EM Implosion Memos

Memo 6

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Focal Waveforms for Various Source Waveforms Driving a Prolate- Spheroidal IRA

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Abstract

This paper considers time domain characteristics of some analytic source waveforms used for determining the waveform characteristic of a prolate- spheroidal IRA near the second focus.

1 Introduction

This paper is a analytical calculation of a prolate-spheroidal IRA that is based on [1],[2],[3]. The analytical waveforms for 2-TEM-Feed-Arm, 45^{o} 4-TEM-Feed-Arm and 60^{o} 4-TEM-Feed-Arm cases for x, y, z axis variations near the second focus are calculated.

1.1 Description of geometry

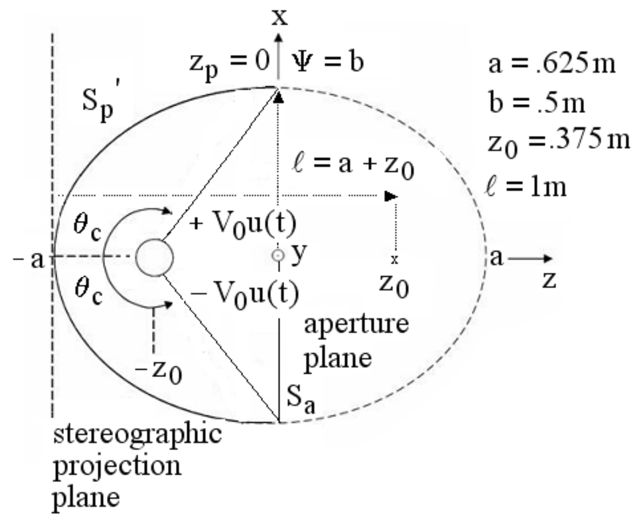

Figure 1.1 IRA Geometry [1]

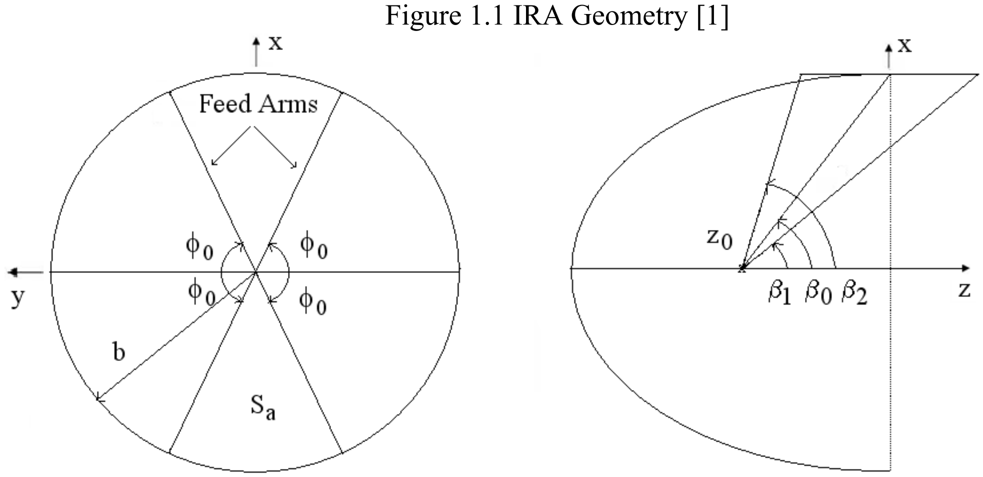

Figure 1.2 Feed Arm Geometry [1]

For our design, we choose a special case of the prolate-spheroidal IRA's geometric parameters as [3]

$$z_p = 0,\ b = \Psi_0 = .5\,m,\ a = .625\ m\ ,\ z_0 = .375\,m, \ell = 1\,m \tag{1.1}$$

ϕ_o is the angle from y-axis to the feed arm and β_0 ,β_1 , β_2 are the angles from the z-axis to the electrical center, the first edge and the second edge of the feed arms as in Fig. 1.2

