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HIGH ALTITUDE CONDUCTIVITY MODELS FOR ELECTROMAGNETIC PULSE CALCULATIONS

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The conduction current in an applied electric field, $\bar{E}$, can be written as

$$\bar{J}(\bar{x},t) = e\, n(\bar{x},t) \langle \bar{v}(\bar{x},t) \rangle \tag{1}$$

where $n(\bar{x},t)$ is the free electron density and $\langle \bar{v}(\bar{x},t) \rangle$ is the average electron velocity found by integrating over the electron velocity distribution. If the field has been applied for a sufficiently long time such that

$$f(\bar{v},\bar{x},t) / \int d^3v\, f(\bar{v},\bar{x},t) = c(\bar{v},\bar{x}) \tag{2}$$

where $f(\bar{v},\bar{x},t)$ is the electron distribution function then the system is said to be in equilibrium with respect to the distribution. An important point to note is that the total electron density still may be a function of time; thus, if Eq. (2) is true, then $\langle \bar{v}(\bar{x},t) \rangle$ is constant in time, but $n(\bar{x},t)$ may be changing. For most electromagnetic pulse (EMP) applications, spatial homogeneity can be assumed. Thus, for convenience, the spatial dependence will be dropped.

If the system is in equilibrium as defined above, then it is conventional to define the electron mobility

$$u = |\langle \bar{v} \rangle| / |\bar{E}| \tag{3}$$

where u is the mobility and $\bar{E}$ is the applied electric field. Eq. (1) can be re-written as

$$\bar{J} = e\, n(t)\, u\, \bar{E} \tag{4}$$

where it is assumed that electric field vector is parallel to the average velocity vector. From Ohm's law then

$$\sigma(t) = e\, n(t)\, u \tag{5}$$

where $\sigma(t)$ is the electrical conductivity. In most EMP calculations, equilibrium is assumed. The justification will be given later in the discussion on nonequilibrium treatments. For a given applied field in equilibrium and no external electron source, velocity (drift velocity) is only a function of E/N where N is the particle density. Thus it follows that

$$uN/N_o = f(E/N) \tag{6}$$

where N_o is the ambient particle density at the earth's surface. This is the form most used for mobilities. Data fits of the form of (6) are calculated from experimental drift velocity measurements and calculations. In a series of papers (refs. 1-5), A. V. Phelps and others developed methods for calculating the drift velocity, characteristic energy, ionization coefficient, and electron attachement coefficient for various mixtures of gases including N_2, O_2, H_2O, CO_2, and other species. Basically, they first collect all available experimental information for each species such as the drift velocity and characteristic energy as a function of E/N, the elastic and inelastic electron cross sections, and the ionization and electron attachment cross sections. These known cross sections, along with estimates of the unknown cross sections, are inserted in a program which solves the Boltzman equation for the equilibrium electron distribution. From this distribution function, the drift velocity, characteristic energy, and other desired quantities can be calculated. The unknown cross sections are manipulated until all the theoretically calculated quantities match the experimental quantities. Thus, for each species, as a function of electron energy, a set of cross sections is developed which reproduce the available experimental data. The relevant properties of mixtures, then, are easily calculated from the cross sections of the individual species. In the one mixture calculation for N_2 and H_2O seen by this author, where experimental data also were present, the agreement with data was accurate to 20 percent at the worst, and generally within 10 percent. It is the opinion of this author that the drift velocity results used to calculate mobilities for present EMP calculations are generally not in error by more than 25 percent for either dry or saturated air.

For dry air in cgs units a typical fit is

$$uN/N_o = 4.5 \times 10^6 \left(1 + 4.97 \times 10^{-4} (EN_o/N)^4\right)^{.0505} /$$

$$\left(\left(1 + 2.07 \times 10^9 (EN_o/N)^{5.3}\right)^{.089} \left(1 + 1.25 \times 10^{-8} (EN_o/N)^{2.75}\right)^{.0887}\right) \quad (7)$$

This fit is based primarily on air calculations by Phelps (ref. 6). Another fit by the author for three percent water vapor from the Phelps results can be found in the appendix. These fits match the data to better than 20 percent everywhere.

