Theoretical Notes
Note 307

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) EARLY GROUND BLOW-OFF OF NUCLEAR SURFACE BURSTS		5. TYPE OF REPORT & PERIOD COVERED Topical Report
		6. PERFORMING ORG. REPORT NUMBER MRC-R-533
7. AUTHOR(s) Joseph A. Ball Conrad L. Longmire		8. CONTRACT OR GRANT NUMBER(s) F29601-79-C-0035
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mission Research Corporation 735 State Street, P.O. Drawer 719 Santa Barbara, California 93102		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Weapons Laboratory Kirtland Air Force Base, NM 87117		12. REPORT DATE 26 October 1979
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Nuclear Surface Burst
Radiation Diffusion
Shock Waves
Ground Blow-Off
Nuclear Burst Craters
Thermal Wave
Self-Similar Solutions
Gamma Rays
Neutron Capture

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

To determine ground sources of gamma rays in near surface atmospheric nuclear bursts densities of early blown-off ground material are calculated by a sequence of analytic approximations for: atmospheric fireball temperature, ground temperature and thermal wave penetration, and blow-off densities behind ground shock. An analytic expression for ground material density as a function of position and time, valid for times of 0.01 and 10 msec in regions beyond a few weapon diameters away from burst is derived. Efects of weapon debris are not accounted for.

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SECTION 1
INTRODUCTION

This report addresses the problem of the blow-off of the ground in the immediate vicinity of a near-surface nuclear burst in the megaton range. This problem is important because blow-off at times early enough and to an extent great enough can seriously influence the gamma ray source originating from neutron capture in ground material. Specifically, decrease in density of ground material can slow the rate of neutron capture with the result that the gamma ray source is less intense and more spread out in time than would be expected if, as is presently assumed in EMP source models, the ground remained in place. This mechanism has been suggested to account for the anomalous gamma ray dose rate (approximately 10 times weaker and 10 times longer than normal) measured in the Mike test at time of a few milliseconds. Our discussion is not restricted, however, to any specific test. The question is analyzed from a general point of view, with use of a hypothetical 1 MT shot for purposes of illustration.

The presentation of an analytic solution for the density of blown-off radiatively heated ground, ignoring debris slap, constitutes the body of this report.

GENERAL PROBLEM ASPECTS

Blow-off of the ground layer in which neutron capture occurs depends critically upon the details of energy and momentum coupling into the ground at early times during the radiative expansion phase of the atmospheric fireball.

At the earliest times a thermal wave, propagated by radiation diffusion, heats a layer of ground. When the thermal wave front slows to near the isothermal sound velocity in the heated ground, hydrodynamic motions become significant and a strong shock is driven into the ground. Because of the large density difference, ground pressures both within the thermal wave and behind the strong shock exceed the air pressure by factors of 100 to 1000. Consequently the ground surface blows off immediately, accelerated by a hydrodynamic rarefaction which propagates into the ground and eventually overtakes and weakens the strong shock. The density and location of ground material within this rarefaction fan is the focus of our interest, but this simple picture is far from complete.

At some time during these events the ground surface is impacted by the high velocity ($\sim 10^8$ cm/sec) material of the bomb debris. From the surface of impact, shocks are driven both into the ground material and back through the bomb debris. The shock into ground material will recompress it and initially reduce or reverse its escape velocity into the air. Only when the other shock, driven back through the bomb debris, emerges into the air can another rarefaction propagate back through the bomb debris into the ground material and re-establish a rarefaction fan of blow-off material. One would expect the effect of debris slap to delay initial blow-off but perhaps to enhance it later through the additional energy delivered to the ground.

Considered in all detail the problem is one of coupled hydrodynamics and radiation transport in two-dimensional axially symmetric geometry. The events described qualitatively above have different time scales and magnitudes dependent upon location with respect to burst point. Our approach chooses to simplify as many of these complications as possible, consistent with a description sufficient for the purpose. We use common analytic approximations for radiation transport and one-dimensional hydrodynamics (motion normal to ground surface) which is parameterized by radial distance along the surface away from the burst point. In this way we have secured an approximate analytic expression for the density of blow-off ground material at positions not too near (2 meters) the burst point in the absence of debris slap. This partial solution constitutes the body of this report.

The modifications of this solution caused by debris slap and a more careful analysis of events very near the burst point remain to be undertaken.

METHOD OF PARTIAL SOLUTION (NO DEBRIS SLAP)

We calculate the details of the early radiation expansion phase of the atmospheric fireball on the basis of an energy balance in a hemisphere. This calculation provides boundary and initial conditions at the ground surface from which the thermal wave penetrating into the ground may be estimated. (More detailed numerical codes show that less than 10 percent of the total device yield is coupled into the ground. So the ground heating is considered to be driven by the atmospheric fireball, which behaves as a heat reservoir whose temperature is unaffected by energy interchange with the ground.) The ground receives energy at early times, but cools more slowly than the air, so later the ground radiates into the air.

When the velocity of the front of the thermal wave into the ground slows to the isothermal sound speed it "shocks up" driving a shock into the ground. This transition from radiation transport to hydrodynamics occurs much earlier in the ground than in the gaseous atmosphere. On the basis of planar shock dynamics we analyze this ground shock and the expansion behind it. The solution is continued to late times by fitting it to the self-similar solution for a shock wave resulting from an impulse at the surface. This procedure permits an estimate of densities in the ground material blown off from the surface at different surface positions encompassed by the atmospheric fireball.

Since this analysis ignores effects of weapon and case debris impact upon the ground it is valid only when this is a minor effect.

ANALYTIC APPROXIMATIONS

The calculation requires tractable analytic approximations to describe material equations of state and radiative diffusion. Below are listed the approximations we have used. Units are cgs and temperature is measured in kilovolts.