2 Analytical Focal Waveforms

2.1 Double exponential excitation

Let us use the commonly used wave-forms which is the difference between two exponentials times a unit step function instead of unit step function, so our excitation is

$$V(t) = V_0\, f(t) \;,\quad f(t) = \left[e^{-\beta t} - e^{-\alpha t}\right]\; u(t) \tag{2.1}$$
$$\alpha = t_\delta^{-1} \;,\; t_\delta = 100\,\text{ps} \;,\; 50\,\text{ps} \quad ,\quad \beta = t_d^{-1} \;,\; t_d = 1\,\text{ns} \;,\; 2\,\text{ns}$$

where t_δ is the rise and t_d is the decay time constant. The peak of the waveform is give by (2.14) in [4] as

$$f_{max} = \max_t \big(f(t)\big) = \zeta^{\frac{1}{1-\zeta}}\left[\frac{1}{\zeta} - 1\right] = f(t_{max}) \tag{2.2}$$
$$\zeta = \beta / \alpha$$

Where t_{max} is the time that maximum occurs and it can be found by taking the derivative of (2.1) as zero, so

$$t_{max} = \frac{1}{\alpha - \beta} \ln\left(\frac{\alpha}{\beta}\right) \tag{2.3}$$

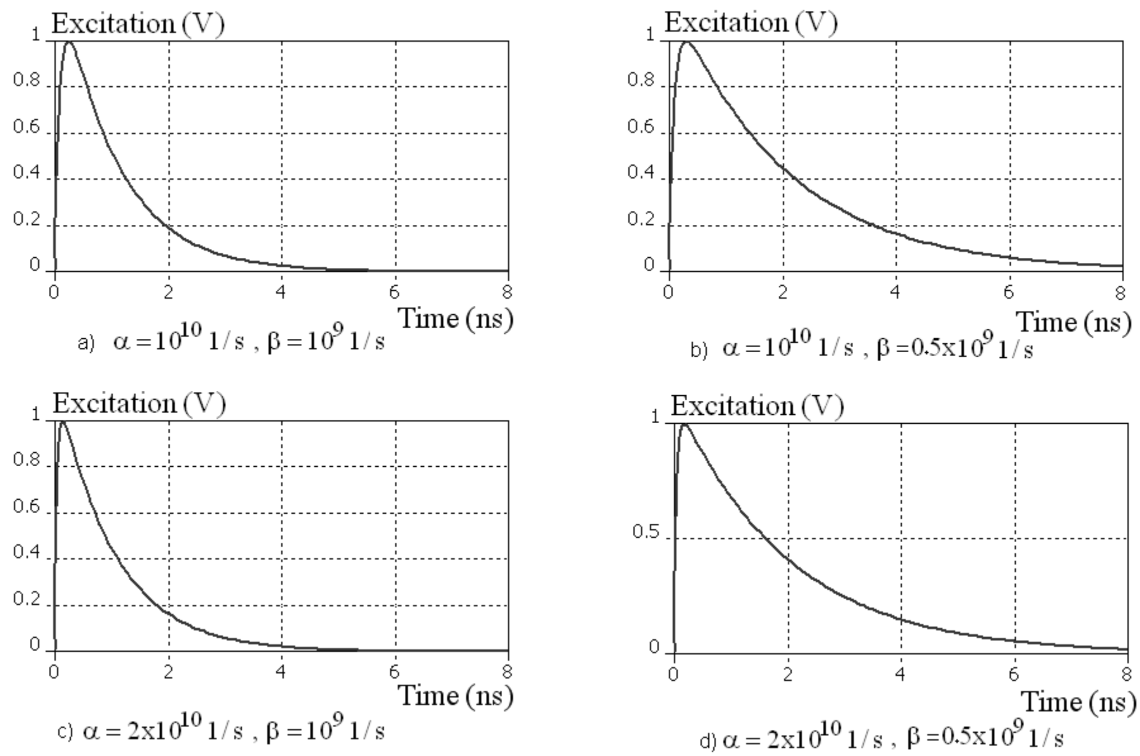

Figure 2.1 Double exponential excitation for $f(t)/f_{max}$ for different α and β

f_{max}	t_{max} (ns)	α (1/s)	β (1/s)
0.671	.26	10^{10}	10^{9}
0.81	.32	10^{10}	$5x10^{8}$
0.81	.16	$2x10^{10}$	10^{9}
0.887	.19	$2x10^{10}$	$5x10^{8}$

Table 2.1 f_{max} and t_{max} values for different α and β

2.2 Analytical focal waveforms

The analytical focal waveforms for ramp rising step excitation are from [1,2]. The excitation is a 1 Volt ($V_0 = .5$ Volt) step, rising as a ramp function lasting 100 ps.

