Theoretical Notes

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SGEMP Coupling through a General Aperture

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Abstract

In this report SGEMP coupling through apertures is defined. Electric and magnetic fields due to moving electric (magnetic) point charges are derived in time and frequency domains. Quasistatic representations of the field due to a moving charge are discussed. Symmetry decomposition is discussed for sources. Babinet's principle is exhibited in the presence of point sources. Quasistatic representation of the equivalent problem for a thin slot is also shown.

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1. Introduction

The photons released by a nuclear explosion above the atmosphere travel great distances. If a space system such as a space craft, satellite or the space shuttle is in the line of sight of the nuclear explosion, electrons will be ejected from the surfaces of these space systems. The electromagnetic pulse created by the movement of these electrons is called the system generated electromagnetic pulse[1] or SGEMP.

Reviewing some of the earlier work in the SGEMP area, Longmire[2] has developed a model for estimating currents and voltages induced on the external structure of satellites by short pulses of X-rays and γ-rays. Lee and Marin[3] have treated the problem of a charged particle orbiting a perfectly conducting sphere. In a recent report these same authors[4] calculated the currents induced on a thin wire by the motion of a charged particle. For present purposes only one charged particle near the space system is considered. All reaction forces of the charged particles are either ignored or neglected. These approximations would allow one to assume uniform motion of the charged particle in a prescribed orbit thus simplifying the problem.

The present report deals with the coupling of SGEMP through an aperture. In general, the apertures of interest are in bodies of various shapes. However, we will only consider apertures in an infinite plane screen. Fields due to moving electric and magnetic charges that are developed in this report are in general applicable to uniform motion of a charge. Using simple rotation of the coordinate system, these can be extended for arbitrary direction of motion with a uniform velocity.

2. Fields Due to a Moving Charge

Consider a charge Q moving at a constant velocity v in the positive z direction as shown in figure 1. Resulting electric and magnetic fields can be calculated by using the Lorentz transformations for the field. The fields due to the charge in a moving coordinate system can be calculated and transformed into those in an inertial frame of reference. We now assume that any external force does not affect the motion of the charge and calculate the fields for both electric and magnetic charges. Although a magnetic charge is not known to physically exist, its introduction simplifies a class of problems. The coordinates of the source point P' can be represented by $\vec{R}_o$ given by

$$\vec{R}_o = x_o\vec{1}_x + y_o\vec{1}_y + vt\vec{1}_z$$

2.1 Fields Due to a Moving Electric Charge

Using Coulomb's law and the field transformations,[5] the electric field at the field point P due to an electric charge at P' can be written as

$$\vec{E}(x,y,z,t) = \frac{Q_e}{4\pi\epsilon_o}\left[\frac{(x-x_o)\vec{1}_x + (y-y_o)\vec{1}_y + \gamma(z-vt)\vec{1}_z}{\left\{(x-x_o)^2 + (y-y_o)^2 + \gamma^2(z-vt)^2\right\}^{3/2}}\right] \quad (2.1.1)$$

where Q_e is the electric charge at the source point and

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad (2.1.2)$$

where c is the speed of light in free space. Similarly the magnetic field at the field point is given by

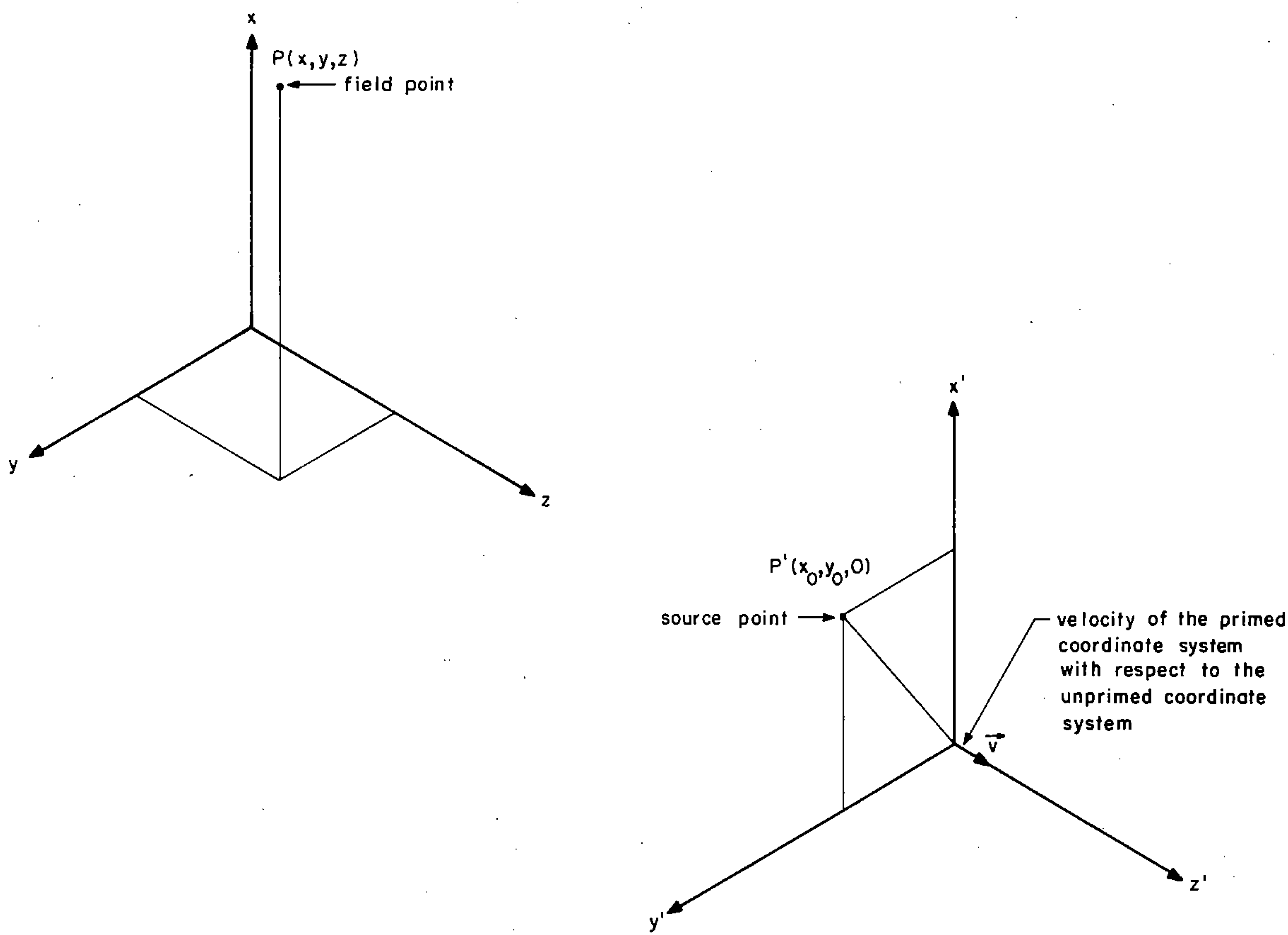

Figure I. Coordinate Systems for the Field and Source Points

$$\vec{H}(x,y,z,t) = \frac{Q_e v}{4\pi} \gamma \left[\frac{-(y-y_o)\vec{1}_x + (x-x_o)\vec{1}_y}{\left\{(x-x_o)^2 + (y-y_o)^2 + \gamma^2(z-vt)^2\right\}^{3/2}} \right] \quad (2.1.3)$$

The time coordinate is assumed to be zero when the primed and the unprimed coordinates coincide. Using equations 2.1.1 and 2.1.3, the time history of the electric and magnetic fields at the field point can be constructed. Notice that in both the electric and magnetic fields, singularities occur when the field point and the source point coincide. If the time domain integral equation approach is used, one can use equations 2.1.1 and 2.1.3 as the incident fields.

If the singularity expansion method (SEM) or some other frequency domain method is used, one has to express equations 2.1.1 and 2.1.3 in the Laplace or Fourier domain. Express the bilateral Laplace transform of a function F(t) as

$$\tilde{F}(s) = \int_{-\infty}^{+\infty} F(t)\, e^{-st} dt \quad (2.1.4)$$

where the tilde (~) denotes Laplace transformed quantity. Hence we can write

$$\tilde{\vec{E}}(x,y,z) = \frac{Q_e}{4\pi\epsilon_o} \int_{-\infty}^{+\infty} \left[\frac{(x-x_o)\vec{1}_x + (y-y_o)\vec{1}_y + \gamma(z-vt)\vec{1}_z}{\left\{(x-x_o)^2 + (y-y_o)^2 + \gamma^2(z-vt)^2\right\}^{3/2}} \right] e^{-st} dt \quad (2.1.5)$$

$$\tilde{\vec{H}}(x,y,z) = \frac{Q_e v}{4\pi} \gamma \int_{-\infty}^{+\infty} \left[\frac{-(y-y_o)\vec{1}_x + (x-x_o)\vec{1}_y}{\left\{(x-x_o)^2 + (y-y_o)^2 + \gamma^2(z-vt)^2\right\}^{3/2}} \right] e^{-st} dt \quad (2.1.6)$$

Making a change of variable

$$\frac{\gamma(z-vt)}{\rho_o} = -\beta \tag{2.1.7}$$

and setting

$$\rho_o = \left[(x-x_o)^2 + (y-y_o)^2\right]^{1/2} \tag{2.1.8}$$

we have

$$t = \frac{1}{v}\left(z + \frac{\rho_o\beta}{\gamma}\right) \tag{2.1.9}$$

$$dt = \frac{\rho_o}{\gamma v}\, d\beta \tag{2.1.10}$$

Hence we can write equations 2.1.5 and 2.1.6 as

$$\tilde{\vec{E}}(x, y, z) = \frac{Q_e}{4\pi\epsilon_o}\,\frac{1}{\gamma v\rho_o^2}\, e^{-\frac{sz}{v}} \int_{-\infty}^{+\infty} \left[\frac{(x-x_o)\vec{1}_x + (y-y_o)\vec{1}_y - \rho_o\beta\vec{1}_z}{\left(1+\beta^2\right)^{3/2}}\right] e^{-\frac{\rho_o s}{\gamma v}\beta}\, d\beta \tag{2.1.11}$$

and

$$\tilde{\vec{H}}(x,y,z) = \frac{Q_e v}{4\pi}\,\frac{1}{\rho_o^2}\,\frac{e^{-\frac{sz}{v}}}{v} \int_{-\infty}^{+\infty} \left[\frac{-(y-y_o)\vec{1}_x + (x-x_o)\vec{1}_y}{\left(1+\beta^2\right)^{3/2}}\right] e^{-\frac{\rho_o s}{\gamma v}\beta}\, d\beta \tag{2.1.12}$$

