Theoretical Notes

Note 313

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# A Geometrical Instability In A Soil-Filled Coaxial Structure

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## Abstract

This note discusses an instability which can exist in a soil-filled coaxial structure. This instability is characterized by the soil breaking down beyond a critical radius to the outer conductor in an uncontrolled manner. A method for removing the instability is described. This structure is used in experiments relevant to the MX power line resistive link and MF antenna.

It has been pointed out that an instability will exist in a coaxial structure filled with soil which we plan to use in certain experiments relating to the R-cable. It is the purpose of this note to illuminate the origin of this instability and show how it may be removed.

The coaxial structure is to be used in a continuing set of experiments relating to the R-cable. Specifically, we can measure the radial extent of streamers, under the conditions of soil breakdown, for a given applied voltage. Also, the details of the geometrical configuration of streamers can be investigated. Finally, we can answer the all-important question of whether streamers produced by a strong radial electrical field will enhance soil conductivity in the longitudinal direction.

The coaxial structure is shown in Fig. 1. It consists of a metal inner rod of radius a and an outer metal cylinder of radius b. The length of the coax is $\ell$ and it is filled with soil of conductivity $\sigma$. The inner rod is grounded while the outer cylinder is connected through a resistor $R_0$ to a pulser which applies a peak voltage $V_0$. The voltage of the outer cylinder is $V_R$.

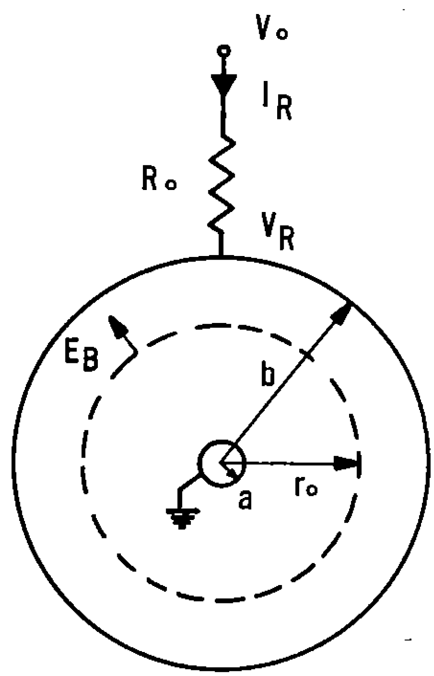


Figure 1. Coaxial geometry; structure is filled with soil from $a \geq r \leq b$.

The model that we use assumes that the voltage $V_R$ is large enough to break down the soil out to a radius $r_o$ and does so with cylindrical symmetry. For $r < r_o$, the soil conductivity, at least in the radial direction, is infinite. We assume that there is a critical electric field $E_B$ which will cause the soil to break down. Thus, the radial electric field at $r_o$ is $E_B$.

According to Liew, et al. [1], $E_B$ should be from 100 to 300 KV/m. Note that this is the breakdown voltage at the tips of the streamers in the broken down soil. In the absence of streamers, e.g., at the inner rod surface at $r = a$ before the soil is broken down, the breakdown voltage will be much higher. Typically, this breakdown voltage will be about 2000 KV/m [2].

From Gauss' Law and other elementary relations, the radial electric field at $r \geq r_o$ is

$$E_r = \frac{V_R}{r \,\ell n(b/r_o)} \tag{1}$$

Thus,

$$E_B = \frac{V_R/b}{(r_o/b)\, \ell n(b/r_o)} \tag{2}$$

Calling $x = r_o/b$,

$$-x \,\ell n\, x = \frac{V_R/b}{E_B} \tag{3}$$

The function $-x \,\ell n\, x$ is plotted in Fig. 2. It has a maximum at $x = 1/e$ with the function having a maximum value of $1/e$.

