Theoretical Notes
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A SIMPLE MODEL FOR ESTIMATING SGEMP REPLACEMENT CURRENTS AND EVALUATING SGEMP SIMULATION DIFFICULTIES

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SECTION 1

INTRODUCTION

For many purposes, it is quite useful to have a simple analytic model for estimating SGEMP effects. Any simple model will undoubtedly be somewhat inaccurate yet such a model may still be quite valuable in pointing out general trends and in promoting a basic understanding of SGEMP phenomena. This report discusses one such simple model.

This simple model is based upon a sphere emitting electrons in an azimuthally symmetric pattern over some range of polar angles. By assuming that surface charge immediately redistributes over the sphere, one can use the continuity equation to write an analytic expression for the surface replacement current in terms of the electron emission current density. These expressions will be derived and the results then compared to more accurate numerical calculations.

These analytic expressions for replacement current will then be used to point out possible difficulties with SGEMP replacement current simulation techniques where only a limited number of drive points are used to excite the test object. It is shown that it may be quite difficult to obtain the proper skin currents on the emitting surface of the test object unless the entire surface is excited, as in the case of SGEMP. For skin currents in the shadowed region, however, the effect of discrete drivers is not nearly so important.

SECTION 2

A SIMPLE MODEL FOR ESTIMATING REPLACEMENT CURRENTS

The basic geometry considered is spherical, as indicated in Figure 1. Azimuthal symmetry with respect to the z-axis is assumed. Some radial emission current density, $\vec{J}$, is also assumed; i.e.,

$$\vec{J} = J_0(r,t)[U(\theta_1) - U(\theta_2)]\,\hat{r} \quad , \tag{1}$$

where $U(\theta)$ is the unit step function and $\hat{r}$ is a unit vector in the radial direction.

The basis for this simple model is the continuity equation,

$$\frac{\partial\rho}{\partial t} + \nabla \cdot \vec{J} = 0 \quad , \tag{2}$$

which, when calculated at the surface of the sphere becomes

$$-\frac{d\sigma(\theta)}{dt} = J_0(r=a,\theta) + \frac{1}{a\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, K(\theta)\right) \quad , \tag{3}$$

where σ is the surface charge density (coul/m^2) and K is the surface current density (amps/m). Note that σ and K are just the basic SGEMP response parameters.

In general, one must solve both Maxwell's equations and the equations of motion of the emitted electrons to solve for the σ and K parameters in Equation 3. Several simplifications to this process are possible, however. First of all, one can specify the spatial current density, $\vec{J}$. This is

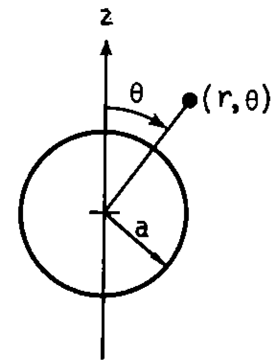

Figure 1. Basic geometry.

equivalent to the assumption that electron trajectories are fixed and will not be influenced by the resultant electromagnetic fields. Further simplification is possible if one assumes the problem is quasi-static. One can then solve Poisson's equation for the electric field, given the boundary conditions and electron positions in space. The surface charge density, σ, can then be calculated from the electric field normal to conducting surfaces and Equation 3 then gives the surface current density, K.

Note that an accurate quasi-static calculation includes the effects of electrons that have been emitted from the test object but have not yet had time to move very far from the object. The presence of these nearby electrons tends to hold down surface currents since they repel any electrons flowing on the surface to replace the emitted charge. [One can also describe this process by saying that the image charge has not yet moved away from the emission surface.]

A worst case estimate of skin currents can thus be obtained by assuming that all emitted electrons immediately move to infinity. In this case, the resultant skin currents are indeed the so-called "replacement currents" since charge will immediately flow to replace emitted electrons.

Under these conditions, the surface charge density is just that which would exist if the object were DC charged. For the conducting sphere, this charge density is uniform; i.e.,

$$\sigma(\theta) = \frac{Q}{4\pi a^2} \quad , \tag{4}$$

where Q is the total charge on the sphere. This equation can be rewritten as

$$\frac{d\sigma}{dt} = -\frac{I_E}{A_T} = \frac{J_0 A_e}{A_T} \; , \tag{5}$$

where I_E is the total emission current, A_T is the total area of the sphere ($4\pi a^2$) and A_e is area of that part of the sphere that is emitting electrons.