Air

Caloric equation of state (normal density, $\rho_{10} = 1.293(10)^{-3}\text{gm/cm}^3$)

$$e_1(\text{ergs/cm}^3) = c_1 T(\text{kv})$$

$$= 9.363(10)^{11} T(\text{kv}) \quad ; \quad T > 5(10)^{-3}\ \text{kv}$$

$$e_1'(\text{ergs/gm}) = 7.241(10)^{14} T(\text{kv}) \quad ; \quad T > 5(10)^{-3}\ \text{kv}$$

$$p_1 = 2/3\ e_1 \quad . \qquad (1)$$

Rossland mean free path (normal density)

$$\ell_1(\text{cm}) = 3.85(10)^3 \qquad T > 0.5\ \text{kv}$$

$$\ell_1(\text{cm}) = b_1' T^{m_1}$$

$$= 2.81(10)^4 [T(\text{kv})]^{2.868} \qquad 0.01\ \text{kv} < T < 0.5\ \text{kv} \ . \qquad (2)$$

Radiation Diffusivity (normal density)

$$D_1(\mathrm{cm}^2/\mathrm{sec}) = c\ell_1/3 \qquad c = \text{velocity of light}$$

$$= b_1 T^{m_1} \qquad (3)$$

Aluminum (We use aluminum values to approximate both case material and ground material)

Caloric equation of state (normal density, $\rho_{20} = 2.699\ \mathrm{gm/cm}^3$)

$$e_2(\mathrm{ergs/cm}^3) = c_2 T(\mathrm{kv})$$

$$= 2.159(10)^{15} T(\mathrm{kv}) \quad ; \quad T > 0.177\ \mathrm{kv}$$

$$e_2'(\mathrm{ergs/gm}) = 8.0(10)^{14} T(\mathrm{kv}) \quad ; \quad T > 0.177\ \mathrm{kv} \qquad (4)$$

Caloric equation of state (low temperature, variable density)

$$e_2(\mathrm{ergs/cm}^3) = 3.760(10)^{16} T^{1.32} (\rho_{20}/\rho_2)^{0.056} \quad ; \quad 0.177\ \mathrm{kv} > T > 0.01\ \mathrm{kv}$$

$$e_2'(\mathrm{ergs/gm}) = 1.393(10)^{15} T^{1.32} (\rho_{20}/\rho_2)^{0.056} \quad ; \quad 0.177\ \mathrm{kv} > T > 0.01\ \mathrm{kv}$$

$$p_2 = 2/3\ e_2 \quad .$$

Rossland mean free path (normal density)

$$\ell_2(\mathrm{cm}) = b_2' T^{m_2}$$

$$= 0.0783[T(\mathrm{kv})]^{2.307} \quad ; \quad 4\ \mathrm{kv} > T > 0.05\ \mathrm{kv} \ . \qquad (5)$$

Radiation diffusivity (normal density)

$$D_2(\text{cm}^2/\text{sec}) = c\ell_2/3 \qquad c = \text{velocity of light}$$

$$= b_2 T^{m_2} \quad . \tag{6}$$

Radiation

Energy density

$$e_r(\text{ergs/cm}^3) = a\,T^4$$

$$= 1.37(10)^{14}[T(\text{kv})]^4$$

$$p_r = 1/3\ e_r \quad . \tag{7}$$

Black body flux

$$F(\text{ergs/cm}^2\text{-sec}) = \frac{c\ a}{4}\,T^4 = \sigma T^4 \qquad c = \text{velocity of light.} \tag{8}$$

SECTION 2
ATMOSPHERIC FIREBALL

For illustration we will consider throughout a hypothetical weapon whose mass (including case) is 1 metric ton = 10^6 gm and whose yield is 1 MT. We model the burst by the instantaneous release of the yield energy in a solid aluminum sphere of this mass (radius r_o = 44.6 cm) resting directly upon the ground. In general, however, results are expressed as a function of yield in the megaton range.

FIREBALL TEMPERATURE VS. RADIUS

We assume that only a fraction of the total yield is available as thermal energy for heating of the early fireball, the rest appearing as kinetic energy of case material. We denote this thermal energy by E^1, and assume in the 1 MT case that $E^1 = 0.75E$. The energy balance of the fireball consisting of a hemisphere of air and the aluminum sphere can be written,

$$(2\pi/3)(R^3 - r_o^3)(aT^4 + c_1T) + (4\pi/3)r_o^3(aT^4 + c_2T) = E^1$$

where R is the radius of the isothermal fireball. We have used (1), (4), and (7). The resulting fireball temperature as a function of fireball radius for the 1 MT case is shown in Figure 1. Thermal energy in the ground is neglected in this calculation. At small radii most of the energy is in the radiation field and $T \sim R^{-0.75}$. Near a radial distance $R_1 = 4.39\ (10)^3$ cm at which the material energy density equals the energy density of radiation a transition in slope of the curve occurs after which $T \sim R^{-3}$ and material energy is dominant.

The temperature vs. radius of the fireball during this early radiation expansion may be approximated by piecing together two power law relations, each accounting for only one form of energy. Numerical evaluation gives the following formulas:

$$T(\text{kv}) = 1.098(10)^2[E^1(\text{MT})]^{1/4}[R(\text{cm})]^{-3/4} \quad ; \quad R < R_1 \qquad (9)$$

$$T(\text{kv}) = 2.137(10)^{10}E^1(\text{MT})[R(\text{cm})]^{-3} \quad ; \quad R_1 < R \qquad (10)$$

$$R_1(\text{cm}) = 4.829(10)^3[E^1(\text{MT})]^{1/3} \quad . \qquad (11)$$

These formulas are in error both near R_1 and at very small radii. For example by (9) the initial temperature of the aluminum sphere $R = 44.6$ cm is 5.9 kv, while its value by more accurate calculation is 4.8 kv. Both here and subsequently we have chosen simple power law expressions despite their approximate nature, because of the requirements of later analysis.

RADIAL VELOCITY OF FIREBALL FRONT

The initial fireball expansion, or "burn-out" is not limited by radiation diffusion since the fireball radius is less than the Rossland mean free path at the prevailing temperature, but as the temperature drops radiation diffusion limits the front velocity. These two phases are calculated separately.

