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Spatially Limited Exponential Lens Design for Better Concentrating an Impulse

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Abstract

A spatial limited exponential lens design is discussed and an analytical formulation has been used to examine the pulse droop to minimize it.

1 Introduction

A formulation in [1] has been used to examine the pulse droop for a transmission line with an exponentially tapered impedance profile. We wish to minimize this droop, or ask how long the transmission line should be for a given droop. The exponentially tapered transmission line has an optimal transfer function in terms of early time voltage gain and improved droop characteristics.

We apply this result to an exponentially tapered dielectric constant of a focusing lens. We find the required lens dimensions for a given droop.

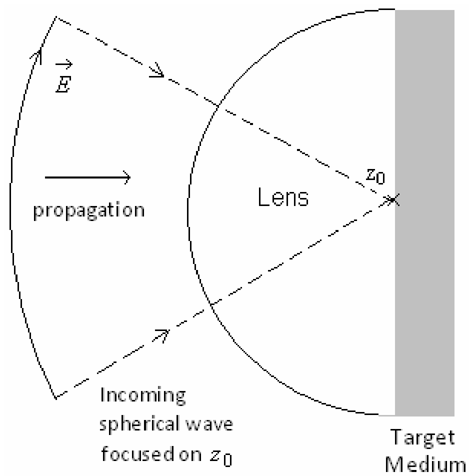

Figure.1 Lens Geometry

The focal point is $z_0 = 37.5$ cm and the other parameters of a prolate-spheroidal IRA are defined in [2,3].

Our calculations are based on a one-dimensional plane-wave approximation (Fig.2).

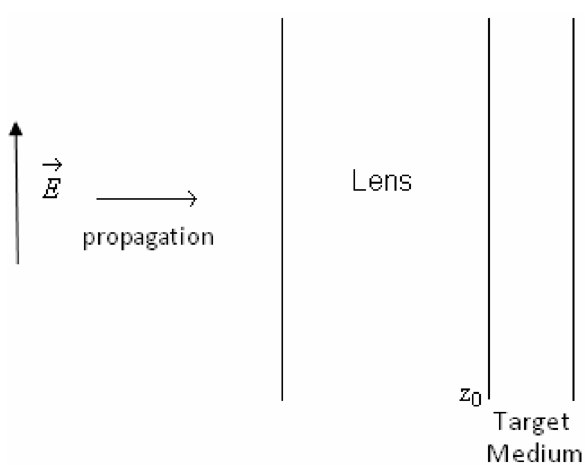

Figure.2 Equivalent Plane Wave Geometry

This will not directly give an estimate of spot size, only transmission/reflection by the lens. Other considerations also apply [4].

2. Equivalent Transmission-Line Model (One Dimensional) of Lens

As discussed in [1], the exponentially tapered lens has a minimized droop and optimal transfer function for the case of uniform propagation speed. Here we adapt this solution to a dielectric lens, noting that the propagation speed slows as the wave propagates inti higher-permittivity media. This model does not include any calculations about spot sizes.

We can define the lens wave impedance as follows $z = space\ coordinate$, $\zeta = modified\ space\ coordinate$, so we have a new coordinate where the wave propagates with a constant v_1 speed and has an exponential wave impedance variation through the lens. We use a plane wave approximation and this approximation is valid up to the case that the wavelength is still small compared to the cross section of the beam.

$$\frac{\zeta}{c} = transit\ time\ to\ z\ \&\ hence\ \zeta \tag{1}$$

So let

$$Z(\zeta) = Z_1\, e^{-\zeta/\zeta_0} \tag{2}$$

where Z_1 is the wave impedance at the beginning of the lens; it is $Z_0 = 377\,\Omega$ in our case.

$$Z_2 = Z_1\, e^{-\zeta_{max}/\zeta_0} \tag{3}$$

Z_2 is the wave impedance at the end of the lens.

$$Z(\zeta) = \left[\frac{\mu_0}{\varepsilon(\zeta)}\right]^{1/2} = Z_1\, \varepsilon_r^{-1/2}(\zeta) \tag{4}$$

The propagation speed can be defined as

$$v = \frac{1}{[\mu_0\, \varepsilon(\zeta)]^{1/2}} = v_1\, \varepsilon_r^{-1/2}(\zeta) \tag{5}$$

where v_1 is the propagation speed before the lens, which is typically c.