If Equations 3 and 5 are combined, remembering that negative charge (i.e., electrons) is emitted, it can be shown that

$$I_K(\theta) = -\frac{|J_0| A_e}{A_T} 2\pi a^2 (1 - \cos\theta) \quad \text{for} \quad 0 \le \theta \le \theta_1 \; , \tag{6}$$

$$I_K(\theta) = -\frac{A_1}{A_T} A_e \, |J_0| + |J_0| \left(1 - \frac{A_e}{A_T}\right) 2\pi a^2 (\cos\theta_1 - \cos\theta)$$

$$\text{for } \theta_1 \le \theta \le \theta_2 \; , \tag{7}$$

and

$$I_K(\theta) = \frac{|J_0| A_e}{A_T} 2\pi a^2 (1 + \cos\theta) \text{ for } \theta_2 \le \theta \le \pi \; , \tag{8}$$

where

$$A_1 = 2\pi a^2 (1 - \cos\theta_1) \; , \tag{9}$$

$$A_e = 2\pi a^2 (\cos\theta_1 - \cos\theta_2) \; , \tag{10}$$

and

$$I_K(\theta) = 2\pi a \sin\theta\, K(\theta) \quad . \tag{11}$$

Note that $I_K(\theta)$ is just the total replacement current as a function of angle θ.

This general case can be simplified if it is assumed that $\theta_1 = 0$. Then

$$I_K(\theta) = |J_0|\left(1 - \frac{A_e}{A_T}\right) 2\pi a^2 (1 - \cos\theta) \quad \text{for } 0 \le \theta \le \theta_2 \ , \tag{12}$$

and

$$I_K(\theta) = \frac{|J_0| A_e}{A_T} 2\pi a^2 (1 + \cos\theta) \quad \text{for } \theta_2 \le \theta \le \pi \ . \tag{13}$$

These two equations indicate several very interesting facts. First of all, the peak skin current will occur at $\theta = \theta_2$, the edge separating the emitting and non-emitting regions of the surface. The peak current is then

$$I_{K_{PEAK}} = |J_0| \left(1 - \frac{A_e}{A_T}\right) 2\pi a^2 (1 - \cos\theta_2) \ . \tag{14}$$

However,

$$A_e = 2\pi a^2 (1 - \cos\theta_2) \ , \tag{15}$$

so that

$$I_{K_{PEAK}} = \left(1 - \frac{A_e}{A_T}\right) |J_0| A_e$$

$$= I_0 \left(1 - \frac{A_e}{A_T}\right) \ , \tag{16}$$

where I_0 is the total emitted current.

Equation 16 says that the peak replacement current, for a given emission current, just depends upon the fraction of the surface area that is emitting electrons! Furthermore, when the whole surface is emitting, replacement currents go to zero. [This result is quite reasonable since the problem becomes spherically symmetric when the entire surface is emitting and it is well known that for spherical symmetry only radial electric fields are created; i.e., no magnetic fields or skin currents are produced.] It is also obvious that the maximum value of replacement current, for a given emission current, is achieved when the emission area is small (i.e., when the emission current is emitted along a filament along the z-axis).

For most SGEMP cases, one would expect about half the external surface to be emitting electrons (assuming the body is not highly convoluted). Therefore, the peak replacement current can be very roughly estimated by

$$I_{K_{PEAK}} \lesssim \frac{I_0}{2} \quad . \tag{17}$$

One should again be reminded that this is a worst case estimate and that actual replacement currents will depend on electron emission spectra and the average speed that the emitted electron cloud moves away from the emission surface.

