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EFFECTS OF NUCLEAR SCATTERING AND ENERGY LOSS ON SMALL SIGNAL HIGH-ALTITUDE ELECTROMAGNETIC PULSE CALCULATIONS

L. A. Wittwer, Capt, USAF
James E. Brau, Capt, USAF
G. H. Canavan, Maj, USAF

Air Force Weapons Laboratory
Kirtland AFB, New Mexico 87117

ABSTRACT

The effects of nuclear scattering and ionization energy loss on the electromagnetic fields from high-altitude bursts are examined using Monte Carlo techniques to calculate electron trajectories. Maxwell's equations and the Lienard-Wiechert potentials are both used to evaluate the fields. The obliquity model is demonstrated to be an adequate treatment of such effects in electromagnetic pulse calculations.

CONTENTS

Section		Page
I	INTRODUCTION	4
II	FIELD EQUATIONS	5
III	MONTE CARLO PROCEDURE	9
IV	RESULTS	12
	REFERENCES	19

ILLUSTRATIONS

Figure		Page
1	Electric Field Generated at the Ground by 0.7-Mev Electron at 30 km	14
2	Electric Field Generated at the Ground by 0.53-Mev Electron at 30 km	15
3	Electric Field Generated at the Ground by 0.13-Mev Electron at 30 km	16
4	Electric Field Generated at the Ground by 0.7-Mev Electron at 20 km	17
5	Electric Field Generated at the Ground by 0.53-Mev Electron at 20 km	18

SECTION I

INTRODUCTION

The effects of nuclear scattering and energy loss on high-altitude electromagnetic pulse (EMP) calculations were first studied by G. Knutson (ref. 1) in extensive Monte Carlo scattering calculations. This work extends the Knutson work by calculating in a simple geometry the electric radiation fields from Monte Carlo electron calculations similar to those of Knutson. The field calculations are done simultaneously using the high-frequency approximation (ref. 2) and the "particle" formalism (refs. 3, 4). In addition, the Mission Research Corporation (MRC) obliquity model (ref. 5) and a recent modification are examined. Finally, the applicability of using a 0.53-Mev forward-scattered Compton electron to simulate the effects of the Compton angular distribution and nuclear scattering for incident 1.5-Mev gammas is considered.

SECTION II

FIELD EQUATIONS

For a single particle of charge e and velocity, $v/c = \bar{B}$, the electric radiation field at some field point is (ref. 6)

$$E(r,t) = \frac{e}{c}\left[\frac{\hat{n} \times \left[(\hat{n} - \bar{B}) \times \dot{\bar{B}}\right]}{R(1 - \hat{n} \cdot \bar{B})^3}\right]_{RET} \tag{1}$$

where $\hat{n}$ is a unit vector pointing from the moving charge to the field point, R is the separation between the particle and the field point, and $[\ ]_{RET}$ means that the enclosed expression is to be evaluated at time t - R/c where t is the time at the field point of interest. For an initial estimate of scattering, assume a horizontally uniform thin layer of Compton electrons deposited at 30 km at t = 0, initially moving directly downward. If there were no scattering, the electrons would rotate about the magnetic field line with angular frequency $\omega = eH/\gamma m_0 c$ where H is the ambient magnetic field and $\gamma = (1 - v^2/c^2)^{-1/2}$. At a given time t, then, the field seen at the ground is found by summing equation (1) over all electrons in a disk at 30 km centered directly over the field point. The radius of the disk is

$$r = \left[(3 \times 10^6 + ct)^2 - (3 \times 10^6)^2\right]^{1/2} \text{cm}$$

Thus, an electron at the center of the disk has existed the longest, i.e., t seconds, and an electron at the edge has just been made (Compton scattered).

Scattering causes the electrons to deviate from the circular orbits (ref. 7). This degrades the coherence of the sources and results in a lower net electric field.

From equation (1), the transverse radiation field from a particle with velocity $\bar{B} = \bar{v}/c$ is