The transit time through the lens can be defined as

$$t_\zeta \equiv t_z \equiv \int_0^z v^{-1}(z')\, dz' = \int_0^\zeta v_1^{-1}\, d\zeta = \frac{\zeta}{v_1} \tag{6}$$

Let's take the derivative of both sides of (6). Then we have

$$\frac{dt}{d\zeta} = \mathrm{v_1}^{-1} = \mathrm{v}^{-1}(\mathrm{z})\frac{dz}{d\zeta}$$

$$\frac{dt}{dz} = \mathrm{v}^{-1}(\mathrm{z}) \tag{7}$$

$$\frac{d\zeta}{dz} = \frac{\mathrm{v_1}}{\mathrm{v(z)}} = \varepsilon_r^{1/2}(\zeta) = e^{\zeta_{max}/\zeta_0}$$

So let's find from (7) the space coordinate z, in terms of modified space coordinate ζ

$$z = \int_0^{\zeta} \varepsilon_r^{-1/2}(\zeta') \, d\zeta' \tag{8}$$

From (2) and (4) we can write (8) as

$$z = \int_0^{\zeta} e^{-\zeta'/\zeta_0} \, d\zeta' = \zeta_0\left[1 - e^{-\zeta/\zeta_0}\right] \tag{9}$$

We can see from (9) as $\zeta \to \infty$, $z \to \zeta_0$ and this does not continue to grow. This gives us a spatially limited lens. This is convenient for construction purposes.

The wave propagation can be described by the source-free telegrapher equations[1(2.3)]. We can transform the 1D wave equation to an equivalent ζ space coordinate as

$$\begin{aligned} \frac{dV(\zeta,s)}{d\zeta} &= -s\mu_0 e^{-\zeta/r_{max}} \, I, \\ \frac{dI(\zeta,s)}{d\zeta} &= -s\varepsilon_0 e^{\zeta/r_{max}} \, V. \end{aligned} \tag{10}$$

2. Solution of the Transmission-Line Equations

We solve an equivalent problem of [1] , but instead of an increase in transmission-line impedance we have an decrease in wave impedance but the equations in [1] still apply (Appendix). The impedance is decreasing but there is still a droop.

We can use the exact solution of transfer function in [1(A.8)]

$$\tilde{T} = e^{S+G}\left[cosh\left(\left(S^2 + G^2\right)^{1/2}\right) + \frac{S}{\left(S^2 + G^2\right)^{1/2}} sinh\left(\left(S^2 + G^2\right)^{1/2}\right)\right]^{-1} \tag{11}$$

where S is the normalized complex frequency

$$S = s\, t_{\zeta\; max} = (\Omega + j\omega)\, t_{\zeta\; max} \tag{12}$$

The high-frequency gain is defined in [1(3.4)] as

$$g = e^{G} = \sqrt{Z_2 / Z_1} = \varepsilon_r^{-1/4} \tag{13}$$

One can define the transit, normalized and droop time [1(A.11)] parameters as follows

$$\begin{aligned} t_{\zeta\, max} &\equiv \zeta_{max} / c \qquad \textit{transit time through lens} \\ \tau &\equiv t / t_{\zeta} \qquad \textit{normalized time} \\ \tau_d &= 2\, ln^{-2}(g) \qquad \textit{normalized droop time} \\ &= t_d / t_{\zeta\, max} \end{aligned} \tag{14}$$

t_d is the droop time, the step-response form is defined as [1(A.13)]

$$R(\tau) = g\left[1 + \tau / \tau_d + O(\tau^2)\right] \quad as \quad \tau \to 0 \tag{15}$$

