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Memo 3

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Analytical Calculation for the Focal Waveform of a Prolate-Spheroidal IRA

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Abstract

This paper is focused on an analytical field calculation and the focal waveform produced at the second focus of a prolate-spheroidal reflector.

1 Introduction

This paper is based on analytical interpretation of the formulae developed in [1],[2],[3]. The focal waveform produced at the second focus of a prolate-spheroidal reflector was produced.

2. Fields at Second Focus

Summarizing, we have [1]

$$E_\delta = \frac{E_0}{c}\left[a+z_0\right]\left[1-\left[1+\left[\frac{\Psi_p}{z_0-z_p}\right]^2\right]^{-1/2}\right] = \frac{V_0}{\pi f_g c}\frac{a+c}{a-c}\cot\left(\frac{\theta_c}{2}\right)\left[1-\left[1+\left[\frac{\Psi_p}{z_0-z_p}\right]^2\right]^{-1/2}\right]$$

$$E_s = \frac{E_0}{2}\frac{a+z_0}{z_0-z_p}\left[1+\left[\frac{z_0-z_p}{\Psi_p}\right]^2\right]^{-1} = \frac{V_0}{2\pi f_g}\frac{1}{z_0-z_p}\frac{a+z_0}{a-z_0}\cot\left(\frac{\theta_c}{2}\right)\left[1+\left[\frac{z_0-z_p}{\Psi_p}\right]^2\right]^{-1} \qquad (2.1)$$

$$E_p = \frac{V_0}{2\pi f_g z_0}\tan\left(\frac{\theta_c}{2}\right) \quad , \quad E_{pa} = E_p\Delta t_p = \frac{V_0}{2\pi f_g c}\frac{a-z_0}{z_0}\tan\left(\frac{\theta_c}{2}\right) \text{ time integral or “area” of prepulse}$$

$$E_0 = \frac{V_0}{\pi f_g}\frac{1}{a-z_0}\cot\left(\frac{\theta_c}{2}\right)$$

$$\text{and } \tan\left(\frac{\theta_c}{2}\right) = \left[\frac{a+z_p}{a-z_p}\right]^{1/2} \quad \frac{a+z_0}{b} \quad \text{from[2]} \qquad (2.2)$$

2.1 Prepulse Term E_{p2} after the Impulse

What happens to the pre-pulse term after the impulse, ie after the truncation at the aperture boundary ($\Psi = \Psi_p$, or b for special case)

Let E_{pt} = tangential E field (x component) on S_a due to pre-pulse wave

Then we have

$$E_{p1} = \frac{1}{2\pi c}\frac{\partial}{\partial t}\int \frac{z_0 - z_p}{r_2{}^2} E_{pt}\, dS \qquad (2.3)$$

$$E_{p2} = \frac{1}{2\pi}\int \frac{z_0 - z_p}{r_2{}^3} E_{pt}\, dS$$

After we see the edge of Sa, neglecting diffraction terms from this edge, and approximating E_{pt} by the TEM pre-pulse wave out to this edge we have , for step-function excitation

E_{pt} = time independent pre-pulse field on S_a

$$E_{p1} = 0 \text{ the derivative being zero after the aperture edge is seen} \qquad (2.4)$$

$$E_{p2} = \frac{1}{2\pi}\int \frac{z_0 - z_p}{r_2{}^3} E_{pt}\, dS \text{ = constant , i.e. a step term.}$$

So we need the static E_{pt}. As before, since we are confining ourself to the z-axis we can use a uniform field on the projection plane to give E_{pt} in the above integral.

From Section 2 (2.11), [1] at $r_1 = z_0$ (aperture plane center)

$$E_{pa0} \cong \frac{V_0}{z_0 \pi f_g} \tan\left(\frac{\theta_c}{2}\right) \text{ and,} \tag{2.5}$$

$$\tan\left(\frac{\theta_c}{2}\right) = \left[\frac{a+z_p}{a-z_p}\right]^{1/2} \frac{a+z_0}{b} \quad \text{Memo 1 (2.16)}$$

We need this extended over S_a , since , as we have seen , for the z axis only the uniform field terms (on the projection plane) need be considered (by symmetry).

On the projection plane at z =-a (2.7),[1]

$$V(x_0, y_0) = \frac{V_0}{2} \operatorname{arccos h}^{-1}\left(\frac{\Psi_{c0}}{r_{w0}}\right) \ln\left(\frac{\left(\frac{\Psi_{c0}}{r_{w0}}\right)^2 + 2\frac{\Psi_{c0}}{r_{w0}}\cos(\theta_1) + 1}{\left(\frac{\Psi_{c0}}{r_{w0}}\right)^2 - 2\frac{\Psi_{c0}}{r_{w0}}\cos(\theta_1) + 1}\right) \tag{2.6}$$

