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RADIATION AND HYDRODYNAMIC EFFECTS FROM A BURST IN THE MX TUNNEL

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January 1978

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1. INTRODUCTION

This report presents analytically derived estimates of the radiative and hydrodynamic phenomena that would occur following the burst of a 1 megaton bomb in the MX tunnel. It shows that about 9 kilotons is trapped in each half of the tunnel at times of a few hundred microseconds, after venting in the neighborhood of the burst.

The reason for this effort was two-fold. First, we needed estimates of the hydrodynamic phenomena to use in the prediction of the EMP sources. Second, we thought it would be good to go through the analytical estimates to provide a check on the detailed computer calculations being made at the same time, and to provide some understanding of the results of those calculations to those unable to examine the computer calculations in minute detail.

The reason why a check on the computer calculations is useful is that the phenomena involved are quite complex and significant effects occur in distance and time scales very short compared to overall dimensions and times. It is difficult, especially in two- and three-dimensional computer calculations, to zone fine enough to resolve all of phenomena well. Fortunately, the problem at hand is quite forgiving of errors made at early times, because of continual degradation of that part of the energy which is important for later phenomena of interest.

Our results, insofar as we went, agree fairly well with the computer results. We are indebted to Dr. Bud Pyatt of Systems, Science, and Software, Inc., for going over the results with us. In view of his long experience with problems of a similar type, it is no surprise that we find ourselves in agreement with him; but we, at least, find it conducive of confidence in the answers as far as we went.

2. PROPERTIES OF MATTER AND RADIATION

We collect here some useful formulae relating to matter and radiation at high temperatures. First, the definition of the megaton is

$$1 \text{ megaton} = 10^{15} \text{calories} = 4.18\times10^{15} \text{ Joules} \tag{1}$$

We may assume that about 3/4 of the energy comes out of a bomb in the form of radiation (thermal x rays) in a few tens of nanoseconds, and that about 1/4 of the energy appears as kinetic energy of the expanding bomb debris. Then, if the 1 megaton bomb weighed 1000 Kg, its average expansion speed would be

$$v_{deb} \approx 1.4\times10^{6} \text{ m/sec} , \tag{2}$$

and the outermost debris could be expected to go two or three times faster.

Blackbody radiation in a cavity at temperature T keV has energy content

$$\mathscr{E}_r(\text{Joules/m}^3) = 1.37\times10^{13} T^4 (\text{keV}) . \tag{3}$$

The pressure of radiation is

$$p_r(\text{Newton/m}^2) = \frac{1}{3} \mathscr{E}_r . \tag{4}$$

Air is almost completely ionized at temperatures greater than about 0.1 keV. Concrete (SiO_2) and typical soil are similarly stripped at temperatures above 0.8 keV. In the stripped condition, the internal kinetic energy of matter is

$$E_k(\mathrm{J/kg}) = 7.2\times10^{10} \left(\frac{2(Z+1)}{A}\right) T(\mathrm{keV}) \qquad (5)$$

and the internal potential energy (of ionization) is

$$E_p(\mathrm{J/kg}) = 7.2\times10^{8} Z^{1.37} \left(\frac{2Z}{A}\right), \qquad (6)$$

where Z is the atomic number and A the mass number. The factors involving Z and A are close to unity for all but the lightest elements. In case of mixtures of elements, $Z^{1.37}$ is to be averaged over the mass fractions of the various elements. Thus for air and concrete:

$$\left.\begin{array}{rl} \text{air:} & E_p = 1.1\times10^{10}\ \mathrm{J/kg} \\ \text{concrete:} & E_p = 1.9\times10^{10}\ \mathrm{J/kg} \end{array}\right\} \qquad (7)$$

The internal energy density of matter at mass density $\rho(\mathrm{kg/m^3})$ is

$$\mathcal{E}_m = \rho(E_k+E_p) = \rho E_m , \qquad (8)$$

and the matter pressure is

$$p_m = \frac{2}{3}\rho E_k \qquad (9)$$

By comparing Eq. (3) and (8), one can find the temperature T_c at which radiation and material energy densities are equal. For air and concrete we have:

$$\left.\begin{array}{rll} \text{air:} & \rho = 1.3\ \mathrm{kg/m^3}, & T_c \approx 0.22\ \mathrm{keV} \\ \text{concrete:} & \rho = 2000\ \mathrm{kg/m^3}, & T_c \approx 2.3\ \mathrm{keV} \end{array}\right\} \qquad (10)$$

At higher temperatures the radiation energy is dominant. Note that for both air and concrete T_c is high enough to ensure complete stripping, so that our approximations for the material energies are valid.

3. RADIATIVE PHASE

According to Eq. (2), it will take about 1.5 μsec for the bomb debris to reach the tunnel wall, if the tunnel radius $R = 2.5$ m. In this time the x-rays can travel 450 m. Since the wall material can hardly move faster than the bomb debris, it is clear that there will be a phase, lasting on the order of microseconds, in which significant energy transport occurs by radiation alone. In this section we shall assume that the radiation which falls upon the tunnel wall heats a thin layer of the wall to a temperature such that it emits radiation approximately as fast as it receives. The radiation spectrum will then continue to be approximately Planckian, in equilibrium with the local wall temperature, and radiation will diffuse along the tunnel.

Let us at first ignore the presence of air in the tunnel, and assume that the radiation occupies uniformly a length $2L$ of the tunnel. The half-length L will of course increase with time. Since the cross sectional area of the tunnel is about

$$A = 20 \text{ m}^2 \quad , \tag{11}$$

the volume occupied is

$$V = 40 \; L \text{ m}^3 \; . \tag{12}$$

Putting 0.75 megaton in this volume gives, from Eq. (3), a radiation temperature

$$T(\text{keV}) = 1.55/L^{1/4} \tag{13}$$

This formula leads to the following table:

$$
\left.\begin{array}{lllll} L: & 10 & 20 & 40 & 80\ \text{m} \\ T: & 0.87 & 0.73 & 0.62 & 0.52\ \text{keV} \end{array}\right\} \qquad (14)
$$

Notice that all of these temperatures are greater than T_c(air), so that the energy of the air is negligible compared with that of radiation. Also, the air will be well stripped at these temperatures. We need to consider how the air gets "burned out".

For x-rays at quantum energy $\approx$ 2 keV (appropriate for $T \approx 0.7$ keV), the absorption length in cold air is about 0.002 gm/cm^2, or about 1.5 cm at normal air density. The absorption length remains this short until the K-shell electron disappear, which begins at $T \approx 0.08$ keV, and is essentially complete (for opacity purposes) at $T \approx 0.4$ keV, where the opacity is dominated by Compton scattering. The result is that air exposed to radiation at temperatures of the order of those above is brought to equilibrium with the radiation in times very much less than a microsecond. The shortness of the cold air absorption length has the result that the radiation front which moves into the air is quite thin with a steep temperature gradient.

We can estimate the rate at which the radiation front burns into cold air as follows. Given the radiation energy density $\mathcal{E}_r$ just behind the front, the flux of radiation energy per unit area is twice the black body flux

$$
F_r = 2F_{bb} = 2\left(\frac{c}{4}\,\mathcal{E}_r\right), \qquad (15)
$$

where c is the speed of light. The factor 2 comes from the fact that there is a temperature gradient behind the front. If there is no flux coming back from the front, then the flux due to the gradient equals the black body flux (in diffusion theory approximation).

If the front advances a distance δz in time δt, then F_r has to supply the radiation and material energy in δz, i.e.

$$F_r \delta t = (\mathscr{E}_r + \mathscr{E}_m)\delta z \quad . \tag{16}$$

Therefore, the speed of the front is

$$v_f = \frac{\delta z}{\delta t} = \frac{F_r}{\mathscr{E}_r + \mathscr{E}_m} = \frac{c}{2(1+\mathscr{E}_m/\mathscr{E}_r)} \quad . \tag{17}$$

Using the expressions given above for $\mathscr{E}_m$ and $\mathscr{E}_r$ we obtain the following results:

T	= 0.10	0.15	0.20	0.25	0.30 keV	(18)
$\frac{1}{1+\mathscr{E}_m/\mathscr{E}_r}$	= 0.07	0.24	0.46	0.65	0.78	
λ_R	= 0.8	2.6	5.1	9.6	12.8 m	
λ_P	= 0.01	0.25	1.1	4.5	11.0 m	

This table also gives the Rosseland and Planck mean free paths, λ_R and λ_P, in normal density air at the temperatures indicated.