In equation 2.1.11 and equation 2.1.12, we note that the transforms of $\tilde{E}_x$, $\tilde{E}_y$, and $\tilde{\vec{H}}$ are symmetric while that of $\tilde{E}_z$ is antisymmetric with respect to β. Letting

$$[I] \equiv \int_{-\infty}^{+\infty} \frac{1}{\left(1+\beta^2\right)^{3/2}} e^{-\frac{\rho_o s}{\gamma v}\beta} d\beta \tag{2.1.13a}$$

we have

$$[I] = 2\int_0^{\infty} \frac{1}{\left(1+\beta^2\right)^{3/2}} \cosh\left(\frac{\rho_o s}{\gamma v}\beta\right) d\beta \tag{2.1.13b}$$

which can be written as[6]

$$\int_{-\infty}^{\infty} \frac{1}{\left(1+\beta^2\right)^{3/2}} e^{-\frac{\rho_o s}{\gamma v}\beta} d\beta = \begin{cases} 2i\frac{\rho_o s}{\gamma v} K_1\left(i\frac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] > 0 \\ -2i\frac{\rho_o s}{\gamma v} K_1\left(-i\frac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] < 0 \end{cases} \tag{2.1.13c}$$

Hence, we can write

$$\tilde{E}_x = \begin{cases} \frac{Q_e}{2\pi\epsilon_o}\frac{(x-x_o)}{\rho_o}\frac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_1\left(i\frac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] > 0 \\ -\frac{Q_e}{2\pi\epsilon_o}\frac{(x-x_o)}{\rho_o}\frac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_1\left(-i\frac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] < 0 \end{cases} \tag{2.1.14}$$

$$\widetilde{E}_y = \begin{cases} \dfrac{Q_e}{2\pi\epsilon_o} \dfrac{(y-y_o)}{\rho_o} \dfrac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_1\left(i\dfrac{\rho_o s}{\gamma v}\right) & Im[s] > 0 \\ -\dfrac{Q_e}{2\pi\epsilon_o} \dfrac{(y-y_o)}{\rho_o} \dfrac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_1\left(-i\dfrac{\rho_o s}{\gamma v}\right) & Im[s] < 0 \end{cases} \tag{2.1.15}$$

$$\widetilde{H}_x = \begin{cases} -\dfrac{Q_e v}{2\pi} \dfrac{(y-y_o)}{\rho_o} \dfrac{(is)}{\gamma v^2} e^{-\frac{z}{v}s} K_1\left(i\dfrac{\rho_o s}{\gamma v}\right) & Im[s] > 0 \\ \dfrac{Q_e v}{2\pi} \dfrac{(y-y_o)}{\rho_o} \dfrac{(is)}{\gamma v^2} e^{-\frac{z}{v}s} K_1\left(-i\dfrac{\rho_o s}{\gamma v}\right) & Im[s] < 0 \end{cases} \tag{2.1.16}$$

$$\widetilde{H}_y = \begin{cases} \dfrac{Q_e v}{2\pi} \dfrac{(x-x_o)}{\rho_o} \dfrac{(is)}{\gamma v^2} e^{-\frac{z}{v}s} K_1\left(i\dfrac{\rho_o s}{\gamma v}\right) & Im[s] > 0 \\ -\dfrac{Q_e v}{2\pi} \dfrac{(x-x_o)}{\rho_o} \dfrac{(is)}{\gamma v^2} e^{-\frac{z}{v}s} K_1\left(-i\dfrac{\rho_o s}{\gamma v}\right) & Im[s] < 0 \end{cases} \tag{2.1.17}$$

We note that the frequency spectra of $\widetilde{E}_x$, $\widetilde{E}_y$, $\widetilde{H}_x$, and $\widetilde{H}_y$ are symmetric with respect to β. The frequency spectrum for $\widetilde{E}_z$ is found to be

$$\tilde{E}_z = \begin{cases} -\dfrac{Q_e}{2\pi\epsilon_o} \dfrac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_o\left(i\dfrac{\rho_o s}{\gamma v}\right) & Im[s] > 0 \\ \\ -\dfrac{Q_e}{2\pi\epsilon_o} \dfrac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_o\left(-i\dfrac{\rho_o s}{\gamma v}\right) & Im[s] < 0 \end{cases} \qquad (2.1.18)$$

Noting that[6]

$$\begin{aligned} K_o\left(i\frac{\rho_o s}{\gamma v}\right) &= \frac{\pi i}{2} H_o^{(1)}\left(-\frac{\rho_o s}{\gamma v}\right) = -\frac{\pi i}{2} H_o^{(2)}\left(\frac{\rho_o s}{\gamma v}\right) \\ K_o\left(-i\frac{\rho_o s}{\gamma v}\right) &= \frac{\pi i}{2} H_o^{(1)}\left(\frac{\rho_o s}{\gamma v}\right) \\ K_1\left(i\frac{\rho_o s}{\gamma v}\right) &= -\frac{\pi}{2} H_1^{(1)}\left(-\frac{\rho_o s}{\gamma v}\right) = -\frac{\pi}{2} H_1^{(2)}\left(\frac{\rho_o s}{\gamma v}\right) \\ K_1\left(-i\frac{\rho_o s}{\gamma v}\right) &= -\frac{\pi}{2} H_1^{(1)}\left(\frac{\rho_o s}{\gamma v}\right) \end{aligned} \qquad (2.1.19)$$

we can write equations 2. 1. 14 through 2. 1. 18 in terms of Hankel functions rather than the modified Bessel functions. The modified Bessel function representation is more convenient in the Fourier transform domain while the Hankel function representation is appropriate in the Laplace domain.

Substituting equation 2. 1. 19 into the field equations, we can write them in terms of Hankel functions as

$$\tilde{E}_x = \begin{cases} -\frac{Q_e}{4\epsilon_o}\frac{(x-x_o)}{\rho_o}\frac{(is)}{(\gamma v)^2}e^{-\frac{z}{v}s}H_1^{(2)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] > 0 \\ \frac{Q_e}{4\epsilon_o}\frac{(x-x_o)}{\rho_o}\frac{(is)}{(\gamma v)^2}e^{-\frac{z}{v}s}H_1^{(1)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] < 0 \end{cases} \quad (2.1.20)$$

$$\tilde{E}_y = \begin{cases} -\frac{Q_e}{4\epsilon_o}\frac{(y-y_o)}{\rho_o}\frac{(is)}{(\gamma v)^2}e^{-\frac{z}{v}s}H_1^{(2)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] > 0 \\ \frac{Q_e}{4\epsilon_o}\frac{(y-y_o)}{\rho_o}\frac{(is)}{(\gamma v)^2}e^{-\frac{z}{v}s}H_1^{(1)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] < 0 \end{cases} \quad (2.1.21)$$

$$\tilde{H}_x = \begin{cases} \frac{Q_e v}{4}\frac{(y-y_o)}{\rho_o}\frac{(is)}{\gamma v^2}e^{-\frac{z}{v}s}H_1^{(2)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] > 0 \\ -\frac{Q_e v}{4}\frac{(y-y_o)}{\rho_o}\frac{(is)}{\gamma v^2}e^{-\frac{z}{v}s}H_1^{(1)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] < 0 \end{cases} \quad (2.1.22)$$

$$\tilde{H}_y = \begin{cases} -\frac{Q_e v}{4}\frac{(x-x_o)}{\rho_o}\frac{(is)}{\gamma v^2}e^{-\frac{z}{v}s}H_1^{(2)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] > 0 \\ \frac{Q_e v}{4}\frac{(x-x_o)}{\rho_o}\frac{(is)}{\gamma v^2}e^{-\frac{z}{v}s}H_1^{(1)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] < 0 \end{cases} \quad (2.1.23)$$

and

$$\tilde{E}_z = \begin{cases} -\dfrac{Q_e}{4\epsilon_o} \dfrac{s}{(\gamma v)^2} e^{-\frac{z}{v}s} H_o^{(2)}\left(\dfrac{\rho_o s}{\gamma v}\right) & Im[s] > 0 \\ \\ \dfrac{Q_e}{4\epsilon_o} \dfrac{s}{(\gamma v)^2} e^{-\frac{z}{v}s} H_o^{(1)}\left(\dfrac{\rho_o s}{\gamma v}\right) & Im[s] < 0 \end{cases} \qquad (2.1.24)$$

Equations 2.1.11, 2.1.12, 2.1.14 through 2.1.24 give the field components due to a moving point charge in both time and frequency domains. Depending upon the type of the formulation used, the proper field representation can be used. For instance, if a thin wire on the z axis is of interest, and if the time domain approach is used, one would use $\vec{E}$ from equation 2.1.1 as the incident field. However, if frequency domain approach is used, one would use equation 2.1.18 as the incident field. The fields given by equations 2.1.14 through 2.1.18 can be simply extended for an arbitrary direction of motion of the charge by using rotation of coordinates.

In keeping with our general practice of expressing the fields in the combined field formulation[7], we define the combined field $\vec{E}_q(x,y,z,t)$ as

$$\vec{E}_q(x,y,z,t) = \vec{E}(x,y,z,t) + qi Z_o \vec{H}(x,y,z,t) \qquad q = \pm 1 \qquad (2.1.25)$$

We can write the combined field due to a moving electric charge as

$$\vec{E}_q(x,y,z,t) = \frac{Q_e}{4\pi\epsilon_o} \frac{1}{(\xi)^{3/2}} \left[\left\{(x-x_o) - qi\frac{\gamma v}{c}(y-y_o)\right\} \vec{1}_x + \left\{(y-y_o) + qi\frac{\gamma v}{c}(x-x_o)\right\} \vec{1}_y + \left\{\gamma(z-vt)\right\} \vec{1}_z \right] \qquad (2.1.26)$$

where

$$\zeta = \left\{(x-x_o)^2 + (y-y_o)^2 + \gamma^2(z-vt)^2\right\} \tag{2.1.27}$$

In a similar way, the combined field representation in the frequency domain can be obtained. We can write this as

$$\tilde{E}_{q_x} = \frac{Q_e}{4\epsilon_o}\frac{1}{\rho_o}\frac{(is)}{(\gamma v)}e^{-\frac{z}{v}s}\begin{cases}\left[-\dfrac{(x-x_o)}{(\gamma v)} + q\dfrac{i}{c}(y-y_o)\right]H_1^{(2)}\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] > 0 \\ \\ \left[\dfrac{(x-x_o)}{(\gamma v)} - q\dfrac{i}{c}(y-y_o)\right]H_1^{(1)}\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] < 0\end{cases} \tag{2.1.28}$$

$$\tilde{E}_{q_y} = \frac{Q_e}{4\epsilon_o}\frac{1}{\rho_o}\frac{(is)}{(\gamma v)}e^{-\frac{z}{v}s}\begin{cases}\left[-\dfrac{(y-y_o)}{(\gamma v)} - q\dfrac{i}{c}(x-x_o)\right]H_1^{(2)}\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] > 0 \\ \\ \left[\dfrac{(y-y_o)}{(\gamma v)} + q\dfrac{i}{c}(x-x_o)\right]H_1^{(1)}\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] < 0\end{cases} \tag{2.1.29}$$

$$\tilde{E}_{q_z} = \frac{Q_e}{4\epsilon_o}\frac{s}{(\gamma v)^2}e^{-\frac{z}{v}s}\begin{cases}-H_o^{(2)}\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] > 0 \\ \\ H_o^{(1)}\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] < 0\end{cases} \tag{2.1.30}$$