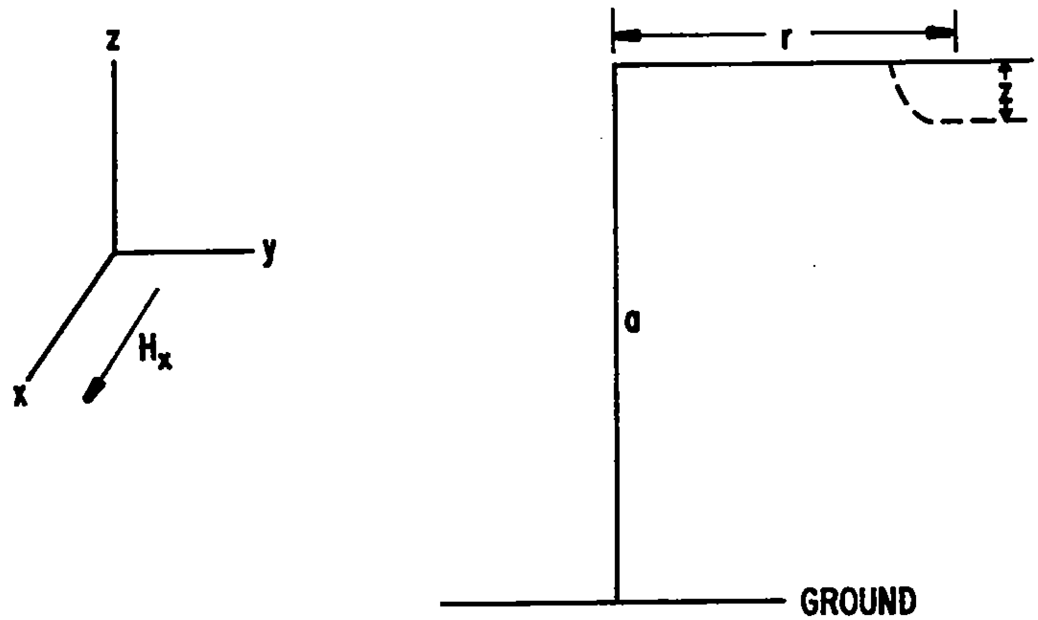
$$E_T(t) = e\Big[n_x\Big(\big(n_x - B_x\big)\big(\dot{B}_y - eH_xB_z/\gamma m_0c\big) - \big(n_y - B_y\big)\dot{B}_x\Big) - n_z\Big(\big(n_y - B_y\big)\big(\dot{B}_z + eH_xB_y/\gamma m_0c\big) - \big(n_z - B_z\big)\big(\dot{B}_y - eH_xB_z/\gamma m_0c\big)\Big)\Big]/\big(cR(1 - \hat{n}\cdot\bar{B})^3\big) \qquad (2)$$

where the coordinate system has the z axis vertically upward, H_x is the earth's magnetic field, and $\dot{R}$ is the distance from the source particle to the ground field point. $\dot{B}_x$, $\dot{B}_y$, and $\dot{B}_z$ are the accelerations from scattering and energy loss. Due to the specific problem, some approximations can be made. R is taken as 30 km, as the diameter of the disk containing contributing radiating electrons is always small (5×10^4 cm for $\tau = t - R/c \leq 2 \times 10^{-8}$ sec) compared to the altitude. Also $1 - \hat{n}\cdot\bar{B} \approx 1 - n_zB_z$. Finally, n_x and n_y can be taken as zero either because of their small size ($n_x, n_y \leq 0.016$) or because the integration over electrons around the disk results in zero for terms in equation (2) that are linear in n_x and n_y. Equation (2) can now be rewritten.

$$E_T(\tau_P) = -e\Big[n_z\Big(B_y\big(\dot{B}_z + eH_xB_y/\gamma m_0c\big) - \big(n_zB_z\big)\big(\dot{B}_y - eH_xB_z/\gamma m_0c\big)\Big)\Big]/\Big(cR\big(1 - n_zB_z\big)^3\Big) \qquad (3)$$

where the expression on the right is evaluated at τ_P. The geometry involved for an electron at some point on the disk is illustrated by the following drawing. a is the altitude, z is the vertical distance traveled by the electron, and r is the distance from the center of the disk of the electron. Assume that the electron was scattered at $\tau = 0$. The proper time at z is $\tau_P = t + z/c$ where t is the real time that has passed in the life of the electron. The effect of the particle at z is felt at

$$\tau = \tau_P + \Big(\big((a + z)^2 + r^2\big)^{1/2} - (a + z)\Big)/c \qquad (4)$$



For a given τ after expanding equation (4)

$$\tau_p \simeq \tau - r^2/2ac \tag{5}$$

$$E_p(\tau) = 2\pi \int_0^{R_{max}} r\rho(r)\ E_T\left(\tau_p\right)\ dr\ \Delta h \tag{6}$$

where

$$R_{max} = (2ac\tau)^{1/2} \quad , \quad \rho(r)$$

is the density of Compton electrons, and Δh is the depth of the deposition layer. $E_p(t)$ is the net field on the ground found by integrating over all contributing electrons. Now $r dr = -ac d\tau_p$, hence

$$E_p(\tau) = 2\pi ac \int_0^{\tau} \rho E_T\left(\tau_p\right)\ d\tau_p\ \Delta h \tag{7}$$

