrt{\epsilon_{r_{\text{hc}}}} + \left[\left[\frac{h_{\text{pv}} - r_{\text{hc}}\cot(\theta)}{\sin(\theta_t)}\right] - [r_{\text{pv}} - r_{\text{hc}}]\right]\sqrt{\epsilon_{r_{\text{pv}}}}}{\sqrt{r_{\text{pv}}^2 + h_{\text{pv}}^2} - r_{\text{pv}}}\right]^2 \tag{2.13}$$

$$\epsilon_{r_{ll}} = \left[\frac{[\csc(\theta) - 1]\, r_{\text{hc}}\sqrt{\epsilon_{r_{\text{hc}}}} + \left[\left[\frac{h_{\text{pv}} - r_{\text{hc}}\cot(\theta)}{\sqrt{\epsilon_{r_{\text{hc}}}/\epsilon_{r_{\text{pv}}}}\cos(\theta)}\right] - [r_{\text{pv}} - r_{\text{hc}}]\right]\sqrt{\epsilon_{r_{\text{pv}}}}}{\sqrt{r_{\text{pv}}^2 + h_{\text{pv}}^2} - r_{\text{pv}}}\right]^2 \tag{2.14}$$

3 Discussion

Equations (2.9) and (2.14) are plotted as a function of r_{pv} in Fig. 3.1. Only specific regions of the h_{pv} and $\epsilon_{r_{ll}}$ curves lead to practically acceptable solutions. In these regions, the curves satisfy the following two constraints,

1. The use of a cylindrical pressure vessel mandates the need of cylindrical (guiding) structures over the switch cones as shown in Fig. 3.2. These cylindrical structures, of height H_{css}, are required to provide structural support to the pressure vessel. They also serve to guide the waves originating from the source. It is evident that H_{css} must be constrained such that $H_{\mathrm{css}} + h_{\mathrm{sw}} \geq h_{\mathrm{pv}}$. For the discussion that follows, consider $H_{\mathrm{css}} + h_{\mathrm{sw}} = h_{\mathrm{pv}}$, i.e., the cylindrical guiding structures end at the edge of the pressure vessel. Further, it is desired that the spherical TEM wave, of rise time $t_\delta = 100$ ps, is guided by the switch cones, cylindrical support structures and the feed arms, i.e., $H_{\mathrm{css}} + h_{\mathrm{sw}} = h_{\mathrm{pv}} < ct_\delta$. If $H_{\mathrm{css}} + h_{\mathrm{sw}} = h_{\mathrm{pv}} > ct_\delta$, the wave will be guided only by the switch cones and the cylinder and not by the feed arms. Let us assume for the calculations that follow that $h_{\mathrm{pv}} \leq 2.0$ cm $= (2/3)ct_\delta$.

2. The medium surrounding the switch, pressure vessel and launching lens is assumed to be oil, $\epsilon_{r_{\mathrm{oil}}} = 2.25$ as shown in Fig. 3.2. For a net increase in the transmission coefficient ("bump-up"), the relative permittivity of the launching lens must be constrained such that $\epsilon_{r_{\mathrm{oil}}} \leq \epsilon_{r_{ll}} \leq \epsilon_{r_{\mathrm{pv}}} \Rightarrow 2.25 \leq \epsilon_{r_{ll}} \leq 3.7$.