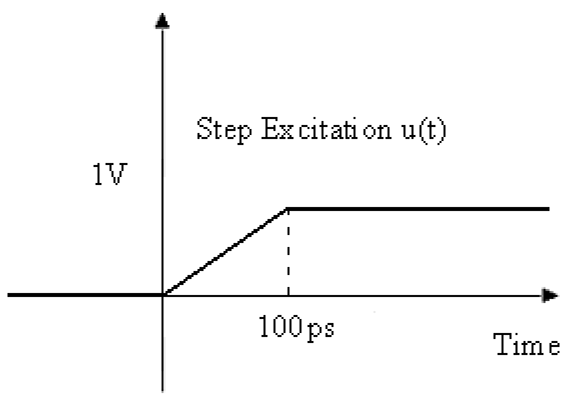

Figure 2.2 Ramp rising step excitation u(t)

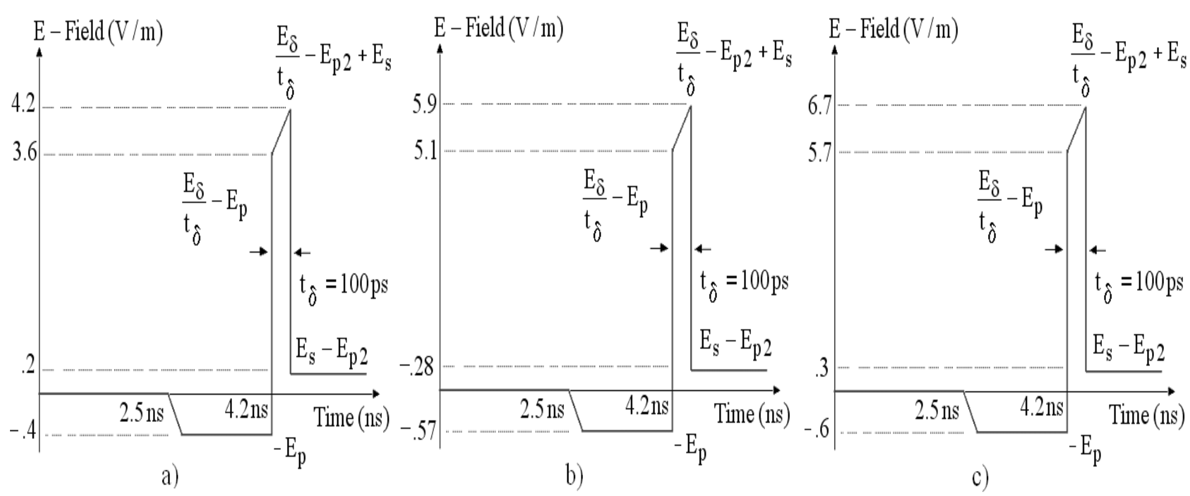

Figure 2.3 Analytical focal waveforms for ramp rising step excitation a) 2-Arm $400\,\Omega$ b) 45^{o} degree 4-Arm $200\,\Omega$ c) 60^{o} degree 4-Arm $200\,\Omega$

