CHEMP: A CODE FOR CALCULATION OF HIGH-ALTITUDE EMP

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ABSTRACT

CHEMP is a code originally designed to examine the effects of self-consistency on high-altitude fields by using a particle pushing model for the current calculation and the high-frequency approximation for calculating the fields. CHEMP has been extended to study the effects of nonforward Compton scattering, varying gamma energies, and elastic nuclear scattering.

CONTENTS

SECTION I

INTRODUCTION

The purpose of this report is to describe the CHEMP code and its usage. CHEMP was originally developed to examine the effects of self-consistency in high-altitude EMP calculations. The code then went through continual development to look at other topics of interest. The present version described here is state of the art in all aspects of the physical model and can perform calculations for all the known high-altitude EMP scenarios. This paper is in four sections. The first is a general description of the algorithm. Subsequent sections deal with specific portions of the methodology in greater detail.

SECTION II

GENERAL ALGORITHM

CHEMP is based on a particle pushing model for the Compton electron dynamics and the high-frequency approximation (ref. 1) for the electromagnetic fields. The fundamental equations are those for the motion of the particles and those which determine the fields. For the particle's motion in retarded time

$$\gamma m_0 \, \partial \overline{v}_k / \partial \tau = \left[\overline{F} - \overline{v}_k (\overline{v}_k \cdot \overline{F}) / c^2 \right] / \left(1 - \hat{\ell}_r \cdot \overline{v}_k / c \right) \tag{1}$$

$$\overline{F} = e\overline{E} + e\left(\overline{v}_k \times \overline{B} \right) / c - \overline{v}_k \left(dW_k / dx \right) / |\overline{v}_k| \tag{2}$$

where $\overline{v}_k$ in the velocity of the kth particle, γ equals $(1 - |\overline{v}_k|^2/c^2)^{-1/2}$, $\hat{\ell}_r$ is a unit vector along the line of sight from the burst, τ is retarded time, and dW_k/dx is the energy loss per unit path length from ionization (appendix II).

For the fields in retarded time

$$\partial \, (r\overline{E}_T) / \partial r = - 2\pi r \overline{J}_T / c \tag{3}$$

$$\partial \, E_r / \partial \tau = -4\pi J_r \tag{4}$$

$$B_r = 0 \, , \quad \overline{B}_T = \hat{\ell}_r \times \overline{E}_T \tag{5}$$

where $\overline{E}_T$, $\overline{B}_T$, and $\overline{J}_T$ are the transverse electric field, magnetic field, and current, and E_r, B_r and J_r are the radial electric field, magnetic field, and current. The currents include the conduction currents.

The fundamental equations are all expressed in retarded time

$$\tau = t - r/c \tag{6}$$

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where τ is retarded time, t is absolute time, and r is the coordinate along ℓ_r. The transform of the equation of motion (1) is seen by differentiating (6) to give

$$\partial\tau/\partial t = 1 - \hat{\ell}_r \cdot \bar{v}_k/c \qquad (7)$$

For the field transform the time derivative in (7) is, at constant r

$$\partial\tau/\partial t = 1 \qquad (8)$$

$$\nabla_t = \nabla_\tau - \frac{1}{c} \hat{\ell}_r \frac{\partial}{\partial\tau} \qquad (9)$$

where (9) is the transform of the gradient operator.

The transform of the charge density is derived from (9) and the invariance of the form of the charge continuity equation as

$$\rho'(r,\tau) = \rho(r,t) - (\hat{\ell}_r \cdot \bar{J})/c \qquad (10)$$

The retarded charge density at r can be expressed as a sum over all particles at r in the form

$$\rho'(r,\tau) = e \sum_k WT'(r_k,\tau_k) \, \Delta\tau \qquad (11)$$

where $\Delta\tau$ in the proper time volume element and $WT'(r_k,\tau_k)$ is the production rate of Compton electrons per unit volume per retarded time at (r_k,τ_k) where the kth particle at (r,τ) orginated and is given by

$$WT'(r_k,\tau_k) = N_\gamma N\sigma_{KN} \exp\left(-\int_0^{r_k} N\sigma_{KN}dx\right) f(\tau_k)/4\pi r_k^2 \qquad (12)$$

where N_γ is the total number of gammas that scatter the type of electron of interest, N is the particle density at r_k, σ_{KN} is the appropriate cross section

for Compton scattering (appendix I), and $f(\tau_k)$ is the retarded time dependence of the gamma source and is normalized to one.