Early Radiation Burn-out

Fireball front velocity in the burn-out is estimated by equating the heating rate of a differential volume at the front to the surface radiation flux (assumed black body) of the hot fireball. Using (1), (7) and (8),

$$\frac{c\, aT^4}{4} 2\pi R^2\, dt = (aT^4 + c_1 T)\, 2\pi R^2\, dR$$

or

$$\dot{R} = \frac{c}{4} \frac{aT^4}{aT^4 + c_1 T} \simeq \frac{c}{4} \qquad \text{(burn-out)} \quad . \tag{12}$$

Except for the small effect of material energy this is the expansion rate of a sphere of radiation in thermal equilibrium.* The early-time burn-out velocity is plotted in Figure 1 for the 1 MT case.

Radiation Diffusion

Both numerical codes and similarity solutions show that within a sphere expanding and cooling by radiation diffusion the divergence of the radial flux of radiation energy is approximately a constant, independent of position. This fact accords with a spatially uniform temperature and cooling rate within the sphere. We therefore model the radiation diffusion within the fireball by a linear increase with radius of radiation energy flux. The radiation flux is the negative of the radiation diffusivity times the gradient of the radiation energy density. Using (3) and (7),

$$-b_1 T^{m_1} \frac{d}{dr} aT^4 = kr$$

where k is the constant of proportionality. Integrating the above between $r = o$ where the temperature is T to the front radius R where the temperature is zero, we obtain kR, the radiation flux at the front.

$$kR = \frac{8ab_1 T^{m_1+4}}{(m_1+4)R} \quad .$$

*A lower limit of velocity, since radiative equilibrium is not possible without scattering. In fact, front velocities of the actual sphere of nonequilibrium radiation may start near c.

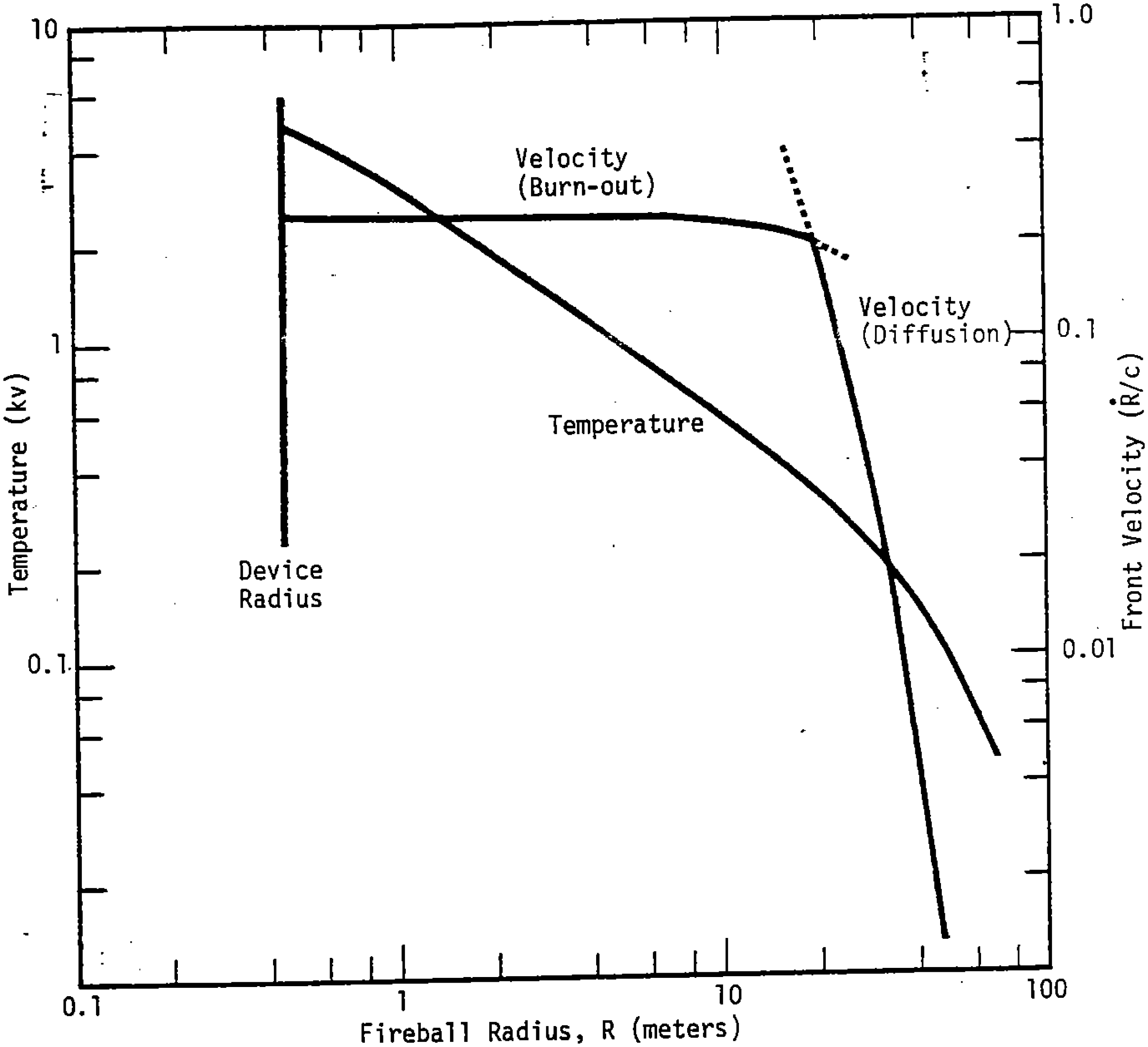


Figure 1. Fireball temperature and front velocity for 1 MT surface burst.