The field calculation is based on the high-frequency approximation (ref. 2). The equation for the transverse field is for a given proper time τ.

$$\frac{\partial\left(rE_F\right)}{\partial r} = -\frac{2\pi}{c} r J \tag{8}$$

where J is the current and is expressed in terms of the electron motion as

$$J = ce \sum_i n_i B_{yi} \, \delta\left(r - r_i\right)/\left(1 - B_{zi}\right) \tag{9}$$

at τ, n_i is the particle weight, and r_i is the particle location with respect to gamma source. Upon integrating equation (8),

$$\int_0^R \frac{\partial\left(rE_F\right)}{\partial r} dr = -\frac{2\pi}{c}\int_0^R rJ \, dr \tag{10}$$

$$RE_F(\tau) = -2\pi e \sum_i r_i n_i \, B_{yi}/\left(1 - B_{zi}\right) \tag{11}$$

$$E_F(\tau) = -2\pi e \sum_i n_i \, B_{yi}\left(r_i/R\right)/\left(1 - B_{zi}\right) \tag{12}$$

The particle calculation is for an infinite source. To do the field calculation in the same scenario, the gamma source is taken to infinity. Finally, for fields

$$E_F(\tau) = -2\pi e \sum_i n_i \, B_{yi}/\left(1 - B_{zi}\right) \tag{13}$$

and for particles

$$E_p(\tau) = 2\pi ac \int_0^\tau \rho E_T(\tau) \, d\tau_p \, \Delta h \tag{14}$$

SECTION III

MONTE CARLO PROCEDURE

The Monte Carlo calculation of the motion of the individual particle includes the magnetic field turning, elastic nuclear scattering, and the electron energy loss from ionization. The algorithm is almost the same as used by Knutson (ref. 1). The nuclear cross section used is

$$d\sigma_{sc} = 2\pi\rho r_0^2\ z^2\ (1 - B^2)\ d\mu/\left(B^4(1 - \mu + N)^2\right) \tag{15}$$

where ρ = atom density, $z = 7.2$, $\mu = \cos\theta$, $r_0 = 2.82 \times 10^{-13}$ cm, and

$$N = (1 - B^2)\ \left(z^{1/3}/(0.885)\ (137)\right)^2\ \left(1.13 + 3.76\ (z/137B)^2\right)/2B^2 \tag{16}$$

The energy loss formula is

$$dE/dx = 4.077 \times 10^{-31}\ \rho_e\left[\ln\left(3.53 \times 10^7\ (\gamma - 1)(\gamma^2 - 1)\right) - B^2\right]/B^2 \tag{17}$$

where ρ_e is the ambient electron density.

To illustrate the Monte Carlo procedure, look at a particle just Compton-scattered forward with velocity B_z and $\gamma = (1 - B_z^2)^{-1/2}$. The total nuclear cross section is calculated.

$$\sigma_T = 1.04 \times 10^{-22}\ \rho(1 - B^2)/(NB^2) \tag{18}$$

A random number r_n is calculated. The length traveled to the first collision is

$$d = -\ln\left(r_n\right)/\sigma_T \tag{19}$$

The real time passed is

$$\Delta t = d/(3 \times 10^{10}\ B) \tag{20}$$

The z coordinate is updated

$$z = z + 3 \times 10^{10}\ B_z\ \Delta t/K \tag{21}$$

where K is the obliquity factor developed by MRC (ref. 5) when implemented. The proper time is also updated

$$\tau = t + z/(3 \times 10^{10}) \tag{22}$$

Next the particle is turned in the magnetic field for a time t and the contribution to the integral in equation (14) is calculated for the particle. Two more random numbers r_1 and r_2 are calculated to determine the scattering angles

$$\cos(\theta) = \left(2r_1(1 + N) - N\right)/(2r_1 + N)$$

$$\theta = 2\pi r_2$$

$$\sin\theta = (1 - \cos^2\theta)^{1/2} \tag{23}$$

The velocity components are now adjusted.

$$a_2 = \cos(\phi)\ \sin(\theta)/\left(B_y^2 + B_z^2\right)^{1/2} \tag{24}$$

$$a_1 = B\ \sin(\phi)\ \sin(\theta)/\left(B_y^2 + B_z^2\right)^{1/2} \tag{25}$$

$$B_z^1 = B_z \cos(\theta) - a_1B_y - a_2B_xB_z \tag{26}$$

$$B_y^1 = B_y \cos(\theta) + a_1B_z - a_2B_xB_y \tag{27}$$

$$B_x^1 = B_x \cos(\theta) + a_2\left(B_y^2 + B_z^2\right) \tag{28}$$

Next, the energy loss is calculated and the total velocity changes due to scattering and energy loss are used to update the particle integral in equation (14). Finally, the accumulated particle result and the field calculation result are accumulated in the proper time bin spanned since the last collision. The MRC obliquity factor is used and updated in the field calculation when desired. Each subsequent collision of the particle is done the same way until 2×10^{-8} sec of proper time has passed. In general, the high-frequency approximation and the "particle" model agreed to 5 percent.