From a step excitation , u(t), we get a waveform as follow

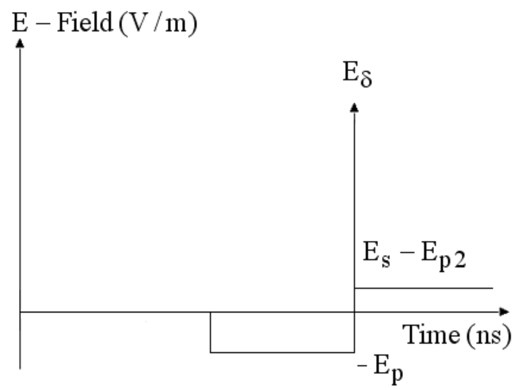

Figure 2.4 Step Response

Let us consider prepulse , impulse and postpulse separately as

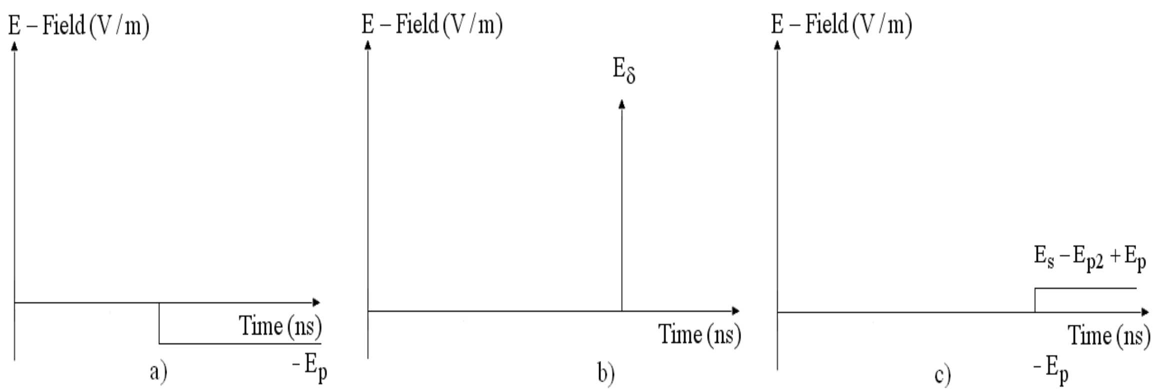

Figure 2.5 Decomposition of step response a)prepulse b)impulse c)postpulse

What is going to happen if we use double exponential excitation function (2.1) instead of step function? We have a response from (2.1) as

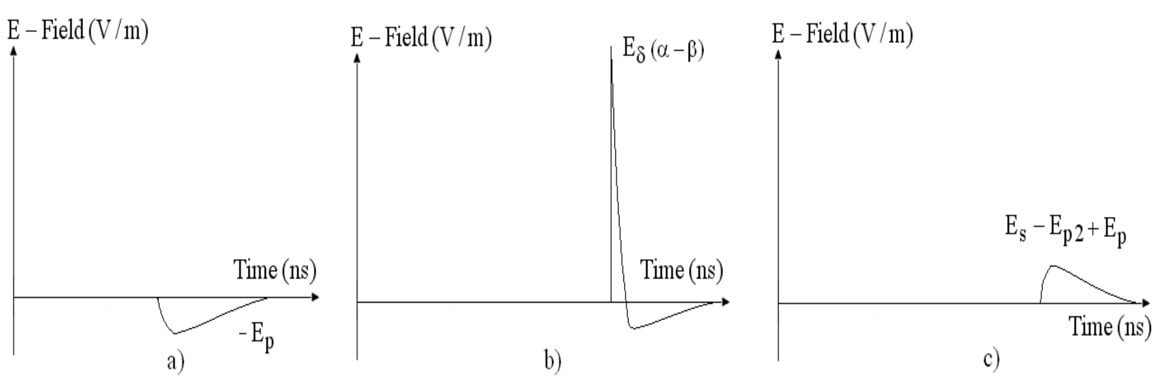

Figure 2.6 decompostion of double exponential response a)prepulse b)impulse c)postpulse

The impulsive part of double exponential excitation can be defined as

$$\frac{E_\delta}{f_{max}}\frac{dV(t-t_2)}{dt} = \frac{E_\delta}{f_{max}}\left[-\beta e^{-\beta(t-t_2)} + \alpha e^{-\alpha(t-t_2)}\right]u(t-t_2) \qquad (2.3)$$

and the peak value is $E_\delta(\alpha-\beta)$, where $t_2 = 4.2$ ns is the time that impulse arrives the second focus. Finally, we obtain response waveforms for (2.1) as