2.2 Fields Due to a Moving Magnetic Charge

Although magnetic currents and charges do not physically exist, they form a useful mathematical tool in a certain class of problems. Babinet's principle has been a useful concept in solving the problems of coupling through apertures. In this principle, the plane scatterer with an aperture can be replaced by its complement, while the electric sources are replaced by the magnetic sources and vice versa.[9] Because of this simplicity achieved by the introduction of magnetic charges, we will calculate the fields due to a magnetic charge. Maxwell's equations are given by

$$\begin{aligned} \nabla \times \tilde{\vec{E}} &= -s\mu_o \tilde{\vec{H}} \\ \nabla \times \tilde{\vec{H}} &= s\epsilon_o \tilde{\vec{E}} + \tilde{\vec{J}} \\ \nabla \cdot \tilde{\vec{B}} &= 0 \\ \nabla \cdot \tilde{\vec{D}} &= \tilde{\rho} \end{aligned} \qquad (2.2.1)$$

when only electric current density and charge density are present and by

$$\begin{aligned} \nabla \times \tilde{\vec{E}} &= -s\mu_o \tilde{\vec{H}} - \tilde{\vec{J}}_m \\ \nabla \times \tilde{\vec{H}} &= s\epsilon_o \tilde{\vec{E}} \\ \nabla \cdot \tilde{\vec{B}} &= \tilde{\rho}_m \\ \nabla \cdot \tilde{\vec{D}} &= 0 \end{aligned} \qquad (2.2.2)$$

when only magnetic current density and charge density are present. The equivalence between these two sets of equations is obtained by

$$\tilde{\vec{E}} \to Z_o \tilde{\vec{H}} \tag{2.2.3}$$

$$\tilde{\vec{H}} \to -\frac{1}{Z_o} \tilde{\vec{E}} \tag{2.2.4}$$

$$\tilde{\vec{J}} \to \frac{1}{Z_o} \tilde{\vec{J}}_m, \ \tilde{\vec{J}}_m \to -Z_o \vec{J} \tag{2.2.5}$$

$$\tilde{\rho} \to \frac{1}{Z_o} \tilde{\rho}_m \tag{2.2.6}$$

$$\tilde{\rho}_m \to -Z_o \tilde{\rho} \tag{2.2.7}$$

This establishes an equivalence between the fields produced by electric and magnetic sources; as a consequence, it is easy to obtain the fields due to a magnetic charge. In figure 1, if the charge at the source point is a magnetic charge Q_m, the field at the field point can be obtained from equations 2.1.14 through 2.1.18 by utilizing equations 2.2.3 through 2.2.7. Noting that the electric point charge $Q_e \to \frac{1}{Z_o} Q_m$ where Q_m is a magnetic point charge, we obtain

$$\tilde{H}_x = \begin{cases} \dfrac{Q_m}{2\pi\mu_o} \dfrac{(x-x_o)}{\rho_o} \dfrac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_1\left(i\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] > 0 \\ \\ -\dfrac{Q_m}{2\pi\mu_o} \dfrac{(x-x_o)}{\rho_o} \dfrac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_1\left(-i\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] < 0 \end{cases} \tag{2.2.8}$$

$$\widetilde{H}_y = \begin{cases} \dfrac{Q_m}{2\pi\mu_o} \dfrac{(y-y_o)}{\rho_o} \dfrac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_1\left(i\dfrac{\rho_o s}{\gamma v}\right) & \text{Im}[s] > 0 \\ -\dfrac{Q_m}{2\pi\mu_o} \dfrac{(y-y_o)}{\rho_o} \dfrac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_1\left(-i\dfrac{\rho_o s}{\gamma v}\right) & \text{Im}[s] < 0 \end{cases} \qquad (2.2.9)$$

$$\widetilde{H}_z = \begin{cases} -\dfrac{Q_m}{2\pi\mu_o} \dfrac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_o\left(i\dfrac{\rho_o s}{\gamma v}\right) & \text{Im}[s] > 0 \\ -\dfrac{Q_m}{2\pi\mu_o} \dfrac{(is)}{(\gamma v)^2} e^{-\frac{z}{v}s} K_o\left(-i\dfrac{\rho_o s}{\gamma v}\right) & \text{Im}[s] < 0 \end{cases} \qquad (2.2.10)$$

$$\widetilde{E}_x = \begin{cases} \dfrac{Q_m v}{2\pi} \dfrac{(y-y_o)}{\rho_o} \dfrac{(is)}{\gamma v^2} e^{-\frac{z}{v}s} K_1\left(i\dfrac{\rho_o s}{\gamma v}\right) & \text{Im}[s] > 0 \\ -\dfrac{Q_m v}{2\pi} \dfrac{(y-y_o)}{\rho_o} \dfrac{(is)}{\gamma v^2} e^{-\frac{z}{v}s} K_1\left(-i\dfrac{\rho_o s}{\gamma v}\right) & \text{Im}[s] < 0 \end{cases} \qquad (2.2.11)$$

$$\widetilde{E}_y = \begin{cases} -\dfrac{Q_m v}{2\pi} \dfrac{(x-x_o)}{\rho_o} \dfrac{(is)}{\gamma v^2} e^{-\frac{z}{v}s} K_1\left(i\dfrac{\rho_o s}{\gamma v}\right) & \text{Im}[s] < 0 \\ \dfrac{Q_m v}{2\pi} \dfrac{(x-x_o)}{\rho_o} \dfrac{(is)}{\gamma v^2} e^{-\frac{z}{v}s} K_1\left(-i\dfrac{\rho_o s}{\gamma v}\right) & \text{Im}[s] > 0 \end{cases} \qquad (2.2.12)$$

or

$$\widetilde{H}_x = \begin{cases} -\dfrac{Q_m}{4\mu_o}\dfrac{(x-x_o)}{\rho_o}\dfrac{(is)}{(\gamma v)^2}\, e^{-\frac{z}{v}s}\, H_1^{(2)}\!\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] > 0 \\ \dfrac{Q_m}{4\mu_o}\dfrac{(x-x_o)}{\rho_o}\dfrac{(is)}{(\gamma v)^2}\, e^{-\frac{z}{v}s}\, H_1^{(1)}\!\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] < 0 \end{cases} \qquad (2.2.13)$$

$$\widetilde{H}_y = \begin{cases} -\dfrac{Q_m}{4\mu_o}\dfrac{(y-y_o)}{\rho_o}\dfrac{(is)}{(\gamma v)^2}\, e^{-\frac{z}{v}s}\, H_1^{(2)}\!\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] > 0 \\ \dfrac{Q_m}{4\mu_o}\dfrac{(y-y_o)}{\rho_o}\dfrac{(is)}{(\gamma v)^2}\, e^{-\frac{z}{v}s}\, H_1^{(1)}\!\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] < 0 \end{cases} \qquad (2.2.14)$$

$$\widetilde{H}_z = \begin{cases} -\dfrac{Q_m}{4\mu_o}\dfrac{s}{(\gamma v)^2}\, e^{-\frac{z}{v}s}\, H_o^{(2)}\!\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] > 0 \\ \dfrac{Q_m}{4\mu_o}\dfrac{s}{(\gamma v)^2}\, e^{-\frac{z}{v}s}\, H_o^{(1)}\!\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] < 0 \end{cases} \qquad (2.2.15)$$

$$\widetilde{E}_x = \begin{cases} -\dfrac{Q_m v}{4}\dfrac{(y-y_o)}{\rho_o}\dfrac{(is)}{\gamma v^2}\, e^{-\frac{z}{v}s}\, H_1^{(2)}\!\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] > 0 \\ \dfrac{Q_m v}{4}\dfrac{(y-y_o)}{\rho_o}\dfrac{(is)}{\gamma v^2}\, e^{-\frac{z}{v}s}\, H_1^{(1)}\!\left(\dfrac{\rho_o s}{\gamma v}\right) & \mathrm{Im}[s] < 0 \end{cases} \qquad (2.2.16)$$

$$
\widetilde{E}_y = \begin{cases} \dfrac{Q_m v}{4} \dfrac{(x-x_o)}{\rho_o} \dfrac{(is)}{\gamma v^2} e^{-\frac{z}{v}s} H_1^{(2)}\left(\dfrac{\rho_o s}{\gamma v}\right) & Im[s] > 0 \\ -\dfrac{Q_m v}{4} \dfrac{(x-x_o)}{\rho_o} \dfrac{(is)}{\gamma v^2} e^{-\frac{z}{v}s} H_1^{(1)}\left(\dfrac{\rho_o s}{\gamma v}\right) & Im[s] < 0 \end{cases} \tag{2.2.17}
$$

in the frequency domain and

$$
\vec{H}(x,y,z;t) = \frac{Q_m}{4\pi\mu_o}\left[\frac{(x-x_o)\vec{1}_x + (y-y_o)\vec{1}_y + \gamma(z-vt)\vec{1}_z}{\left\{(x-x_o)^2 + (y-y_o)^2 + \gamma^2(z-vt)^2\right\}^{3/2}}\right] \tag{2.2.18}
$$

$$
\vec{E}(x,y,z;t) = -\frac{Q_m v}{4\pi}\gamma\left[\frac{-(y-y_o)\vec{1}_x + (x-x_o)\vec{1}_y}{\left\{(x-x_o)^2 + (y-y_o)^2 + \gamma^2(z-vt)^2\right\}^{3/2}}\right] \tag{2.2.19}
$$

in the time domain.