SECTION IV

RESULTS

Figures 1 through 3 demonstrate the results at 30 km for various initial energies. Figure 1 is for 0.7 Mev and is the most similar to the typical energy used in more sophisticated codes. The circular orbit result at 2×10^{-8} sec is about 2.9 times larger than the Monte Carlo scatter, and the obliquity results. The agreement between the Monte Carlo and the obliquity is excellent. Figure 2 is for 0.53 Mev, and the decrease due to scattering from circular orbits is a factor of 0.26. The agreement between the scatter and the obliquity is not as good as for higher energies but is still quite acceptable. The square on figure 2 is a result by Sollfrey (ref. 8) for the same case. Finally, figure 3 is for 0.13 Mev. The fractional decrease from circular orbits is about 0.2. The discrepancy between scatter and obliquity is now more pronounced.

Figures 3 and 4 contain the same results as figures 1 and 2 at 20 km. The agreement between the obliquity factor and the Monte Carlo is still reasonably good.

In addition to comparisons of obliquity to Monte Carlo scatter, other investigations were made. At a 1 November 1973 meeting at AFWL, Conrad Longmire of MRC described a new obliquity model. If the equation of change of the old obliquity factor is represented as

$$dN/dt = F(E,\rho)$$

where E is the particle energy and ρ is the density, then the new equation is

$$dN/dt = NF(E,\rho)$$

Results are plotted in figures 2, 4, and 5. Results were not plotted on figure 1, as the new model did not change significantly from the old. In all cases, the new model is a significant improvement over the old. The effect of using the new model as compared to the old was tested in a full EMP calculation with CHEMP, the AFWL/DYT EMP code, and only very slight differences were noted in the tails of the EM pulses. This means that older calculations with the old model are still useful and also that field results are not very sensitive to fine details in the source.

Plotted in figures 2 and 5 are the results of combining the Klein-Nishina angular distribution from an incident 1.5-Mev gamma with the Monte Carlo scattering and with the new obliquity model. This was done to test the equivalence, as stated by Cullen Crain (ref. 4), between the Klein-Nishina plus scatter and a 0.53-Mev forward-directed electron with scatter alone. At 30 km the agreement is very good. At 20 km at 2×10^{-8} sec, however, the 0.53-Mev electron is about 30 percent below the comparison result. Thus, the use of the 0.53-Mev forward-directed electron to model the currents from a 1.5-Mev gamma pulse would appear to underestimate the fields.

In summary, the obliquity models do an adequate job in simulating the effects of nuclear scattering with the new model being clearly better than the old. These results do not establish the obliquity result over all possible regions, but they do substantiate the much more extensive Monte Carlo results by Knutson (ref. 1).