4. Example

Now we can calculate the lens thickness for a given dielectric target permittivity $\varepsilon_{r\,max}$. Let's set $t / t_d = 0.05 \; and \; 0.1$ and we have a $t = 100\,\text{ps}$ pulse width(maximum time of interest) so from (12)

$$\begin{aligned} t_d &= 2 \text{ ns} \; and \; 1\text{ns} \\ \zeta_{max} &= \frac{2\, t_d}{ln^2(g)} \end{aligned} \tag{16}$$

From (3) and (14)

$$\begin{aligned} \frac{Z_2}{Z_1} &= \varepsilon_{r\,max}^{-1/2} = e^{-\zeta_{max} / \zeta_0} \\ \zeta_0 &= \frac{c\, t_d\, ln^2(g)}{ln(\varepsilon_{r\,max})} \quad \text{(meters)} \end{aligned} \tag{17}$$

Let's substitute (16) in (9); we have

$$z_{max} = \zeta_0 \left[1 - e^{-\zeta_{max} / \zeta_0}\right] = \frac{c\, t_d\, ln^2(g)}{ln(\varepsilon_{r\,max})} \left[1 - \varepsilon_{r\,max}^{-1/2}\right] \quad \text{(meters)} \tag{18}$$

Conclusion

We might design a spatially limited exponential lens based on [1]. This lens is designed for a biological application [5]. From (18) we can find the z_{max} values for different biological tissues.

		Water	Muscle	Tumor	Skin	Fat
	$\varepsilon_{r\,max}$	81	70	50.74	34.7	9.8
	g	0.33	0.34	0.37	0.41	0.56
	τ_d	1.65	1.77	2.07	2.5	6.1
t_d = 1ns	$t_{\zeta\,max}$(ns)	0.6	0.56	0.48	0.39	0.16
	ζ_{max} (cm)	18.1	16.9	14.5	11.8	4.9
	ζ_0 (cm)	8.2	8	7.4	6.7	4.3
	z_{max}(cm)	7.3	7	6.3	5.5	2.9
t_d = 2 ns	$t_{\zeta\,max}$(ns)	1.2	1.1	0.96	0.79	0.33
	ζ_{max} (cm)	36.2	33.8	28.9	23.6	9.8
	ζ_0 (cm)	16.5	15.9	14.7	13.3	8.6
	z_{max}(cm)	14.6	14	12.7	11	5.8

Table.1 Design parameter values for different biological tissues

One can see from Table.1 that if we have lower electric dielectric constant for target biological tissue we need a smaller lens. This is not the only consideration. Higher dielectric constant in the lens exit results is a smaller spot size and higher fields [4]. The smaller spot size concentrates the energy in the vicinity of the cancer.

One can find how ε_r changeS w.r.t. ζ and z from (9) and (17)

$$\varepsilon_r(\zeta) = e^{2\zeta/\zeta_0}$$

$$\varepsilon_r(z) = \left(\frac{\zeta_0}{\zeta_0 - z}\right)^2 \tag{19}$$

Let's consider $t_d = 1$ ns and find $\varepsilon_r(\zeta)$ *and* $\varepsilon_r(z)$ for different dielectric tissues.