This flux equated to the heating rate necessary to warm a layer of cold air at the front to the temperature, T, provides the propagation velocity, by radiation diffusion, of the fireball front.

$$\dot{R} = \frac{8ab_1T^{m_1+4}}{(m_1+4)R(aT^4+c_1T)} \quad \text{(radiation diffusion)} \quad . \qquad (13)$$

This velocity as a function of front position is plotted in Figure 1 for the 1 MT case. The "diffusion" velocity on this plot intersects the "burn-out" velocity at a definite radius R_o. This radius marks the transition between the two propagation mechanisms and is of the order of the Rossland mean free path at the prevailing temperature. For yields in the megaton range R_o is less than R_1, so the transition occurs when radiation energy still dominates material energy in the fireball. We equate (13) with material energy omitted to (12), express $\bar{T}$ in terms of R by (9) and obtain an approximate expression for R_o. When R becomes larger than R_1 the material energy dominates and the radiation energy density may be dropped from the denominator of (13). After these operations are performed and numerical constants introduced we obtain the following approximations for fireball front velocity.

$$R_o(\text{cm}) = 2.140(10)^3[E^1(\text{MT})]^{0.22755}$$

$$\dot{R}(\text{cm/sec}) = c/4 \qquad ; \quad R < R_o$$

$$\dot{R}(\text{cm/sec}) = 2.338(10)^{20}[E^1(\text{MT})]^{0.717}[R(\text{cm})]^{-3.151} \quad ; \quad R_o < R < R_1$$

$$\dot{R}(\text{cm/sec}) = 1.960(10)^{77}[E^1(\text{MT})]^{5.868}[R(\text{cm})]^{-18.604}; \qquad R_1 < R$$

$$= 3.022(10)^{27}[E^1(\text{MT})]^{5.868}[R(\text{m})]^{-18.604}$$

SECTION 3
THERMAL PENETRATION INTO GROUND

The thermal boundary conditions at the ground surface, we assume, are provided by the temperature as a function of time (or front radius) of the atmospheric fireball. We model the early ground heating by a one-dimensional wave of radiation diffusion whose front is parallel to the ground and which propagates normal to the ground surface. Since the radiation mean free path is much shorter (a factor of 10^{-5} or so) in the ground than in the air, the early nondiffusive "burn-in" into the ground is confined to an extremely thin surface layer. We ignore this brief early phase and consider radiation diffusion only. Although ground blow-off would start immediately we delay its inception until the thermal wave develops into a shock. Up to this time ground material is assumed at rest at its normal density. For our purposes this is a conservative assumption, in that ground blow-off is slightly delayed.

GROUND THERMAL WAVE (ONE-DIMENSIONAL)

If we call y the distance from the ground surface, measured downward into the ground, the one-dimensional radiation diffusion equation can be written,

$$\frac{\partial}{\partial t} \varepsilon = \frac{\partial}{\partial y} \left[b_2 T^{m_2} \frac{\partial}{\partial y} aT^4 \right]$$

where the subscript 2 denotes ground material and the quantity in brackets is the negative of the flux of radiation energy. The quantity $\varepsilon = e_2 + e_r$ is the sum of material and radiation energy densities. We multiply the above equation by y and integrate between $y = o$ (the surface) and infinity.

The right hand side of the result may be integrated by parts. The integrated term vanishes at both limits; at the surface $y = o$ and at infinity the flux is zero. There remains,

$$\int_o^\infty y \frac{\partial \varepsilon}{\partial t} dy = -\int_o^\infty 4b_2 a T^{m_2+3} \frac{\partial T}{\partial y} dy = -\int_{T_s}^{T_\infty} 4b_2 a T^{m_2+3} dT$$

where the right hand side is expressed as an integral over T which can be explicitly evaluated. The region of thermal penetration is a region of approximately uniform temperature (radiation diffusion permits no sharp temperature gradients) bounded by a wave front at distance Y, beyond which the temperature is zero. Since y is a variable of integration, integrated between fixed limits, the left hand side is the total time derivative of the first moment of energy density. Also, because the temperature (or energy density is approximately uniform within the wave region and zero beyond,

$$\frac{d}{dt}\left(\frac{\varepsilon Y^2}{2}\right) = \frac{4ab_2}{m_2+4} T_s^{m_2+4}$$

where T_s is the ground surface temperature. The time integral of this equation yields the first moment of the energy density of the heated layer.

$$\frac{\varepsilon Y^2}{2} = \frac{4ab_2}{m_2+4} \int_{t_a}^{t} T_s^{m_2+4} \; dt \quad .$$

At any ground position the integration starts at t_a, the time of arrival of the atmospheric fireball front. The temperature of the ground surface follows the temperature of the atmospheric fireball which in the 1 MT case falls as $t^{-3/4}$ at distances within 20 meters of the bomb. Since m_2+4 is 6.307 almost the entire contribution to the integral comes at times immediately after t_a. We therefore make little error by extending the upper limit to infinity and considering each ground position to receive a thermal impulse at t_a given by the integral which fixes a constant value of the first moment of energy density at that point. Hence,

$$\frac{\varepsilon Y^2}{2} = \frac{4ab_2}{m_2+4} P \quad \text{where} \quad P = \int_{t_a}^{\infty} T_s^{m_2+4} \, dt \quad . \tag{14}$$

A similarity solution exists for a one-dimensional thermal wave with a constant energy moment. However, the essential features of this similarity solution may be obtained by assuming the radiation flux to vary linearly with depth through the heated region. (Compare the previous discussion of radial radiation diffusion in the air.) One assumes the flux to equal zero at $Y/2$ and to be negative at the ground surface.* (The ground radiates into the air which is assumed at zero temperature once the thermal pulse is over.) Equating the radiation flux to this linear function,

$$- 4ab_2 T^{m_2+3} \frac{dT}{dy} = k(y - \frac{Y}{2})$$

and integrating between $Y/2$ where the temperature is T_g, the ground temperature, to the wavefront Y where the temperature is zero gives,

$$\frac{4ab_2 T_g^{m_2+4}}{m_2+4} = (k/2) \ (Y/2)^2 \quad . \tag{15}$$

Now assume times are late enough and temperature low enough that the radiation contribution to ground energy is negligible. Using (4), Equation 14 can be expressed in terms of the ground temperature, T_g.

$$\frac{c_2 T_g Y^2}{2} = \frac{4ab_2}{m_2+4} \ P \quad . \tag{16}$$

Equations 15 and 16 determine the constant of proportionality k in terms of P and either T_g or Y. Knowing k the radiation flux at the thermal front can be equated to the energy required to warm a layer of thickness dY in time dt to the temperature T_g, and the velocity of the thermal front obtained. The velocity can be integrated to obtain the

*The exact similarity solution suggests that a flux distribution symmetric about the center of the heated layer is a good approximation.

front position Y as a function of time and (16) used to express T_g as a function of time.