SECTION 3

COMPARISON WITH MORE ACCURATE CALCULATIONS

Skin currents on a 1 meter radius sphere, as calculated by the simple scheme described here and as obtained from the LFLUX code[1], are compared in Figure 2. The prescribed emission current had a $\sin^2\left(\frac{t}{T}\pi\right)$ time history with T = 10 ns and a peak emission current density of 1.0 $amps/m^2$. The LFLUX calculation assumed all electrons were emitted radially from half the spherical surface and that the electron spectrum was mono-energetic, corresponding to a velocity of 1.0×10^8 m/sec. Note that the LFLUX code solves Maxwell's equations numerically, given a prescribed current density.

A comparison of the currents predicted by Equation 12 and by LFLUX (assumed here to be the baseline for evaluating the accuracy of our simple model) shows that the simple model does give replacement currents several times larger than more accurate calculations. The reason for this error is the fact that emitted electrons are assumed to immediately move to infinity, thus allowing complete charge redistribution immediately after electron emission. On the other hand, the LFLUX calculation includes the fact that the electrons take some finite time to move away from the sphere, thus giving a lower peak current and a current pulse stretched out in time.

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[1] Stettner, R., and D. Higgins, X-ray Induced Currents on the Surface of a Metallic Sphere, Mission Research Corporation, MRC-N-111, DNA 3612T, April 1975.