Expressing the fields due to a moving magnetic charge in terms of the combined field, we have

$$
\vec{E}_q(x,y,z;t) = -\frac{Q_m}{4\pi}\frac{\gamma v}{(\xi)^{3/2}}\left[\left\{-(y-y_o) - qi\frac{c}{\gamma v}(x-x_o)\right\}\vec{1}_x + \left\{(x-x_o) - qi\frac{c}{\gamma v}(y-y_o)\right\}\vec{1}_y - qi\frac{c}{v}(z-vt)\vec{1}_z\right] \tag{2.2.20}
$$

$$\widetilde{E}_{q_x}(x,y,z) = -\frac{Q_m}{4}\frac{1}{\rho_o}\frac{is}{\gamma v}e^{-\frac{z}{v}s}\begin{cases}\left[(y-y_o) + qi\frac{c}{\gamma v}(x-x_o)\right]H_1^{(2)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] > 0\\ -\left[(y-y_o) + qi\frac{c}{\gamma v}(x-x_o)\right]H_1^{(1)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] < 0\end{cases} \quad (2.2.21)$$

$$\widetilde{E}_{q_y}(x,y,z) = \frac{Q_m}{4}\frac{1}{\rho_o}\frac{is}{\gamma v}e^{-\frac{z}{v}s}\begin{cases}\left[(x-x_o) - qi\frac{c}{\gamma v}(y-y_o)\right]H_1^{(2)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] > 0\\ \left[-(x-x_o) + qi\frac{c}{\gamma v}(y-y_o)\right]H_1^{(1)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] < 0\end{cases} \quad (2.2.22)$$

$$\widetilde{E}_{q_z}(x,y,z) = -qi\frac{Q_m}{4}\frac{sc}{(\gamma v)^2}e^{-\frac{z}{v}s}\begin{cases}H_o^{(2)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] > 0\\ -H_o^{(1)}\left(\frac{\rho_o s}{\gamma v}\right) & Im[s] < 0\end{cases} \quad (2.2.23)$$

As in the case of the fields due to an electric charge, the fields due to a magnetic charge can also be calculated for arbitrary direction of motion of the charge by a simple rotation of the coordinates.

3. Quasistatic Representation of the Field

There exists some confusion as to what quasistatic really means and what is really necessary. It is clear that the fields due to a moving charge can be expanded in terms of either the velocity or frequency using some series expansion. If the velocity of the charge is small compared to the velocity of light c, the field can be approximated quasistatically. However, it is also possible to expand the integral operators associated with a scatterer in terms of the frequency which leads to a quasistatic operator formulation. As one can clearly see, this leads to confusions if not errors in the formulation of the problem. Table 1 indicates the different combinations one can have.

	Dynamic Operator	Quasistatic Operator
Dynamic Incident Field	Dynamic Incident Field and Dynamic Operator	Dynamic Incident Field and Quasistatic Operator
Quasi-static Incident Field	Quasistatic Incident Field and Dynamic Operator	Quasistatic Incident Field and Quasistatic Operator

NOTE: Dynamic operator as used above is either EFIE, HFIE or a like quantity.

Table 1

The confusion which probably was only enhanced by table 1 would lead one to ask what is really quasistatic? There appears to be no definite answer to that question. If one uses the dynamic field with the dynamic operator and expands the resulting quantity in some series form, the quasistatic response function would be obtained. This would yield numerically accurate results; however, for simplicity one of the other combinations in table 1 should be considered.

3.1 Quasistatic Representation of the Field Due to a Moving Electric Charge

The electric and magnetic fields due to a moving charge are given by equations 2.1.1 and 2.1.3 in the time domain. Noting that

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{3.1.1}$$

and $v < c$, we can expand γ in the binomial series as

$$\gamma = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \frac{5}{16}\frac{v^6}{c^6} + \cdots\cdots \tag{3.1.2}$$

Neglecting terms of order v^4/c^4 and substituting it into equations 2.1.1 and 2.1.3 we obtain

$$\vec{E}(x, y, z, t) = \frac{Q_e}{4\pi\epsilon_o}\left[\frac{(x-x_o)\vec{1}_x + (y-y_o)\vec{1}_y + \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)(z-vt)\vec{1}_z}{\left\{(x-x_o)^2 + (y-y_o)^2 + \left(1 + \frac{v^2}{c^2}\right)(z-vt)^2\right\}^{3/2}}\right] \tag{3.1.3}$$

and

$$\vec{H}(x, y, z, t) = \frac{Q_e}{4\pi} v\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)\left[\frac{-(y-y_o)\vec{1}_x + (x-x_o)\vec{1}_y}{\left\{(x-x_o)^2 + (y-y_o)^2 + \left(1 + \frac{v^2}{c^2}\right)(z-vt)^2\right\}^{3/2}}\right] \tag{3.1.4}$$

If we neglect terms of order v^2/c^2 in equations 3.1.3 and 3.1.4, we obtain the simple Galilean transformations[5] of the field which are applicable if $v << c$. These can also be obtained by simply setting $\gamma = 1$ in equations 2.1.1 and 2.1.3. It is interesting to note that the magnetic field is similar to that due to a uniform z directed current element along the entire z axis whose current is $Q_e v$. Higher order terms in equations 3.1.3 and 3.1.4 seem to have no physical analogue and can simply be considered as correction terms.

Using the frequency spectra given by equations 2.1.20 through 2.1.24, if the velocity of the charge $v << c$, we can write

$$\tilde{E}_x \simeq \begin{cases} -\dfrac{Q_e}{4\epsilon_o}\dfrac{(x-x_o)}{\rho_o}\dfrac{is}{v^2}\left(1+\dfrac{zs}{c}\dfrac{c}{v}\right)H_1^{(2)}\left(\dfrac{\rho_o s}{v}\right) & \text{Im}[s]>0 \\ \dfrac{Q_e}{4\epsilon_o}\dfrac{(x-x_o)}{\rho_o}\dfrac{is}{v^2}\left(1+\dfrac{zs}{c}\dfrac{c}{v}\right)H_1^{(1)}\left(\dfrac{\rho_o s}{v}\right) & \text{Im}[s]<0 \end{cases} \tag{3.1.5}$$

$$\tilde{E}_y \simeq \begin{cases} -\dfrac{Q_e}{4\epsilon_o}\dfrac{(y-y_o)}{\rho_o}\dfrac{is}{v^2}\left(1+\dfrac{zs}{c}\dfrac{c}{v}\right)H_1^{(2)}\left(\dfrac{\rho_o s}{v}\right) & \text{Im}[s]>0 \\ \dfrac{Q_e}{4\epsilon_o}\dfrac{(y-y_o)}{\rho_o}\dfrac{is}{v^2}\left(1+\dfrac{zs}{c}\dfrac{c}{v}\right)H_1^{(1)}\left(\dfrac{\rho_o s}{v}\right) & \text{Im}[s]<0 \end{cases} \tag{3.1.6}$$

$$\tilde{H}_x = \begin{cases} \dfrac{Q_e v}{4}\dfrac{(y-y_o)}{\rho_o}\dfrac{is}{v^2}\left(1+\dfrac{zs}{c}\dfrac{c}{v}\right)H_1^{(2)}\left(\dfrac{\rho_o s}{v}\right) & \text{Im}[s]>0 \\ -\dfrac{Q_e v}{4}\dfrac{(y-y_o)}{\rho_o}\dfrac{is}{v^2}\left(1+\dfrac{zs}{c}\dfrac{c}{v}\right)H_1^{(1)}\left(\dfrac{\rho_o s}{v}\right) & \text{Im}[s]<0 \end{cases} \tag{3.1.7}$$

$$\tilde{H}_y = \begin{cases} -\dfrac{Q_e v}{4}\dfrac{(y-y_o)}{\rho_o}\dfrac{is}{v^2}\left(1+\dfrac{zs}{c}\dfrac{c}{v}\right)H_1^{(2)}\left(\dfrac{\rho_o s}{v}\right) & \text{Im}[s] > 0 \\ \dfrac{Q_e v}{4}\dfrac{(y-y_o)}{\rho_o}\dfrac{is}{v^2}\left(1+\dfrac{zs}{c}\dfrac{c}{v}\right)H_1^{(1)}\left(\dfrac{\rho_o s}{v}\right) & \text{Im}[s] < 0 \end{cases} \tag{3.1.8}$$

$$\tilde{E}_z = \begin{cases} -\dfrac{Q_e}{4\epsilon_o}\dfrac{s}{v^2}\left(1+\dfrac{zs}{c}\dfrac{c}{v}\right)H_o^{(2)}\left(\dfrac{\rho_o s}{v}\right) & \text{Im}[s] > 0 \\ \dfrac{Q_e}{4\epsilon_o}\dfrac{s}{v^2}\left(1+\dfrac{zs}{c}\dfrac{c}{v}\right)H_o^{(1)}\left(\dfrac{\rho_o s}{v}\right) & \text{Im}[s] < 0 \end{cases} \tag{3.1.9}$$

It is interesting to note that the electric and magnetic fields as shown above are simply the Coulomb and Biot-Savart law fields, respectively, due to a moving charge. We will now expand the fields given by equations 3.1.5 through 3.1.9 about $s = 0$ for $v > 0$ but $v << c$. If we only consider the principle branch associated with the Hankel functions, i.e., the imaginary part of the argument is greater than zero, the determining factor for the singularities associated with the Hankel functions of order greater than zero is not the logarithmic term. Hence we can write for $\text{Im}[s] > 0$, $\lim\ s \to 0$,

$$\tilde{E}_x \simeq -\frac{Q_e}{2\pi\epsilon_o}\frac{(x-x_o)}{v\rho_o^2} \tag{3.1.10}$$

$$\tilde{E}_y \simeq -\frac{Q_e}{2\pi\epsilon_o}\frac{(y-y_o)}{v\rho_o^2} \tag{3.1.11}$$

$$\tilde{H}_x \simeq \frac{Q_e v}{2\pi} \frac{(y-y_o)}{v\rho_o^2} \tag{3.1.12}$$

$$\tilde{H}_y \simeq -\frac{Q_e v}{2\pi} \frac{(x-x_o)}{v\rho_o^2} \tag{3.1.13}$$

and

$$\tilde{E}_z \simeq -i \frac{Q_e}{2\pi\epsilon_o} \frac{s}{v^2} \ell n \left(\frac{2v}{\rho_o s}\right) \tag{3.1.14}$$

From equations 3.1.12 and 3.1.13, the magnetic field components are similar to those produced by a uniform z directed current along the entire z axis carrying a current $Q_e v$ in the $-z$ direction. The electric field and the magnetic field are related by $\vec{\tilde{H}} = \epsilon_o \vec{v} \times \vec{\tilde{E}}$. A more detailed analysis of this is considered outside the scope of this report.