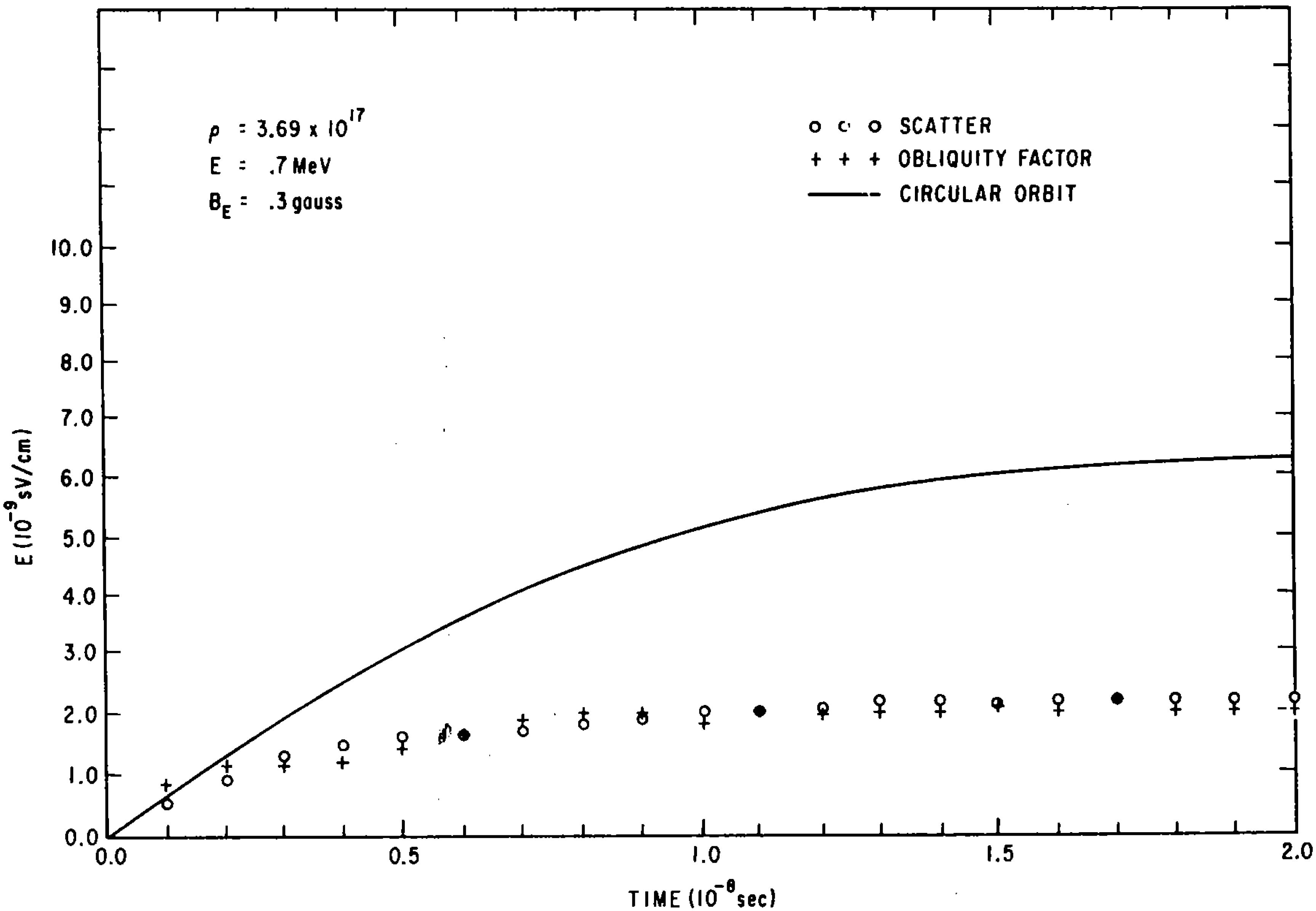


Figure 1. Electric Field Generated at the Ground by 0.7-Mev Electron at 30 km

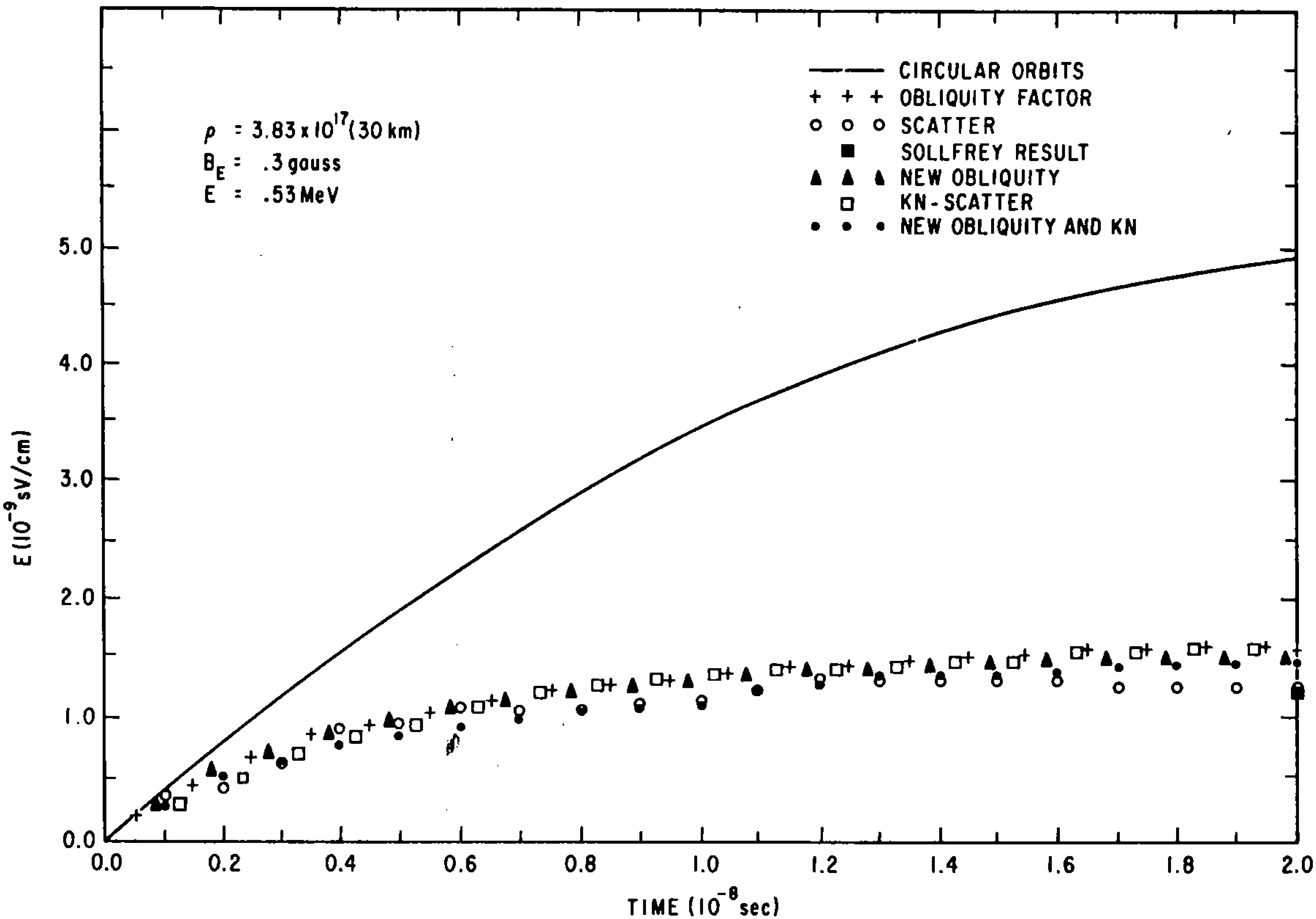


