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A Monte Carlo Treatment of Secondary Electrons In High Altitude EMP Calculations*

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ABSTRACT

A Monte Carlo treatment of the secondary electron currents suitable for use in benchmark calculations with high altitude line-of-sight approximation EMP computer codes has been developed. The model presented here includes the effects of Lorentz forces, Coulomb scattering from ions, molecular scattering, and avalanching, and (if desired) could also include recombination. While the model is limited by the lack of good experimental data on differential cross-sections for electron-molecule interactions, it represents a significant improvement over existing models because of its accurate treatment of the Lorentz forces which are the source of strong dispersive effects at higher altitudes where the gyration period for electrons in the geomagnetic field is short compared to the mean collision time.

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INTRODUCTION

Since the pioneering work of Karzas and Latter[1], many major improvements have been made in the modeling of the generation of the electromagnetic pulse (EMP) from a high altitude nuclear burst. The modeling of the secondary electron currents used in present day EMP computer codes, however, has lagged behind that used for other parts of the problem. In this report, I will present a brief critique of some of these models and describe a Monte Carlo treatment which better represents the behavior of the secondary electrons in the high altitude environment.

EMP SOURCE MODELS

The early time EMP is due to the interaction with the atmosphere of the pulse of gamma rays emitted by the nuclear explosion. For photons having energies greater than about 35 keV, Compton scattering is the predominant mechanism for energy loss in air.[2] A large fraction of the photon energy can be transferred to the recoil electron, which is usually called a Compton electron.

The Compton electrons recoil in directions clustered about the original direction of the gamma ray, executing a generally helical trajectory in the geomagnetic field, all the while dissipating kinetic energy through electromagnetic interactions with the medium according to the well-known range-energy relation, and undergoing multiple small-angle Rutherford scattering

from the nuclei of air atoms. The electromagnetic interactions with the air molecules result in the production of large numbers of electron-ion pairs, the newly freed electrons having initial kinetic energies of order 50 eV.

It is at this point that the physical processes become so complicated that major simplifications are made for computational models. The subsequent behavior of these low energy "secondary" electrons, of course, determines the "conduction current" which must be used together with the current attributable to the high energy Compton electrons as the source in Maxwell's equations for the EMP.

As one example of our profound ignorance of the detailed processes involved, there is no experimental information about (let alone any adequate theoretical model for) the distribution in direction of the newly ejected secondary electrons. In the absence of any such information, this distribution is explicitly assumed to be isotropic in the model I will describe below. It should be noted, however, that if the actual distribution were strongly peaked in the forward direction of motion of the Compton electron, then at altitudes above about 60 or 70 km the secondary electron current would reinforce the Compton electron current for up to 100 or 200 nanoseconds after ejection of the secondary electrons (i.e., during the first quarter of an orbit in the geomagnetic field), not at all the effect the secondary electrons have in present day EMP computer codes.

In the simplest treatment of the secondary electorns which is actually used in some EMP codes, the mean drift velocity of the secondary electrons is given by an empirical function of the ratio of the electric field strength to the density (or, equivalently, pressure) of the atmosphere.[3] This is basically the Drude model of electrical conductivity[4], and it can be represented by the two equations of motion

$$\frac{dn_e}{dt} = C(r,t) - L(r,t) \qquad (1\text{-a})$$

$$\vec{v}_D = f(|\vec{E}|/P)\,\vec{E} \qquad (1\text{-b})$$

where n_e is the secondary electron number density, $C(r,t)$ is the creation rate for secondary electrons, $L(r,t)$ is the loss rate for secondary electrons, $\vec{v}_D$ is the mean drift velocity, $\vec{E}$ is the total electric field vector, P is the ambient atmospheric pressure, and $f(|\vec{E}|/P)$ is an empirically determined mobility function.

Underlying (1-b), however, are several tacit assumptions which severely restrict the legitimate applicability of this model to atmospheric EMP calculations.

The model begins by visualizing the electrons to be freely accelerated by the electric field to a speed large compared to the initial random speeds of the electrons until a collision with an air atom or molecule occurs, at which point the random velocity of the electrons is reduced to its initial small value, and the whole process begins anew. If the mean time between such collisions is very short compared to other time scales relevant to the problem, then the mean drift velocity of the electrons equilibrates to a terminal velocity proportional to the ratio of the accelerating force (i.e., the electric field) to the density of collision targets times a suitably averaged collision cross-section.

This model is clearly inadequate in several regions of interest in high altitude EMP calculations. As mentioned above, the secondary electrons

are born with a kinetic energy of some 50 eV, an energy which cannot be neglected unless the electrons can very rapidly dissipate most of it. Only two energy loss mechanisms are significant for such electrons in the very weakly ionized gas many kilometers away from the burst point. (1) Inelastic collisions with N_2, O_2, and other polyatomic molecules can transfer a few tenths of an electron volt per collision into molecular vibrational or rotational states.[5] (2) If the kinetic energy of the electron is greater than the ionization energy for the air atoms or molecules, then some fraction of inelastic collisions with air molecules will dislodge a valence electron ("cascading" or "avalanching"), removing a sizable fraction of the kinetic energy, and producing an additional secondary electron. (For air, the ionization energies range from 12.5 eV for O_2 to 15.68 eV for A, with 15.51 eV for N_2, 13.55 eV for O, and 12.56 eV for H_2O.[6]) Thus several hundred collisions are required to effectively thermalize the secondary electrons in the absence of accelerating electric fields. Since the mean time between energy transfer collisions for electrons in the range of 5 to 50 eV exceeds 10^{-11} seconds at altitudes above 20 km, thermalization cannot occur at these altitudes in less than a few nanoseconds, a time scale in which the electric fields and the secondary electron number density can change significantly. (Indeed, above 100 km, the thermalization time can approach milliseconds.)