3.2 Quasistatic Representation of the Field Due to a Moving Magnetic Charge

Quasistatic electric and magnetic fields associated with a moving magnetic charge can be simply obtained by transforming those due to a moving electric charge by using equations 2.2.3 through 2.2.7. If we assume that the velocity $v \ll c$, we obtain

$$\vec{E}(x, y, z, t) = \frac{Q_m v}{4\pi} \left[\frac{(y-y_o)\vec{1}_x - (x-x_o)\vec{1}_y}{\left\{(x-x_o)^2 + (y-y_o)^2 + (z-vt)^2\right\}^{3/2}} \right] \tag{3.2.1}$$

and

$$\vec{H}(x, y, z, t) = \frac{Q_m}{4\pi\mu_o} \left[\frac{(x-x_o)\vec{1}_x + (y-y_o)\vec{1}_y + (z-vt)\vec{1}_z}{\left\{(x-x_o)^2 + (y-y_o)^2 + (z-vt)^2\right\}^{3/2}} \right] \tag{3.2.2}$$

The electric field as shown in equation 3.2.1 is similar to that produced by an infinite magnetic current element. In the limit $s \to 0$, $\mathrm{Im}[s] > 0$ and $v \ll c$, we have

$$\tilde{H}_x \simeq -\frac{Q_m}{2\pi\mu_o}\frac{(x-x_o)}{v\rho_o^2} \tag{3.2.3}$$

$$\tilde{H}_y \simeq -\frac{Q_m}{2\pi\mu_o}\frac{(y-y_o)}{v\rho_o^2} \tag{3.2.4}$$

$$\tilde{H}_z \simeq -i\frac{Q_m}{2\pi\mu_o}\frac{s}{v^2}\ell n\left(\frac{2v}{\rho_o s}\right) \tag{3.2.5}$$

$$\tilde{E}_x \simeq -\frac{Q_m v}{2\pi}\frac{(y-y_o)}{v\rho_o^2} \tag{3.2.6}$$

$$\tilde{E}_y \simeq -\frac{Q_m v}{2\pi}\frac{(x-x_o)}{v\rho_o^2} \tag{3.2.7}$$

The electric fields are similar to those produced by an infinite magnetic current element carrying a current $Q_m v$ in the $-z$ direction.

Quasistatic fields as derived here can in some cases be used as the incident fields. Depending upon the integral operator and the approximations associated with it, the incident field can be chosen to fit some error criterion.

4. Symmetry Decomposition

4.1 The Concept of Symmetry Decomposition

The concept of symmetry with respect to a plane is quite useful.[8] It has been shown in an earlier note[9] that the principle of symmetry simplifies the analysis of plane scatterers. Consider a symmetry plane S as shown in figure 2 with $\vec{r}$ being the position vector to an arbitrary point P while $\vec{r}_I$ is the position vector to its image point P_I. Defining a reflection dyadic $\overleftrightarrow{R}$ such that

$$\overleftrightarrow{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{4.1.1}$$

we have

$$\vec{r}_I = \overleftrightarrow{R} \cdot \vec{r} \qquad \text{or} \qquad \vec{r} = \overleftrightarrow{R} \cdot \vec{r}_I \tag{4.1.2}$$

Corresponding to the reflection of $\vec{r}$ to $\vec{r}_I$, we define image quantities for the fields, currents and charges with a subscript I. These can be written as

$$\tilde{\vec{E}}_I(\vec{r}) = \overleftrightarrow{R} \cdot \tilde{\vec{E}}(\vec{r}_I) \tag{4.1.3a}$$

$$\tilde{\vec{D}}_I(\vec{r}) = \overleftrightarrow{R} \cdot \tilde{\vec{D}}(\vec{r}_I) \tag{4.1.3b}$$

$$\tilde{\vec{B}}_I(\vec{r}) = -\overleftrightarrow{R} \cdot \tilde{\vec{B}}(\vec{r}_I) \tag{4.1.3c}$$

$$\tilde{\vec{H}}_I(\vec{r}) = -\overleftrightarrow{R} \cdot \tilde{\vec{H}}(\vec{r}_I) \tag{4.1.3d}$$

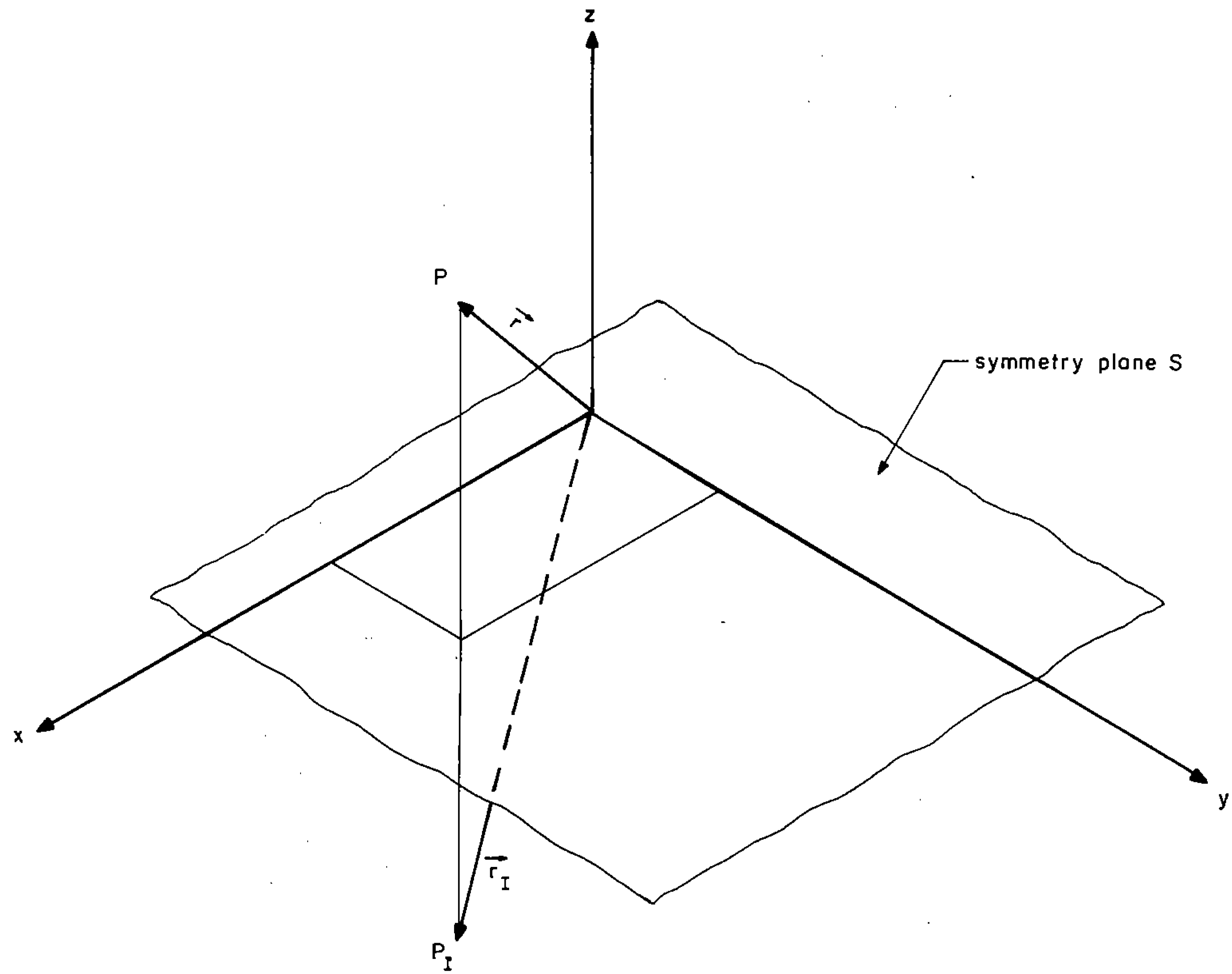

Figure 2. Symmetry Plane

$$\tilde{\vec{J}}_I(\vec{r}) = \vec{\vec{R}} \cdot \tilde{\vec{J}}(\vec{r}_I) \tag{4.1.3e}$$

$$\tilde{\vec{J}}_{m_I}(\vec{r}) = -\vec{\vec{R}} \cdot \tilde{\vec{J}}_m(\vec{r}_I) \tag{4.1.3f}$$

$$\tilde{\rho}_I(\vec{r}) = \tilde{\rho}(\vec{r}_I) \tag{4.1.3g}$$

$$\tilde{\rho}_{m_I}(\vec{r}) = -\tilde{\rho}_m(\vec{r}_I) \tag{4.1.3h}$$

We now define a symmetric (antisymmetric) quantity as one half the sum (difference) of the original quantity and its image quantity. Representing the symmetric and antisymmetric quantities with subscripts sy and as, respectively, for some arbitrary vector or scalar quantity $\tilde{\psi}(\vec{r})$, we can write

$$\tilde{\psi}_{\substack{sy \\ as}}(\vec{r}) = \frac{1}{2}\left[\tilde{\psi}(\vec{r}) \pm \tilde{\psi}_I(\vec{r})\right] \tag{4.1.4}$$

where $\tilde{\psi}_I(\vec{r})$, the image quantity, can be found from equation 4.1.3 by using the appropriate equation. Readers are referred to earlier works[8, 9] for a more detailed study of symmetry decomposition. Simply stated, the symmetric part corresponds to reflection through an infinite magnetic sheet while the antisymmetric part corresponds to reflection through an infinite electric sheet.