Figure 2. Electric Field Generated at the Ground by 0.53-Mev Electron at 30 km

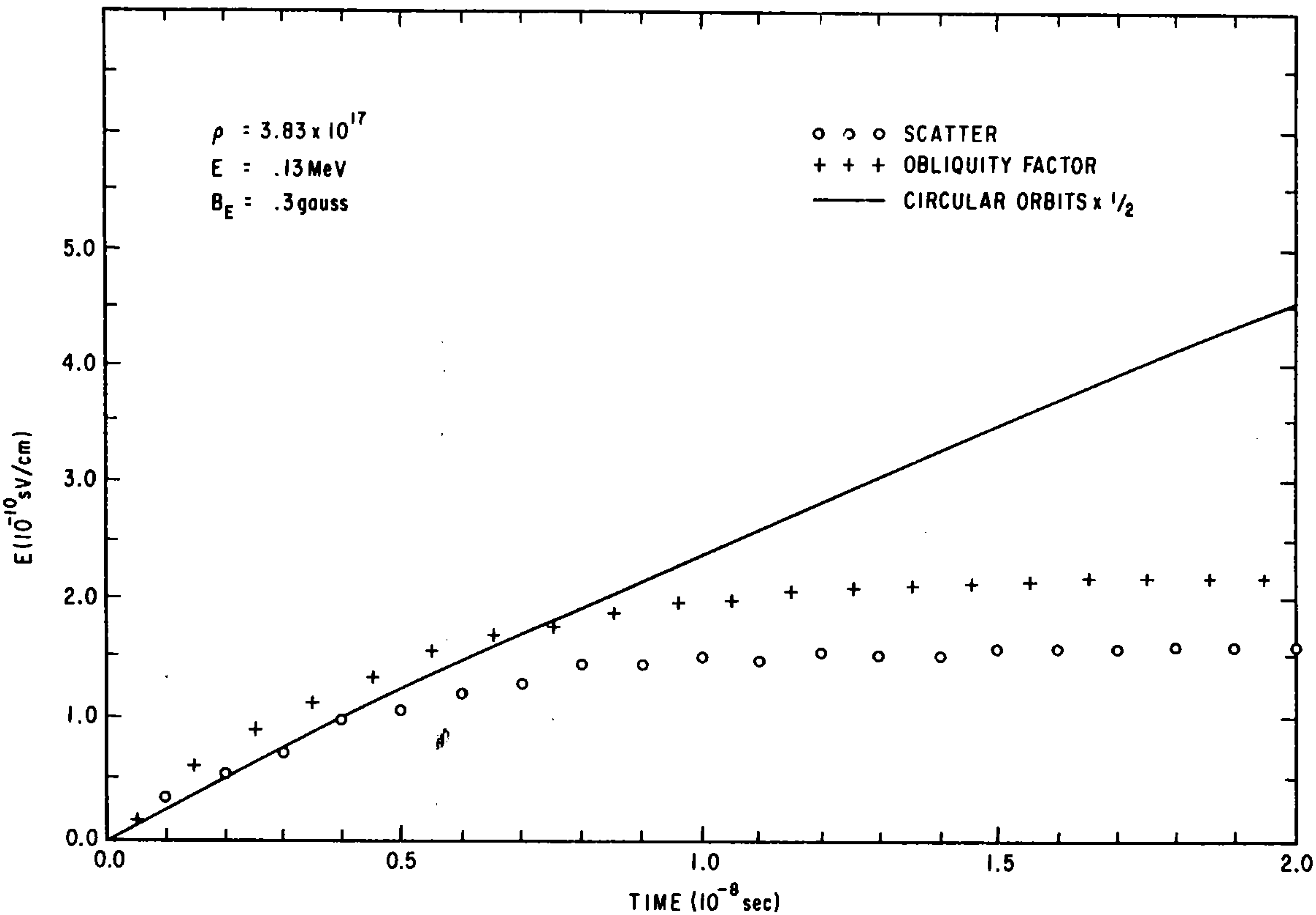


Figure 3. Electric Field Generated at the