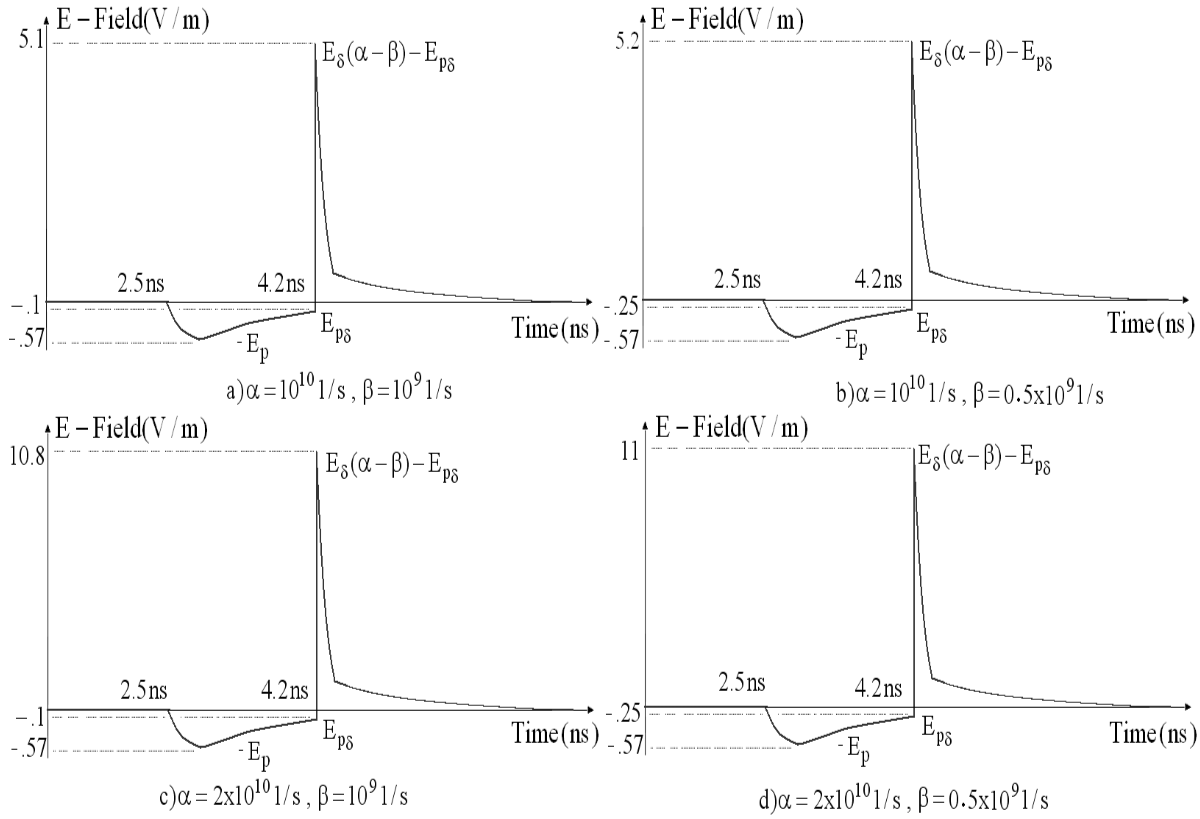

Figure 2.7 Double Exponential Excitation Responses for different α and β

where $E_{p\delta}$ is the value of E_p at the time the impulse starts

$$E_{p\delta} = \frac{E_p}{f_{max}}\left[e^{-\beta(t_2-t_1)} - e^{-\alpha(t_2-t_1)}\right] \qquad (2.4)$$

where $t_1 = 2.5$ ns is the time that prepulse arrives at the second focus.

45^o degree 4-Arm $200\,\Omega$ and 60^o degree 4-Arm $200\,\Omega$ case are just $\sqrt{2}$ and 1.606 times of these waveforms.

$E_{p\delta}$	α (1/s)	β (1/s)
0.1	10^{10}	10^{9}
0.25	10^{10}	$0.5x10^{9}$
0.1	$2x10^{10}$	10^{9}
0.25	$2x10^{10}$	$0.5x10^{9}$

Table 2.2 $E_{p\delta}$ values for different α and β

6.Conclusion

We are obtaining higher impulse values for double exponential excitation because the prepulse value $E_{p\delta}$ is lower than the regular E_p value that we have from step excitation. One can see in Fig. 2.7 a) that we have a decrease from $-0.57\,V/m$ to $-0.1\,V/m$. So we have around 82% decrease. If we compare $E_{p\delta}$ with the E_p value for step excitation from Fig. 2.3 a) , we have a decrease from $-0.4\,V/m$ to $-0.1\,V/m$. So we have a 75% decrease. If we look at the impulse for step and double exponential excitation and compare Fig. 2.3 a) with Fig. 2.7 a), one can see that we have an increase from $4.2\,V/m$ to $5.1\,V/m$. So we have a 22% increase in the peak focal field. The postpulse also decays toward zero. This type of waveform is convenient because of its simplicity and it may better model the pulser.

References

1. S. Altunc and C. E. Baum, "Extension of the Analytic Results for the Focal Waveform of a Two-Arm Prolate-Spheroidal Impulse-Radiating Antenna (IRA)", Sensor and Simulation Note 518, Nov 2006.
2. S. Altunc and C. E. Baum, "Comparison of Analytical and Numerical Results for a Prolate-Spheroidal Impulse-Radiating Antenna (IRA)", Sensor and Simulation Note 519, Nov 2006.
3. C. E. Baum, "Focal Waveform of a Prolate-Spheroidal IRA", Sensor and Simulation Note 509, February 2006.
4. C. E. Baum, "Some considerations concerning Analytical EMP Criteria Waveforms ", Theoretical Note 285, Oct 1976.