The essential results, suppressing for the moment multiplicative constants, are as follows:

$$Y \sim [P^{m_2+3}(t-t_a)]^{\frac{1}{2(m_2+4)}} \qquad (17)$$

$$T_g \sim [P/(t-t_a)]^{\frac{1}{m_2+4}} \quad . \qquad (18)$$

The depth and temperature of the heated ground layer depends upon position through the thermal impulse P and weakly through t_a , the time the position is reached by the atmospheric fireball. Note the ground temperature decreases less rapidly in time than the atmospheric fireball temperature.

EVALUATION OF THERMAL IMPULSE

For yields in the 1 MT range of near surface bursts the estimates of Section 2 indicate that at radial distances less than 20 meters the atmospheric fireball temperature is given by (9) and the front velocity is approximately c/4. Blow-off within this distance is our prime concern, so these expressions are used to evaluate the thermal impulse. Two transformations of the integral (14) are useful. We replace time as a variable of integration by fireball front radius X measured horizontally along the ground from ground zero.

$$\int_{t_a}^{\infty} T_s^{m_2+4}\, dt = \int_{x}^{\infty} \frac{[T(X)]^{m_2+4}\, dX}{\dot{X}} \quad .$$

The variable x is ground position. Change variables again by letting $\omega = R^2 = X^2 + x_o^2$ where R is the fireball radius and x_o is the vertical distance above the ground of the bomb center.

$$\int_{t_a}^{\infty} T_s^{m_2+4} dt = \frac{2}{c}\int_{x^2+x_o^2}^{\infty} [T(\omega^{1/2})]^{m_2+4} \omega^{-1/2} d\omega \quad .$$

Using (9) the integral, complete with constant factors, gives,

$$P = 2.6502(10)^2 (E^1)^{1.578} (x^2+x_o^2)^{-1.8651} \tag{19}$$

with distances in centimeters, and E^1 in megatons. The dimensions of P are $(kv)^{6.307}$-sec.

THERMAL WAVE-SHOCK TRANSITION

A strong hydrodynamic shock develops from the thermal wave front when the front velocity drops to the isothermal sound speed in the heated ground. On the assumption that the strongly heated ground has an effective heat capacity ratio, γ, of 5/3 the isothermal sound speed, c_A, is,

$$c_A^2 = 2/3 \; c_2 T \quad .$$

Using the previous analysis we may equate the isothermal sound speed to the thermal front velocity and determine the time and depth of the shock wave origin as a function of ground position. We denote the time as t_s and the depth as Y_s, both of which are power law functions of P, the thermal impulse. After evaluation of constants the results are:

$$t_s - t_a = 1.9997(10)^{-4} (E^1)^{0.6398} (x^2+x_o^2)^{-0.7568} \tag{20}$$

$$Y_s = 1.0735(10)^5 (E^1)^{0.7141} (x^2+x_o^2)^{-0.8447} \tag{21}$$

E^1 is the thermal energy of the atmospheric fireball in megatons, distances are in centimeters and time in seconds. The distance x_o is the height above ground of the bomb center and

$$t_a = \frac{4}{c}[(x^2+x_o^2)^{1/2}-x_o] \quad , \tag{22}$$

is the arrival time of the fireball front at position x.

These results are plotted in Figure 2 for the case of 1 MT yield (E^1=0.75 MT) at the standoff distance x_o = 44.6 cm. It is interesting to note that the ground shock develops first at a distance of 2-5 meters from the bomb at a time less than 10^{-7} sec. At larger x the shock develops very soon after arrival of the fireball while nearer the bomb the ground shock is significantly delayed. Owing to our planar shock approximation these results have only qualitative validity near ground zero. Larger standoff distance, x_o , would give better answers in this region.