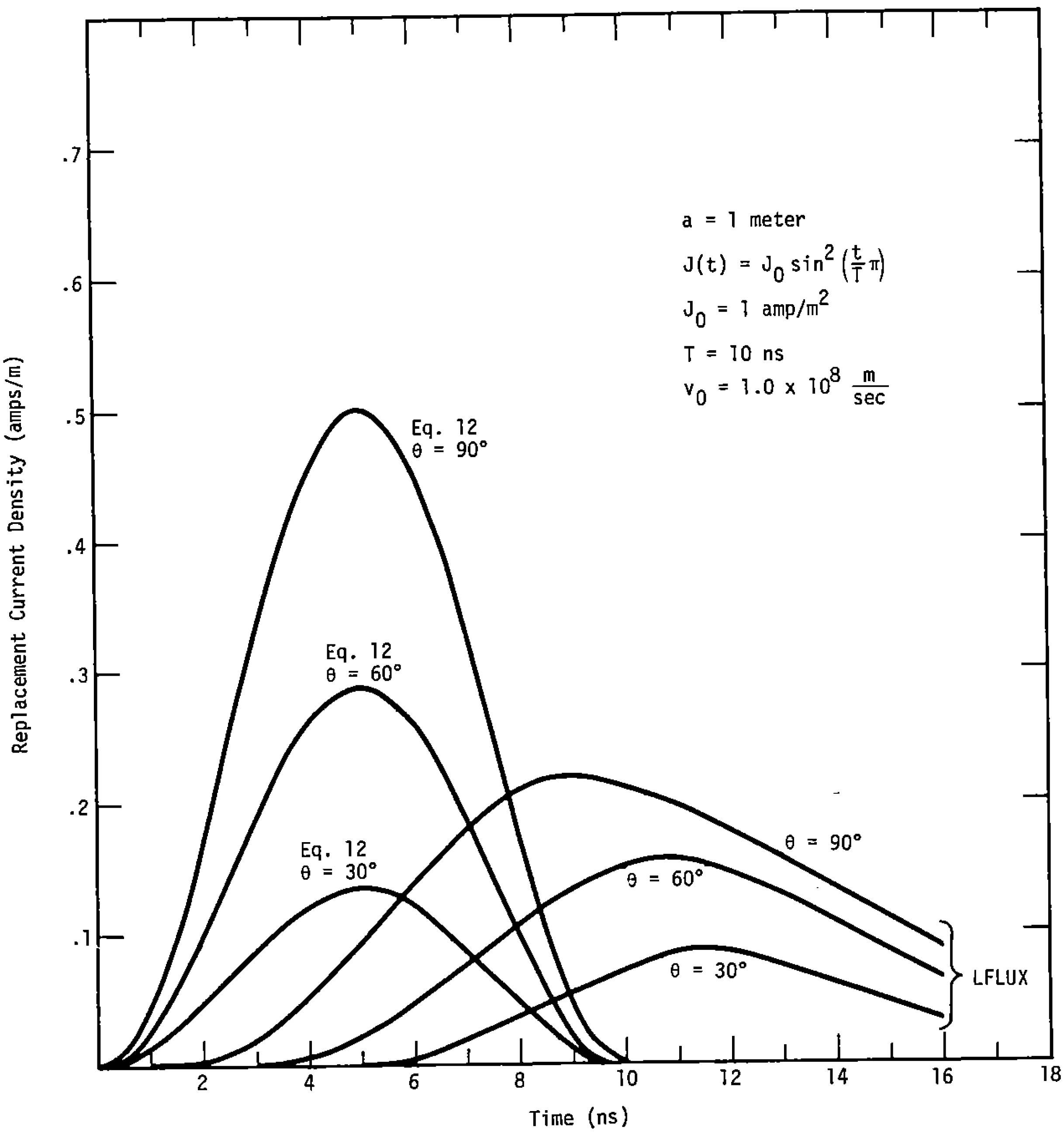

Figure 2. A comparison of replacement current densities as calculated from Equation 12 and from the LFLUX code.

Note that the total replacement charge (i.e., the time integral of the replacement current) will be the same for LFLUX and the simple analytic model used here as long as all of the emitted electrons eventually escape the sphere and move to infinity. Thus, the simple model developed here will always give a replacement current with too high of a peak value and too short of a pulse width. The product of pulse height and pulse width will, however, be roughly the same as that of a more accurate calculation (at least for triangle-type time histories where this product is just proportional to the total replacement charge). In mathematical terms

$$I_t T_t \simeq I_0 T_0 \propto Q \ , \qquad (18)$$

where

I_t is the peak current as given by Equation 12 or 13

T_t is the pulse width of the emission current (e.g., FWHM)

I_0 is the actual peak replacement current flowing past some location

T_0 is the actual pulse width of the replacement current pulse

and

Q is the total replacement charge that flows past the specified location.

Note that for $\theta = 90°$, Figure 2 gives

$I_t = 3.14$ amps

$T_t = 5$ ns

$I_0 = 1.31$ amps

$T_0 = 9.8$ ns

so that Equation 18 is valid to within about 20 percent.

Another way of looking at the simple analytic model being considered here is to compare it to a circuit model that treats the space outside the sphere as a single capacitor with a parallel current generator representing the electron emission current. It is known[2] that such a model gives replacement currents from two to ten times larger than a more accurate circuit which uses numerous radial capacitors and takes the finite electron velocity into account.

To correct for this effect, one can also calculate an "effective" capacitance. If at some time t, the "center" of the emitted electron cloud is some distance b from the center of a sphere of radius a, the effective capacitance is just

$$C_{eff} \simeq \frac{4\pi\varepsilon_0 ab}{b - a} . \tag{19}$$

[C_{eff} is just the capacitance of concentric spheres of radii a and b.]

If the electrons had moved to infinity, however, the capacitance is given by

$$C_\infty = 4\pi\varepsilon_0 a . \tag{20}$$

The amount of charge redistribution is then just proportional to the ratio

$$\frac{C_\infty}{C_{eff}} = \frac{b - a}{b} . \tag{21}$$

Note that this ratio equals .5 when b = 2a; i.e., half of the total charge

2 Wenaas, E. P.,"Lumped-Element Modeling of Satellite SGEMP Excitation," IEEE Trans. Nucl. Sci., Vol. NS-21, December 1974.

replacement will have occurred when the "average" electron has moved one spherical radius away from the emission surface.

One can thus roughly estimate the actual replacement current pulse width T_0 (FWHM) by

$$T_0 \simeq \frac{a}{v_0} \quad , \tag{22}$$

where v_0 is the average electron velocity. [This formula is valid only for emission time histories with pulse widths less than T_0.]

Equation 18 can be combined with Equation 22 to give an estimate of the actual peak replacement current,

$$I_0 \simeq I_t \left(\frac{T_t}{T_0}\right) . \tag{23}$$

Equations 22 and 23 agree fairly well with the data shown in Figure 2.