Besides the above objection to the Drude model, we note that at altitudes above 70 to 80 km the mean time between collisions for electrons of about 50 eV exceeds the gyration period of the electron in the geomagnetic field -- about 0.7 microseconds. The Drude model does not include the component of

the mean drift velocity which is transverse to the electric field in the presence of a magentic field: in general, if the collision time is longer than a small fraction of a gyration period, the secondary electron current will not be parallel to the electric field.[7]

Higgins, Longmire, and O'Dell[8] have proposed a swarm theory as an improvement over the Drude model. In this model, the secondary electron distribution is characterized by three parameters: the number density n_e, the drift velocity $\vec{v}_D$, and the characteristic energy $U = kT_e$, where T_e is the effective temperature of the electrons. The equations of motion for the swarm are then written as

$$\frac{dn_e}{dt} = C(r,t) - L(r,t) \tag{2-a}$$

$$\frac{d(n_e \vec{v}_D)}{dt} = n_e \frac{e}{m} \vec{E} - n_e \nu_m \vec{v}_D \tag{2-b}$$

$$\frac{d(n_e U)}{dt} = \frac{2}{3} n_e |e| \vec{E} \cdot \vec{v}_D - n_e \nu_w (U - U_o) + S(r,t) \tag{2-c}$$

where $C(r,t)$ and $L(r,t)$ are the creation and loss rates, respectively, as before; ν_m is the momentum transfer collision frequency, e is the electron charge, ν_w is the energy transfer collision frequency, U_o is the characteristic energy for electrons in thermal equilibrium with the medium, and $S(r,t)$ is the rate at which newly created secondary electrons add energy to the swarm. (The 2/3 factor converts kinetic energy $\frac{3}{2} kT_e$ to characteristic energy kT_e.)

While this is somewhat more refined than the Drude model and probably extends upward the altitude range of legitimate applicability by its inclusion of the finite acceleration time for electrons in an electric field in (2-b), it is still subject to the same criticism discussed above, viz., at higher altitudes, the swarm model is inadequate because it, too, ignores the effects of the geomagnetic field. A further criticism is that the experimentally determined collision frequencies ν_m and ν_w in (2-b) and (2-c) are, in effect, based on microscopic cross-sections averaged over Maxwellian distributions of "hot" electrons interacting with a Maxwellian distribution of "cold" air molecules.[9] But as we shall see in the discussion below, the secondary electron distribution in the high-altitude EMP environment will not be Maxwellian, and since the microscopic cross-sections can vary rapidly with the electron kinetic energy, this approximation cannot be justified a priori.

THE MONTE CARLO APPROACH

Given the deficiencies of existing models, how can we make a better model? Since the time span of interest is at most a few microseconds, no electron (either Compton or secondary) can travel a distance greater than a few hundred meters in that time, a distance which is small compared to the scale height of the atmosphere or to distances over which the gamma ray or x-ray flux from the nuclear burst changes significantly. Thus, as far as secondary electron processes are concerned, we may ignore all spatial variations and discuss, at each calculational point along a line of sight from the burst, the infinite homogeneous medium problem, so that only the velocity components of the secondary electrons are relevant coordinates.

Consider, therefore, the distribution of secondary electrons in velocity space at any given point of the atmosphere as a function of time after the

arrival of the leading edge of the gamma ray or x-ray pulse from the burst ("retarded time"). Just for a concrete example, if the magnetic field is in the z direction, and the electric field is (instantaneously) in the x direction, then the Lorentz forces on a set of electrons A, B, C, etc., (see Figure 1) produce (in the absence of collision processes) a rigid motion in velocity space. This rigid motion consists of a translation parallel to the v_x axis (the acceleration by the electric field) and a rotation about the v_z axis at the gyration frequency $\omega = \frac{|e|}{m} |\vec{B}|$ (the deflection by the magnetic field $\vec{B}$).

The object is to compute the net current as a function of time for this distribution of electrons. Assuming at first for simplicity that there are no collisions, let us consider the scenario underlying typical line-of-sight EMP codes[3,10] and imagine successive clumps of electrons injected at successive time steps of the calculation at a given spatial point. (Ignore for the moment the details of the shape of these clumps, i.e., the distribution of the secondary electrons at creation. It suffices to note that these clumps will initially lie at the origin of velocity space.) If we track the rigid motion of each clump under the influence of the Lorentz force, after a number of time steps the electron distribution in velocity space will resemble that shown in Figure 2. Clumps injected at different times will, of course, contain different numbers of electrons, depending on the time dependence of the Compton electron density and thus on the time dependence of the gamma ray output of the nuclear device. The exact shape of the gerrymander-like distribution shown in Figure 2 will depend on the entire previous history of the electric field.

Figure 1. (a) The effect of Lorentz forces on a distribution of electrons in velocity space during a time step Δt. The solid lines connect the particles A, B, C, D, and E at the beginning of the time step; the dotted lines connect corresponding particles at the end of the time-step. (b) The effect of the electric field alone is a rigid translation of the figure. (c) The effect of the magnetic field alone is a rigid rotation.