Ground by 0.13-Mev Electron at 30 km

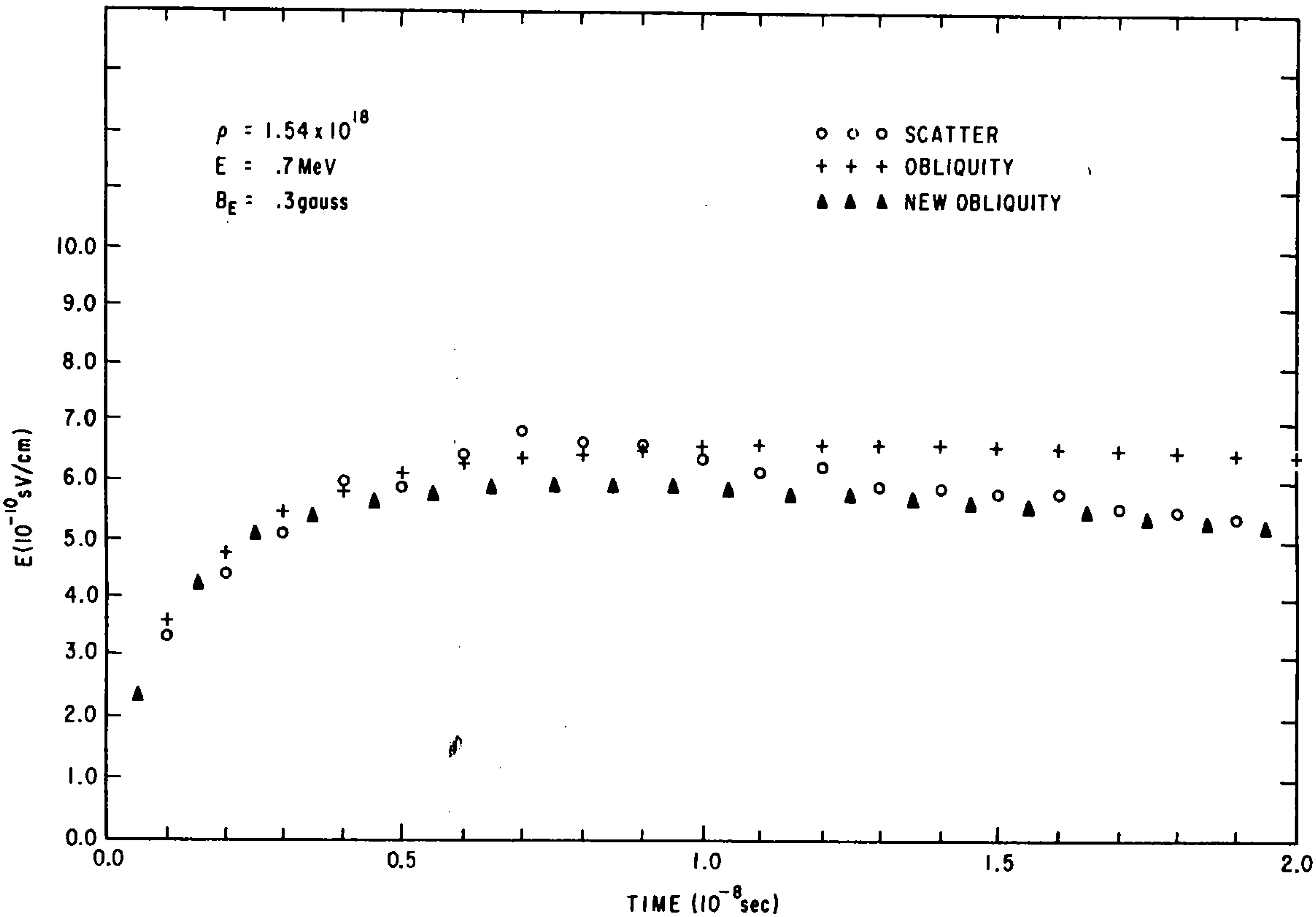


Figure 4. Electric Field Generated at the Ground by 0.7-Mev Electron at 20 km