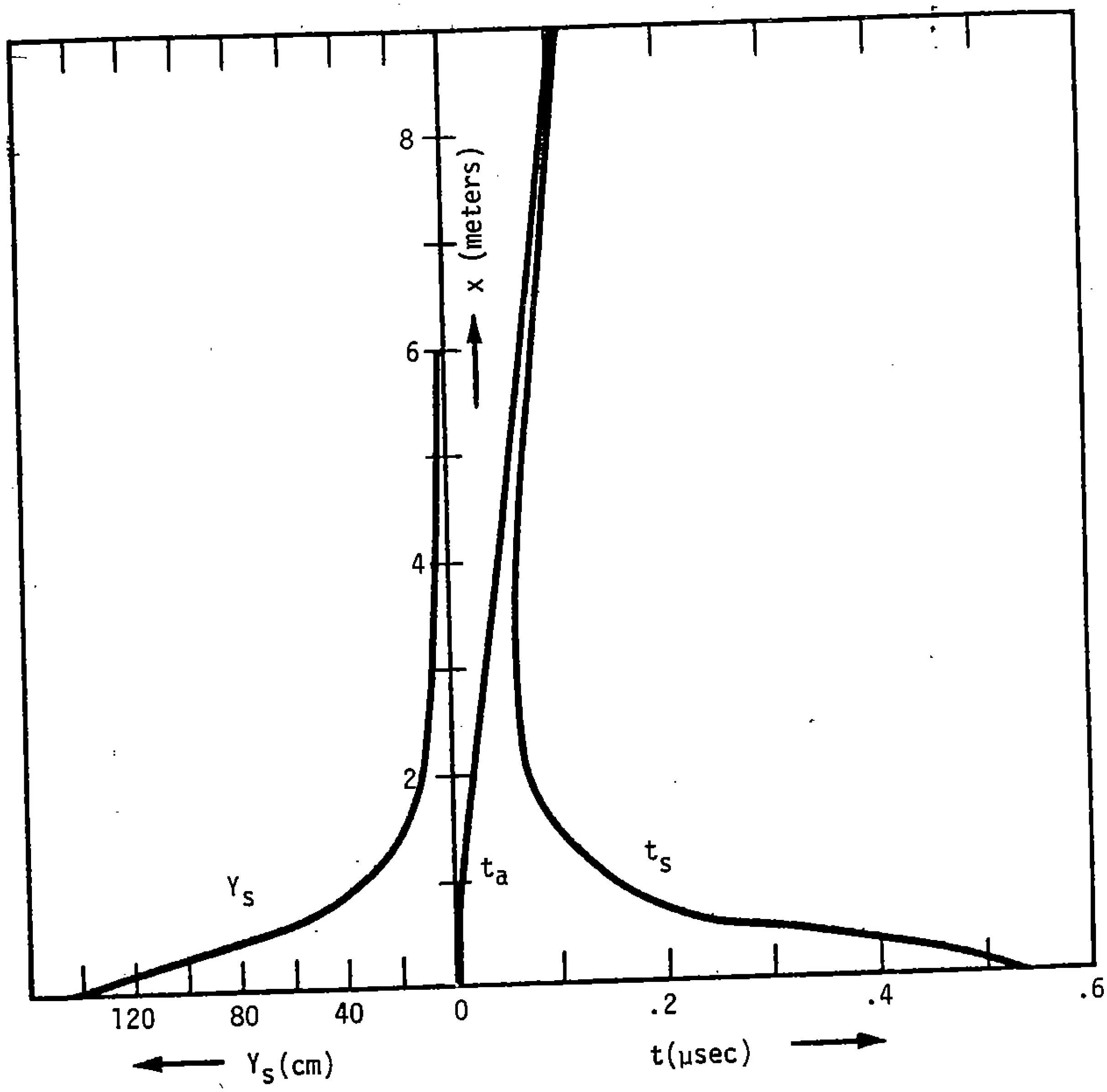


Figure 2. Thermal wave-shock transition in ground of 1 MT surface burst. (Depth, Y_s, and time, t_s, of transition as function of x, distance from ground zero. Fireball front arrival at t_a.)

SECTION 4
LATE TIME GROUND BLOW-OFF

In the present context late time means times from 1 μsec to 10 msec when the thermal wave has effectively stopped and the expansion fan of underdense ground material is developing behind the ground shock. At these times the hydrodynamic structure, of a plane shock, can be described by a similarity solution given by Zeldovitch and Raizer (1967)* for the case of an impulsive pressure applied to a plane material interface with a vacuum. For the case of $\gamma = 7/5$ this solution can be given in a closed analytic form. For convenience we model the late time hydrodynamics with this solution, although $\gamma = 7/5$ for ground material is inconsistent with our earlier assumption of 5/3. The shock position Y_1 is given by,

$$Y_1 = At^{3/5}$$

in terms of which the similarity variable $\xi = y/Y_1$ is defined. The density, velocity, and pressure at values of $\xi<1$ are then given by,

$$\rho = 6\rho_o(5-4\xi)^{-5/2} \; ; \quad u = -5/6\dot{Y}_1(1-2\xi) \; ; \; p = 5/6\rho_o\dot{Y}_1^2(5-4\xi)^{-3/2}$$

where ρ_o is the unshocked initial density of the material.

Unfortunately the constant, A , cannot be determined from simple parameters of the initial impulse such as momentum or energy, but depends

*Zeldovitch, Ya B., and Y. P. Raizer, Physics of Shock Waves and High Temperature Hydrodynamic Phenomena, Vol. II, Academic Press, New York, 1967, p. 829.

upon the details of the starting impulse. This is the reason we have been forced to analyze initial conditions in the previous sections. In this section we continue the analysis to a later time at which the hydrodynamic structure may be matched to the similarity solution. The matching determines, A , and the subsequent hydrodynamics.

EARLY GROUND SHOCK

When a one-dimensional hydrodynamic shock develops from the thermal wave front it is driven by the pressure of an approximately isothermal expansion fan within the heated ground which is produced by a rarefaction moving back toward the ground surface. Both the material velocity and the pressure must have the same value at the contact surface between the shock heated and the radiation heated ground. We can express the pressure in the isothermal expansion fan of radiation heated ground in terms of material speed u and sound speed c_A. The pressure balance with material accelerated by a strong shock gives,

$$\exp(-u/c_A) = (6/5)(u/c_A)^2$$

for $\gamma = 7/5$. From this relation one finds the material velocity u behind the shock and the shock speed v in terms of the sound speed,