$$\Psi_0 = 2[a - z_0]\tan\left(\frac{\theta_1}{2}\right)$$

On the projection plane at $\Psi_0 = 0$ the E-field has only an x component

$$V \cong \frac{V_0}{2} \operatorname{arccos h}^{-1}\left(\frac{\Psi_{c0}}{r_{w0}}\right) \ln \frac{2\frac{x}{\Psi_{c0}} + 1}{-2\frac{x}{\Psi_{c0}} + 1} \cong \frac{V_0}{2} \operatorname{arccos h}^{-1}\left(\frac{\Psi_{c0}}{r_{w0}}\right) 2\frac{x}{\Psi_{c0}}$$

$$E_{3,0} = -\frac{2V_0}{\Psi_{c0}} \operatorname{arccos h}^{-1}\left(\frac{\Psi_{c0}}{r_{w0}}\right) = \frac{2V_0}{\pi f_g}\frac{1}{\Psi_{c0}} \tag{2.7}$$

Choose a potential (uniform)

$$V = -E_{3,0}\, x = -\frac{2V_0}{\Psi_{c0}} \operatorname{arccos h}^{-1}\left(\frac{\Psi_{c0}}{r_{w0}}\right) \Psi_0 \cos(\Phi_0) = -E_{30}\, \Psi_0 \cos(\Phi_0) \tag{2.8}$$

Map this back onto the $\vec{r}_1$ system

$$V = -E_0\, 2[a - z_0]\tan\left(\frac{\theta_1}{2}\right)\cos(\phi) \tag{2.9}$$

$$\Psi_{c0} = 2[a - z_0]\tan\left(\frac{\theta_c}{2}\right) = 2[a - z_0]\left[\frac{a + z_p}{a - z_p}\right]^{1/2} \frac{a + z_0}{b}$$

$$V = -E_{3,0}\, 2[a - z_0]\tan\left(\frac{\theta_1}{2}\right)\cos(\phi)$$

Now on Sa we have

$$\vec{E}_3 = -\frac{1}{r_1}\nabla\theta_1\phi_1 V$$

$$= -E_{3,0}\, 2\frac{[a - z_0]}{r_1}\left(\sec^2\left(\frac{\theta_1}{2}\right)\frac{1}{2}\cos(\phi)\,\vec{1}_{\theta_1} - \frac{\tan\left(\frac{\theta_1}{2}\right)}{\sin(\theta_1)}\sin(\phi)\,\vec{1}_{\phi_1}\right) \tag{2.10}$$

The tangential part is

$$\vec{E}_{3t} = -E_{3,0}\frac{[a - z_0]}{r_1}\left(\sec^2\left(\frac{\theta_1}{2}\right)\frac{1}{2}\cos(\theta_1)\cos(\phi_1)\,\vec{1}_{\Psi} - 2\frac{\tan\left(\frac{\theta_1}{2}\right)}{\sin(\theta_1)}\sin(\phi)\,\vec{1}_{\phi_1}\right) \tag{2.11}$$

$$= E_{3,0}\frac{[a - z_0]}{r_1}\left(\frac{2\cos(\theta_1)\cos(\phi_1)}{1 + \cos(\theta_1)}\vec{1}_{\Psi} - 2\frac{\sin(\phi_1)}{1 + \cos(\theta_1)}\vec{1}_{\phi_1}\right)$$

$$r_1[1 + \cos(\theta_1)] = r_1 + z_0 + z_p \tag{2.12}$$

$$r_1 = \left[\Psi_0^{\,2} + [z_0 + z_p]^2\right]^{1/2}$$

$$\cos(\theta_1) = \frac{z_0 + z_p}{r_1}$$

The x component is

$$E_{3x} = E_{3,0}\frac{[a - z_0]}{r_1 + z_0 + z_p}\left[2\frac{z_0 + z_p}{r_1}\cos^2(\theta_a) + 2\sin^2(\phi_a)\right] \tag{2.13}$$

$$E_{p2} = \frac{E_{30}}{2\pi}[z_0 - z_p][a - z_0]\int_{S_a}\frac{1}{r_2^{\,3}}E_{3,0}\frac{1}{r_1 + z_0 + z_p}\left[2\frac{z_0 + z_p}{r_1}\cos^2(\phi_a) + 2\sin^2(\phi_a)\right]dS$$

$$= E_{30}[z_0 - z_p][a - z_0]\int_0^{\Psi_p}\frac{1}{r_2^{\,3}}\frac{1}{r_1 + z_0 + z_p}\left[\frac{z_0 + z_p}{r_1} + 1\right]d\Psi$$

$$= E_{30}[z_0 - z_p][a - z_0]\int_0^{\Psi_p}\frac{1}{r_2^{\,3}}\frac{1}{r_1}d\Psi$$