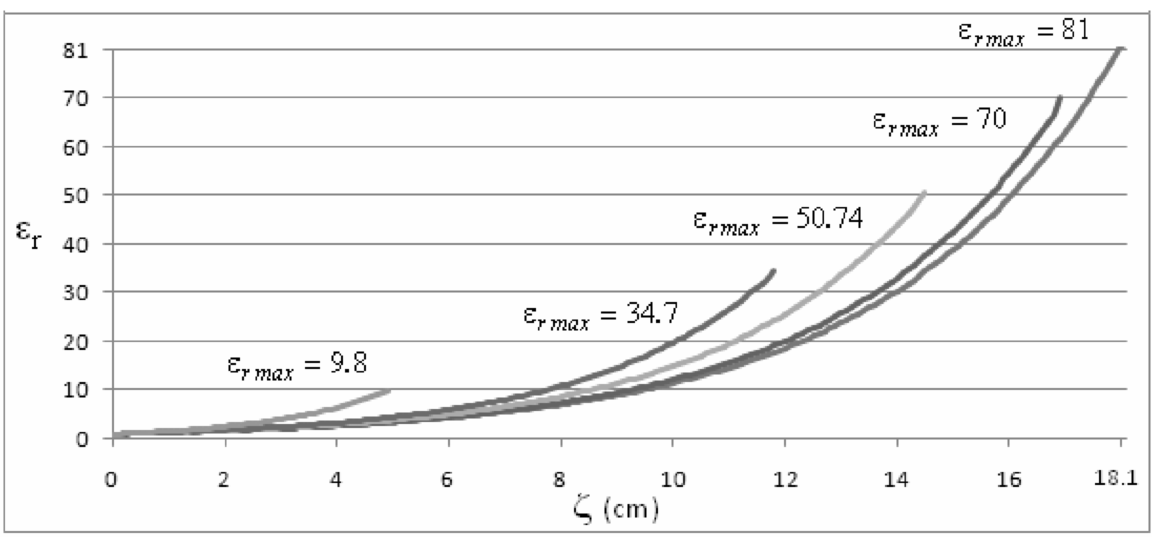

Figure.3 $\varepsilon_r(\zeta)$ values for different dielectric tissues

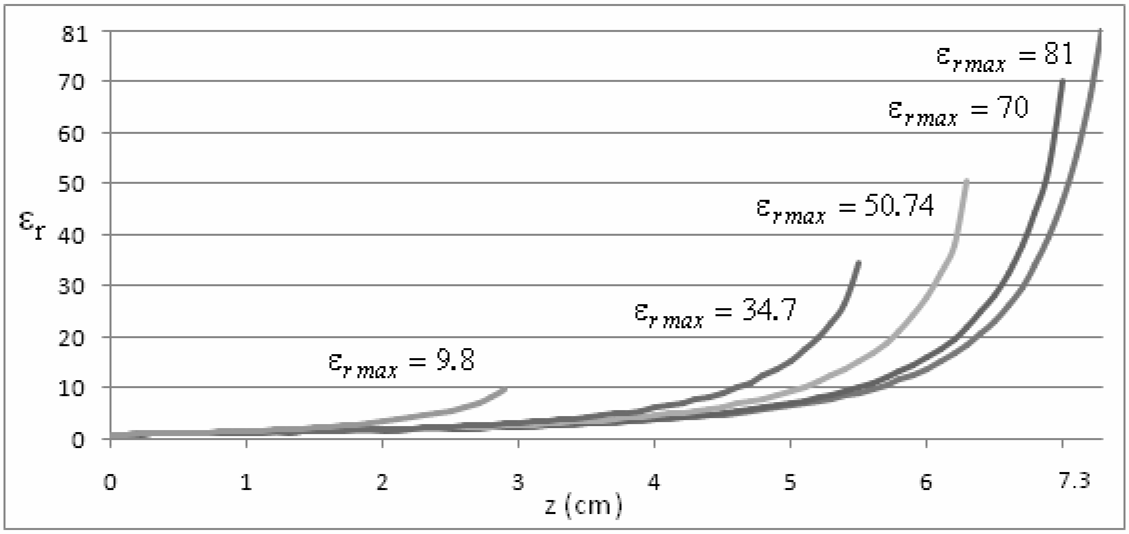

Figure.4 $\varepsilon_r(z)$ values for different dielectric tissues

We can show the compression of the coordinates for $t_d = 1$ ns and $\varepsilon_{\text{r max}} = 81$ as in Fig.5