$$u/c_A = 0.6572$$

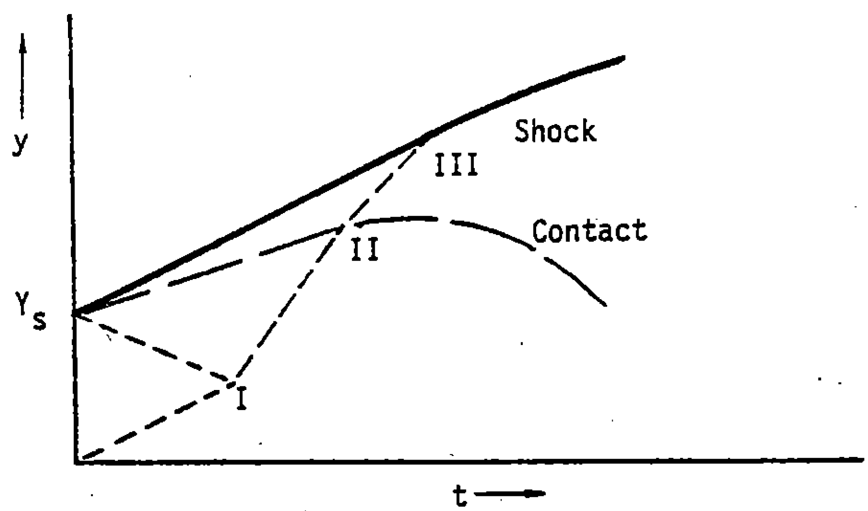
$$v/c_A = 0.7886 \quad .$$

Until the ground shock is overtaken by a rarefaction wave propagating from the surface, these velocities remain constant. (We assume the sound speed constant, since it is a very slowly varying function of time.) At the time and shock position at which the shock is overtaken we match it to the similarity solution. The time and position can be estimated by considering the wave diagram following. The heads of rarefaction waves are indicated by dotted lines. Prior to point 1 a rarefaction from the surface propagates into the heated layer at the sound speed c_A until met by a rarefaction propagating from the shock origin. Between I and II this rarefaction propagates through the remaining heated ground at a speed assumed to be c_A+u. Beyond the contact surface, between II and III the rarefaction propagates through shocked material at the speed c_B+u , where c_B is the ordinary adiabatic sound speed. By piecing together these segments one finds,

$$Y_{III} = 2.684\ Y_s$$

$$t_{III} = 2.136\ (Y_s/c_A)$$

for the coordinates at which the ground shock begins to attenuate by feeling the effects of the open surface. At this time between the shock and the contact surface the material velocity decreases linearly with distance in a manner described by a simple adiabatic expansion wave.



FITTING TO SIMILARITY SOLUTION

If we assume the material velocity behind the shock follows the adiabatic expansion wave relation even behind the contact surface, we find the velocity passes through zero at a position, $y = 2.048\ Y_s$. We note that the similarity solution gives a velocity of zero at one-half the distance to the shock front. Consequently fitting the similarity solution at this time requires a shift of coordinate origin. It also requires a shift of the time origin. We look for a similarity solution in the form,

$$Y_1 - Y_o = A(t-t_o)^{3/5} \quad .$$

The consideration above already establishes $Y_o = 1.411 Y_s$. From the ratio $(Y_1-Y_o)/\dot{Y}$ and the shock velocity, $0.7886\ c_A$, $t_o = 1.167\ (Y_s/c_A)$. Finally, matching of the shock position requires,

$$A = 1.298\ (c_A)^{3/5}\ Y_s^{2/5} \quad .$$

The late time shock position, Y_1, can be expressed by the similarity solution,

$$Y_1 - 1.411 Y_s = 1.298 (c_A)^{3/5} Y_s^{2/5}\ (t-t_1)^{3/5} \tag{23}$$

where $t_1 = t_o + t_s$ to include the time delay in the heating before the shock starts.

We define the origin-shifted similarity variable ξ_1 as,

$$\xi_1 = \frac{y-Y_o}{Y_1-Y_o} < 1$$

in terms of which the ground density ratio, $\eta = \rho/\rho_o$, for all points behind the ground shock is,

$$\eta = 6(5-4\xi_1)^{-5/2} \quad . \tag{24}$$

BLOW-OFF GROUND DENSITY

We have now completed all the analysis required, and it remains only to put it together into a form from which the density of blow-off ground material can be computed. To implement the similarity solution requires the values of Y_s , and t_s as a function of distance from the burst point which are given by Equations 20, 21, and 22. The additional quantity needed is the isothermal sound speed at the time t_s when the ground shock starts. The numerical formula for this speed is found to be,

$$c_A = 5.466\,(10)^7 \;(E^1)^{0.0743}\;(x^2+x_o^2)^{-0.0879}$$

where c_A is in cm/sec if E^1 is megatons and distances x, x_o are cm.

We have used this method to calculate the density distribution of ground material at several late times. An example is shown in Figure 3 which gives density contours as a function of distance from ground zero, x , and vertical distance y at 1 msec after a 1 MT burst (E^1=0.75 MT), at a standoff distance x_o = 44.6 cm. The calculated ground shock position Y_1 is included. The dotted line is the locus of positions at which the material velocity changes direction from into the ground to out of the ground. (Note that in Figure 3 positive y is upward, contrary to our usual convention.)

Evidently the plane wave approximation of the shock front at this time begins to fail seriously at distances within 5 meters, since the shock front there is not even approximately parallel to the original ground