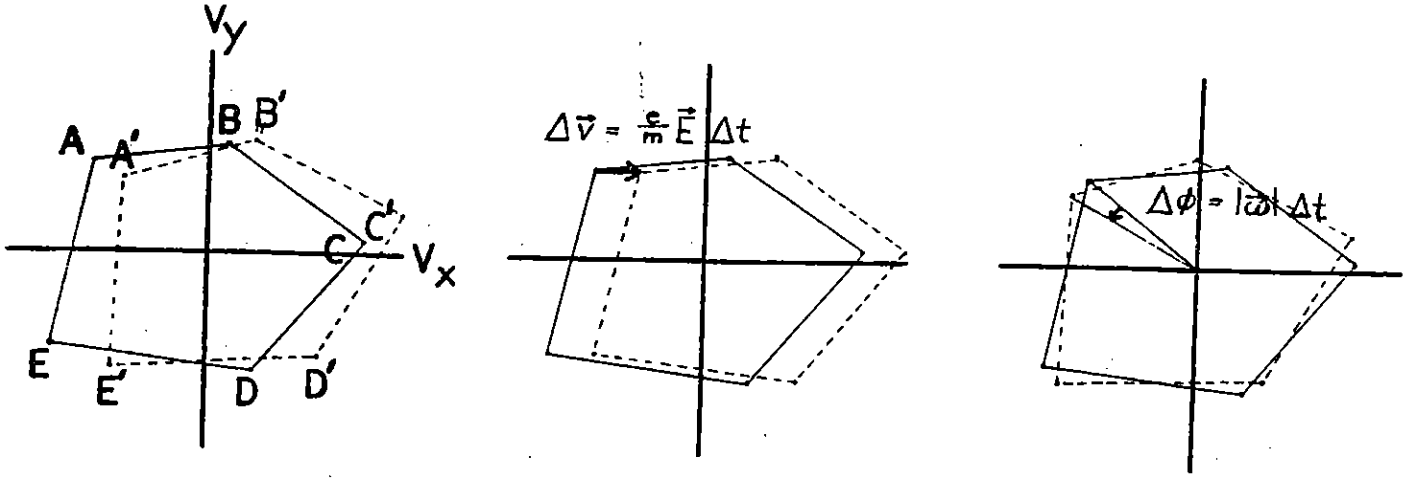
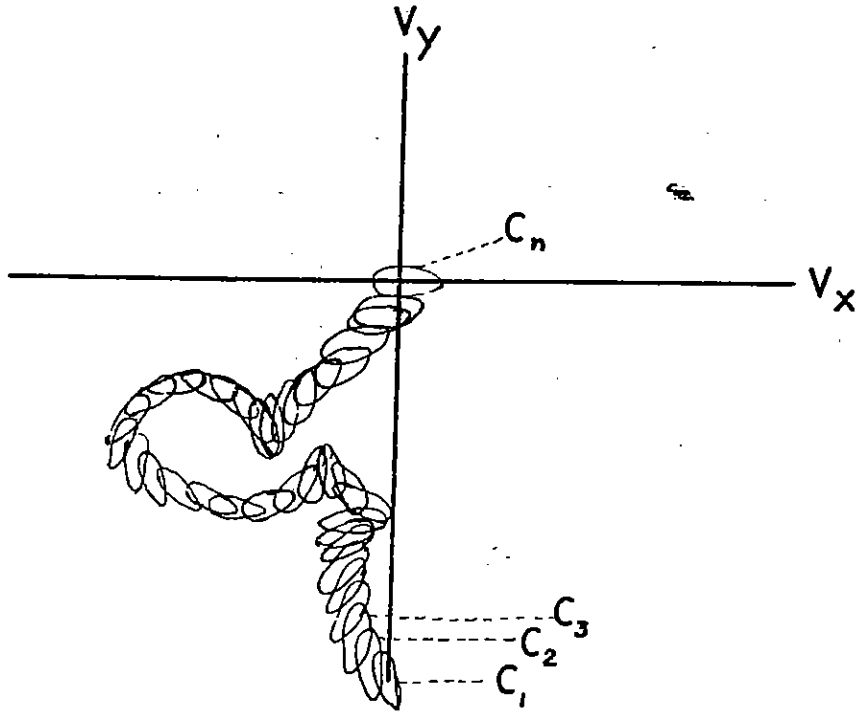


Figure 2. The electron distribution function at the n-th time step in the absence of collisions. The clump labeled C_1 was injected during the first time step and has moved from the origin to the position shown under the influence of a time-varying electric field and a constant magnetic field. The clump labeled C_n has just been injected.



If we know the number of secondary electrons in each clump, and if we know the exact shape of the gerrymander, we can determine the centroid of the distribution (which is just the mean drift velocity) and the net current due to the secondary electrons. But since the history of the electric field depends on the net secondary electron current itself (and thus on the time dependence of the gamma ray output and on the previous history of the electric field), the history of the net secondary electron current is describable (in principle) by a non-linear integro-differential equation, an equation which is difficult enough to write down explicitly (even in the collisionless case) and virtually impossible to solve by analytic or finite difference techniques except for certain very restrictedspecial cases.

If collisional processes are included, of course, the situation becomes even more hopeless. Scattering processes smear out the gerrymander; recombination and attachment reactions reduce the density according to the velocity-dependent cross-sections; avalanching relocates high energy particles to the vicinity of the origin, while creating new particles at low energy.

In the face of all these effects, one is inexorably forced to consider, even for the collisionless case, a Monte Carlo scheme as the basis for an improved, more credible model for computing the secondary electron current in the high altitude EMP environment, and the rest of this report will present in some detail such a Monte Carlo scheme. One cannot, of course, track every single physical electron in the problem. The essence of the Monte Carlo approach is to instead track some modest number of sample electrons, assigning to each a weight representing some number of physical electrons

which have approximately the same physical parameters as the sample electron. The sample electrons must then be chosen in such a way that the distribution of the sample electrons will closely approximate the true distribution of the physical electrons. (In particular, the zero-th and first moments, i.e., the number density and the current density of the sample electrons, should be very close to the true secondary electron density and current density.)