We see from the table that v_f is close to $c/2$ for driving temperature greater than 0.3 keV. Since the energy to drive the front must diffuse from the region of the burst up to the front, it cannot support a front that moves at speed too close to $c/2$.

Let us examine the diffusion along the tunnel. Table (18) shows that for temperatures greater than about 0.25 keV, the radiation mean free path in air is greater than the tunnel diameter. It can be shown that the effective mean free path in an empty cylinder, the walls of which are black, is the tunnel diameter $D = 2R$. Thus, when the matter energy is negligible, $\mathscr{E}_r$ satisfies the diffusion equation

$$\frac{\partial \mathcal{E}_r}{\partial t} = \frac{cD}{3} \frac{\partial^2}{\partial x^2} \mathcal{E}_r \ . \tag{19}$$

The solution of this equation is

$$\mathcal{E}_r = \mathcal{E}_0 \exp\left(-\frac{3x^2}{4Dct}\right) , \tag{20}$$

where $\mathcal{E}_0$ is the value at $x = 0$,

$$\mathcal{E}_0 = \frac{B}{\sqrt{ct}} , \quad B \text{ a constant.} \tag{21}$$

The radiation flux is, according to diffusion theory

$$F_r = -\frac{cD}{3} \frac{\partial}{\partial x} \mathcal{E}_r = \frac{x}{2t} \mathcal{E}_r \ . \tag{22}$$

The maximum value of the gradient flux is that from a black body, given in Eq. (15). Equating (22) to F_{bb}, we find

$$\frac{x}{2t} = \frac{c}{4} \quad \text{or} \quad x = \frac{ct}{2} \ . \tag{23}$$

Thus, without the necessity of heating material, diffusion can bring forward energy at a sufficient rate to make the radiation front move at speed $c/2$. With material present, the burnout front will move at speed close to $c/2$, provided $\mathcal{E}_m$ is small compared with $\mathcal{E}_r$ at the temperature given by diffusion theory at the front. For our case, the constant B in Eq. (21) is

$$B = 3.43\times10^{13} \frac{J}{m^3} \sqrt{m} \tag{24}$$

From Eqs. (3) and (20) one then obtains a formula for the temperature as a function of x and t,

$$T(\text{keV}) = \frac{1.26}{(ct)^{1/8}} \exp\left(- \frac{3x^2}{16Dct}\right) \tag{25}$$

A graph of T versus x at various times is shown in Figure 1. The Gaussian is truncated at $x = ct/2$, which is valid as long as the temperature at the point of truncation is such that $\mathcal{E}_m$ is small compared with $\mathcal{E}_r$. Since $\mathcal{E}_m$ and $\mathcal{E}_r$ are equal at 0.22 keV, and the truncation temperature is equal to this value at 0.4 μsec, the burnout front will move at speeds less than $c/2$ after about this time. This will give the main portion of the radiation energy time to catch up with the burnout front, so that the temperature distribution will become flatter. At $x = 0$ the temperature will continue to fall as

$$T \approx \frac{1.26}{(ct)^{1/8}} \tag{26}$$

for some time, in fact, until energy loss into the walls becomes important.

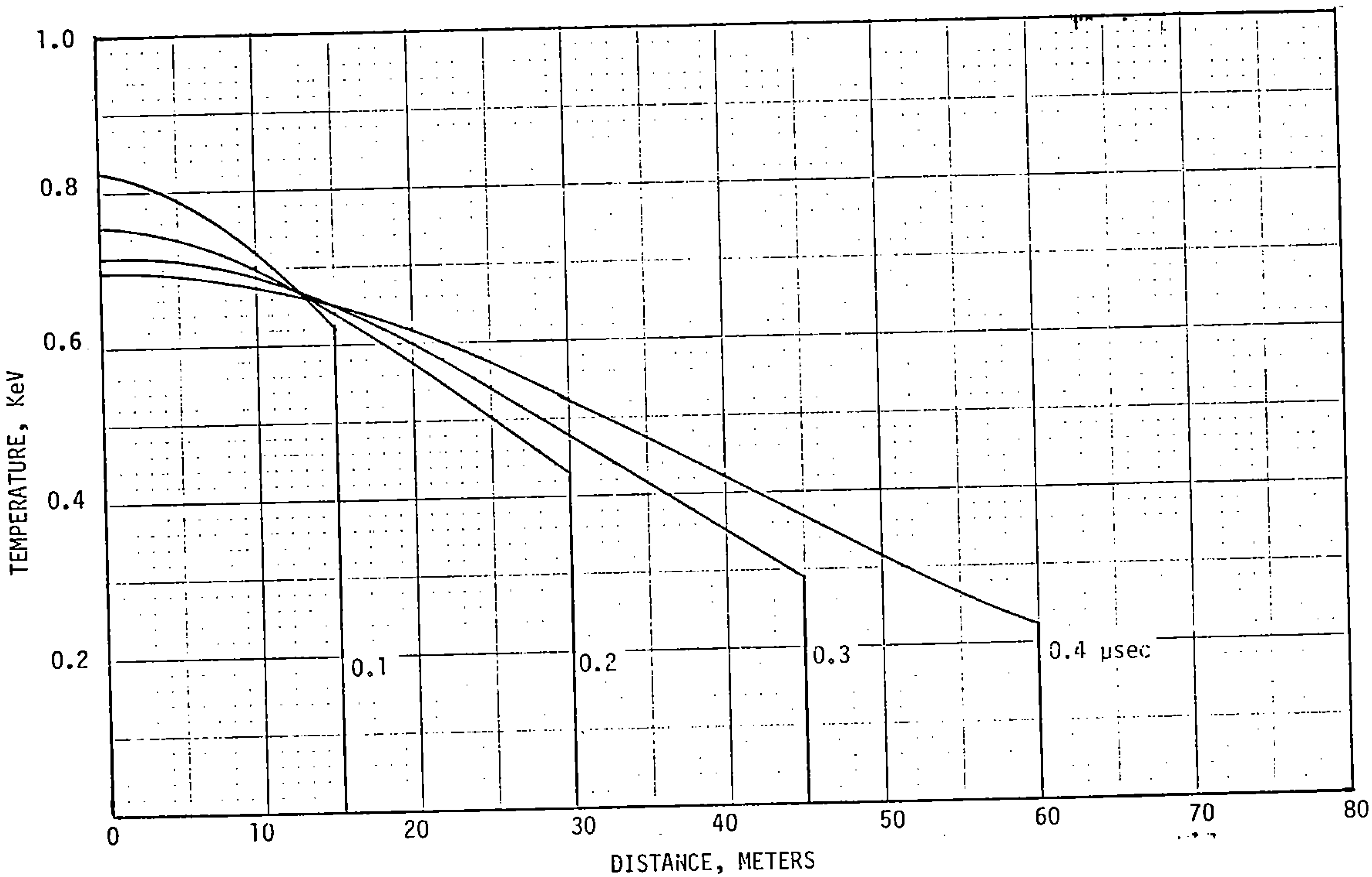

Figure 1. Temperature versus distance from the burst point at various times in the radiation phase.

4. RADIATION DIFFUSION INTO WALL: EARLY PHASE

In this section we examine the energy loss into the tunnel wall at early times, during which this loss is small compared with the energy in the tunnel and the motion of the wall material can be neglected. After we have performed our calculation, we shall see how long these two conditions remain valid.

The flux of radiation energy in the wall is

$$F_r = - \frac{c}{3\rho K} \frac{\partial}{\partial y} \mathscr{E}_r \quad , \tag{27}$$

where y is the depth into the wall and K ($\frac{m^2}{kg}$) is the opacity. The opacity depends on temperature and density, and can be fitted quite well over substantial regions of these variables by the formula

$$K = K^* \eta^m / T^n \quad , \tag{28}$$

where K^* is a constant, η is the ratio of the mass density to, say, the normal density and T is again the temperature in keV. The exponent m is not far from unity over the range of interest to us. The exponent n is about 4 in the temperature range 0.5 to 1 keV, and about 1 between 0.2 and 0.5 keV. With the form (28) of the opacity, the flux can be written

$$F_r = - \frac{c}{3\rho K^* \eta^m} T^n \frac{\partial}{\partial y} \mathscr{E}_r = - \frac{4}{4+n} \frac{c}{3\rho K^* \eta^m} \frac{\partial}{\partial y} T^n \mathscr{E}_r \quad . \tag{29}$$

Here we have used the fact that $\mathscr{E}_r \sim T^4$.