In Fig. 2, it is seen that $-x \,\ell n\, x$ is a doubly valued function of $x$, i.e., for a given value of $V_R$, there are two values of $x$ which will satisfy (3). Presumably, to move the outside radius of the streamers $r_o$ from $r = a$ to

$r = b$, one would increase $V_R/bE_B$ from $-(a/b)\ \ln(a/b)$ (neglecting the large initial breakdown voltage) to 1/e and then to zero again if the streamers should choose to track on that portion of the $-x \ln x$ curve with the negative slope. Suppose we are at the operating point 2 in Fig. 2. Further, let the resistance $R_0$ be very small compared to the resistance of the unbroken down portion of the soil R. We hold $V_0$ fixed. Suppose that there is a small noise fluctuation in the system which causes the current $I_R$ to increase slightly. $V_R$ will then drop, and according to Fig. 2, x will increase, decreasing R. $I_R$ will then increase further, etc., with the result that x will continue to increase until x = 1, at which time the soil is broken down all the way across the cylinder. We see that points on the negative slope portion of Fig. 2 are unstable operating points leading to an instability which will cause the soil to rapidly break down until $r_0 = b$ is reached.

On the other hand, if we are at the operating point 1, if $I_R$ increases and $V_R$ decreases, according to Fig. 2, x will decrease and R will increase. Thus, the resistance of the soil R changes in such a way as to tend to damp out noise fluctuations in $I_R$, and point 1 is a stable operating point.

We call this instability a <u>geometric instability</u>, since it primarily depends on the cylindrical geometry of the coaxial structure.

The stable region of operation is thus for x from approximately zero to 1/e = .3679. This means that only about 13% of the volume of the soil may contain streamers. Thus, if we wish to develop streamers out to a radius of 50 cm, we would need a cylinder 2.7 m in diameter. For a meter-long cylinder, the volume of the soil required to fill it is 5.8 $m^3$, weighing about 8 tons! The corresponding volume of the soil containing the 50 cm long streamers is .78 $m^3$, weighing about 1 ton.

Since the 8 ton structure may be logistically unmanageable, it is highly desirable to devise a method of eliminating the geometric instability so that only .78 $m^3$ of soil is required.

Consider now the case where $R_o$ is large compared to R, and we are again at the operating point 2 in Fig. 2. A noise fluctuation causes $I_R$ to first increase; $V_R$ drops; x increases; R decreases. Next, however, $I_R$ will increase only a small percent of this first increase because R is small compared to $R_o$. As the process continues, $I_R$ will increase by only a small percent of its previous increase. Thus, we may expect $I_R$ and x to reach limiting values. We see that by adding a sufficiently large resistor, we have removed the instability.

To calculate the required value of $R_o$, we let

$$R_c = (2\pi\sigma\ell)^{-1} \tag{4}$$

and note that

$$R = R_c \ \ell n(1/x) \tag{5}$$

We have

$$V_o = I_R R_o + V_R = V_R(R_o/R) + V_R \tag{6}$$

Using (3), (4), and (5) in (6), we can obtain

$$\frac{V_o b}{E_b} = (-x \ \ell n \ x)(1 - \frac{R_o/R_c}{\ell n \ x}) \tag{7}$$

The function on the left side of (7) is plotted in Fig. 3 for $R_o/R_c = 1$. In Fig. 3, we see that for any choice of $V_o$, the slope of the function is positive, leading to stable operating points. Thus, $R_o = R_c$ will remove the instability.