4.2 Symmetry Decomposition in Planar Apertures

Let us now consider an aperture A in an infinite, perfectly conducting plane S as shown in figure 3. The aperture region may be loaded by a dyadic sheet admittance $\vec{\vec{Y}}_S$ which, of course, would include open apertures as well. Let us assume that the incident field is incident from the

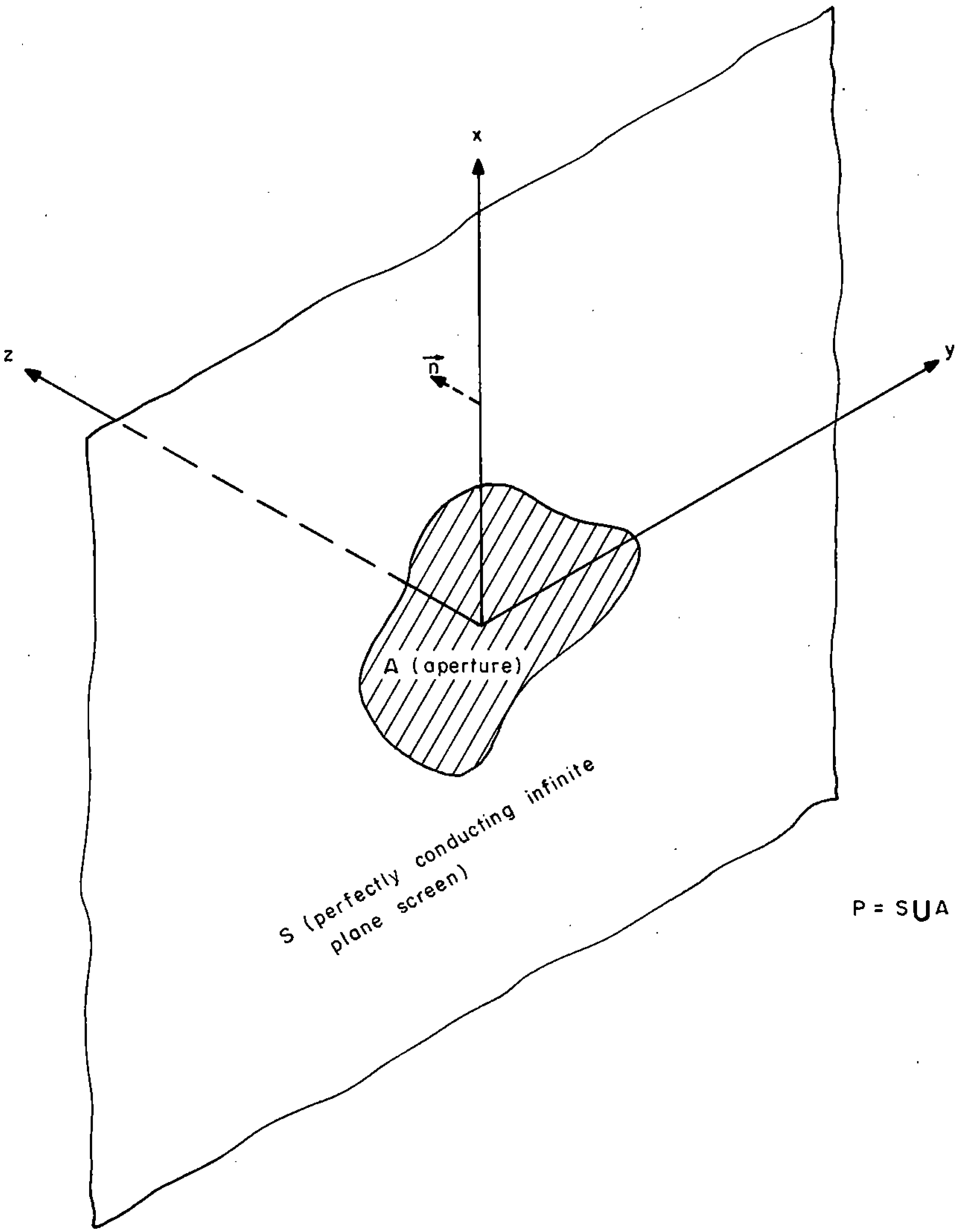

Figure 3. Perfectly Conducting Infinite Sheet with an Aperture

$z < 0$ direction. We denote the fields on the $z > 0$ and $z < 0$ sides of the screens by subscripts + and - respectively. We denote the incident field by the subscript inc, scattered by an infinite perfectly conducting screen by c, scattered by a screen by sc and the aperture or perturbation fields by a.

Using the results obtained in an earlier note[9], we have the scattered field to be symmetric with respect to $P = S \cup A$. This conclusion implies that $\tilde{\vec{E}}_c$, $\tilde{\vec{E}}_a$, $\tilde{\vec{H}}_c$ and $\tilde{\vec{H}}_a$ are symmetric. Hence

$$\tilde{\vec{E}}_{sy_-} = \tilde{\vec{E}}_{inc_{sy}} + \tilde{\vec{E}}_c + \tilde{\vec{E}}_a \tag{4.2.1}$$

$$\tilde{\vec{H}}_{sy_-} = \tilde{\vec{H}}_{inc_{sy}} + \tilde{\vec{H}}_c + \tilde{\vec{H}}_a \tag{4.2.2}$$

$$\tilde{\vec{E}}_{as_-} = \tilde{\vec{E}}_{inc_{as}} = -\tilde{\vec{E}}_{as_+} \tag{4.2.3}$$

$$\tilde{\vec{H}}_{as_-} = \tilde{\vec{H}}_{inc_{as}} = \tilde{\vec{H}}_{as_+} \tag{4.2.4}$$

$$\tilde{\vec{E}}_{sy_+} = \tilde{\vec{E}}_a \tag{4.2.5}$$

$$\tilde{\vec{H}}_{sy_+} = \tilde{\vec{H}}_a \tag{4.2.6}$$

$$\tilde{\vec{E}}_{sc_{as_\pm}} = \vec{0} = \tilde{\vec{H}}_{sc_{as_\pm}} \tag{4.2.7}$$

Clearly, these equations imply that the antisymmetric part of the field is unaffected by the aperture and the screen. The aperture surface current density $\tilde{\vec{J}}_s$ is given by

$$\tilde{\vec{J}}_s \equiv \tilde{\vec{\vec{Y}}}_s \cdot \tilde{\vec{E}}_{a_\pm} = \tilde{\vec{J}}_{s_{s.c.}} \pm 2\vec{n} \times \tilde{\vec{H}}_{a_\pm} \tag{4.2.8}$$

where

$$\tilde{\vec{J}}_{s_{s.c.}} \equiv -2\vec{n} \times \tilde{\vec{H}}_{inc_{sy}} \tag{4.2.9}$$

For the special case of $\vec{\vec{Y}}_s = \vec{\vec{0}}$, we have

$$\tilde{\vec{J}}_{s_{s.c.}} \pm 2\vec{n} \times \tilde{\vec{H}}_{a_\pm} = \vec{0} \tag{4.2.10}$$

which implies that the presence of the screen with an unloaded aperture does not modify the tangential component of the magnetic field in the aperture.

4.3 Complementary Fields, Currents, Charges and the Generalized Babinet's Principle

The transformation of fields, currents and charges between electric and magnetic quantities forms the basis for Babinet's principle. Corresponding to the original fields, currents and charges, etc., given by

$$\tilde{\vec{E}},\ \tilde{\vec{H}},\ \tilde{\vec{J}},\ \tilde{\vec{J}}_m,\ \tilde{\rho},\ \tilde{\rho}_m,\ \tilde{\vec{E}}_q,\ \tilde{\vec{J}}_q,\ \tilde{\rho}_q$$

we define the transformed or complementary quantities as

$$\tilde{\vec{E}}',\ \tilde{\vec{H}}',\ \tilde{\vec{J}}',\ \tilde{\vec{J}}'_m,\ \tilde{\rho}',\ \tilde{\rho}'_m,\ \tilde{\vec{E}}'_q,\ \tilde{\vec{J}}'_q,\ \tilde{\rho}'_q$$

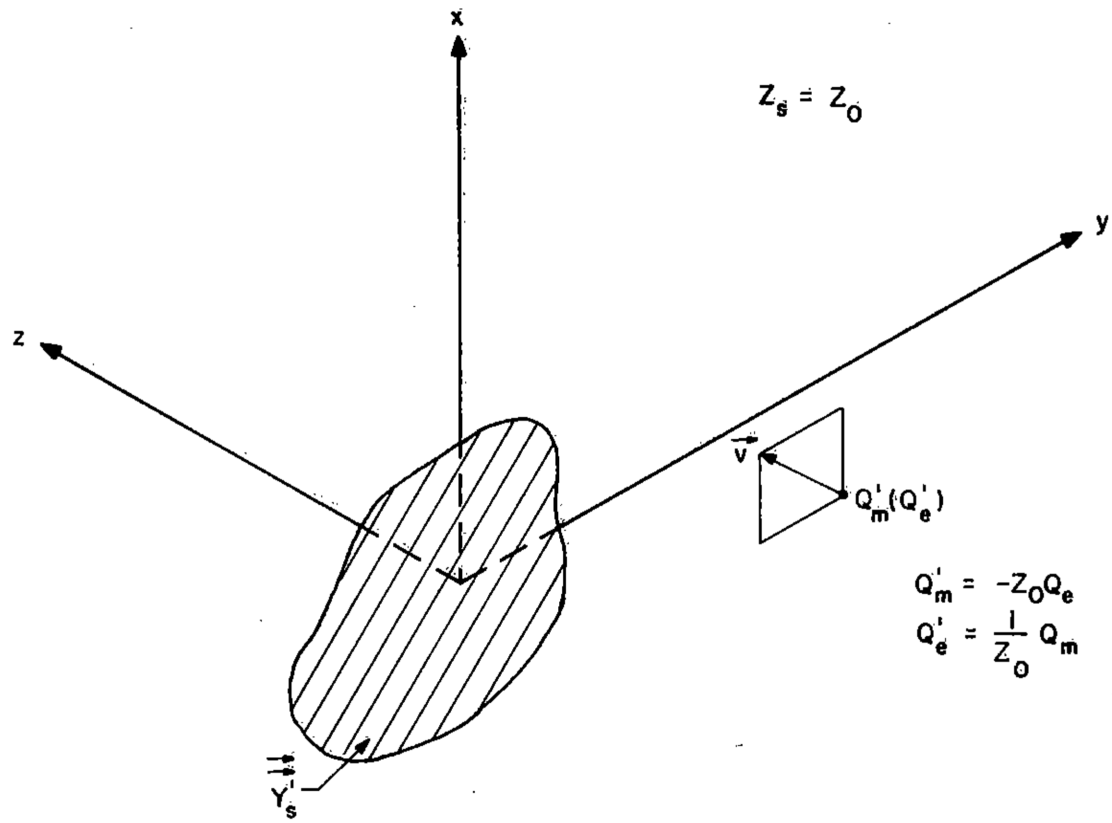

Figure 5. Complementary Disk with Complementary Source

unaffected by the $z = 0$ scatterer, only symmetric parts of the fields have to be considered. With this in mind, the problem for the symmetric parts of the field is as shown in figure 6, where the symmetric field is constructed by using symmetry decomposition of the fields and sources. The complementary problem for this case is as shown in figure 7. The complementary screen is obtained by replacing the perfectly conducting plane screen with an aperture by its complement given by a disk. Complementary sources are obtained by using equations 4.3.2 and 4.3.3. If the aperture in figure 6 is rectangular or circular, the complementary disk will also be rectangular or circular.