The constant flux approximation assumes that the flux is constant in y, from the surface of the wall to the position y_1 of the head of the radiation wave, while at the head the flux is used up in heating the material to a temperature approximately equal to the driving temperature

T_0 in the tunnel. Under the assumption of constant flux, if ρ is constant, we would find the temperature profile by integrating Eq. (29),

$$T = T_0 \left(1 - \frac{y}{y_1}\right)^{1/(4+n)} \quad . \tag{30}$$

Thus the temperature varies only slowly with y up to points very close to y_1, where it drops shortly to zero. Since the material energy E_m is roughly proportional to T, E_m also is nearly constant in y up to points quite close to y_1, and then drops shortly to zero. Thus indeed the material receives most of its energy near the head of the radiation wave, and the constant flux approximation is well justified.

Since the constant flux approximation implies that $T^n \mathscr{E}_r$ is a linear function of y, the flux into the wall is

$$F_r = \frac{4}{4+n} \frac{c}{3\rho K_0} \frac{\mathscr{E}_{r0}}{y_1} \quad . \tag{31}$$

Here K_0 and $\mathscr{E}_{r0}$ are the values of K and $\mathscr{E}_r$ at the tunnel temperature T_0. Thus the opacity at the lower temperatures in the radiation wave affects the flux only through the exponent n in the factor $4/(4+n)$. In our case

$$\begin{aligned} \frac{4}{4+n} &\approx 0.5 \quad \text{for} \quad 0.5 < T < 1 \text{ keV}, \\ &\approx 0.8 \quad \text{for} \quad 0.2 < T < 0.5 \text{ keV} \quad . \end{aligned} \tag{32}$$

We now equate the flux to the rate at which matter is heated at the head of the wave,

$$\mathscr{E}_{m0} \frac{dy_1}{dt} = F_r \quad . \tag{33}$$

Using Eq. (31) and integrating in time, we find

$$y_1 = \left[\frac{8}{4+n}\,\frac{\mathscr{E}_{r0}}{\mathscr{E}_{m0}}\,\frac{ct}{3\rho K_0}\right]^{1/2} \tag{34}$$

Since $y_1 \sim \sqrt{t}$, $dy/dt \sim 1/\sqrt{t}$, and the speed of the radiation wave is arbitrarily large at early times (though not, of course, larger than c). Thus at sufficiently early times it is justified to neglect the wall motion

At T = 0.7 keV, we have

$$\begin{aligned} \mathscr{E}_{r0} &= 3.3\times10^{12}\ \mathrm{J/m^3}\ , \\ \mathscr{E}_{m0} &= 1.3\times10^{14}\ \mathrm{J/m^3}\ , \\ 3\rho K_0 &= 1.5\times10^{4}/\mathrm{m}\quad . \end{aligned} \tag{35}$$

With these values, we have

$$\begin{aligned} y_1 &= 0.9\times10^{-3}\sqrt{ct}\quad \mathrm{m}\quad , (T_0 = 0.7\ \mathrm{keV}) \\ &= 0.5\ \mathrm{cm}\quad \mathrm{at}\quad t = 0.1\ \mu\mathrm{sec}\quad . \end{aligned} \tag{36}$$

The ratio of the energy W_w in the wall to the energy W_t in the tunnel is

$$\frac{W_w}{W_t} = \frac{2\pi R y_1\ \mathscr{E}_{m0}}{\pi R^2\ \mathscr{E}_{r0}} = \frac{2y_1}{R}\,\frac{\mathscr{E}_{m0}}{\mathscr{E}_{r0}}\quad , \tag{37}$$

$$\begin{aligned} &= 0.071\sqrt{ct}\quad (T_0 = 0.7\ \mathrm{keV}), \\ &= 0.4\ \mathrm{at}\quad t = 0.1\ \mu\mathrm{sec}\quad . \end{aligned} \tag{38}$$

Thus W_w and W_t would be equal at $t \approx 0.6$ μsec, for $T_0 = 0.7$ keV. The temperature dependence of W_w/W_t can be found from Eq.s (37) and (34),

$$\frac{W_w}{W_t} \sim \left[\frac{\mathcal{E}_{m0}}{\mathcal{E}_{r0}} \frac{ct}{K_0}\right]^{1/2} .$$

Since $\mathcal{E}_{m0}$ is approximately proportional to T, this is

$$\frac{W_w}{W_t} \sim T_0^{(n-3)/2} \sqrt{ct} . \tag{39}$$

This quantity is almost independent of temperature over the temperature range of interest here. Thus over the time frame 0-0.4 μsec of the diffusion problem of Sec. 3, a fair approximation is to regard the heat capacity of the tunnel as being about 1.6 times larger than we assumed. This would lower the temperatures in Fig. 1 by a factor $(1.6)^{1/4} \approx 1.12$ and increase the times labeling the curves by a factor 1.6.

The heated wall material will blow off with velocity of the order of

$$v_m \approx \sqrt{E_m} = \sqrt{\mathcal{E}_m/\rho} \approx 2.5\times10^5 \text{ m/sec} \tag{40}$$

This speed should be compared with the speed of the radiation wave, which is from Eq. (36),

$$\dot{y}_1 = \frac{1}{2} \frac{0.9\times10^{-3}c}{\sqrt{ct}} = \frac{1.3\times10^5}{\sqrt{ct}} \text{ m/sec} \tag{41}$$

(Both speeds are for $T_0 = 0.7$ keV.) We see that $\dot{y}_1$ falls below v_m at very early time, and the material motion is never really negligible in our problem. We consider the effect of material motion in the next section. We shall see that the effect is not great.

5. RADIATION DIFFUSION INTO WALL: RADIATION-HYDRODYNAMIC PHASE

The results just obtained suggest that, after a negligibly short time in our problem, the wall material will blow off as rapidly as it is heated by the radiation. Thus we must account for the changing mass density in the diffusion problem. In addition a shock wave runs ahead of the radiation wave into the wall, increasing the initial density as seen by the radiation wave by a factor

$$\eta_1 = \frac{\gamma+1}{\gamma-1} \approx 5 \quad . \tag{42}$$

Here γ is the constant that appears in the equation of state

$$p_m = (\gamma-1)\, \mathscr{E}_m, \tag{43}$$

and is approximately

$$\gamma \approx 1.5 \tag{44}$$

in our temperature and density range.

The fact that the material is moving suggests that we change the independent variable in Eq. (29) for the radiation flux from y to a new variable z determined by

$$\rho_a \, dz = \rho\eta^m \, dy \quad , \tag{45}$$

where ρ_a is the original, unshocked density of the wall. All of the density varying terms have been included in the variable z, and Eq. (29) with this variable reads

$$F_r = -\frac{4}{4+n}\frac{c}{3\rho_a K^*}\frac{\partial}{\partial z}T^n \mathscr{E}_r \quad . \tag{46}$$

Again, the arguments for the constant flux approximation are valid, and the flux is to good approximation

$$F_r = \frac{4}{4+n}\frac{cT_0^n}{3\rho_a K^*}\frac{\mathscr{E}_{r0}}{z_1} \quad , \tag{47}$$

where z_1 is the value of z at the head of the radiation wave. Now we can relate z_1 to the mass M_1 of wall material, per unit area of wall, that has been penetrated by radiation. Let us assume that, due to blow off, the density distribution of the heated material is

$$\eta = \eta_1 \, e^{-(y_1-y)/\lambda} \quad , \tag{48}$$

where λ is some scale length. Then, from Eq. (45),

$$z_1 = \int_{-\infty}^{y_1} \eta^{m+1} dy = \eta_1^{m+1}\lambda/(m+1) \tag{49}$$

But with the same assumption (48), the mass penetrated is

$$M_1 = \int_{-\infty}^{y_1} \rho_a \, \eta dy = \rho_a \, \lambda , \tag{50}$$

so that

$$\rho_a z_1 = \frac{\eta_1^m}{m+1} \; M_1 \tag{51}$$

The flux, Eq. (47), therefore becomes

$$F_r = \frac{4}{4+n}\frac{(m+1)c}{3\eta_1^m \, K_0}\;\frac{\mathscr{E}_{r0}}{M_1} \tag{52}$$

Here K_0 is the opacity at the driving temperature T_0 and the original unshocked density ρ_a, as before.