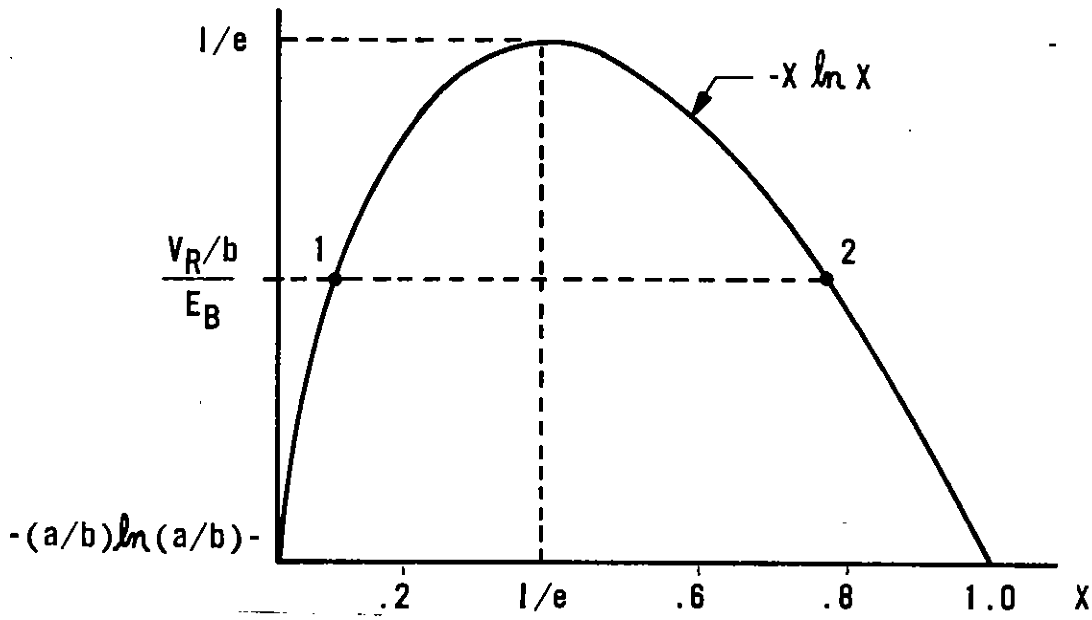


Figure 2. Plot of $-x \ln x$ vs. x

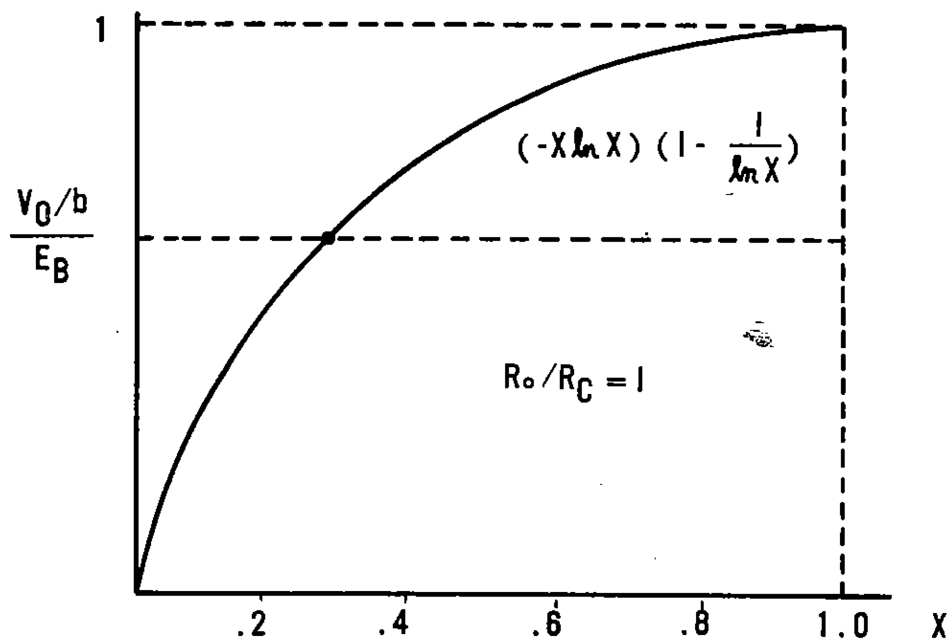


Figure 3. Plot of $(-x \ln x)(1 - \frac{Ro}{R_C \ln x})$ vs. x for $R_o/R_c = 1$

Had we chosen $R_0/R_c < 1$, we would find that the function on the left side of (7) would have a maximum at $x < 1$, and that there would be a range of x for which the system is unstable. For $R_0 = R_c$, this maximum occurs at $x = 1$; for $R_0 > R_c$, the maximum occurs for $x > 1$, and the system is unconditionally stable. We call $R_c$ the critical resistance.

For $\sigma \doteq 10^{-3}$ mho/m and $\ell = 1$ m, $R_c = 159$ ohm. We note that $R_c$ is independent of a and b.

Acknowledgement

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## REFERENCES


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2. Mallon, C., R. Denson, R. E. Leadon, and T. M. Flanagan, "Electrical Breakdown Characteristics of Soil Samples," Report 200-81-218/2148, Jaycor, San Diego, CA, January 12, 1981.