Let us now consider a thin slot in a perfectly conducting infinite plane. A symmetry decomposed charge $Q_e/2$ $(Q_m/2)$ is assumed to travel by the slot, parallel to the perfectly conducting plane with a velocity v as shown in figure 8. The complementary problem with symmetry decomposition is as shown in figure 9. For simplicity, we will only consider the case of the unloaded aperture whose complement is a perfectly conducting strip. If the strip is of width w and length ℓ, in the quasi-static approximation[11], we can consider the complementary problem to be a thin wire of radius $w/4$ and length ℓ as shown in figure 10. It is clear that this approximation simplifies the analysis considerably. Similar simplifications may be made in other aperture problems[12].

We have only considered the case where the electric or magnetic charge is traveling by the aperture and the charge has existed for all time. However, in the SGEMP problems the charge is emitted at some time $t = t_o$ in some direction to the scatterer. Let us assume that the charge is emitted toward the $z>0$ direction. Using the symmetry decomposition, the sources that produce symmetric fields are as shown in figure 11. The complementary problem for this is as shown in figure 12. It is interesting to note that the complementary problem requires the ejection of null total magnetic charge from free space.

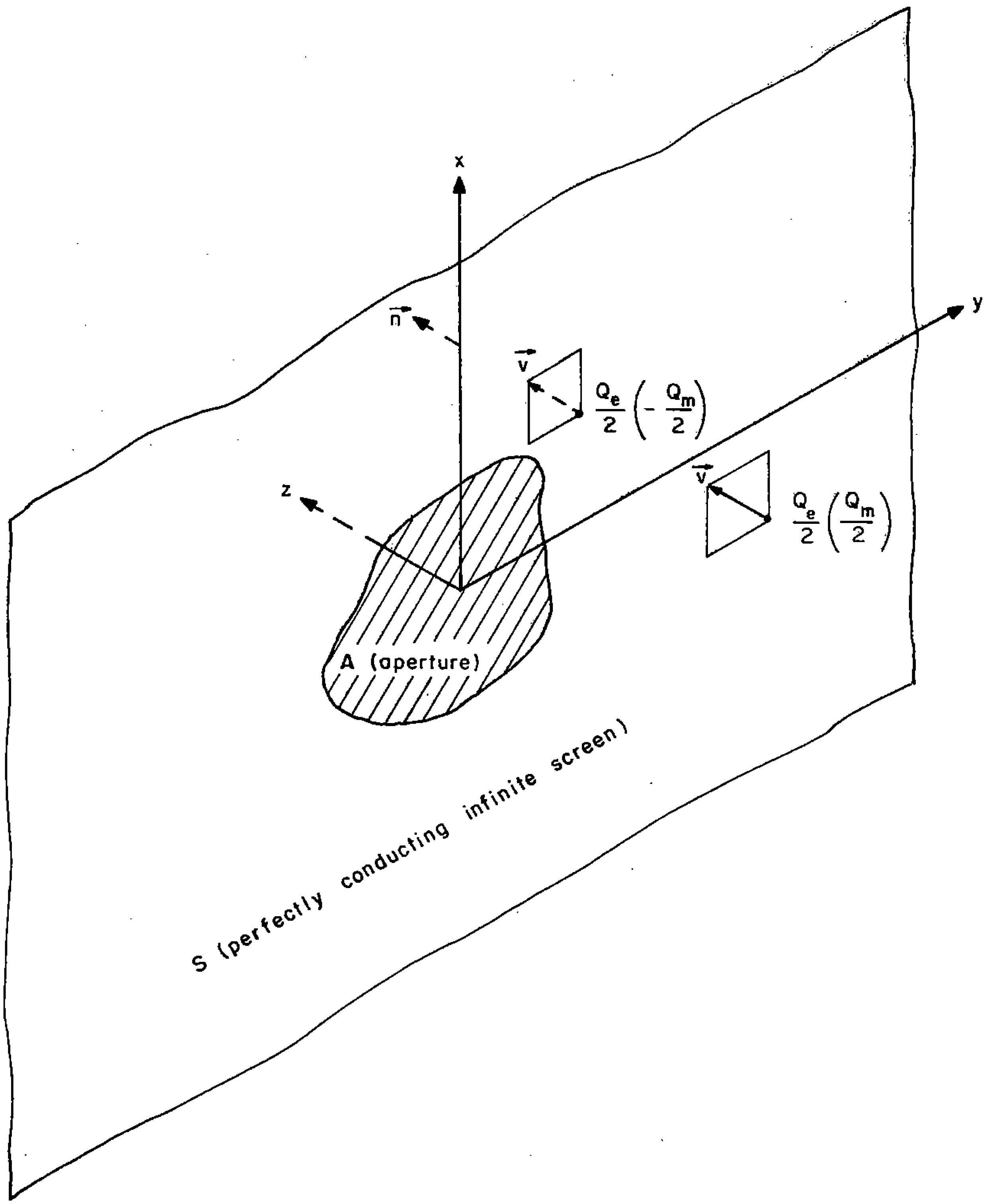

Figure 6. Symmetric Sources for a Plane Screen with an Aperture

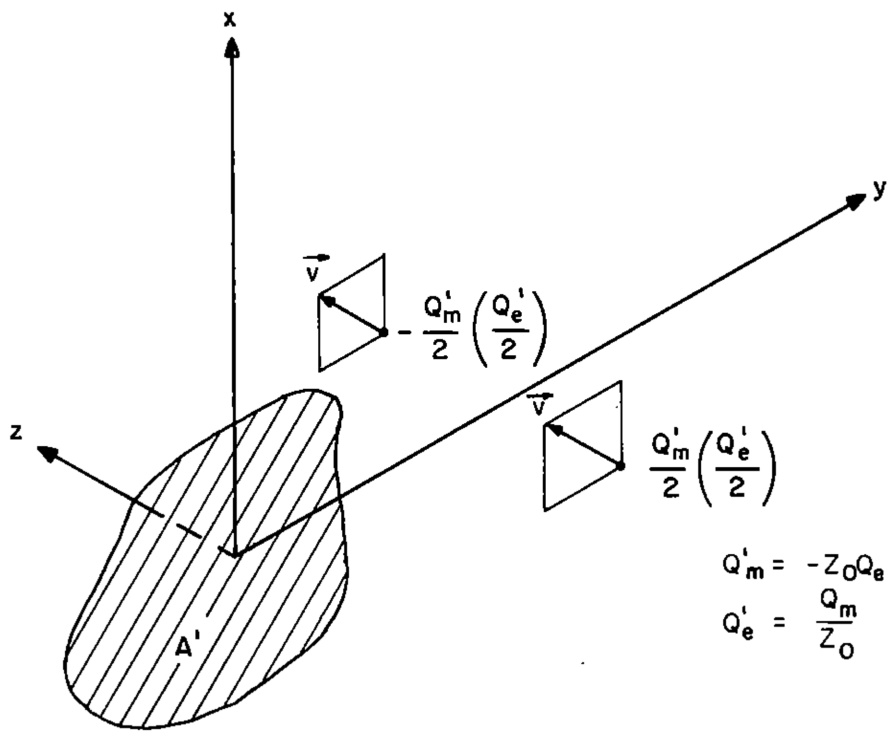

Figure 7. Symmetric Sources for the Complementary Problem

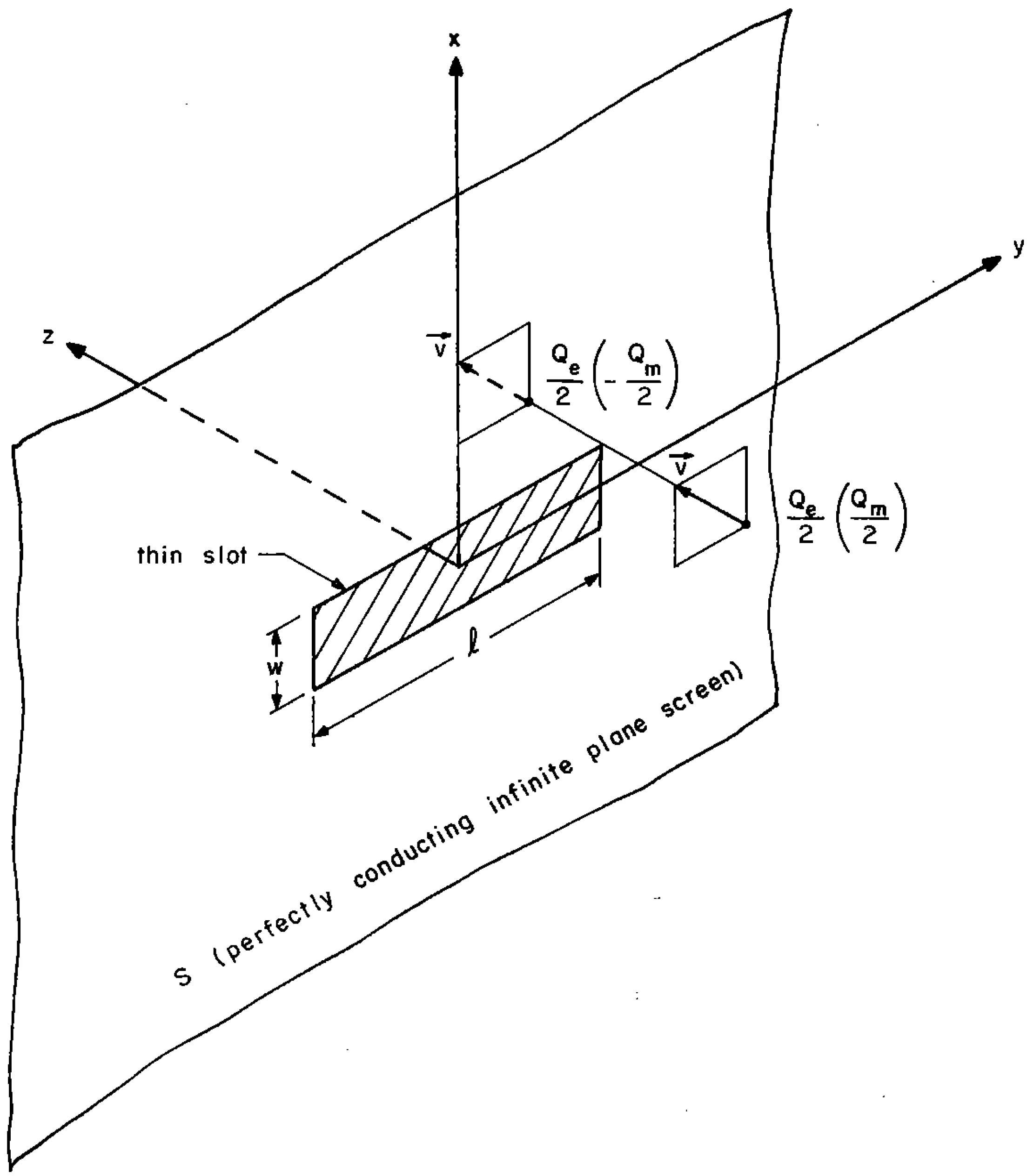

Figure 8. Symmetric Sources for a Thin Slot in a Perfectly Conducting Screen