SECTION 4

SIMULATION CONSIDERATIONS

Now let us use Equations 6, 7, and 8 to look at the distribution of surface replacement currents around the sphere. Figure 3 shows the normalized replacement current, $I_K/2\pi$, as a function of polar angle θ on a 1 meter radius sphere with an emission current density J_0 of 1 amp/m^2. Four different cases are shown: Case 1 is for emission in the range $0° \leq \theta \leq 90°$, Case 2 for $0° \leq \theta \leq 30°$, Case 3 for $30° \leq \theta \leq 60°$, and Case 4 for $60° \leq \theta \leq 90°$.

Note that the current for Case 1 is just the summation of the currents for Cases 2, 3, and 4, as expected since the superposition of these emission regions is just the emission region of Case 1. Note, however, that there is significant cancellation of currents when Cases 2, 3, and 4 are summed in the range $0° \leq \theta \leq 75°$. This is due to the fact that, for example, emission from 60° to 90° only gives negative skin currents for $0° \leq \theta \leq 60°$ while half sphere emission gives positive currents in this region. In Case 4, electrons will flow from the front of the sphere to replace those emitted, while in Case 1, electron emission from the front of the sphere will dominate this effect thus giving rise to currents of the opposite sign.

This cancellation effect creates some interesting questions regarding various simulation techniques that may be used to test satellites for SGEMP response. For example, current sources may be attached to a satellite at a number of locations with thin wires to simulate electron emission. Since only a finite number of current generators would be used to simulate electron emission over an entire exposed surface, some question occurs as to the accuracy of the simulation technique; i.e., how many drive points are required and what timing requirements are necessary to reproduce the threat situation?

For example, instead of individual wires, let us assume we have azimuthally symmetric current sources and that we wish to simulate half-sphere emission ($0° \le \theta \le 90°$) by two drivers, one extending over $0° \le \theta \le 30°$ and the other in the range $60° \le \theta \le 90°$. For a 1 meter radius sphere, the surface area for $\theta_1 \le \theta \le \theta_2$ is

$$A(\theta_1 \le \theta \le \theta_2) = 2\pi(\cos\theta_1 - \cos\theta_2) \quad . \tag{24}$$

Therefore

$$\begin{aligned} A(0° \le \theta \le 30°) &= .84 \text{ m}^2 = A_1 \\ A(60° \le \theta \le 90°) &= 3.14 \text{ m}^2 = A_2 \\ A(0° \le \theta \le 90°) &= 6.28 \text{ m}^2 = A_3 \end{aligned} \tag{25}$$

Let us assume that the emitted currents are scaled by the ratio

$$\frac{A_3}{A_1 + A_2} = 1.58 \quad , \tag{26}$$

so that proper total emitted current is obtained. One can then multiply the appropriate curves in Figure 3 by this ratio and add the results to obtain the simulated skin current distribution. The result of this operation is shown in Figure 4.

An examination of Figure 4 reveals several interesting facts. First of all, the "simulated" skin current distribution does a good job of reproducing the actual skin current distribution on the back half of the sphere; i.e., for $90° \le \theta \le 180°$. On the front surface ($0° \le \theta \le 90°$), however, simulation is quite poor, with the "simulated" current exceeding