We are now ready to rewrite the energy equation (33). This time the radiation flux must supply enough energy to heat the material and give it the blow-off kinetic energy. The kinetic energy is developed in the ravefaction wave that follows the radiation wave. We shall show in the next section that the kinetic energy per unit mass is approximately

$$KE = \frac{13}{4} (\gamma-1) E_{m0}. \tag{53}$$

Thus the total energy that must be supplied by the radiation flux is

$$KE + E_{m0} = \alpha E_{m0} \tag{54}$$

where

$$\alpha = \frac{13}{4} (\gamma-1) + 1 \approx 2.625 \tag{55}$$

The energy equation is therefore

$$\alpha E_{mo} \frac{dM_1}{dt} = \frac{4}{4+n} \frac{(m+1)c}{3\eta_1^m K_0} \frac{\mathscr{E}_{r0}}{M_1} \tag{56}$$

Integrating, we obtain

$$M_1 = \left[\frac{8}{4+n} \frac{(m+1)}{\alpha\, \eta_1^m} \frac{\mathscr{E}_{r0}}{E_{m0}} \frac{ct}{3K_0}\right]^{1/2} . \tag{57}$$

We can define an equivalent depth of penetration,

$$y_{eq} \equiv \frac{M_1}{\rho_a} = \left[\frac{8}{4+n} \frac{(m+1)}{\alpha\, \eta_1^m} \frac{\mathscr{E}_{r0}}{\mathscr{E}_{m0}} \frac{ct}{3\rho_a K_0}\right]^{1/2} , \tag{58}$$

where $\mathscr{E}_{m0}$ is evaluated at the original density ρ_a. This result can be compared with Eq. (34) for assumed constant density. The only difference is in the factor $(m+1)/\alpha\eta_1^m$ inside the brackets in Eq. (58). Since $m \approx 1$, we have

$$\left[\frac{m+1}{\alpha\ \eta_1^m}\right]^{1/2} \approx \left[\frac{2}{2.625\times 5}\right]^{1/2} = 0.39 \quad . \tag{59}$$

Therefore,

$$y_{eq}(t) \approx 0.39\ y_1(\text{previous}) \quad . \tag{59}$$

At a given time, the radiation penetrates only 0.39 as much mass as we found in Sec. 4. The energy in the wall now contains the factor α, to account for the kinetic energy. Thus Eq. (38) is replaced by

$$\begin{aligned} \frac{W_w}{W_t} &= 0.39\ \alpha \times (\text{previous result}) \\ &= 1.02 \times (\text{previous result}) \end{aligned} \tag{60}$$

Thus there is very little change in the net energy loss to the wall. The scaling of the temperature curves of Fig. 1 that was stated in Sec. 4 is therefore still appropriate: scale the temperatures down by a factor 1.12 and the times up by a factor 1.6. This scaled solution is valid until about $t = 0.6$ μsec, when the wall and tunnel energies are about equal.

6. PROPERTIES OF THE BLOW-OFF

The head of the radiation wave brings each element of the wall material to approximately T_0 in a very short time. This rise in temperature occurs at approximately constant pressure, since the sound speed is greater than the speed of the radiation wave. Thus some expansion of the material occurs in the radiation wave head. But even after this expansion, the material pressure is still higher than the radiation pressure in the tunnel, so the material expands into the tunnel in a rarefaction wave. This wave keeps up with the radiation wave, which is subsonic. We shall find a similarity solution for the rarefaction wave.

In this wave the temperature and material internal energy are approximately constant. Thus the hydrodynamic equation of motion for the material velocity v is

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = - \frac{C^2}{\rho} \frac{\partial \rho}{\partial x} \quad , \tag{61}$$

where C is the isothermal speed of sound

$$C = \sqrt{(\gamma-1)E_{m0}} \quad . \tag{62}$$

The conservation of mass is expressed by

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \rho v = 0 \quad . \tag{63}$$

In these equations x is the distance measured from the radiation wave head into the tunnel.

We are looking for a solution in which the total mass in the rarefaction wave is proportional to $\sqrt{t}$. Since the thickness of the wave is likely to be proportional to t, we expect the density to be proportional to $1/\sqrt{t}$. Accordingly, we look for a solution of the form

$$v(x,t) = Cu(\xi) \quad , \tag{64}$$

$$\rho(x,t) = \sqrt{\frac{t_1}{t}}\, g(\xi) \quad , \tag{65}$$

where the similarity variable is

$$\xi = \frac{x}{Ct} \quad , \tag{66}$$

and t_1 is a constant. Substituting these forms into Eq.s (61) and (63), we obtain

$$(u-\xi)\,\frac{du}{d\xi} = -\,\frac{1}{g}\,\frac{dg}{d\xi} \quad , \tag{67}$$

$$-\,\frac{1}{2}\,g - \xi\,\frac{dg}{d\xi} + \frac{d}{d\xi}\,gu = 0 \quad . \tag{68}$$

On working out the derivative of gu in Eq. (68), we find this equation can be put in the form

$$\frac{1}{g}\,\frac{dg}{d\xi} = -\,\frac{\frac{du}{d\xi} - \frac{1}{2}}{u-\xi} \quad . \tag{69}$$

We can use this equation to eliminate g from Eq. (67) with the result

$$\frac{du}{d\xi} = \frac{1}{2}\,\frac{1}{1-(u-\xi)^2} \quad . \tag{70}$$

The general solution of Eq.s (69) and (70) could be discussed rather easily, but we shall find a particular simple solution adequate for our purposes. This solution of Eq. (70) is

$$u = \frac{1}{\sqrt{2}} + \xi \quad . \tag{71}$$

With use of this relation, Eq. 69 becomes

$$\frac{1}{g}\frac{dg}{d\xi} = -\frac{1}{\sqrt{2}} \quad ,$$

from which we obtain

$$g = g_1 e^{-\xi/\sqrt{2}} \tag{72}$$

Thus the density falls exponentially with increasing ξ or x. The complete solution is

$$\left.\begin{aligned} \rho &= g_1 \sqrt{\frac{t_1}{t}}\, e^{-\xi/\sqrt{2}} \\ v &= C\left(\frac{1}{\sqrt{2}} + \xi\right) \end{aligned}\right\} \quad \xi = \frac{x}{Ct} \tag{73}$$

Let us calculate the total kinetic energy in the rarefaction wave. This is

$$KE = \int_0^\infty \frac{1}{2}\rho v^2 dx = \frac{1}{2} g_1 \sqrt{t_1 t}\, C^3 \int_0^\infty \left(\frac{1}{\sqrt{2}} + \xi\right)^2 e^{-\xi/\sqrt{2}}\, d\xi$$

$$= \frac{13}{2\sqrt{2}} g_1 \sqrt{t_1 t} \quad C^3 \quad . \tag{74}$$

The time derivative of the kinetic energy is

$$\frac{d}{dt}\,KE = \frac{13}{4}\,g_1\sqrt{\frac{t_1}{t}}\;\frac{C}{\sqrt{2}}\,C^2 \quad . \tag{75}$$

Now $g_1\sqrt{t_1/t}$ is, according to Eq. (73), the value ρ_1 of ρ at $x = 0$, i.e., just at the back of the radiation wave head. Also, $C/\sqrt{2}$ is the value v_1 of v at $x = 0$. Now

$$\rho_1 v_1 = \dot{M}_1 \equiv \frac{dM_1}{dt} \quad , \tag{76}$$

the rate of increase of the mass of wall material heated by the radiation wave. According to Eq. (57) of Sec. 5, $\dot{M}$ is proportional to $1/\sqrt{t}$, as our solution provides. Thus we can write Eq. (75) as

$$\frac{d}{dt}\,KE = \frac{13}{4}\,C^2\,\dot{M}_1 = \frac{13}{4}\,(\gamma-1)E_{m0}\dot{M}_1 \quad . \tag{77}$$

This shows that the kinetic energy given per unit mass engulfed by the radiation wave is $(13/4)(\gamma-1)E_{m0}$, as stated in Sec. 5.