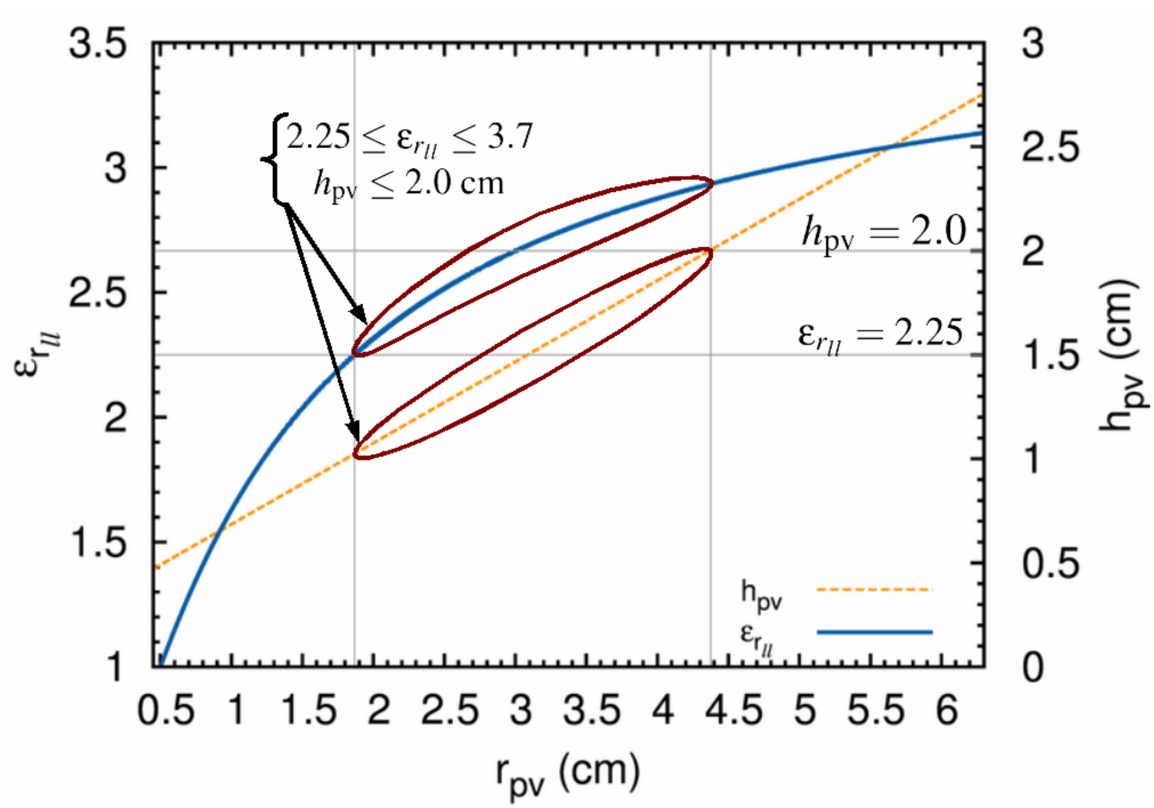

Figure 3.1: $\epsilon_{r_{ll}}$ and h_{pv} as a function of r_{pv}.