SECONDARY ELECTRON INJECTION

When a relativistic electron passes through air, it loses kinetic energy to the medium via electrical polarization of the atoms and molecules, resulting in an ionization trail along its trajectory. At first (i.e., within an atomic time scale of about 10^{-18} second after passage of the fast electron), the resultant density of electron-ion pairs corresponds to one pair for each 86 eV of kinetic energy lost by the relativistic electron. On the average, about 50 eV of this energy appears as kinetic energy of the ejected secondary electron. In the absence of external fields, however, that 50 eV will be redistributed as the secondary electron collides with other atoms and molecules, resulting eventually in an electron-ion pair density corresponding to one pair for each 34 eV of kinetic energy lost by the relativistic electron. This subsequent build-up of ionization occurs on a time scale which depends on the density and composition of the medium and on the original kinetic energy of the secondary electron. At higher altitudes, this time scale can be several shakes or more.

This delayed ionization has been modeled in some EMP codes by injecting, in effect, one electron-ion pair for each 34 eV of energy lost by the Compton

electrons, but spreading out the injection according to a universal function whose time dependence grows with diminishing air density. This "ionization lag" model does not include the effects of any acceleration by the electric field of the secondary electrons, so that, if the lag time were more than a few nanoseconds (as it is for the higher altitudes), the ionization build-up from the original 50 eV secondary electrons may be seriously in error.

To avoid this pitfall, we shall simply inject one secondary electron with a mean energy of 50 eV for every 86 eV lost by Compton electrons during a given time-step, and then calculate the cascading produced by each secondary electron during each subsequent time step. This will correctly account for the ionization lag if the time steps taken by the EMP code are reasonably short compared to the mean time to an ionizing collision, a condition which will usually be satisfied for altitudes above about 40 kilometers.

If during the time step from time t^n to time $t^{n+1} = t^n + \Delta t$, the Compton electrons lose a kinetic energy corresponding to the creation of N_s secondary electrons with a mean energy of 50 eV, then we shall generate some number N_{new} of new sample electrons by sampling $N_{new}/2$ speeds from an inverted cumulative distribution function (in tabular form) of speed versus total number. For each of these speeds, we add two sample particles to the census, each having weight N_s/N_{new}, moving in opposite, randomly chosen, directions. One more random number is selected to determine just when during the time step this pair of electrons is injected. (The assumption that the N_s electrons are injected uniformly over the time step introduces negligible errors if $d(\log N)/dt \ll 1/\Delta t$, where N is the total number of secondary electrons injected up to time t^n.) The above algorithm guarantees that the distribution function for newly injected electrons is precisely isotropic. It must be

remembered, however, that this assumption -- made in our ignorance of what really happens -- will result in underestimating the EMP electric fields which would be generated if secondary electrons were in fact ejected preferentially forward, in the direction of motion of the Compton electrons.

The secondary electron cumulative distribution function referred to above is stored as an array of 100 speeds such that I-percent of all newly created secondary electrons have speeds less than or equal to the I-th speed in the table. To select M speeds, we pick a random number Z between zero and 100/M, and interpolate in the table to find the speed below which Z-percent of all secondary electrons occur. We then successively increment Z by (100/M), repeating the above procedure until we have selected M speeds, uniformly spaced in number of particles across the cumulative distribution function ("stratified sampling").

A few more words are in order concerning this secondary electron distribution function. Far from the burst point, the Compton electrons will -- on the average -- have higher energies than the Compton electrons closer to the burst point, because the lower energy gamma rays have a shorter mean free path in air. Consequently, the secondary electrons may have higher energies at creation at points farther out along the line of sight than do the secondary electrons closer in to the burst point. Further investigations should be made to determine how significant this effect might be. If it were a significant effect, the cumulative distribution function table could be recomputed for each new point along the line of sight before beginning the integration with respect to time at that point.

The ionosphere, of course, has a low level of background ionization prior to disturbance by a nuclear explosion. The ambient free electrons have a kinetic temperature of order 1000 to 2000 °K, which corresponds to kinetic energies of a small fraction of an electron volt. Typically, their number density is swamped by ionization due to Compton electrons at rather early times in the EMP problem. But it is quite simple to include a modest number of sample electrons representing these ambient ionospheric electrons from the beginning of the calculation at each new spacial point along a line-of-sight. Some number of speeds are sampled randomly from a Maxwellian distribution[11] corresponding to the electron temperature at that altitude, and a pair of sample electrons with opposite directions (randomly oriented) are generated for each of these speeds. The total of the particle weights (all equal) corresponds to the ambient electron density at that altitude. In a typical EMP problem, these sample electrons will be "pruned" from the census (as described in a later section) fairly early in the problem, but they will have contributed to the "conduction currents" at very early times, as they should.

EQUATIONS OF MOTION FOR THE SECONDARY ELECTRONS

A secondary electron in the high altitude EMP environment is subjected to three types of influences: (1) the Lorentz forces due to the geomagnetic field and the EMP fields; (2) distant Coulomb scattering from ions and other electrons; (3) collisions with air molecules. It can be shown that, because of the much smaller mass and the much higher temperature of the secondary electrons, the Coulomb scattering from other secondary electrons and from Compton electrons may be neglected in comparison with the Coulomb scattering from ions.[12] It is convenient to treat the effects of Coulomb scattering

and of molecular collisions in two parts. First we will write the equation of motion for the electrons as if these two processes changed only the magnitude (but not the direction) of a secondary electron. We will then superimpose on the resultant position in velocity space of the secondary electron after one time step a small rotation to account for the scattering in direction due to these collisional processes.