It can be shown that the simple solution found above is the only solution that behaves well for large ξ. Further, if one starts at large ξ with ρ and u which deviate from this solution and integrates towards $\xi = 0$ (i.e., towards later times for the same x), the deviant solution approaches the simple solution. Thus it appears that the simple solution is hydrodynamically stable, and is the correct solution.

We can calculate the material pressure at $x = 0$. This is

$$p_m = (\gamma-1)\rho_1 E_{m0} = (\gamma-1)E_{m0}\,\frac{\dot{M}_1}{v_1}$$

$$= \sqrt{2}\,(\gamma-1)E_{m0}\,\frac{\dot{M}_1}{C} = \sqrt{2}\,C\dot{M}_1 \quad . \tag{78}$$

This pressure is supplied by recoil from the blowing-off mass. The total pressure at the radiation wave head, which drives the shock wave ahead of the radiation wave, is the sum of this pressure and the radiation pressure $\mathcal{E}_{r0}/3$. We can also write Eq. (78) as

$$\frac{p_m}{(\gamma-1)\rho_a E_{m0}} = \sqrt{2}\ \frac{\dot{M}_1}{\rho_a C} = \sqrt{2}\ \dot{y}_{eq}/C \quad . \tag{79}$$

Thus the ratio of p_m to the pressure, which would exist in material at temperature T_0 and the original density ρ_a, decreases as the speed of the radiation wave becomes smaller compared with the speed of sound.

For $T_0 = 0.7$ keV, we find the following numerical results

$$\begin{aligned} p_{m0} &= (\gamma-1)\rho_a E_{m0} = 6.5\times10^{13} \text{ Newtons/m}^2, \\ p_{r0} &= \frac{1}{3}\ \mathcal{E}_{r0} = 1.1\times10^{12} \text{ Newtons/m}^3, \\ C &= 1.8\times10^5 \text{ m/sec}, \\ (p_m/p_{m0}) &= 0.40/\sqrt{ct} = 0.073 \text{ at } t = 0.1\ \mu\text{sec}, \\ p_m &= p_{r0} \text{ at } t = 1.9\ \mu\text{sec}. \end{aligned} \tag{80}$$

In our energy equation we neglected the energy put into the pre-radiation shock wave. This energy per unit mass is small compared with E_{m0} because p_m is small compared with p_{m0}. The speed of the shock wave is roughly the geometric mean $\sqrt{\dot{y}_{eq}C}$.

7. CHOKE-OFF OF LONGITUDINAL RADIATION FLOW

The wall material that blows off into the tunnel impedes the flow of radiation along the tunnel. The opacity of the wall material is such that a density of about 6 kg/m^3 reduces the effective photon mean free path in the tunnel by a factor of 2 when $T_0 = 0.7$ keV. At lower temperatures the critical density required for the factor 2 reduction is roughly proportional to T_0.

In Sec. 6 we found the density profile of the blow-off material for a planar wall. In the cylindrical tunnel, the material at the toe of the rarefaction wave is compressed by convergence, and speeded up to some extent. We shall assume that the critical density is reached on the average at that time when the amount of mass at distances $x > 1$ meter in the planar solution reaches the critical density when spread uniformly over a cylinder of radius $R-1 = 1.5$ meters. Using the formulae developed in previous sections, we find the following times to reach the critical density:

T_0	= 0.2	0.3	0.4	0.5	0.6	0.7 keV
Time	= 7.3	5.7	4.7	4.0	3.3	2.8 μsec

(81)

These times are considerably longer than the times labeling the curves in Fig. 1, even when scaled up by the factor 1.6. Thus we could consider extending those curves to later times, up to the time at which the critical density is reached. However, as Eq. (38) shows, the energy loss to the wall is equal to the energy in the tunnel by $t \approx 0.6$ μsec. Thus Fig. 1 cannot be extended to later times without taking into account the effect of the wall loss in reducing $\mathcal{E}_{r0}$. We do this in the next section.

8. WALL PENETRATION WITH ENERGY CONSERVATION

We now take account of the material energy in reducing the radiation energy and flux. Previously we assumed that most of the energy was in radiation in the tunnel. Now we assume that most of the energy is in the material that has been heated by radiation flow. Thus the energy is still in the tunnel, because of the blow off of the heated material. At first the hot material is fairly close to the wall, and a substantial part of the energy is in the form of kinetic energy of the inward-moving material. Later, the blow-off material reaches the tunnel axis, creating one or more back shocks which move out toward the walls and convert the kinetic energy back into internal energy. We need to distinguish these two phases.

In the first phase, the energy in kinetic energy is not available to drive the radiation flux. The total internal energy available to drive the flux is, per unit length of tunnel,

$$\text{internal energy} = \frac{1}{\alpha} W \tag{82}$$

where α is the parameter defined by Eq.s (54) and (55), and W is the total energy per unit length of tunnel. The relation between W and the quantity $\mathscr{E}_{r0}$ previously used is

$$W = \pi R^2 \mathscr{E}_{r0} \quad . \tag{83}$$

We shall continue to use $\mathscr{E}_{r0}$ for the radiation energy density if all of the energy were in radiation in the tunnel, and introduce the symbol $\mathscr{E}_{rt}$ for the actual radiation energy density in the blown-off material in the tunnel. We also let E_{mt} be the internal energy per unit mass of the blown-off material and K_t be the opacity at density ρ_a and temperature T_t in the blown-off material

The energy equation (56) now becomes, for the new problem,

$$\alpha E_{mt} \frac{dM_1}{dt} = \frac{4}{4+n} \frac{(m+1)c}{3\eta_1^m K_t} \frac{\mathscr{E}_{rt}}{M_1} \quad . \tag{84}$$

The conservation of energy requires that

$$2\pi R M_1 \alpha E_{mt} = W = \pi R^2 \mathscr{E}_{r0} \quad ,$$

so that

$$\alpha E_{mt} = \frac{W}{2\ RM_1} = \frac{R\mathscr{E}_{r0}}{2M_1} \quad . \tag{85}$$

Thus, Eq. (84) becomes

$$\begin{aligned} \frac{dM_1}{dt} &= \frac{8}{4+n} \frac{(m+1)c}{3\eta_1^m R K_t} \frac{\mathscr{E}_{rt}}{\mathscr{E}_{r0}} \\ &= \frac{8}{4+n} \frac{(m+1)c}{3\eta_1^m R K_0} \left(\frac{T_t}{T_0}\right)^{4+n} \quad . \end{aligned} \tag{86}$$

Here T_0 is the temperature that would exist in the tunnel if all the energy were in radiation and K_0 is, as before, the opacity at density ρ_a and temperature T_0. Now because most of the energy is in material and the internal energy is approximately proportional to temperature, we can write

$$\alpha M_1 T_t = \alpha M_0 T_0 \quad , \quad \text{or} \quad \frac{T_t}{T_0} = \frac{M_0}{M_1} \quad , \tag{87}$$

where M_0 is the mass per unit area of wall required to hold all of the energy (kinetic and internal) at temperature T_0. Thus M_0 is the value reached by M_1 at $t \approx 0.6$ μsec in the previous calculations, when the wall energy became equal to the radiation energy originally in the tunnel. Eq. (86) can then be written as

$$\left(\frac{M_1}{M_0}\right)^{4+n} \frac{d}{dt}\left(\frac{M_1}{M_0}\right) = \frac{8}{4+n} \; \frac{(m+1)c}{3\eta_1^m R K_0 M_0} \quad . \tag{88}$$

Integrating this equation from the time $t_0 = 0.6$ μsec to arbitrary later time $t = t_0 + \Delta t$, we obtain

$$\frac{M_1}{M_0} = \left[1 + \frac{8(5+n)}{4+n} \; \frac{(m+1)}{3\eta_1^m K_0 M_0} \; \frac{c\Delta t}{R}\right]^{1/(5+n)} \quad . \tag{89}$$