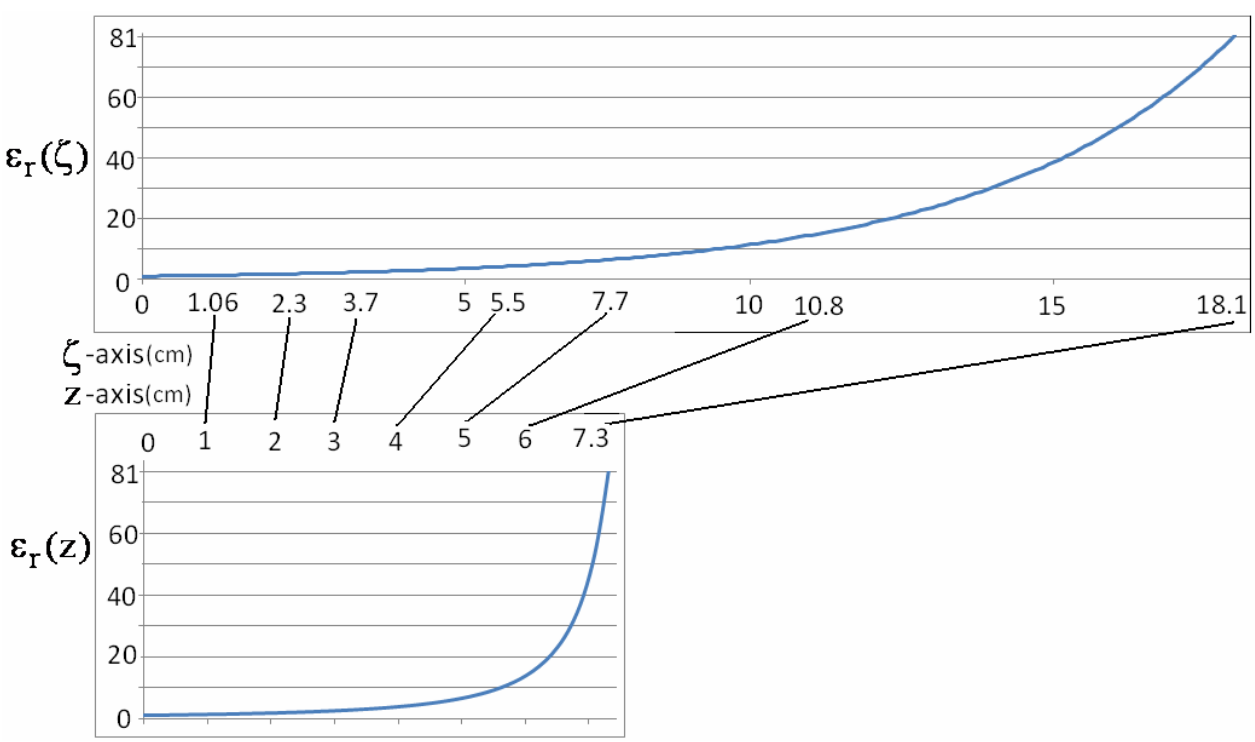

Figure.5 Compression of the coordinates for $t_d = 1$ ns and $\varepsilon_{\text{r max}} = 81$

Appendix

One can define the transmission coefficient for high and low frequency as follows

$$T_h = \left[\frac{Z_2}{Z_1}\right]^{1/2} \quad , \quad T_\ell = \frac{2Z_2}{Z_1 + Z_2} \tag{A1}$$

Let's calculate the difference between these two coefficients as

$$T_h - T_\ell = \frac{Z_2^{1/2}\left[Z_1 + Z_2\right] - 2Z_2 Z_1^{1/2}}{Z_1 + Z_2} = \frac{Z_2^{1/2}\left[Z_1^{1/2} - Z_2^{1/2}\right]^2}{Z_1 + Z_2} > 0 \tag{A2}$$

This is always positive except at $Z_1 = Z_2$. Thus there is a droop(positive, i.e. a decrease) from initial to final value for both increasing and decreasing impedance.

References

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