Neglecting the scattering in direction, we may then write the equation of motion for the electron velocity in rationalized MKS units as

$$\frac{d\vec{v}}{dt} = -\frac{|e|}{m}\vec{E} + \vec{\omega} \times \vec{v} - C(v)\,\vec{v} - D(v)\,\vec{v} \quad . \tag{3}$$

The first two terms on the right side of (3) are the acceleration of the electron by the electric field and the turning of the electron by the magnetic field, where $\vec{\omega} = \frac{|e|}{m}\vec{B}$. The third term is the effective drag force due to Coulomb scattering from the ions; the Fokker-Planck theory[11] gives for the drag coefficient

$$C(v) = N_{ion}\frac{4\pi e^4}{m^2}\left(1 + \frac{m}{M_{ion}}\right)\ln\Lambda\left\{\frac{1}{v^3}\Phi\left(v\sqrt{\frac{M_{ion}}{2kT_{ion}}}\right) - \frac{1}{v^2}\sqrt{\frac{2\,M_{ion}}{\pi kT_{ion}}}\,e^{-v^2\sqrt{M_{ion}/(2kT_{ion})}}\right\} \tag{4}$$

where N_{ion} is the density of ions (assumed equal to the density of free electrons), M_{ion} is the mean ion mass, T_{ion} is the kinetic temperature of the ions (assumed equal to the kinetic temperature of the undisturbed air molecules), $\ln\Lambda$ (the "Coulomb logarithm") is given by

$$\ln \Lambda = \ln \frac{3}{\sqrt{8\pi e^3}} + \frac{3}{2} \ln(KT_{ion}) - \frac{1}{2} \ln N_{ion} \qquad (5)$$

and $\Phi(x)$ is the error function. (The underlying assumption made in deriving (4) is that the ions have a Maxwellian distribution. Since the Compton electrons can transfer only a small momentum to the air molecules during an ionization event, the recoil speeds of the ions are very small, so that the velocity distribution of the ions will differ insignificantly from that of the air molecules.)

The last term in (3) represents the effective drag due to inelastic collisions with polyatomic molecules such as N_2, O_2, H_2O, etc., in which part of the electron kinetic energy is transferred into various molecular excitation energies (rotational and vibrational states). Since detailed information on these processes is lacking, we shall adopt a phenomenological model for estimating the drag coefficient D(v). If we assume that, on the average, an electron loses an energy $\frac{1}{2} m q^2$ in each such collision, then it would take $N_E = v^2/q^2$ collisions to completely deplete its kinetic energy, given an initial speed v. During a time step Δt, however, an electron with speed v would be expected to undergo $N_C = N_{mol}\, v\sigma_q(v)\, \Delta t$ collisions, where $\sigma_q(v)$ is the cross-section for molecular excitations by an electron with speed v. Thus an electron with velocity $\vec{v}$ would, on the average, suffer a velocity change (neglecting for the moment the change in direction) given by

$$\Delta\vec{v} = \frac{1}{2} \frac{\Delta E}{E} \vec{v} = \frac{1}{2} \frac{N_C}{N_E} \vec{v} = \frac{1}{2} \frac{N_{mol}\sigma_q(v)\, q^2}{v} \vec{v}\, \Delta t$$

so that the effective drag coefficient D is

$$D(v) = \frac{1}{2} \frac{N_{mol} \sigma_q(v)\, q^2}{v} \qquad (6)$$

This expression, of course, is valid only for $v \geq q$, since an electron cannot excite vibrational or rotational states if its kinetic energy is less than the spacing between these molecular states.

We now adopt as our difference equations in lieu of the differential equation (3)

$$\frac{\vec{v}^{n+1} - \vec{v}^{n}}{\Delta t^{n+\frac{1}{2}}} = - \frac{|e|}{m} \vec{E} + \frac{|e|}{m} \vec{B} \times \frac{(\vec{v}^{n+1} + \vec{v}^{n})}{2} - G(\tilde{v}^{n+\frac{1}{2}}) \frac{(\vec{v}^{n+1} + \vec{v}^{n})}{2} \qquad (7)$$

where $\vec{v}^{n+1}$ is the velocity computed at time t^{n+1} neglecting changes in direction due to scattering; $G \equiv C + D$; $\Delta t^{n+\frac{1}{2}} \equiv t^{n+1} - t^{n}$; and $\tilde{v}^{n+\frac{1}{2}}$ is our best estimate of $\frac{1}{2}(|\vec{v}^{n+1}| + |\vec{v}^{n}|)$. The electric and magnetic fields E and B are assumed to be evaluated at $t^{n+\frac{1}{2}}$. For the case of constant magnetic field and no electric field or collisions, it can be shown that (7) is accurate to all orders in $\Delta t^{n+\frac{1}{2}}$; for slowly varying electric fields and for well-behaved functions G(v), it can be shown that (7) is accurate to second-order terms in $\Delta t^{n+\frac{1}{2}}$.

Introducing the abbreviations

$$\alpha_i \equiv \frac{1}{2} \omega_i \Delta t = \frac{1}{2} \frac{|e|}{m} B_i \Delta t$$

$$\beta \equiv \frac{|e|}{m} \Delta t \qquad (8)$$

$$\gamma \equiv - \frac{1}{2} G(\tilde{v}^{n+\frac{1}{2}}) \Delta t$$

the difference equations (7) may be expanded and solved for the components of $\vec{v}^{n+1}$, giving the results

$$v_x^{n+1}\cdot\Delta \doteq v_x^n\left\{(1+\gamma)[\alpha_x^2 + (1-\gamma)^2] - (1-\gamma)(\alpha_y^2 + \alpha_z^2)\right\}$$

$$+ 2v_y^n[\alpha_x\alpha_y + \alpha_z(1-\gamma)] + 2v_z^n[\alpha_x\alpha_z - \alpha_y(1-\gamma)]$$

$$+ E_x\beta[\alpha_x^2 + (1-\gamma)^2] + E_y\beta[\alpha_x\alpha_y + \alpha_z(1-\gamma)] + E_z\beta[\alpha_x\alpha_z - \alpha_y(1-\gamma)]$$

$$v_y^{n+1}\cdot\Delta = v_y^n\left\{(1+\gamma)[\alpha_y^2 + (1-\gamma)^2] - (1-\gamma)(\alpha_x^2 + \alpha_z^2)\right\} \qquad (9)$$