We see that the blown-off mass increases quite slowly in this phase. Over most of this phase the parameters n and m are $n \approx 1 \approx m$. The product $K_0 M_0$ is of the order of 100 (dimensionless) for T_0 between 0.4 and 0.6 keV. Numerically, Eq. (89) becomes

$$\frac{M_1}{M_0} = \left[1 + \frac{c\Delta t}{200m}\right]^{1/6} \; , \quad 0.4 \le T_0 \le 0.6 \text{ keV} \quad . \tag{90}$$

By the time $\Delta t \approx 4$ μsec required for the blow-off material to reach the critical density (see Eq. 81) we have

$$\frac{M_1}{M_0} \approx 1.4 \quad . \tag{91}$$

Now, the present solution provides less blow-off material at each time than the solution of Sec. 6 which led to the numbers in the Table (81). Therefore, we should check whether the present solution provides enough mass to reach the critical density. If the mass M_1 is averaged over the tunnel it leads to a density $2M_1/R$. From the definitions of M_0 and the critical density (Sec. 7) we compute the following numbers at $\Delta t \approx 4$ μsec:

$$\begin{aligned} T_0 &= 0.4 \quad 0.5 \quad 0.6 \text{ keV} \\ M_0 &= 4.0 \quad 9.0 \quad 15 \text{ kg/m}^2 \\ 2M_1/R &= 4.5 \quad 10 \quad 17 \text{ kg/m}^3 \\ \rho_{crit} &= 3.4 \quad 4.3 \quad 5.1 \text{ kg/m}^3 \end{aligned} \qquad (92)$$

We conclude that radiation will continue to diffuse along the tunnel until about $\Delta t = 4$ μsec, and somewhat longer in the wings of the distribution. Let us estimate the degree to which the last temperature profile in Fig. 1 spreads in this time interval. Actually, we shall estimate directly the spreading of the profile of W, the total energy per unit length of tunnel. W satisfies the diffusion equation

$$\frac{\partial W}{\partial t} = \frac{cD}{3} \frac{\partial^2}{\partial x^2} W_r \quad , \qquad (93)$$

where W_r is the total radiation energy per unit length. By the definitions of quantities given above,

$$W_r = W \left(\frac{T_t}{T_0}\right)^4 = W \left(\frac{M_0}{M_1}\right)^4 \quad . \qquad (94)$$

Now, if we write Eq. (90) as

$$\frac{M_1}{M_0} = \left[1 + \frac{c\Delta t}{S}\right]^{1/6} , \qquad (95)$$

we find that Eq. (93) becomes

$$\left[1 + \frac{c\Delta t}{S}\right]^{2/3} \frac{\partial W}{\partial t} = \frac{cD}{3} \frac{\partial^2}{\partial x^2} W \quad . \qquad (96)$$

In writing the last three equations, we have assumed that the radial and longitudinal diffusion can be decoupled approximately. Eq. (96) takes the form of the standard diffusion equation if the time variable is changed to

$$\tau = \int_{\Delta t=0}^{\Delta t} \frac{dt}{\left[1+\frac{c\Delta t}{S}\right]^{2/3}} = \frac{3S}{c}\left[\left(1+\frac{c\Delta t}{S}\right)^{1/3}-1\right] \qquad (97)$$

By analogy with Eq.s (19) and (20), the solution is

$$W = W_0 \exp[-x^2/\ell^2] \quad , \qquad (98)$$

where

$$\ell = 2\sqrt{DS}\left[\left(1+\frac{c\Delta t}{S}\right)^{1/3}-1\right]^{1/2} \qquad (99)$$

Evaluating this expression at $\Delta t = 4$ μsec, S = 200 m, gives

$$\ell = 60 \text{ m}.$$

Actually, to include the width $\ell_0 = 28$ m of the energy distribution already present in Fig. 1, we should set

$$\ell = [(28)^2 + (60)^2]^{1/2} = 66 \text{ m} \quad . \qquad (100)$$

We see that ℓ increases only very slowly with time. The blown-off material will stop the longitudinal flow of radiation at a value of ℓ quite close 66 m.

In the wings of the distribution, S is somewhat less than the value 200 m. On the other hand, diffusion goes on somewhat longer here (see Table (81)), because the blow-off occurs more slowly. The result is that the Gaussian distribution holds approximately down to those temperatures at which the energy content of the air in the tunnel dominates the radiation energy, i.e., at temperatures less than about 0.2 keV (or $W \approx 1.0\times10^{12}$ J/m), according to Table (18). The temperature drops rapidly with distance in this range.

Normalizing the energy distribution (98) to a total energy of 1 megaton leads to

$$W_0 = 3.6\times10^{12} \text{ Joules/meter} \quad . \tag{101}$$

We have included the kinetic energy of the bomb debris here because, within the present time interval, the debris will run into the wall and give its kinetic energy to internal energy.

The final distribution of energy as left by the longitudinal radiation diffusion is graphed in Fig. 2. At this point we could also calculate the distribution of temperature and of blown-off mass at time Δt = 4 μsec. It will be more useful, however, to prepare for the final phase of the blow-off to be discussed in the sections to follow. For that purpose it is convenient to redefine the temperature $T_0(x)$ to be that which would exist if all of the energy W, of Eq. (98) and Fig. 2, were in radiation, and to redefine the mass $M_0(x)$ per unit wall area as that required to contain all of the energy W as internal energy at temperature T_0. These quantities are determined from the equations

$$W = \pi R^2 \mathscr{E}_{r0} = \pi R^2 \times 1.37\times10^{13} T_0^4 \tag{102}$$