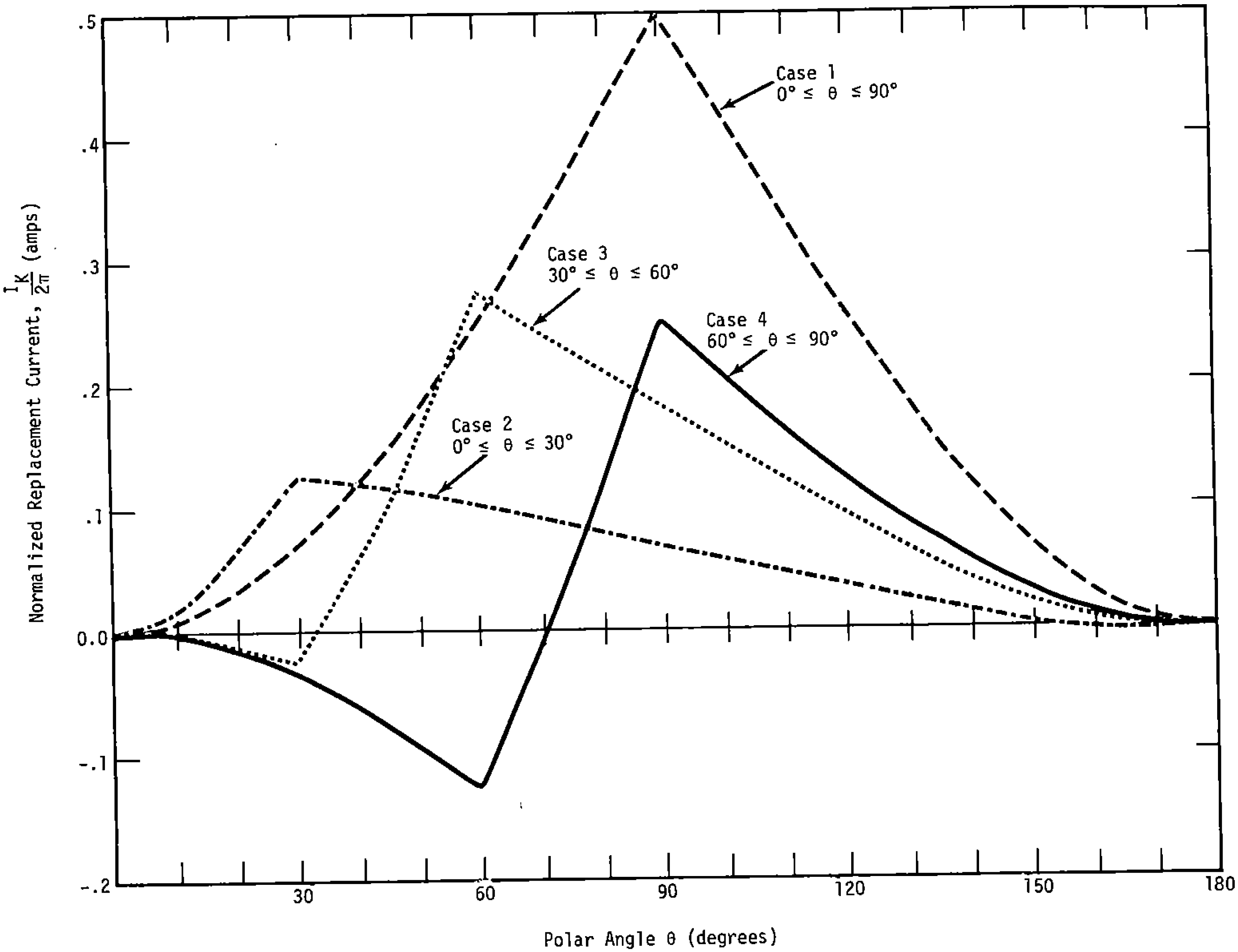

Figure 3. Normalized replacement current as a function of polar angle for various ranges of emission angle: $J_0 = 1.0$ amp/m^2.