$$+ 2v_x^n[\alpha_x\alpha_y - \alpha_z(1-\gamma)] + 2v_z^n[\alpha_y\alpha_z + \alpha_x(1-\gamma)]$$

$$+ E_x\beta[\alpha_x\alpha_y - \alpha_z(1-\gamma)] + E_x\beta[\alpha_y^2 + (1-\gamma)^2] + E_z\beta[\alpha_y\alpha_z + \alpha_x(1-\gamma)]$$

$$v_z^{n+1}\cdot\Delta = v_z^n\left\{(1+\gamma)[\alpha_z^2 + (1-\gamma)^2] - (1-\gamma)(\alpha_x^2 + \alpha_y^2)\right\}$$

$$+ 2v_x^n[\alpha_x\alpha_z + \alpha_y(1-\gamma)] + 2v_y^n[\alpha_y\alpha_z - \alpha_x(1-\gamma)]$$

$$+ E_x\beta[\alpha_x\alpha_z + \alpha_y(1-\gamma)] + E_y\beta[\alpha_y\alpha_z - \alpha_x(1-\gamma)] + E_z\beta[\alpha_z^2 + (1-\gamma)^2]$$

where the determinant Δ is just

$$\Delta \equiv (1-\gamma)[(1-\gamma)^2 + \alpha_x^2 + \alpha_y^2 + \alpha_z^2] \qquad . \qquad (10)$$

In principle, we could use an iterative scheme, using v^n as a first guess for $v^{n+\frac{1}{2}}$; compute a first approximation to v^{n+1} from (9); use that value

to compute a new $\underset{\sim}{v}^{n+\frac{1}{2}} = \frac{1}{2}(\underset{\sim}{v}^{n+1} + \underset{\sim}{v}^{n})$; and repeat the whole process until the difference between successive approximations to $\underset{\sim}{v}^{n+1}$ become arbitrarily small. In practice, the first and second approximations to $\underset{\sim}{v}^{n+1}$ rarely differ by more than one percent of the difference between $\underset{\sim}{v}^{n}$ and the first approximation to $\underset{\sim}{v}^{n+1}$, so little is to be gained by iterating.

We now must adjust the newly computed velocity $\underset{\sim}{v}^{n+1}$ to account for the scattering in direction due to Coulomb collisions with ions and due to collisions with molecules. We assume that these two processes are independent, so that each process can be treated by a separate rotation of the vector $\underset{\sim}{v}^{n+1}$ to represent the changes in direction. The phenomenological model we adopt is as follows: the net effect of a series of scatterings is assumed to be a randomization of the direction of motion into a cone of directions centered about the original direction of motion, the half-angle of the cone being a monotonically increasing function of the number of scattering collisions experienced.

For Coulomb scattering, the Fokker-Planck theory provides an estimate of the "deflection time", i.e., the time required for the half-angle of the cone of directions to open out to 180°:

$$\frac{1}{\tau_D} = N_{ion} \frac{4\pi e^4}{m^2} \ln \Lambda \left\{ \left[\frac{M_{ion}}{kT_{ion} v^3} - \frac{1}{v^5} \right] \Phi\left(v\sqrt{\frac{M_{ion}}{2kT_{ion}}} \right) + \frac{1}{v^4} \sqrt{\frac{2M_{ion}}{\pi kT_{ion}}}\, e^{-v^2 \frac{M_{ion}}{2kT_{ion}}} \right\} \tag{11}$$

where ln Λ was defined in (5), and $\phi(x)$ is the error function. During a time-step Δt, the physical electrons represented by a sample electron with speed v will not, in general, have scattered out to 180°, but only out to the fraction $(\Delta t/\tau_D)$ of 180°. Thus we shall randomly scatter our sample electron into a cone of directions with half-angle

$$\theta_{Coul}^{max} = \text{lesser of } [\pi \ , \ \pi \frac{\Delta t}{\tau_D}] \tag{12-a}$$

about the original direction. The angle of Coulomb scatter θ_{Coul} is chosen by selecting a random number x between 0 and 1 and computing

$$\theta_{Coul} = \cos^{-1}[1 - x(1 - \cos \theta_{Coul}^{max})] \tag{12-b}$$

The azimuth ϕ_{Coul} is computed by selecting another random number y in the range 0 to 1 and computing

$$\phi_{Coul} = 2\pi \cdot y \tag{12-c}$$

For molecular scattering, there is no such tidy treatment available. However, if we consider the quantum mechanical theory of partial wave scattering in the Born approximation[13], we can form a semi-quantitative model. Electrons with kinetic energy E will have significant partial wave contributions up to order $\ell \sim E/E_{char}$, where E_{char} is a characteristic atomic or molecular energy scale, and typically (except at resonant incident energies) the highest order partial wave will dominate the differential scattering cross-section. For the ℓ-th partial wave, the bulk of the scattering will lie

within a maximum scattering angle of order

$$\theta_\ell \sim \frac{\pi}{3\sqrt{\ell}} \quad . \tag{13-a}$$

For $E < E_{char}$, we assume that only $\ell = 0$ contributes, so

$$\theta_{\ell=0} = \pi \quad . \tag{13-b}$$

To completely randomize the direction of motion of an electron with speed v by such molecular collisions then requires π/θ_ℓ such collisions, on the average. In time Δt, however, such an electron can be expected to have undergone only $N_{mol}\, v\sigma_{sc}(v)\, \Delta t$ collisions, where $\sigma_{sc}(v)$ is the total scattering cross-section for electrons with speed v. Thus the maximum scattering angle allowed for our sample electron is

$$\theta_{mol}^{max} = \text{lesser of } [\pi \ , \ N_{mol}\theta_\ell v\sigma_{sc}(v)\, \Delta t] \tag{13-c}$$