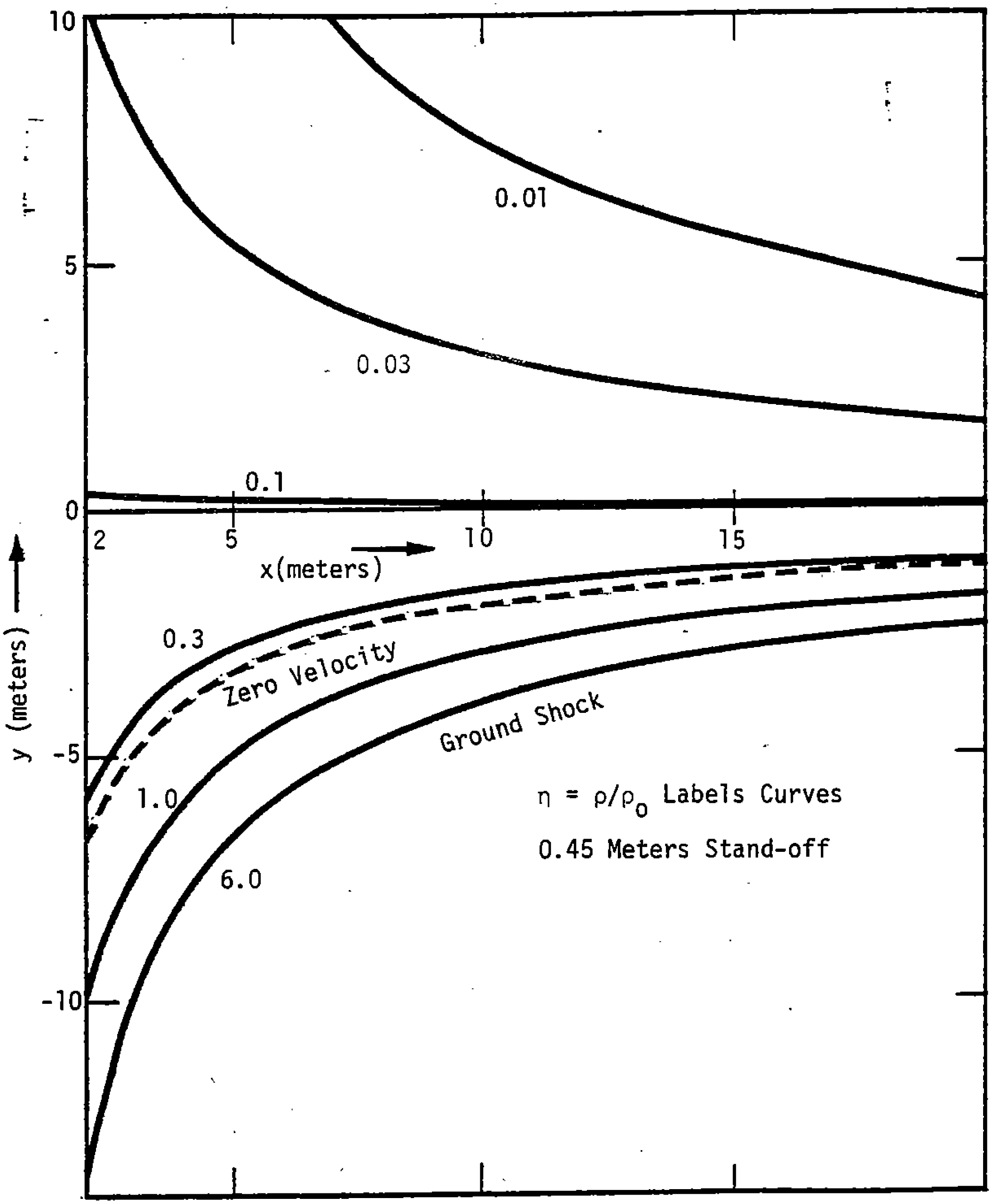


Figure 3. Density contours of ground material 1 msec after 1 MT surface burst. (Radial distance, x , and vertical distance, y , from ground zero.)

surface. The situation is worse at distances less than 2 meters which are not included in the figure for this reason. Physically, near the burst point one expects a quasi-hemispherical shock front which at this time would have engulfed a portion of the planar shock propagating from the surface. These failings are less serious as the standoff distance increases beyond the very small one chosen for this example.

One finds by calculation with the similarity solution that shifts of spatial and temporal origin introduced in the fitting process become insignificant at late times. For example, in the 1 MT case for $x>200$ cm, Y_o is less than 15 cm and t_1 is less than 1 μsec. Consequently for times greater than 10 μsec the origin shifts may be dropped from the similarity solution without appreciable error. A functional relation for the similarity constant A is the single requirement. This is given by,

$$A = 5.8625(10)^6 (E^1)^{0.3302} (x^2+x_o^2)^{-0.3906}$$

with the variables in the same units as before. With this value the similarity variable $\xi = y/At^{3/5}$ may be constructed and substituted into (24) to obtain an explicit formula for the blowoff ground density ratio as a function of spatial position, time, and stand-off distance and yield of the device.

$$\rho/\rho_o = \eta = 6(5-4z)^{-5/2} \text{ where } z = \frac{2.48y(x^2+x_o^2)^{0.3906}}{(E^1)^{0.3302} t^{0.6}} \qquad (25)$$

In this formula distances x, x_o, y are meters, E^1 is megatons, and t is in μsec. The ground shock position corresponds to z=1 and vertical position y is positive downward. This formula provides a convenient estimate of ground material densities when used with proper recognition of its limitations.

SECTION 5
SUMMARY REMARKS

The analysis presented in this report has achieved an approximate analytic expression for ground material densities of the early blow-off in the immediate vicinity of a surface nuclear burst with which the gamma-ray source from neutron capture in ground material may be computed. This result is qualified by the analytical approximations and methods used, but may provide sufficient detail for estimates of ground gamma sources in circumstances of minimal debris effects.

However, these results are incomplete through their failure to incorporate the effects of case debris and through the inadequacy of plane-wave approximations very near the burst point. Both of these problems have a common aspect, the strongly spherically divergent nature of physical processes very near the burst. We have examined these problems, but have been unable to complete analysis of them within the limitations of this effort. They appear to be amenable to a combination analytical and numerical analysis in one-dimensional spherically symmetric geometry. The results of this treatment can be fitted to the present plane-wave approximation (as modified by debris effects) in the region farther from the burst. In our view, this approach is a much needed alternative or complement to full two-dimensional coupled radiation-hydrodynamic codes for obtaining an adequate solution of this problem.

GROUND CRATERS

A final remark concerns the relation of our analysis to an entirely different outstanding problem, the failure of numerical codes to reproduce the observed final shape and volume of ground craters from nuclear surface bursts. Observed craters are wider and shallower than predicted and have volumes 10 to 100 times greater than code results. We suggest that a contributing cause may be omission from the codes of the strong quasi-planar ground shock propagating downward at distances of tens of meters from the burst. This is the shock which we have analyzed by a similarity solution. The fine grids required by numerical codes for adequate resolution at early times may not extend far enough away from the burst point to include the inception and development of these shocks, an omission which excludes the physics of their eventual interaction with the quasi-hemispherical shock centered at the burst point. This remark is conjectural since we are not familiar with cratering codes, but may illustrate an insight available from an analytical solution. The insight could be used to improve the computer codes.