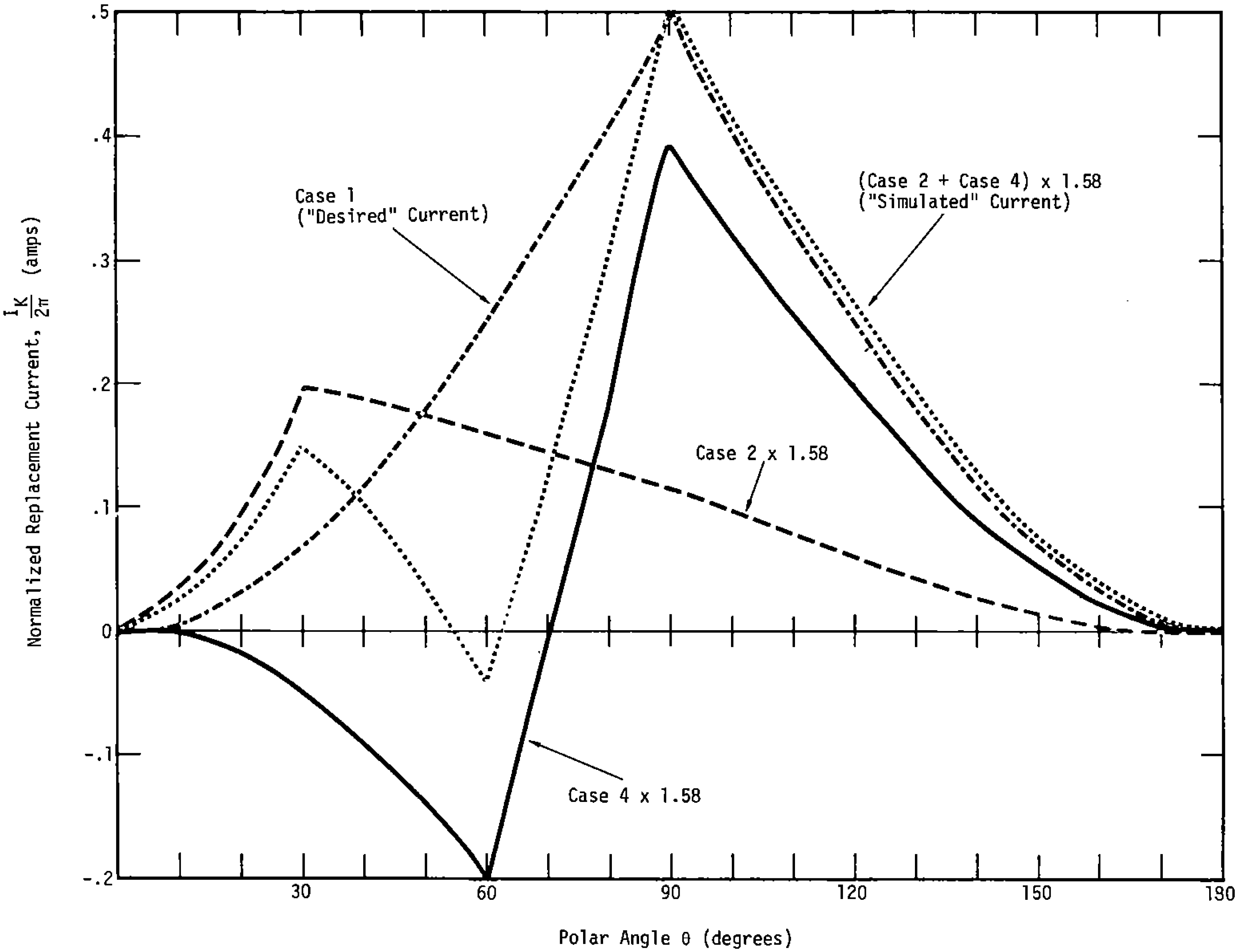

Figure 4. Comparison of replacement current distributions for Case 1 (half sphere emission) and "simulated" excitation.

the actual value for $0° \le \theta \le 40°$ and dropping below the actual value for $40° \le \theta \le 90°$. At $\theta = 60°$ the "simulated" current even has the wrong sign!

The importance of such deviations will depend upon the system being tested. If an aperture or antenna was located at the $\theta = 60°$ location, this simulation technique might give considerable error in evaluating the amount of coupling to the inside of a system. If the important points-of-entry were on the back of the sphere, however, relatively good test data should be attainable, however.

The effect of having a local minima or null on the surface between two drive points is expected to be a real physical phenomena rather than one dependent upon the simple model used here to demonstrate the effect. Since any two drive points will be simulating electron emission from the surface, it is quite reasonable to expect some point on the surface between the two sources that is equally influenced by each, resulting in a null! One must thus be quite careful in applying SGEMP simulation techniques to areas that are actually emitting electrons.

SECTION 5

CONCLUSIONS AND SUMMARY

In this report, a simple analytic model for estimating SGEMP replacement currents, given the emission current, is derived and compared with a more accurate numerical calculation. The simple model is shown to overestimate the peak replacement current magnitude and underestimate the pulse width. Despite these limitations, this simple model is still quite useful for making rough estimates and for promoting a general understanding of replacement current phenomena.

This simple model was then used to demonstrate a potential problem with SGEMP simulation techniques that drive only discrete locations rather than an entire surface. Such drive techniques are shown to give good simulation of replacement currents on the rear (undriven) part of the test object, but unrealistic local skin current minima are shown to be produced at some point between injection points.