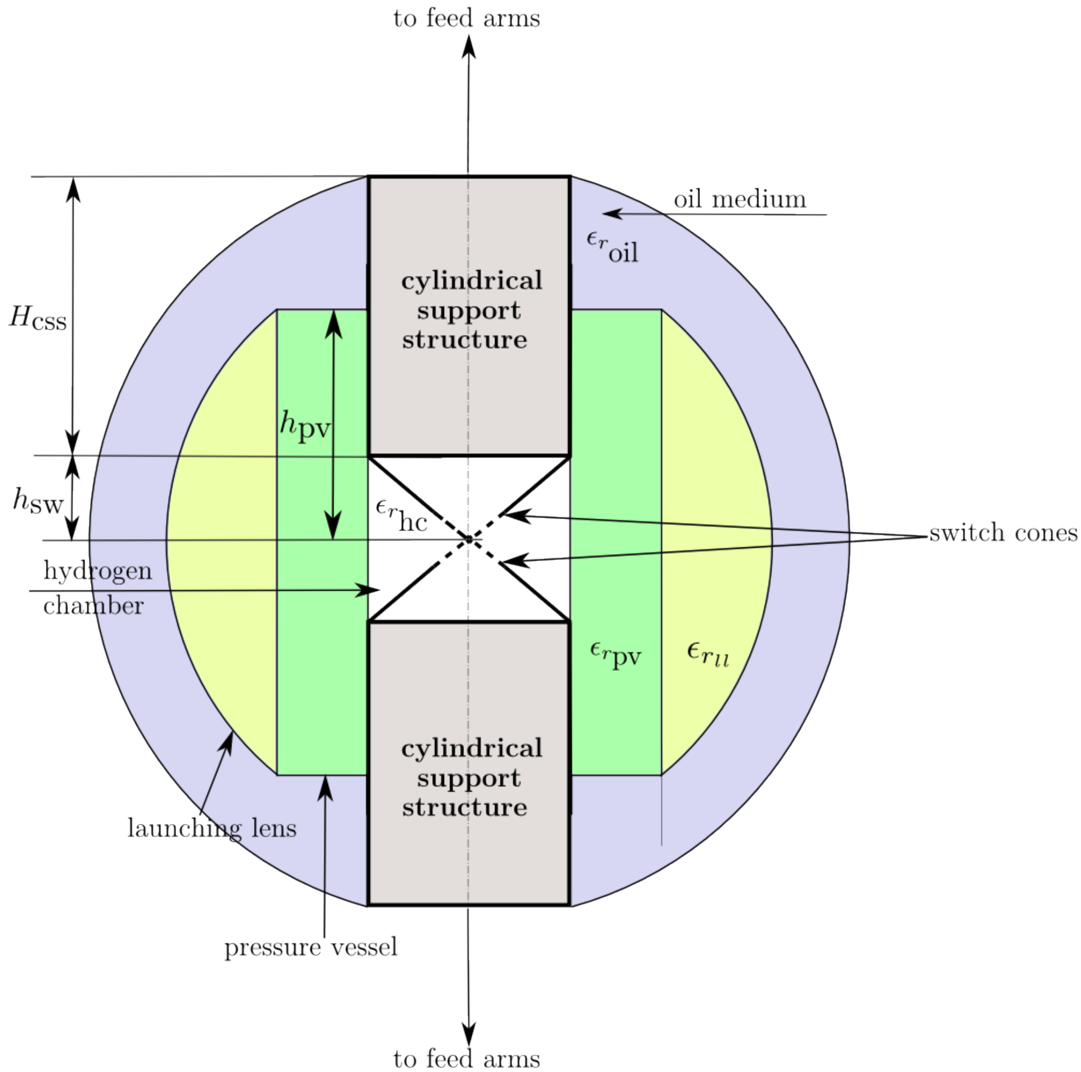

Figure 3.2: Cylindrical pressure vessel showing the need for a cylindrical guiding structure on top of the switch cones.

In Fig. 3.1 one notes that,

- A larger r_{pv} leads to a larger h_{pv} and $\epsilon_{r_{ll}}$.
- At $r_{\text{pv}} = 1.867$ cm, $h_{\text{pv}} = 1.024$ cm and $\epsilon_{r_{ll}} = 2.25$, i.e., the surrounding oil medium can be used as the launching lens. These dimensions of the pressure vessel are attractive from a fabrication point of view.

The curves in Fig. 3.1 are for $Z_c = 100\ \Omega$. From (2.9) and (2.14), h_{pv} and $\epsilon_{r_{ll}}$ are also a function of the bicone impedance, θ. For example, for $Z_c = 75\ \Omega$, $\epsilon_{r_{ll}} = 2.25 \Rightarrow r_{\text{pv}} = 1.764$ cm and $h_{\text{pv}} = 0.742$ cm, i.e., for a smaller bicone impedance a smaller pressure vessel is required.

4 Transmission coefficient for a given $\epsilon_{r_{ll}}$

The transmission coefficient of a wave travelling from the hydrogen chamber to the free space surrounding the oil medium can be determined using the transmission line model shown in Fig. 4.1.

The transmission coeffcient, T_1, of a wave travelling from the hydrogen chamber to the pressure vessel is

$$T_1 = \frac{2Z_{\text{pv}}}{Z_{\text{hc}} + Z_{\text{pv}}} = \frac{2\epsilon_{r_{\text{pv}}}^{-1/2}}{\epsilon_{r_{\text{hc}}}^{-1/2} + \epsilon_{r_{\text{pv}}}^{-1/2}} \tag{4.1}$$

since $Z \propto \epsilon_r^{-1/2}$. Similarly, the transmission coefficient, T_2, from the pressure vessel to the launching lens is

$$T_2 = \frac{2Z_{ll}}{Z_{\text{pv}} + Z_{ll}} = \frac{2\epsilon_{r_{ll}}^{-1/2}}{\epsilon_{r_{\text{pv}}}^{-1/2} + \epsilon_{r_{ll}}^{-1/2}} \tag{4.2}$$

