Theoretical Notes
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Transient Reflected Signals from an Anisotropic Plasma

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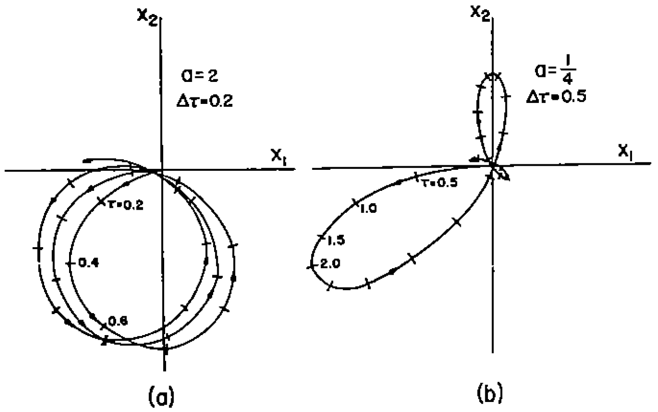

Fig. 1. Locus of the tip of the electric field vector as a function of time. $a = \Omega/2\Pi$, $\Pi t = \tau$. (a) Strong magnetic field. (b) Weak magnetic field.

Transient Reflected Signals from an Anisotropic Plasma

Transient reflected signals from plasmas have recently been used as a diagnostic tool. In particular, reflected nanosecond pulses have been used to measure the plasma frequency and the cyclotron frequency in an ionized gas confined in a coaxial waveguide with a static magnetic field present.[1-3] In these experiments, polarization effects of the electromagnetic pulses were not of interest.

For transient reflected signals from a polarization sensitive medium there is, in addition to characteristic oscillations at the natural frequencies, a marked effect on the instantaneous polarization of the pulse.

The case of a very short pulse reflected from a semi-infinite anisotropic plasma is considered here. The incident pulse is linearly polarized along the x_1 axis and propagation is in the direction of the magnetic field which is along the x_3 axis. The reflected pulse then travels in the negative x_3 direction. The following transformation is used.

$$\mathcal{E}^{\pm} = \frac{1}{2}\left[\mathcal{E}_1 \pm i\mathcal{E}_2\right] \tag{1}$$

In the plasma the characteristic modes $\mathcal{E}^{\pm}$ have the relative dielectric constants

$$K^{\pm} = 1 + \frac{\Pi^2}{s(s \mp i\Omega)} \tag{2}$$

where s is the complex angular frequency of the electric field, Π is the electron plasma frequency, and Ω is the electron cyclotron frequency both in radians per second. The reflection coefficient for the field components

$$R^{\pm} = \frac{1-\sqrt{K^{\pm}}}{1+\sqrt{K^{\pm}}} = \frac{\sqrt{\left(s \mp i\frac{\Omega}{2}\right)^2 + \frac{\Omega^2}{4}} - \sqrt{\left(s \mp i\frac{\Omega}{2}\right)^2 + \frac{\Omega^2}{4} + \Pi^2}}{\sqrt{\left(s \mp i\frac{\Omega}{2}\right)^2 + \frac{\Omega^2}{4}} + \sqrt{\left(s \mp i\frac{\Omega}{2}\right)^2 + \frac{\Omega^2}{4} + \Pi^2}}. \tag{3}$$

The inverse Laplace transform of the reflection coefficient gives the reflected response for a delta function of incident field. Then the reflection of any signal can be synthesized using time convolution. Using the following Laplace transform identities[4]

$$\mathcal{L}^{-1}[F(s-a)] = e^{at}\mathcal{L}^{-1}[F(s)]$$

$$\mathcal{L}^{-1}[F(\sqrt{s^2+\beta^2})] = f(t) - \beta\int_0^t f(\sqrt{t^2-u^2})J_1(\beta u)du$$

$$\mathcal{L}^{-1}\left[\frac{s-\sqrt{s^2+\Pi^2}}{s+\sqrt{s^2+\Pi^2}}\right] = \mathcal{L}^{-1}\left[\frac{-\Pi^2}{(s+\sqrt{s^2+\Pi^2})^2}\right] = -\frac{2J_2(\Pi t)}{t} \tag{4}$$

one obtains

$$\mathcal{L}^{-1}[R^{\pm}] = \Pi f(\tau)\exp\{\pm ia\tau\} \tag{5}$$

where

$$(\tau) = \left[-\frac{2J_2(\tau)}{\tau} + 2a\int^{\tau}\frac{J_2(\sqrt{\tau^2-x^2})J_1(ax)dx}{\sqrt{\tau^2-x^2}}\right] \tag{6}$$

and

$$\tau = \Pi t, \qquad x = \Pi u, \qquad a = \frac{\Omega}{2\Pi}.$$

For a narrow incident pulse of unit amplitude and of length T which is short compared to the periods of the plasma and cyclotron frequencies, the reflected components are

$$\mathcal{E}_R^{\pm} = \Pi T f(\tau)\exp\{\pm ia\tau\}. \tag{7}$$

Then, using the transformation in (1),

$$E_1 = \frac{\mathcal{E}_{1R}}{2\Pi T} = f(\tau)\cos(a\tau)$$

$$E_2 = \frac{\mathcal{E}_{2R}}{2\Pi T} = f(\tau)\sin(a\tau) \tag{8}$$

where $\mathcal{E}_{1R}$ and $\mathcal{E}_{2R}$ are, respectively, the components of the reflected signal along the x_1 and x_2 axes. Equation (6) has been evaluated by numerical integration for values of the parameter a equal to 1/4, 1/2, $1/\sqrt{8}$, and 2. Using these results together with (8), graphs of E_2 vs. E_1 have been obtained and are illustrated in Figs. 1 and 2. The cross marks denote equal intervals of normalized time τ. The curves display the transient polarization behavior of the reflected electric field. Note that the instantaneous polarization curves seem rather sensitive to the parameter a, which is one-half the ratio of the cyclotron frequency to the plasma frequency.