To solve this integral consider the special case $z_p = 0, \Psi_p = b, r_1 = r_2$ Then from 509 (4.2)&(4.3)

$$E_{p2} = E_{3,0}\, z_0 \left[a - z_0\right] b \int_0^b \frac{\Psi d\Psi}{{r_2}^4}$$

$$= \frac{E_{3,0}}{2} \frac{\left[a - z_0\right]}{z_0} \left[1 + \left[\frac{z_0}{b}\right]^2\right]^{-1} \tag{2.14}$$

$$E_{p2} = \frac{V_0}{\pi f_g} \frac{1}{\Psi_{c0}} \frac{\left[a - z_0\right]}{z_0} \left[1 + \left[\frac{z_0}{b}\right]^2\right]^{-1}$$

$$\Psi_{c0} = 2\left[a - z_0\right] \tan\left(\frac{\theta_c}{2}\right) = 2\left[a - z_0\right] \frac{\left[a + z_0\right]}{b}$$

At the end we get

$$E_{p2} = \frac{V_0}{2\pi f_g} \frac{1}{z_0} \frac{b}{a + z_0} \left[1 + \left[\frac{z_0}{b}\right]^2\right]^{-1} \tag{2.15}$$

3. Analytical Waveform

We take the simple case from [1,2] for which,

$$z_p = 0,\ b = \Psi_0 = .5\,\text{m},\ a = .625\ \text{m}\ ,\ z_0 = .375\,\text{m}, t_\delta = 100\,\text{ps} \tag{3.1}$$

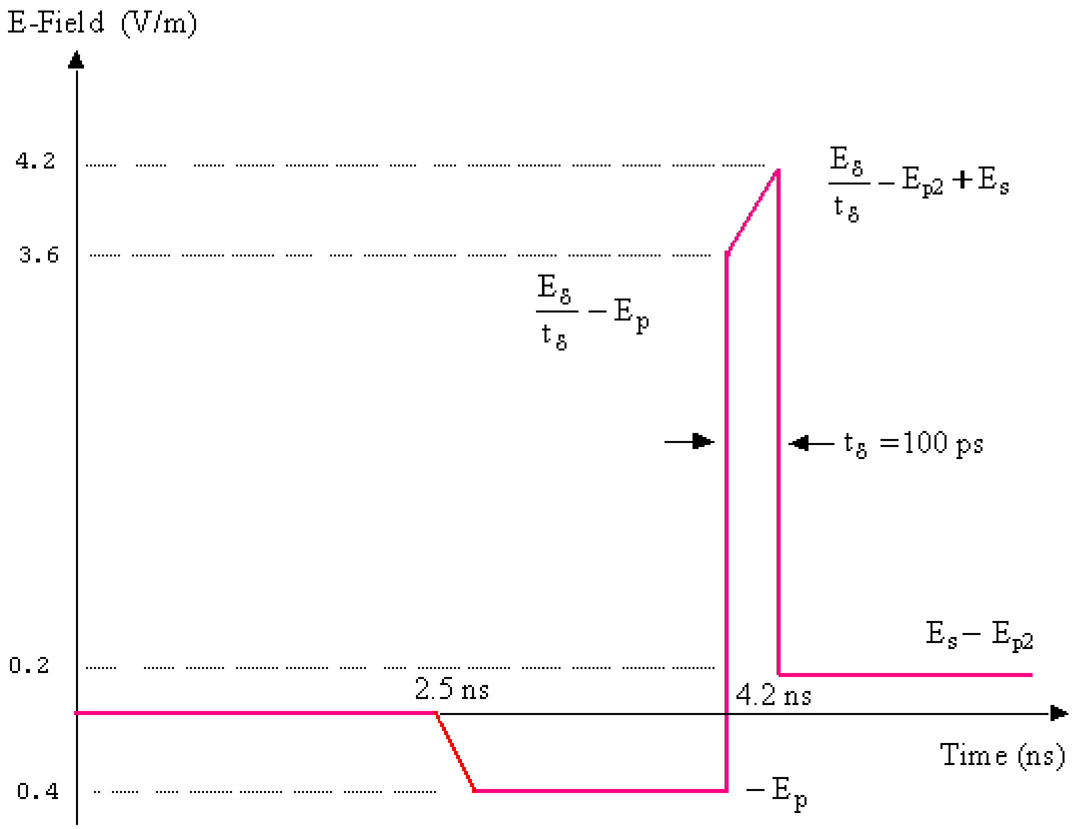

Figure 3.1 Analytic Waveform at the Second Focus

4.Conclusions

We now have an analytic approximate waveform at the focus for the interesting (and useful) case of $z_p = 0$. This includes the “post-pulse” which we should not extend very far to later time, since other scattering begins to have an effect. This can be used for comparison to experimental results and more general numerical computations.

References

1. C. E. Baum, “Focal Waveform of a Prolate-Spheroidal IRA”, Sensor and Simulation Note 509, February 2006.
2. S.Altunc and C.E.Baum. “Parameter Study for Prolate-Spheroidal IRA”,EM Implosion Memos, Memo 1, May 2006.
3. S.Altunc and C.E.Baum. “Analytic Calculation of Fields on z- Axis Near Second Focus”,EM Implosion Memos, Memo 2, June 2006.