for T_0, and

$$W = 2\pi R M_0 E_{m0} = 2\pi R M_0 \times 9.1\times10^{10} T_0 \tag{103}$$

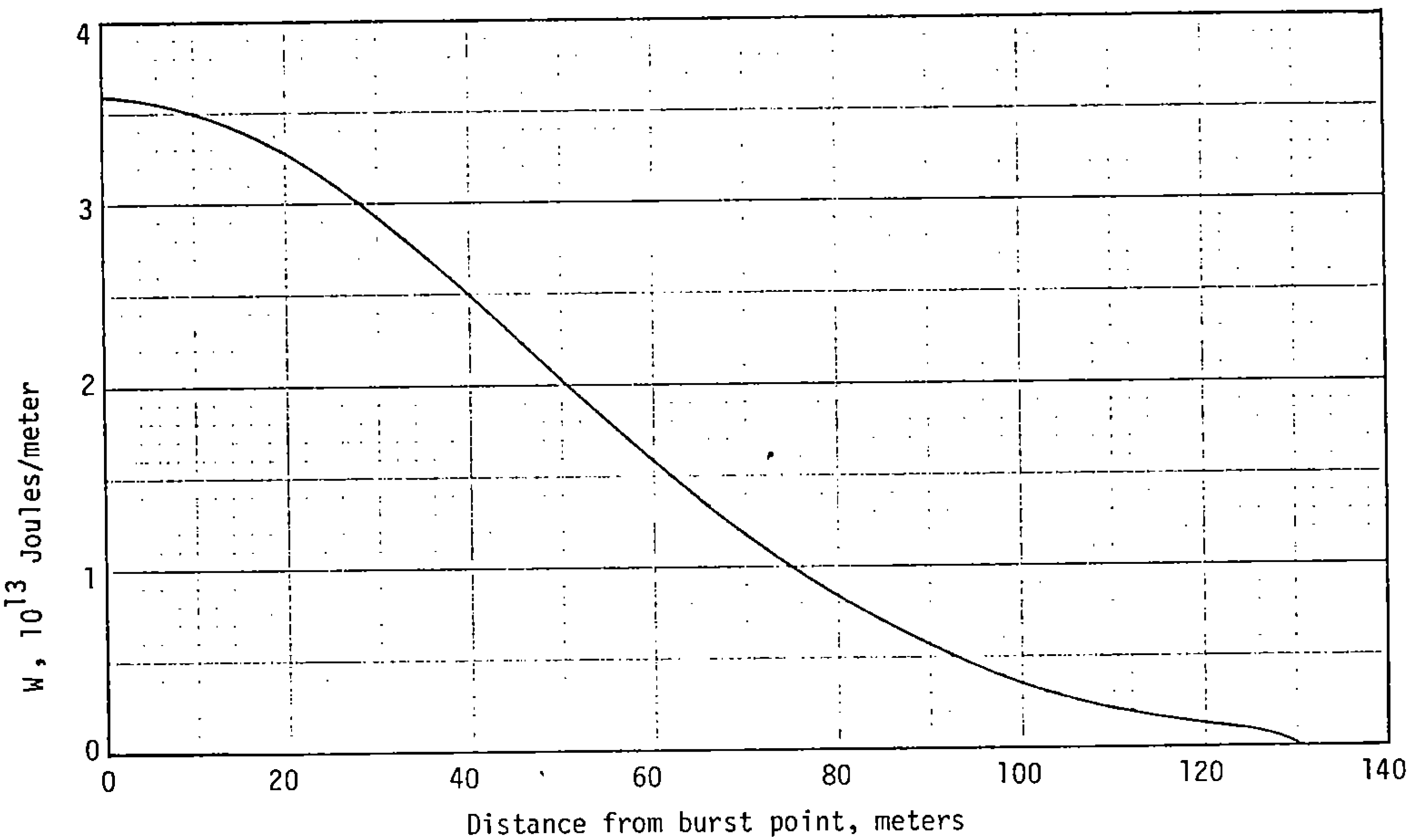

Figure 2. Longitudinal distribution of energy in the tunnel after completion of longitudinal radiation diffusion.

for M_0 once T_0 is known. The quantities T_0 and M_0 are graphed in Fig. 3. It can be seen from Eq.s (102) and (103) that

$$M_0 \sim W^{3/4} \quad . \tag{104}$$

Note that in our redefinition of M_0 we have assumed that the blow-off kinetic energy has been reconverted to internal energy. This reconversion occurs shortly after $\Delta t = 4$ μsec, which is early in the final phase of blow-off.

The actual temperature and mass distributions $T(x)$ and $M(x)$, at a time just after the reconversion of blow-off kinetic energy, can be estimated from the formulae

$$T \approx T_0/1.4 \quad , \quad M \approx 1.4\, M_0 \quad . \tag{105}$$

However, it is the quantity W, and T_0 and M_0 derived from it, that are critical in determining the final phase of blow-off and tunnel venting.

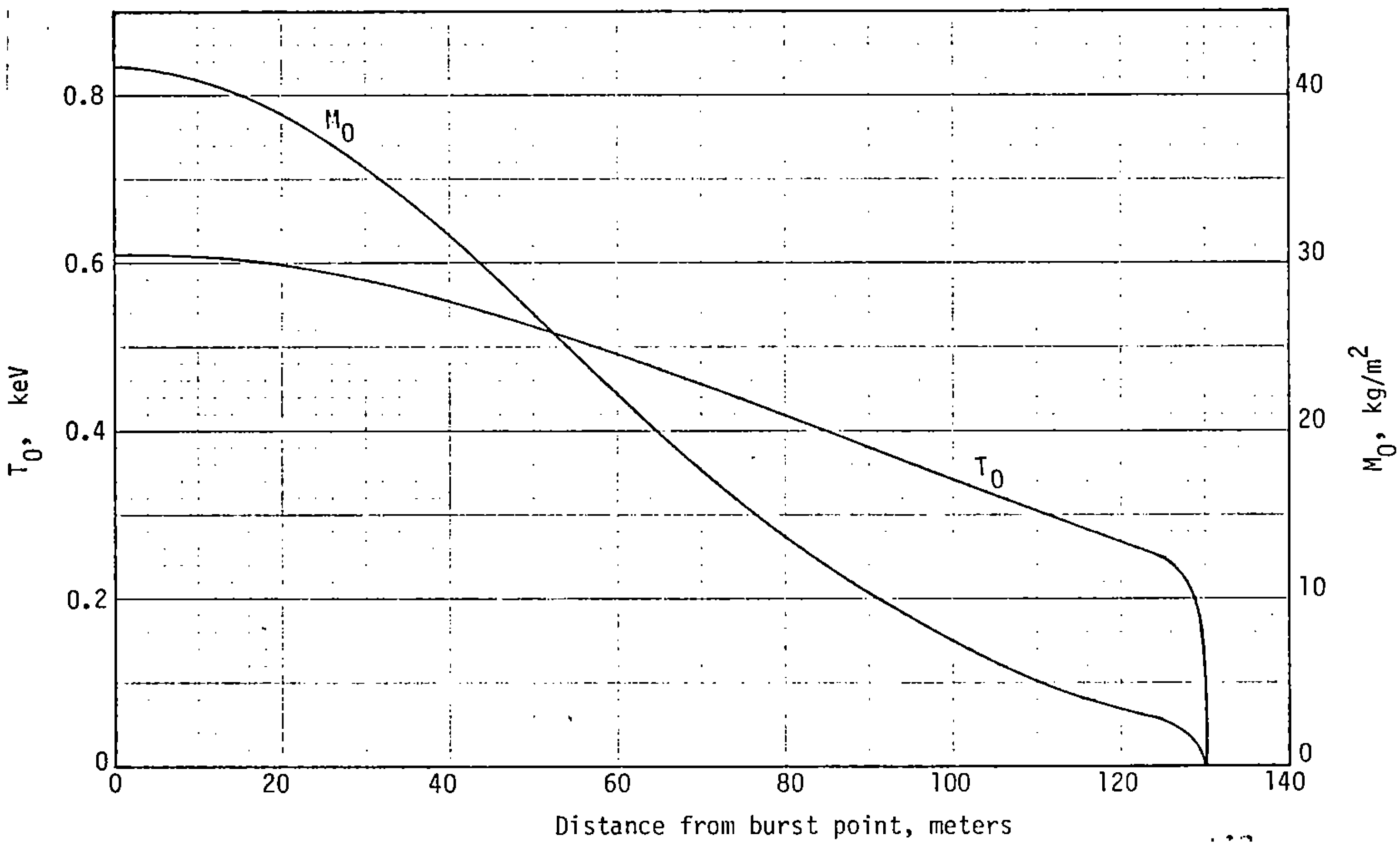

Figure 3. The scaling temperature T_0 and blow-off mass M_0, defined in the text, for the energy distribution of Figure 2.

9. VENTING OF THE TUNNEL

From the energy distribution (98) left by radiation flow, we can estimate the pressure in the tunnel. This pressure causes two principal effects. First, it drives a shock wave into the walls of the tunnel. The upward going shock wave will eventually reach the ground surface, after which the roof of the tunnel will lift off and the pressure in the tunnel will be relieved (venting). Second, the pressure also accelerates the blown-off material along the tunnel, before venting, and the momentum developed will drive a shock wave along the tunnel, even after venting. The chief purpose of this report has been to ascertain the properties of this longitudinal shock wave.

Let us first estimate the time of venting. The initial pressure in the tunnel is

$$p_0 = (\gamma-1)\,\frac{W}{\pi R_0^2}\ , \tag{106}$$

where $\gamma \approx 1.5$, W is given by Eq. (94), and R_0 is the initial tunnel radius. As the tunnel radius expands, the pressure falls adiabatically as

$$p = p_0\left(\frac{R_0^2}{R^2}\right)^{\gamma} = p_0\,\frac{R_0^3}{R^3}\quad . \tag{107}$$

The material speed in the ground shock is

$$\frac{dR}{dt} = \sqrt{\frac{2}{\gamma+1}\,\frac{p}{\rho_a}} = \frac{1}{R^{3/2}}\sqrt{\frac{2p_0R_0^3}{(\gamma+1)\rho_a}}\quad . \tag{108}$$

Integrating this equation, we obtain

$$\frac{2}{5}\left[\left(\frac{R}{R_0}\right)^{5/2}-1\right] = \sqrt{\frac{2(\gamma-1)\,W}{(\gamma+1)\pi\rho_a R_0^2}}\;\frac{t}{R_0}\quad . \tag{109}$$

Setting $R = 2.5+1.5 = 4.0m$, we find the time of roof lift off to be

$$t(\text{lift off}) = 120\ \mu\text{sec} \quad \text{at} \quad x = 0 \quad . \tag{110}$$

At other values of x, lift off will occur approximately at

$$t(\text{lift off}) \approx 120\ e^{x^2/2\ell^2}\ \mu\text{sec} \quad . \tag{111}$$

We have assumed here that venting at $x = 0$, and release of the pressure there, does not delay much the time of lift off at other points. This is a plausible assumption out to at least $x \approx \ell$, since most of the momentum of tunnel wall motion will be established before lift off time at $x = 0$.