From equation (10) for the kth electron

$$e\Delta\tau WT'(r_k,\tau_k) = \rho_k(r_k,\tau_k)(1-\hat{\ell}_r\cdot\overline{v}_k/c) \tag{13}$$

Thus

$$\rho(r,t) = e\Delta\tau\sum_k WT'(r_k,\tau_k)/(1 - \hat{\ell}_r\cdot\overline{v}_k/c) \tag{14}$$

and

$$J(r,t) = \Delta\tau e\Sigma_k WT'(r_k,\tau_k)\overline{v}_k/(1 - \hat{\ell}_r\cdot\overline{v}_k/c) \tag{15}$$

All of the quantities in equation (1) through (4) have been accounted for except for the conduction portion of the current. The conduction current is expressed as

$$\overline{J}_c = en_e < \overline{V} > \tag{16}$$

where $< \overline{V} >$ is the drift velocity of the conduction electrons. The properties of the conduction electron, the drift velocity, density, and mean energy are calculated with the following swarm (ref. 2) equations

$$\frac{d< \overline{V} >}{dt} = \frac{e\overline{E}}{m} - \nu_m < \overline{V} > - \frac{< \overline{V} >}{n_e} \frac{dn_e}{dt} \tag{17}$$

$$\frac{d< U >}{dt} = e\overline{E} \cdot < \overline{V} > - \nu_u(< U > - U_0) + S$$

$$- \frac{< U >}{n_e} \frac{dn_e}{dt} \tag{18}$$

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$$\frac{dn_e}{dt} = n_e[\gamma_T - \gamma_e] + N_s$$

$$+ \sum_k \left(\partial W_k / \partial X\right) |\overline{v}_k| \rho_k\left(r_k, t_k\right) / E_e \tag{19}$$

where ν_m, ν_u, γ_T, γ_e are the momentum exchange collision frequency, energy exchange collision frequency, Townsend air breakdown coefficient, and electron recombination coefficient. All are functions of the particle density and mean energy $< U >$ (see appendix I). U_0 is the ambient thermal energy. S is an energy source term which includes energy lost due to recombination and air breakdown and that gained from ionization by primary Compton electrons. E_e is the energy lost per primary ionization event by the Compton electrons (appendix I). Subsequent ionization by secondary electrons can be handled by an appropriate ionization lag model. N_s represents the source of later ionization from the secondaries.

This completes the discussion of the fundamental equation used. The remainder of this section will examine the structure of the numerical algorithm used in solving the relevant equation on a space-retarded time grid.
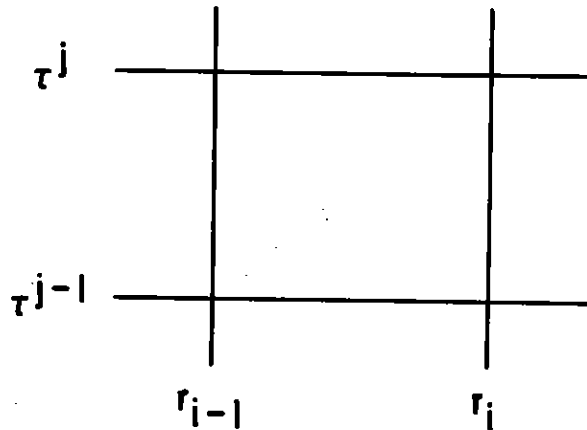
The quantities $\overline{E}$, $\overline{J}$, and $\overline{B}$ are known at (i-1, j-1), (i-1, j), and (i, j-1). In addition n_e, $< \overline{V} >$, $< U >$, and $\overline{V}_k$ for all particles created before τ^{j-1} are known at (i, j-1). It is assumed that a particle created at r_j always is close enough so that it can be assumed to be at r_j. First the appropriate quantities are initialized at (i, j), i.e.,