We then select the angle of molecular scattering θ_{mol} by picking a random number z between 0 and 1 and computing

$$\theta_{mol} = \cos^{-1}[1 - z(1 - \cos\theta_{mol}^{max})] \quad . \tag{13-d}$$

The azimuth ϕ_{mol} is then computed by selecting yet another random number w in the range 0 to 1 and computing

$$\phi_{mol} = 2\pi \cdot w \quad . \tag{13-e}$$

Finally, we perform the consecutive rotations specified by the angles $(\theta_{Coul}, \phi_{Coul})$ and $(\theta_{mol}, \phi_{mol})$ on the vector $\underset{\sim}{v}^{n+1}$ computed in (9) to obtain v^{n+1}:

$$v_x^{n+1} = A_1 v_{\sim x}^{n+1} + A_2 \frac{|\vec{v}_{\sim}^{n+1}|}{v_{\sim r}^{n+1}} v_{\sim y}^{n+1} + A_3 \frac{v_{\sim x}^{n+1}}{v_{\sim r}^{n+1}} v_{\sim z}^{n+1}$$

$$v_y^{n+1} = A_1 v_{\sim y}^{n+1} - A_2 \frac{|\vec{v}_{\sim}^{n+1}|}{v_{\sim r}^{n+1}} v_{\sim x}^{n+1} + A_3 \frac{v_{\sim y}^{n+1}}{v_{\sim r}^{n+1}} v_{\sim z}^{n+1} \qquad (14\text{-a})$$

$$v_z^{n+1} = A_1 v_{\sim z}^{n+1} - A_3 v_{\sim r}^{n+1}$$

where

$$A_1 \equiv \cos\theta_{Coul} \cos\theta_{mol} - \sin\theta_{Coul} \sin\theta_{mol} \cos\phi_{mol}$$

$$A_2 \equiv -\sin\theta_{Coul} \cos\theta_{mol} \sin\phi_{Coul} - \cos\theta_{Coul} \sin\theta_{mol} \sin\phi_{Coul} \cos\phi_{mol}$$

$$- \sin\theta_{mol} \cos\phi_{Coul} \sin\phi_{mol} \qquad (14\text{-b})$$

$$A_3 \equiv \sin\theta_{Coul} \cos\theta_{mol} \cos\phi_{Coul} + \cos\theta_{Coul} \sin\theta_{mol} \cos\phi_{Coul} \cos\phi_{mol}$$

$$- \sin\theta_{mol} \sin\phi_{Coul} \sin\phi_{mol}$$

$$v_{\sim r}^{n+1} \equiv \left[(v_{\sim x}^{n+1})^2 + (v_{\sim y}^{n+1})^2 \right]^{\frac{1}{2}}$$

$$|\vec{v}_{\sim}^{n+1}| \equiv \left[(v_{\sim r}^{n+1})^2 + (v_{\sim z}^{n+1})^2 \right]^{\frac{1}{2}}$$

The model presented above for treating molecular scattering is admittedly crude; in the absence of experimentally determined differential scattering cross-sections for electron energies up to a few hundred electron volts, it would be difficult to do any better. But since the secondary electron currents at higher altitudes are dominated by Lorentz force effects,

the details of the scattering model used in that altitude regime should not make a significant difference.

Avalanching, however, would still be expected to remain important even if scattering does not: since there is a threshold energy for the avalanching process, avalanching can distort the gerrymander-like distribution function more effectively than elastic collisions or non-ionizing inelastic collisions can. This process can be treated by reducing the weight of each sample particle in proportion to the fraction of the physical electrons represented by each sample electron with energy above threshold which would have suffered an ionizing collision during the time step Δt,

$$\Delta w_i = -N_{mol} \cdot w_i v_i \sigma_{aval}(v_i) \cdot \Delta t \qquad (15\text{-a})$$

where the subscript indexes sample particles, and $\sigma_{aval}(v)$ is the total cross-section for ionizing collisions. Then new sample electrons must be generated at lower energies to represent N_{aval}, the number of physical electrons created,

$$N_{aval} = N_{mol} \cdot \sum_i 2 w_i v_i \sigma_{aval}(v_i) \cdot \Delta t \qquad (15\text{-b})$$

(Note that the original, ionizing electron, now demoted in energy, is treated as a "new" electron. Properly, we should have a factor somewhat larger than 2 in the above equation to account for multiple ionization, but this is a small correction which is partially compensated by attachment collisions which create negative ions.) The total energy to be distributed among the newly created avalanche electrons is then

$$E'_{aval} = N_{mol} \cdot \sum_i \left(\tfrac{1}{2} m v_i^2 - E_{ion}\right) w_i v_i \sigma_{aval}(v_i) \cdot \Delta t \qquad (15\text{-}c)$$

where E_{ion} is the average ionization energy for air molecules, about 14.9 eV. In the absence of any detailed experimental information, we may again assume that these electrons are emitted isotropically. For practicality, provided only that the time step Δt is sufficiently small, we may defer injecting these particles until the next time step, and then use a procedure similar to that outlined in the previous section for the secondary electrons produced by the Compton electrons.

Recombination of electrons with ions can be treated in the same fashion, by computing the change in weight for each sample electron during a time step Δt. However, the lifetime for recombination at higher altitudes rapidly grows so long compared to the time scale of the entire EMP calculation that this process is neglected in this model.

PRUNING THE CENSUS

In a Monte Carlo method such as described above, one obviously wishes at each time step in the problem to represent the distribution function for the secondary electrons as accurately as possible, i.e., to keep track of as many sample electrons as possible. But since we need to inject new particles into the problem continuously, and since the lifetime of the physical electrons represented is very long compared to the time step (or, indeed, the length of the problem), the census list must be "pruned" to make room in the computer's memory for the new particles. This process must be done in such a way that the overall shape of the distribution function is not distorted, i.e., that the moments of the distribution

function are unchanged. (Strictly speaking, if the Monte Carlo method is to replicate the actual physical processes faithfully, not only the moments of the distribution function, but also the integrals of the distribution function multiplied by each relevant cross-section, would have to be accurately preserved, but this is impracticable.) The method I have adopted is one in which the zero-th and first moments of the pruned distribution function are exactly equal to those moments of the un-pruned distribution function, that is, the secondary electron number density and current density are faithfully preserved.