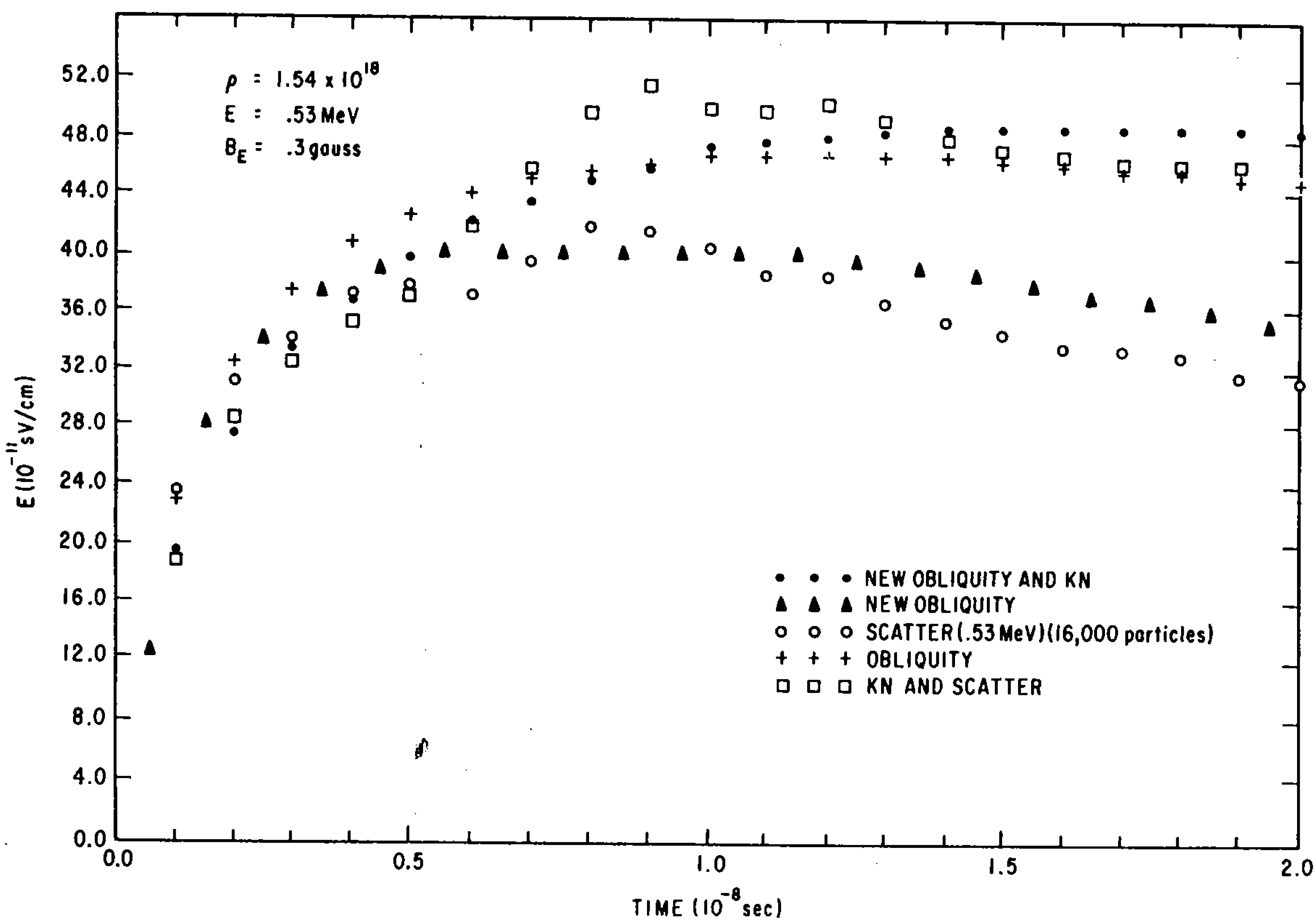


Figure 5. Electric Field Generated at the Ground by 0.53-Mev Electron at 20 km

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