$$\bar{E}_i^j = \bar{E}_{i-1}^j$$

$$\bar{B}_i^j = \bar{B}_{i-1}^j$$

$$< U >_i^j = < U >_{i-1}^j$$

$$< \bar{V} >_i^j = < \bar{V} >_{i-1}^j$$

and the necessary particles are created at (i, j-1). The weight of new particles created at (i, j-1) is

$$\text{Weight} = N \, N_\gamma \, \sigma_{KN} \, \exp \left(- \int_0^{r_i} N\sigma_{KN} \, dr \right) f(\tau^{j-1})(\tau^j - \tau^{j-2})/(8\pi \, n_p \, r_i^2)$$

where n_p is the number of particles created. If n_p is one, generally the particle has the average scatter energy and is directed forward. The option also exists to create several particles at a point to simulate the Compton electron distribution. This will be discussed in greater detail in the source section.

Now

$$\bar{v}_k^j = \bar{v}_k^{j-1}$$

where k includes the newly created particles. Let $\bar{B}_e$ be the earth's magnetic field. The basic iteration loop proceeds as follows.

1. Update $\bar{v}_k^j$ for all k by solving equation (1). The solution, obtained iteratively, is time centered in all terms including those nonlinear in the velocity. The total magnetic field is that from the particles plus $\bar{B}_e$. A one-percent convergence criteria in $|\bar{v}_k^j|$ is used. If desired, the MRC obliquity factor (ref. 3) can be used in this step.

2. Calculate the contribution to $\bar{J}_i^j$ from the Compton electron by using equation (15), using the appropriate weight and the just calculated $\bar{v}_k^j$'s.

3. Update $n_{e_i}^j$ and $< U >_i^j$ by solving equations (18) and (19) iteratively. The solutions are time centered in all quantities, and a one-percent convergence criteria is used on $< U >_i^j$.

4. Update $< \overline{V} >_j^i$ by solving equation (17) where all quantities are time centered, using the new data calculated in step 3.

5. Sum the conduction and Compton currents for $\overline{J}_i^j$

6. Update $\overline{E}_i^j$ and $\overline{B}_i^j$ by using equations (3), (4), and (5) where again all quantities are centered.

7. Test the magnitude of the new $\overline{E}_i^j$ against the old. If the variation is less than one percent, the cell solution is complete. If not, return to step 1 and continue with the updated quantities until convergence is reached.

The above procedure is used to solve for all time the space cells in the grid. The algorithm calculates for all time at a particular space point before proceeding on to the next space point. This procedure is the most efficient. This completes the general description of the algorithm.

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SECTION III

CURRENT SOURCE

The current source package defines the location, time, energy, and direction of all the Compton particles at creation. Only primary Compton events are considered. There are several options available in CHEMP. The simplest option, used primarily when absolute field strengths are unimportant, is producing one particle each time step with the average Compton scattering energy and directed forward. The gamma spectrum is monoenergetic. If σ_{KN} is the total Compton cross section and σ_S is the Compton scattering cross section, then the average Compton electron energy is

$$E = \left(1 - \frac{\sigma_S}{\sigma_{KN}}\right)h\nu$$

where $h\nu$ is the gamma energy. The weight of each particle created is determined by the $\dot{\gamma}$ curve and the attenuation in the atmosphere

$$\dot{\gamma} = N_\gamma f(\tau)$$

where N_γ is the total number of gammas. $f(\tau)$ is the time dependence of the gamma source and is normalized to one. The gamma flux at R and τ is

$$F = N_\gamma \exp\left(-\int_0^R N\sigma_{KN}dr\right)f(\tau)/4\pi R^2$$

The deposition rate per unit time and volume is

$$R = NN_\gamma \sigma_{KN} \exp\left(-\int_0^R N\sigma_{KN}dr\right)f(\tau)/4\pi R^2$$