Having tracked all the sample particles through one time step, the zero-th, first, and second moments of the electron distribution with respect to velocity are computed:

$$
\begin{aligned}
{}^{0}M &\equiv \sum_i w_i = N_e \\
{}^{1}M_\alpha &\equiv \sum_i w_i v_\alpha^i = \frac{1}{e} j_\alpha = N_e v_{D\alpha} \\
{}^{2}M_{\alpha\beta} &\equiv \sum_i w_i v_\alpha^i v_\beta^i
\end{aligned}
\tag{16}
$$

where $\alpha, \beta = x, y, z$; and j_α is the secondary electron current density. The quotient of the first moment divided by the zero-th moment gives the mean drift velocity of the secondary electrons, and the distance in velocity space of each sample particle from the mean drift velocity is then computed. The particles are then sorted in order of increasing distance from this velocity centroid, and divided into ten groups of particles, each group having nearly

the same total "importance", where the "importance" of each sample particle is arbitrarily chosen to be

$$I_i \equiv w_i |\vec{v}_i - v_D| \tag{17}$$

These groups thus form ten concentric shells of particles about the velocity centroid $\vec{v}_D$, and particles to be eliminated from the census are now selected with probability inversely proportional to the "importance" of each particle. (Subdivision of the population of sample particles into ten groups serves to striate the population, helping to ensure that the eliminated particles will be spread out in velocity space. The "importance" function defined above makes particles with large weights and in distant regions of velocity space least likely to be eliminated.)

We now wish to assign adjusted weights to the surviving sample particles in a manner which preserves the zero-th and first moments, that is, we require

$$\begin{aligned} {}^0\tilde{M} &\equiv \sum_S \tilde{w}_i = {}^0M \\ {}^1\tilde{M}_\alpha &\equiv \sum_S \tilde{w}_i v^i_\alpha \equiv {}^1M_\alpha \end{aligned} \tag{18}$$

where $\sum_S$ denotes summation over the surviving sample particles only. A prescription for the new weights $\tilde{w}_i$ can be obtained by the variational principle that $\sum_S \tilde{w}_i^2$ shall be extremal subject to the constraints (18). The solution to this variational problem is

$$\tilde{w}_i = A + B\, v_x^i + C\, v_y^i + D\, v_z^i \tag{19}$$

where A, B, C, and D are Lagrangian multipliers which satisfy the simultaneous linear equations

$$\begin{vmatrix} \sum_s 1 & \sum_s v_x^i & \sum_s v_y^i & \sum_s v_z^i \\ \sum_s v_x^i & \sum_s v_x^{i\,2} & \sum_s v_x^i v_y^i & \sum_s v_x^i v_z^i \\ \sum_s v_y^i & \sum_s v_x^i v_y^i & \sum_s v_y^{i\,2} & \sum_s v_y^i v_z^i \\ \sum_s v_z^i & \sum_s v_x^i v_z^i & \sum_s v_y^i v_z^i & \sum_s v_z^{i\,2} \end{vmatrix} \cdot \begin{vmatrix} A \\ B \\ C \\ D \end{vmatrix} = 2 \begin{vmatrix} 0_M \\ 1_{M_x} \\ 1_{M_y} \\ 1_{M_z} \end{vmatrix} \tag{20}$$

(The idea of exactly preserving more than one quantity characterizing the sample population is apparently novel to the art of Monte Carlo calculations, but since it is of little additional computational expense compared to the computer time expended in tracking individual sample particles, it should have extensive applicability to other problems as well, serving to reduce stochastic effects in the quantities of interest. In the present application, one could also preserve the effective temperature tensor, i.e., $^2M_{\alpha\beta}$, but there seems to be little purpose here for doing so, especially in view of the large uncertainties in the scattering cross-sections which have a large effect in thermalizing the electrons.)

The solution (19) has one potential drawback, however, and that is the possibility that some sample particles may be assigned negative weights by that algorithm. If this should occur, a different method of redistributing

weights while preserving number density and current density is needed. This can be done by computing the total number of physical electrons and the total physical current density contributed by the particles which were just eliminated, finding the velocity centroid of those particles, and then searching the surviving particles for a random set of four particles whose positions form a tetrahedron in velocity space enclosing that velocity centroid. The total weight of the eliminated particles is then distributed to those four surviving particles in such a way that the centroid of this redistribution coincides with the centroid of the eliminated particles. (If additional moments were to be preserved, more than four surviving particles would have to be found, forming a polyhedron enclosing the centroid of the eliminated particles. In principle, the problem would be the same, but the search algorithm would of course be much more difficult and time consuming.)

CONCLUSION

The Monte Carlo approach to modeling secondary electron physics in the high altitude EMP environment is feasible, although it requires large amounts of computer time, so that its use will probably be restricted to benchmark calculations. Such calculations are in progress at AFWL and at LLL, and will be reported on separately. Because of inadequate experimental data on the differential cross-sections for electron production by Compton electrons, for electron scattering, and for electron production by ionizing electron-molecule collisions, this model is not complete. Its accurate treatment of the effects of the Lorentz forces, however, make it much more credible in the higher altitude regions (where collision effects are far less significant) than other models in use at this time. Development of

this model has highlighted certain areas of present ignorance which could have significant effect on EMP strengths, most importantly, the question of the distribution in direction of the secondary electrons produced directly by passage of a Compton electron through the medium.

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