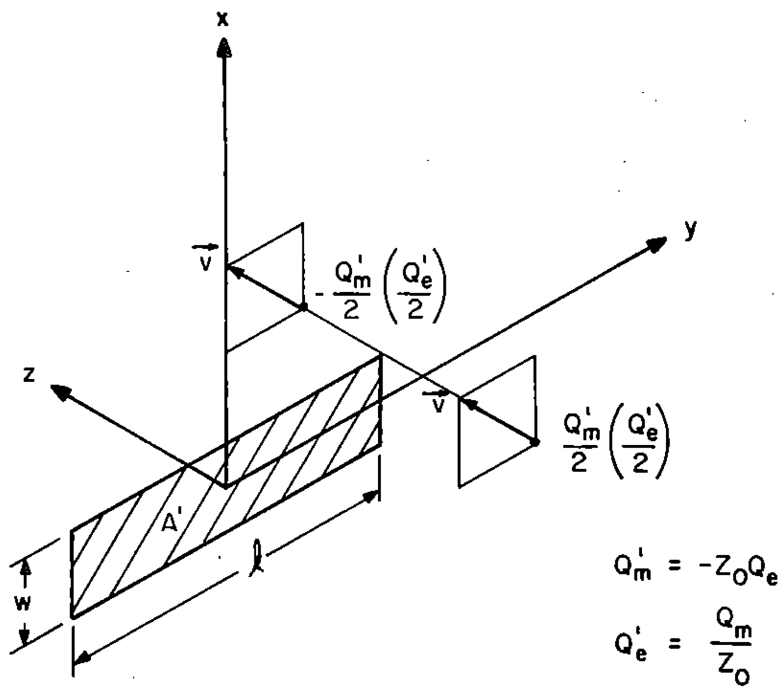

Figure 9. Symmetric Sources for the Complementary Problem

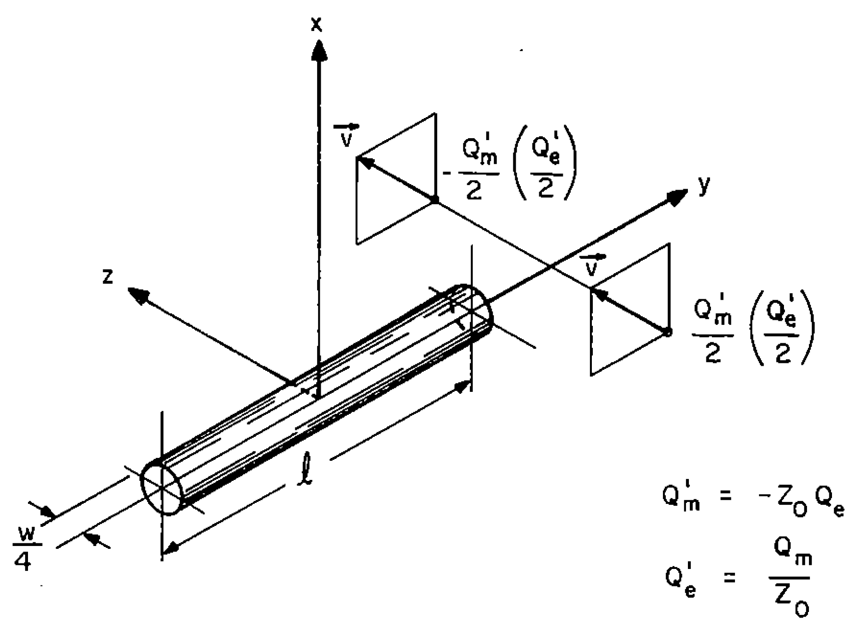

Figure 10. Quasistatic Approximation ($v \ll c$) for the Complementary Problem with Symmetric Sources

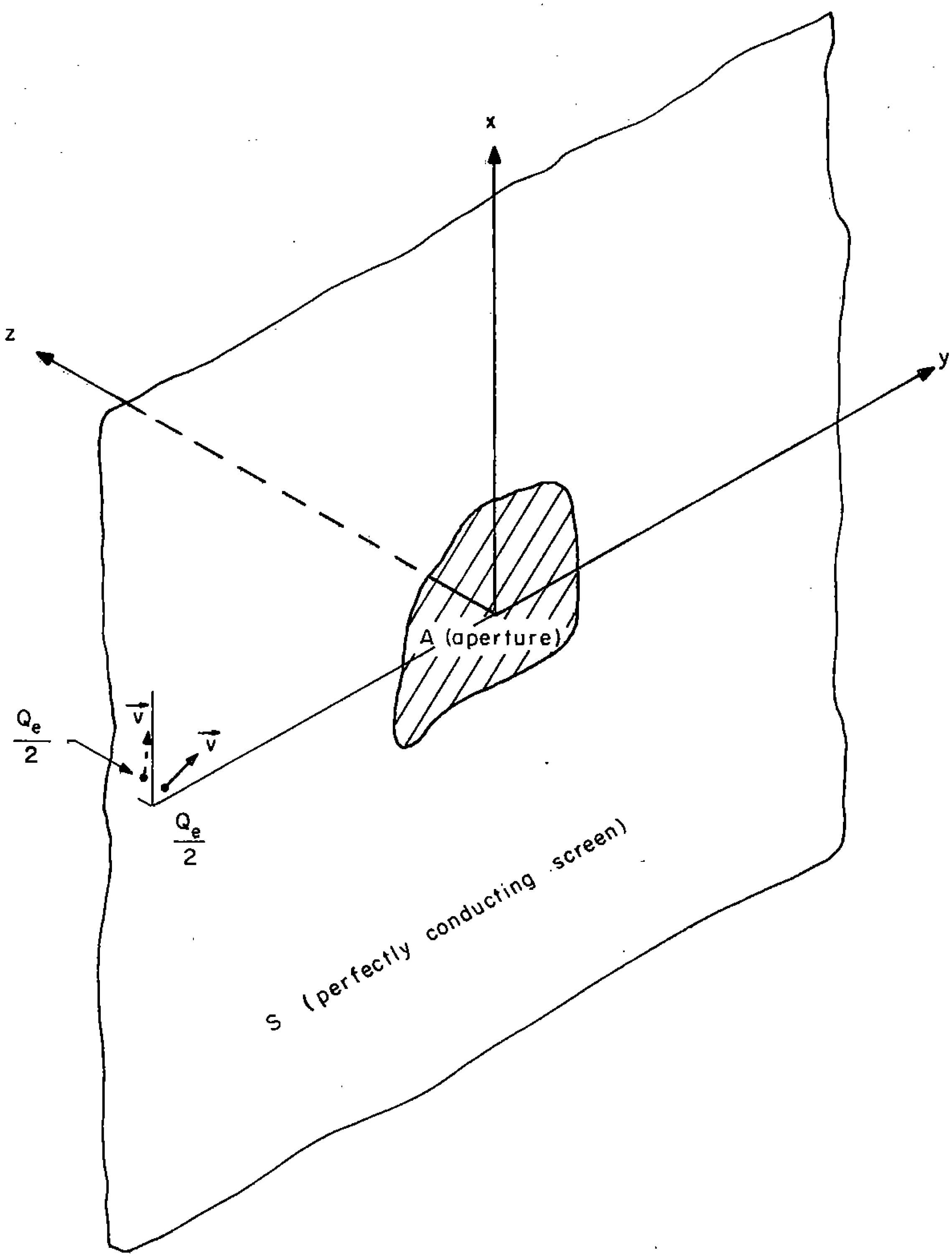

Figure 11. Symmetric Sources of a Charge Emitted from the Screen

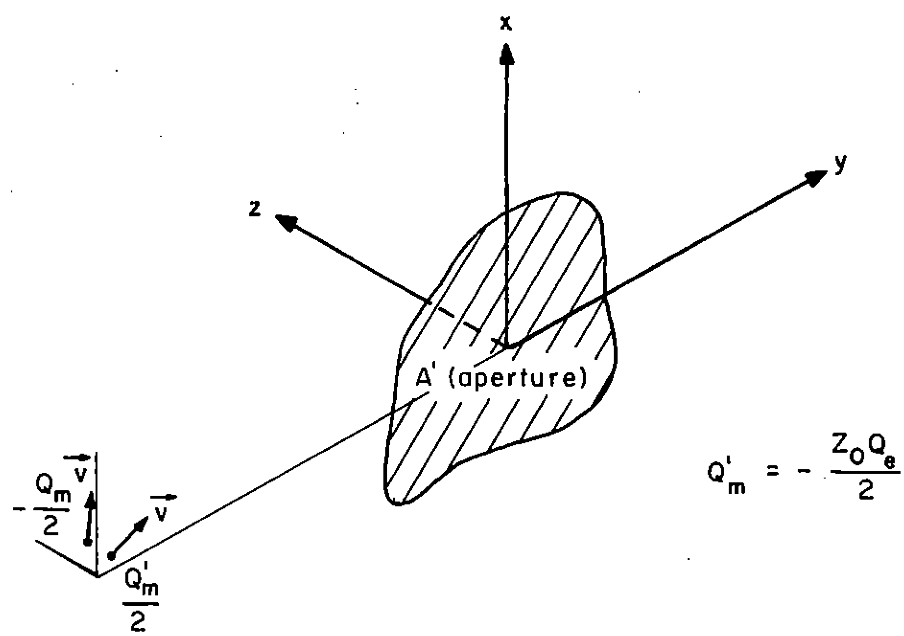

Figure 12. Symmetric Sources for the Complementary Problem

6. Conclusions

In this report we have derived equations for the field due to moving electric and magnetic charges. These are shown in both time domain and frequency domain. Quasistatic representation of these fields is shown for $v \ll c$ and $s \to 0$. These equations can be simply extended for an arbitrary direction of motion of the charge by simple rotation of the coordinate system.

Symmetry decomposition of fields and sources with respect to a magnetic plane is discussed. The scattered fields due to an aperture are shown to be symmetric with respect to the plane of the aperture. Using the symmetry decomposition of the sources and the Babinet's principle, it is shown that the problem of SGEMP coupling through apertures can be simplified. In this formulation of the complementary problem charge conservation is automatically satisfied. For the case of SGEMP coupling through a slot, in the quasistatic approximation, the electric currents induced on a wire of length ℓ and radius a are the same as the magnetic currents in a slot of length ℓ and width $4a$. Numerical results for this problem will be presented in a later report.

7. References

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