The position of venting as a function of time is found by inverting Eq. (111),

$$x = \sqrt{2}\ \ell[\ell n(t/120\ \mu\text{sec})]^{1/2} \quad . \tag{112}$$

We can also obtain the speed of venting,

$$\begin{aligned} \dot{x} &= \frac{1}{1.2\times10^{-4}} \frac{\ell^2}{x}\ e^{-x^2/2\ell^2} \\ &= 5.5\times10^5\ \frac{\ell}{x}\ e^{-x^2/2\ell^2}\ \text{m/sec} \quad . \end{aligned} \tag{113}$$

10. FINAL PHASE OF BLOW-OFF

We shall now calculate the additional mass blown off the wall in the period after longitudinal radiation diffusion stops and before venting. We use the quantities T_0 and M_0 defined at the end of Section 8. We also define, as before, $M_1(x,t)$ to be the mass blown off per unit area, and E_{mt} and T_t to describe actual conditions in the tunnel. Eq.s (84) and (85) apply again, except that αE_{mt} in those equations has to be replaced by E_{mt}, since the blow-off kinetic energy has been reconverted to internal energy. In Eq. (85), W should fall adiabatically as R_0/R as the tunnel expands, but this is a small effect since most of the blow off develops early in the expansion. We shall treat R as if it remained equal to its initial value. Then again the energy equation takes the form of Eq. (99), and the solution is again given by Eq. (89). Again $m \approx 1$ and $n \approx 1$, but we must reevaluate the product $K_0 M_0$, for the range of temperature T_0 in Fig. 3. Doing this, we find, in analogy to Eq. (90),

$$\frac{M_1}{M_0} = \left[1 + \frac{c\Delta t}{S(x)}\right]^{1/6} , \qquad (114)$$

where

$$\begin{array}{lcrrrrrl} S(x) & = & 220 & 210 & 180 & 135 & 85 & 50 \text{ m} \\ \text{at } x & = & 0 & 25 & 50 & 75 & 100 & 125 \text{ m} \end{array} \qquad (115)$$

We shall evaluate Eq. (114) at Δt equal to the lift-off time of each x, as obtained from Eq. (111). We then find the following numbers:

$$\begin{array}{lcrrrrrl} \text{at } x & = & 0 & 25 & 50 & 75 & 100 & 125 \text{ m} \\ \Delta t & = & 120 & 129 & 160 & 230 & 380 & 720\ \mu \\ M_1/M_0 & = & 2.34 & 2.38 & 2.54 & 2.83 & 3.32 & 4.04 \end{array} . \qquad (116)$$

It turns out that, within a few percent, the ratio M_1/M_0 is fitted by the formula

$$M_1/M_0 = 2.34 \exp\left[0.15\left(\frac{x}{\ell}\right)^2\right] , \tag{117}$$

where ℓ = 66 meters. Therefore, noting Eq.s (98) and (104), we have $M_1 \sim W^{0.9}$. Therefore, noting the values of M_0 in Fig. 3, we write

$$M_1 = 98 \left(\frac{W}{W_0}\right)^{0.9} \text{kg/m}^2 \quad . \tag{118}$$

11. BLAST ENERGY TRAPPED IN THE TUNNEL

We can now calculate the longitudinal acceleration of the blown-off mass in the tunnel. The pressure in the tunnel is given by Eqs. (106) and (107). The longitudinal momentum P developed per unit length of tunnel is the time integral

$$P = \int \pi R^2 \frac{\partial p}{\partial x} \, dt$$

$$\approx (\gamma=1) \frac{\partial W}{\partial x} \int \frac{R_0}{R} \, dt$$

$$\approx 0.8 \, (\gamma-1) \frac{\partial W}{\partial x} \, \Delta t \quad . \tag{119}$$

Here Δt is the time of venting at x. The factor 0.8 comes from assuming that R increases linearly with time.

The mass M per unit length in the tunnel at the time of venting is, from Eq. (118),

$$M = 2\pi R_0 M_1 = 1530 \left(\frac{W}{W_0}\right)^{0.9} \text{kg/m} \quad . \tag{120}$$

Therefore, the longitudinal velocity V of the blown-off material at venting time is

$$V = \frac{P}{M} = \frac{0.8(\gamma-1)}{1530} \left(\frac{W_0}{W}\right)^{0.9} \Delta t \, \frac{\partial W}{\partial x}$$

$$= \frac{0.8(\gamma-1)}{1530} \, W_0 \, \Delta t \, \frac{2x}{\ell^2} \left(\frac{W}{W_0}\right)^{0.1} \quad . \tag{121}$$

Putting $\gamma = 1.5$, $W_0 = 3.6\times10^{13}$ J/m, and $\ell = 66$ m, we obtain the numerical result

$$V = 2.85\times10^8 \, \Delta t \, \frac{x}{\ell} \left(\frac{W}{W_0}\right)^{0.1} \quad . \tag{122}$$

This velocity increases with x, whereas the speed of venting, Eq. (113), decreases as x increases. For x greater than some value x_1, the moving material will outrun the venting. On setting V equal to the venting speed, and using Eq. (111) for Δt, we find

$$\left(\frac{x_1}{\ell}\right)^2 \exp(0.9x_1^2/\ell^2) = 16.1 \qquad (123)$$

This relation is satisfied at

$$\frac{x_1}{\ell} \approx 1.48, \quad x_1 = 98 \text{ meters} \quad . \qquad (124)$$

The velocity at $x = x_1$ is

$$V(x_1) = 1.24\times10^5 \text{ m/sec} = 0.124 \text{ m/}\mu\text{sec} \quad .$$

Thus in the $\Delta t \approx 370$ μsec before venting, this material will have moved about 25 meters. This is not enough to invalidate our estimation of the momentum, which neglected the alteration of pressure due to material motion.

The energy in the distribution W(x), Eq. (98), which exists beyond the point x_1 is ($u = x/\ell$, $1.9 = 125/66$)

$$Y_1 \approx W_0\, \ell \int_{1.48}^{1.9} e^{-u^2}\, du = 0.0258\, W_0\, \ell$$

$$= 6.1\times10^{13} \text{ Joules} = 15 \text{ kilotons} \quad . \qquad (125)$$

A substantial fraction of this energy will drive a blast wave down the tunnel, ahead of the venting. This blast wave will attenuate rather rapidly at first, as further energy is lost due to expansion of the tunnel.

In fact, at the point x_1, the energy in the tunnel is already reduced by a factor $R_0/R = 2.5/4 = 0.625$. Thus the energy available to drive the blast wave to greater distances is no greater than about

$$Y_1' \approx 0.625\ Y_1 = 9 \text{ kilotons} \quad . \tag{126}$$

It is not likely that venting at distances less than x_1 will reduce Y_1' by much. Even though the pressure at vented positions must fall by a large factor, there is too much mass at these positions to allow an escape route for the energy in the smaller mass ahead. Even after the original blown-off material escapes into the air, the tunnel will be effectively choked by material rebounding from the shocked walls as soon as the high tunnel pressure is relieved by venting. We therefore conclude that the trapped energy in the contained blast wave will be about 9 kilotons.

12. CONCLUSION

We have found that about 9 kilotons of energy is available to drive a blast wave in the tunnel (in each direction) that outruns venting. This blast wave will attenuate rapidly at first due largely to work done on the walls in expansion of the tunnel. This attenuation could be calculated quite well by further analysis along the lines employed in this report, but we have not had time to pursue the problem further.

Probably the chief cause of uncertainty in our calculations is doubt about the opacity used for the tunnel wall material. However, only the 1/6 power of the opacity enters the critical formulae, so that this does not appear to make a large uncertainty.

At later stages in the blast wave propagation, attenuation will be due primarily to ablation and abrasion, and perhaps due to the rib structure. While these effects pose more difficult problems for theoretical analysis, we believe it would be fruitful to attempt such analysis. At least, such analysis should help in understanding and in scaling of the experimental data.