Rigorously, the above fits should only be used when Eq. (2) is valid over time scales of interest and when there are no external electron sources. Neither of these conditions are always true for typical EMP calculations. A first order correction for violation of the above two conditions is afforded by a swarm theory (ref. 7) treatment for the conduction electrons. In conventional swarm theory, the characteristic energy, drift velocity, and total number of electrons are calculated as a function of time with the following equations:

$$d\langle \bar{v} \rangle dt = e\bar{E}/m - \nu_m \langle \bar{v} \rangle - \alpha_T \langle \bar{v} \rangle \quad (9)$$

$$d\varepsilon/dt = ae\bar{E}\langle \bar{v} \rangle - \nu_\varepsilon (\varepsilon - \varepsilon_o) - \alpha_T \varepsilon + S(t) \quad (10)$$

$$dn/dt = n(\alpha_I - \alpha_r) + R(t) = n\alpha_T \quad (11)$$

where n, ε, and $\langle \bar{v} \rangle$ are the electron density, characteristic energy, and drift velocity. ε_o is the ambient characteristic energy. $S(t)$ and $R(t)$ are the external energy and electron source terms. In typical EMP calculations, they represent the electrons and their energy coming into the swarm from Compton electron ionization. ν_m and ν_ε are the momentum exchange and energy exchange collision frequencies. They are defined by the equilibrium electron distribution with $S(t) = R(t) = 0$.

$$\nu_m/N = \left[(e/m)(E/N) - \alpha_T \langle \bar{v} \rangle / N\right] / (v) \quad (12)$$

$$\nu_\varepsilon/N = [ae(E/N)\langle v\rangle - \alpha_T\varepsilon/N] \,/(\varepsilon - \varepsilon_o) \qquad (13)$$

All quantities on the right side of Eqs. (12) and (13) are taken as the equilibrium values at a given E/N. These collision frequencies are parameterized in terms of the characteristic energy. The variable "a" is the ratio of the characteristic energy to the average electron energy for a given E/N. At very low E/N, where the electron distribution in air, for example, approaches a Boltzman distribution, a = 2/3. At the E/N or ε values characteristic of EMP calculations, a = 1. The Phelps papers previously referenced use a = 1 in calculating the above frequencies. Thus, "a" should be set to one in equation (10). Otherwise, equations (9 through 11) will not relax to the correct equilibrium state. Fits of the collision frequencies for dry air and for air with 3 percent water vapor are found in the appendix. α_I and α_r are ionization and electron attachment coefficients. Fits as a function of characteristic energy are also in the appendix.

The conduction current from swarm theory is given by Eq. (1) with the $\langle\bar{v}(t)\rangle$ and n(t) coming from Eqs. (9) and (11). The basic assumption in using swarm theory is that the real electron distribution does not vary grossly from the equilibrium distribution at the present E/N value. It is extremely difficult to quantify this assumption. One way is to look at the relaxation times of Eqs. (9) and (10). This relaxation times are defined by

$$\tau_\varepsilon \equiv 1/(\nu_\varepsilon + \alpha_T)$$

$$\tau_m \equiv 1/(\nu_m + \alpha_T) \qquad (14)$$

Now $\tau_\varepsilon >> \tau_m$ so only τ_ε need be considered here. In high-altitude EMP scenarios (HOB $\geq$ 60 km), where the line of sight intersects the earth's surface, $\tau_\varepsilon \leq \Delta\tau$ is generally true when the conductivity effects are significant. The only exception is for very low yield ($\leq$ .01KT$_Y$) nuclear devices. Consequently, calculations run with equilibrium mobilities should vary little from those using swarm theory in these scenarios. Several calculational comparisons verify this expectation. Thus, for typical high-altitude scenarios, equilibrium mobility fits are adequate.

The above results strongly imply that if for any altitude or line of sight, $\tau_\varepsilon \leq \Delta\tau$ is true for the times near peak field all along the line of sight, then equilibrium calculations are indicated. The only exception to this statement so far observed occurred in some high-frequency calculations for some low-altitude (HOB $\leq$ 1 km) bursts. Here S(t) dominated Eq. (10) before peak field time and the swarm result varied from the equilibrium result by about 25 percent (ref. 8). This is a situation where even though $\tau_\varepsilon << \Delta\tau$ and Eq. (2) holds, the equilibrium properties are not just a function of E/N but are also a function of S(t) and R(t), the external source functions.