There are two limiting cases where approximate expressions can be obtained for $f(\tau)$ in (6). One case is for a weak magnetic field ($a \ll 1$) and on a time scale long compared to a period of free plasma oscillation ($\tau \gg 1$). The second case is for a strong magnetic field ($a \gg 1$) and on a time scale short compared to the period of free plasma oscillation ($\tau \ll 1$).

For the case of a weak magnetic field, $a \ll 1$, and $\tau \gg 1$

$$f(\tau) \approx -\frac{2^{3/2}}{\sqrt{\pi}\,\tau^{3/2}}\cos\left(\tau - \frac{5\pi}{4}\right)$$

and

$$E_1 \approx -\sqrt{\frac{2}{\pi}}\frac{1}{\tau^{3/2}}\left\{\cos\left(\left[\Pi + \frac{\Omega}{2}\right]t - \frac{5\pi}{4}\right) + \cos\left(\left[\Pi - \frac{\Omega}{2}\right]t - \frac{5\pi}{4}\right)\right\}$$

$$E_2 \approx -\sqrt{\frac{2}{\pi}}\frac{1}{\tau^{3/2}}\left\{\sin\left(\left[\Pi + \frac{\Omega}{2}\right]t - \frac{5\pi}{4}\right)\right.$$

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[1] H. J. Schmitt, "Plasma diagnostics with short electromagnetic pulses," *IEEE Trans. on Nuclear Science*, vol. NS-11, pp. 125–136, January 1964.

[2] ——, "Dispersion of pulsed electromagnetic waves in a plasma," *IEEE Trans. on Microwave Theory and Techniques (Correspondence)*, vol. MTT-13, pp. 472–473, July 1965.

[3] ——, "Pulse dispersion in a gyrotropic plasma," *IEEE Trans. on Antennas and Propagation*, vol. AP-13, pp. 934–942, November 1965.

[4] A. Erdelyi, W. Magnus, F. Oberhettinger, and F. Tricomi, *Tables of Integral Transforms*, vol. I. New York: McGraw-Hill, 1954.

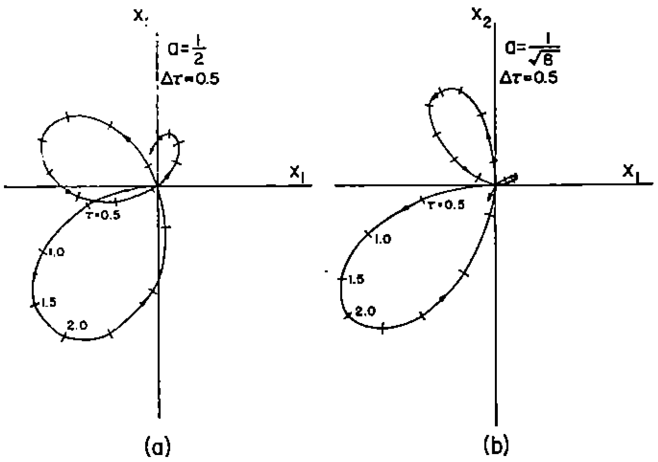

Fig. 2. Locus of the tip of the electric field vector as a function of time. $a=\Omega/2\Pi$, $\Pi t=\tau$. (a) $\Omega=\Pi$. (b) $\Omega=\Pi/\sqrt{2}$.

$$-\sin\left(\left[\Pi-\frac{\Omega}{2}\right]t-\frac{5\pi}{4}\right)\bigg\}. \tag{9}$$

Thus, there is a component rotating counterclockwise about the x_3 axis at an angular frequency $(\Pi+\Omega/2)$ and a clockwise rotating component at an angular frequency $(\Pi-\Omega/2)$. The CCW component arises from frequency components reflected from the right circular wave cutoff at approximately $(\Pi+\Omega/2)$, while the CW component results from reflected components at the left circular wave cutoff at approximately $(\Pi-\Omega/2)$.

For a strong magnetic field $a\gg 1$, and for $\tau\ll 1$

$$f(\tau)\approx -a\sin(a\tau).$$

Then,

$$E_1\approx -\frac{\Pi}{4\Omega}\sin\Omega t$$

$$E_2\approx -\frac{\Pi}{4\Omega}(1-\cos\Omega t). \tag{10}$$

There is thus a counterclockwise rotating component at the angular frequency Ω. This arises from frequency components reflected at the right circular wave cutoff at approximately Ω. The constant term in E_2 is a quasi-dc component on a time scale $t\ll 1/\Pi$. On an expanded time scale this quasi-dc component appears as a clockwise rotating component at the left circular wave cutoff of approximately Π^2/Ω.

The two limiting cases discussed above enable one to analyze the transient polarization as the superposition of right and left circular components of the electric field. When the plasma and cyclotron frequencies are comparable, one must resort to numerical evaluation of (6) along with (8). When this is done, Figs. 1 and 2 indicate that the shape of the polarization curves is very dependent on the parameter a. The time for each lobe of the polarization curve to be traced gives information about the plasma frequency, since the time scale is normalized to the angular plasma frequency. Thus the possibility exists that transient reflected polarized signals could be used as a diagnostic tool in magnetoplasmas.

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13. ABSTRACT

The case of a very short em pulse reflected from a semi-infinite anisotropic plasma is considered. Assuming a linearly polarized incident wave the polarization characteristics of the reflected wave are examined. The analysis indicates the possibility that transient reflected polarized signals can be used as a diagnostic tool in magnetoplasmas.

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