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and the weight of a particle is just the rate integrated over the differential time element.

$$\text{Weight} = \int_{\tau}^{\tau+\Delta\tau} R \, d\tau$$

The next option is generally used for most problems. The gamma spectrum is still monoenergetic, but now several particles are created at each point in time and space. The directions and energies of the particles are created to simulate the initial Compton scattering distribution. As a typical example, to simulate the distribution from a 1.5 MeV source, sixteen particles are created at each point in space and time. Four polar scattering angles, 0.146, 0.357, 0.624, and 1.04 radians and four azimuthal angles, 0, $\pi/2$, π, and $3\pi/2$ are used to specify the direction of the particles. These angles were chosen so that all particles have equal weights. The particle energies are now a function of the polar scattering angle. Let θ_e be the electron scattering angle. Then

$$\theta_\gamma = 2 \tan^{-1}\left(\cot(\theta_e)/(1 + \alpha)\right)$$

where

$$\alpha = h\nu/m_0 c^2$$

$$E_{\theta_e} = h\nu\left(1-1/\left(1+\alpha\left(1-\cos(\theta_\gamma)\right)\right)\right)$$

The weight of a particle is

$$\text{Weight} = (1/16) \, NN_\gamma \, \sigma_{KN} \, \Delta\tau \, \exp\left(-\int_{0}^{R} N\sigma_{KN} \, dr\right) \int_{\tau}^{\tau+\Delta\tau} f(\tau')d\tau'/4\pi R^2$$

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Other weighting schemes with different polar scattering angles are occasionally used to test the accuracy of these sources. The last major option is whether or not to use a gamma spectrum with either of the above two options. To implement an energy spectrum, the above desired option is done for each energy in the gamma energy spectrum. It is presently assumed that one $f(\tau)$ function describes all gamma energies.

Other variations on this source package are available with minimal changes.

SECTION IV

ENERGY CALCULATIONS

The CHEMP code was originally built to study the energetics of high-altitude EMP. Six energy totals are kept for every calculation. The first is the total Compton energy deposited over all time and from $r = 0$ to R. For a monoenergetic spectrum

$$W_c = 4\pi E \int_0^R \int_0^{\tau_{max}} WT'(r,\tau) r^2 dr d\tau$$

where E is the energy of the Compton electrons and $WT'(r,\tau)$ is the source function define in section II

The second energy term is the energy dissipated via the conduction electrons, given by

$$W_d = 4\pi e \int_0^R \int_0^{\tau_{max}} n(r,\tau) \left(<\overline{V}> \cdot \overline{E}(r,\tau) \right) r^2 dr d\tau$$

where $n(r,\tau)$ is the free electron density and $(\overline{V})$ is the drift velocity.

The third term is the radiated energy that has propagated by the spatial point r.

$$W_r = cR^2 \int_0^{\tau_{max}} |\overline{E}_T(R,\tau)|^2 d\tau$$

where c is the velocity of light and $\overline{E}_T$ is the transverse field.

The next energy sum is the longitudinal field energy, given by

$$W_e = \int_0^R |\overline{E}_e(r,\tau_{max})|^2 r^2 dr$$

where $\overline{E}_e$ is the longitudinal field. The kinetic energy remaining in the Compton electrons at τ_{max} from R = 0 to R is found by performing the appropriate sum to get W_k.

The last energy is that lost by the Compton electrons in ionizing collisions. This term, W_i, is also summed as the calculation progresses. If the calculations were exact, then

$$W_c - W_k = W_i + W_d + W_e + W_r$$

In general, the error in total energy in CHEMP is less than one-half percent.

APPENDIX I

PHYSICAL CONSTANTS

1. ENERGY LOSS EQUATION

The energy loss equation (ref. 4) used in CHEMP is

$$\partial W/\partial X = 4.077 \times 10^{-31} \, NZ \left[\ln\left((m_0 c^2)^2 (\gamma^2 - 1)(\gamma - 1)/I^2 \right) - B^2 \right]/B^2$$

where NZ is the total electron density, $B = v/c$, $\gamma = (1 - B^2)^{-1/2}$, and I is taken as 86 eV.