All of the preceding remarks have been concerned with collisionally dominated physical regions, that is, where absorption processes are dominant. Another region of importance is where the collision frequencies are small in Eqs. (9) and (10) as compared to the terms involving the electric field. One such case is when an EMP signal is propagating through the ionosphere. Here dispersion for frequencies above the local plasma frequency and reflection for those below is more important than absorption, although usually some absorption does occur. The swarm equations then are perhaps better looked at as actual dynamic equations for the mean energy and velocity. With this in mind, it is easy to see whey the swarm equations do a quite adequate job in modeling dispersion. In fact, calculations carried out at AFWL/DYT and AFWL/ELP indicate that as long as the collisions do not totally dominate, the swarm theory does well in modeling simultaneously both absorption and dispersion. This conclusion has been reinforced by calculations (ref. 9) where the swarm results were compared to detailed Monte-Carlo calculations of the time-dependent electron distributions.

In summary, for calculations in ordinary high-altitude EMP situations, equilibrium conductivity data are adequate and more elaborate methods such as swarm theory are not necessary. The swarm theory, however, has great utility in propagating signals through the ionosphere where both dispersion and some absorption is occurring.

This author would like to express his appreciation to Phelps and his associates. The fits to the various physical quantitites contained in this paper resulted either directly or indirectly from theoretical calculations or data compiled by Dr. Phelps. In addition, the contributions to the conductivity problem by Dr. Carl Baum (refs. 10 and 11) of AFWL and William T. Wyatt, Jr., (ref. 12) of HDL should be acknowledged.

3% corresponds to 75% relative humidity at 85°F

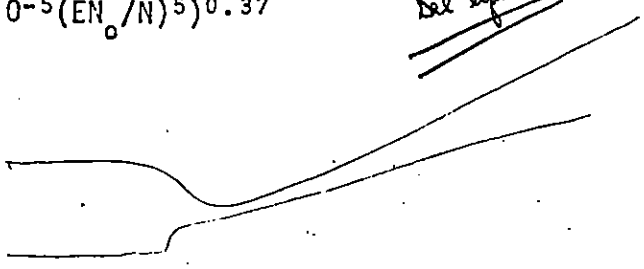
APPENDIX

1. Equilibrium mobilities (cgs units)
   a. Dry air

$$uN/N_o = 2.5 \times 10^{-2}(1 + 0.11\ (EN_o/N)^{3.6})^{0.658}/$$

$$(1 + 5.65 \times 10^{-5}(EN_o/N)^5)^{0.37}$$

del equation 7



   b. Three percent H_2O in air

$$uN/N_o = 6.6 \times 10^5(1 + 1.25 \times 10^{-7}(EN_o/N)^7)^{0.0857}/$$

$$((1 + 8.89 \times 10^{-3}(EN_o/N)^4)^{0.216}(1 + 8.9 \times 10^{-9}(EN_o/N)^{2.75})^{.0887})$$

2. Non-equilibrium collision frequencies
   a. Dry air

$$\nu_m/N = 1.25 \times 10^{-7}U^{0.935}(1 + 0.093\ U^{1.5})^{0.405}/$$

$$(1 + 1.26\ U^{1.67})^{0.373}$$

$$\nu_U/N = 1.3 \times 10^{-11}(1 + 4.4 \times 10^4 U^5)^{0.32}/$$

$$(1 + 1.93 \times 10^3 U^{10})^{0.138} + 4.13 \times 10^{-10} U^{5.22}/$$

$$((1 + 0.06U^7)^{0.456} (1 + 1.68 \times 10^{-3} U^4)^{0.097})$$

b. Three percent H_2O in air

$$\nu_m/N = 3.1 \times 10^{-8} + 6.0 \times 10^{-8} U^{1.08}$$

$$(1 + 0.44 \exp (U-4.4))/(1 + \exp (U-4.4))$$

$$\nu_U/N = 2.04 \times 10^{-10} U^{-0.69}(1 + 4.66U^2)^{0.145}$$

$$+ 4.13 \times 10^{10} U^{5.22}/((1 + 0.06U^7)^{0.456}$$

$$(1 + 1.68 \times 10^{-3} U^4)^{0.097})$$

3. Air breakdown coefficient

$$\alpha/N = 3.76 \times 10^{-15} U^{8.7}/(1 + 2.92 \times 10^{-4}U^5)^{1.412}$$

4. Electron recombination

a. Three body recombination

$$\alpha_3/N^2 = 2.78 \times 10^{-31}/(1 + 12.5U)$$

b. Two body dissociative recombination

$$\alpha_2/N = 4.85 \times 10^{-12}/(1 + 3.45U^{-4.8})^{1.29}$$

In the above, U is the characteristic energy in electron volts. All other units are cgs units.

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