T_3, from the launching lens to the surrounding oil medium

$$T_3 = \frac{2Z_{\text{oil}}}{Z_{ll} + Z_{\text{oil}}} = \frac{2\epsilon_{r_{\text{oil}}}^{-1/2}}{\epsilon_{r_{ll}}^{-1/2} + \epsilon_{r_{\text{oil}}}^{-1/2}} \tag{4.3}$$

T_4, from the oil medium to the surrounding free space

$$T_4 = \frac{2Z_{\text{air}}}{Z_{\text{air}} + Z_{\text{oil}}} = \frac{2\epsilon_{r_{\text{air}}}^{-1/2}}{\epsilon_{r_{\text{air}}}^{-1/2} + \epsilon_{r_{\text{oil}}}^{-1/2}} \tag{4.4}$$

Therefore, the total transmission coefficent is

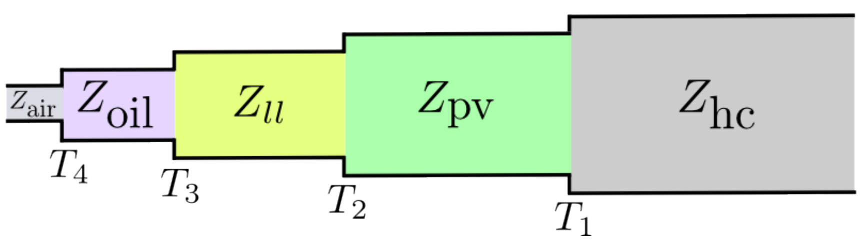

Figure 4.1: Transmission line model of switch system.

$$T_{\text{total}} = T_1T_2T_3T_4 = \left(\frac{2\epsilon_{r_{\text{pv}}}^{-1/2}}{\epsilon_{r_{\text{hc}}}^{-1/2} + \epsilon_{r_{\text{pv}}}^{-1/2}}\right)\left(\frac{2\epsilon_{r_{ll}}^{-1/2}}{\epsilon_{r_{\text{pv}}}^{-1/2} + \epsilon_{r_{ll}}^{-1/2}}\right)\left(\frac{2\epsilon_{r_{\text{oil}}}^{-1/2}}{\epsilon_{r_{ll}}^{-1/2} + \epsilon_{r_{\text{oil}}}^{-1/2}}\right)\left(\frac{2\epsilon_{r_{\text{air}}}^{-1/2}}{\epsilon_{r_{\text{air}}}^{-1/2} + \epsilon_{r_{\text{oil}}}^{-1/2}}\right) \tag{4.5}$$

Substituting $\epsilon_{r_{\text{hc}}} = \epsilon_{r_{\text{air}}} = 1.0,\ \epsilon_{r_{\text{pv}}} = 3.7,\ \text{and}\ \epsilon_{r_{ll}} = \epsilon_{r_{\text{oil}}} = 2.25 \Rightarrow T_{\text{total}} = 0.923$.

Note that the maximum net transmission coefficient with respect to $\epsilon_{r_{ll}}$ for $\epsilon_{r_{\text{hc}}} = \epsilon_{r_{\text{air}}} = 1.0,\ \epsilon_{r_{\text{pv}}} = 3.7,\ \text{and}\ \epsilon_{r_{\text{oil}}} = 2.25$ is

$$\frac{dT}{d(\epsilon_{r_{ll}})} = 0 \Rightarrow \epsilon_{r_{ll}} = 2.89 \Rightarrow T = 0.926 \tag{4.6}$$

5 Concluding Remarks

The surrounding oil medium can be used as the launching lens for $r_{\text{pv}} = 1.867$ cm and $h_{\text{pv}} = 1.024$ cm for a 200 Ω bicone source. From an ease-of-fabrication perspective, the pressure vessel dimensions for $\epsilon_{r_{ll}} = \epsilon_{r_{\text{oil}}} = 2.25$ are very attractive. The formulas presented in this paper can also be used to calculate h_{pv} and $\epsilon_{r_{ll}}$ as a function of the bicone switch impedance.

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