2. COMPTON CROSS SECTION

$$\sigma_{KN} = 2\pi r_0^2 \left\{ (1+\alpha) \left[2(1+\alpha)/(1+2\alpha) - \ln (1+2\alpha)/\alpha \right] /\alpha^2 \right.$$

$$\left. + \ln(1+2\alpha)/2 - (1+3\alpha)/(1+2\alpha)^2 \right\}$$

$$4\pi r_0^2 = 10^{-24} \text{ cm}^2/\text{electron}$$

$$\alpha = h\nu/m_0 c^2$$

$h\nu$ = photon energy

3. AIR BREAKDOWN COEFFICIENT, MOMENTUM EXCHANGE COLLISION FREQUENCY, AND ENERGY EXCHANGE COLLISION FREQUENCY

For a given characteristic electron energy

$$\gamma_T = 3.76 \times 10^{-15} \, NE^{8.7}/(1 + 2.92 \times 10^{-4} \, E^5)^{1.412}$$

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$$\nu_{energy} = N \left(1.3 \times 10^{-11} \, (1 + 4.4 \times 10^4 \, E^5)^{0.32} / (1 + 1.93 \times 10^3 \, E^{10})^{0.138} \right.$$

$$\left. + \, 4.13 \times 10^{10} \, E^{5.22} / \left((1 + 0.06 E^7)^{0.456} (1 + 1.68 \times 10^{-3} \, E^4)^{0.097} \right) \right)$$

$$\nu_{momentum} = 1.25 \times 10^{-7} \, N E^{0.935} \, (1 + 0.093 \, E^{1.5})^{0.405} / (1 + 1.26 \, E^{1.67})^{0.373}$$

Where N is the particle density. There fits are variations of fits used by Higgins, Longmire, and O'Dell (ref. 2).

4. ENERGY LOST PER PRIMARY IONIZATION

For a Compton electron of energy E eV the average energy lost per ionizaton is

E > 250 Kev	$E_e = 86$ eV
250 KeV > E > 0.640 Kev	$E_e = 7.705 \ln(E) - 9.79$
0.640 KeV > E	$E_e = 1.8 \ln(E) + 28.4$

5. ELECTRON RECOMBINATION COEFFICIENT

Three body recombination

$$\gamma_{e_3} = 2.78 \times 10^{-31} \, N^2 / (1 + 12.5 \, E)$$

Two body dissociative recombination

$$\gamma_{e_2} = 4.85 \times 10^{-12} \, N / (1 + 3.45 \, E^{-4.8})^{1.29}$$

Total recombination

$$\gamma_e = \gamma_{e_3} + \gamma_{e_2}$$

APPENDIX II

NUMERICAL METHODS

The general form of the first order differential equations solved is

$$\frac{\partial f}{\partial x} = g(f,x,y) \tag{20}$$

Two numerical forms are used in finding the solution. The first is

$$f_i = f_{i-1} + \Delta x \left(g(f_i,x_i,y_i) + g(f_{i-1},x_{i-1},y_{i-1}) \right)/2 \tag{21}$$

The second is

$$f_i = f_{i-1} + \Delta x \; g(f_{i-1/2},x_{i-1/2},y_{i-1/2}) \tag{22}$$

where

$$f_{i-1/2} = (f_i + f_{i-1})/2$$

$$x_{i-1/2} = (x_i + x_{i-1})/2$$

$$y_{i-1/2} = (y_i + y_{i-1})/2$$

Both of the above equations are solved iteratively. These forms are used primarily because they are two point schemes. Thus, the increment, Δx, can be charged without incurring any loss of accuracy due to loss of centering. Also the implicit nature of the solution ensures a high degree of numerical stability.

The error in equation (21) is $\sim (\Delta x)^3 f'''(\epsilon)/12$

The error in equation (22) is also of the same order.

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