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# COMPUTED ELECTRON DRIFT VELOCITY IN MOIST AIR

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# SUMMARY

The free electron drift velocity in air and water vapor mixtures is calculated by means of the "average electron" method. The results indicate that the electron mobility is decreased by addition of water vapor for low electric fields, and it is increased by addition of water vapor for high electric fields. For still higher electric fields, the change caused by addition of water vapor dwindles. The mobility increases by as much as 43 percent for 6 percent water vapor content, which occurs for an E/p near 2.5; the "crossover" occurs at an E/p of 1.04 for 6 percent water vapor content. The maximum increase is less for smaller water vapor contents, and this increase occurs at lower electric field strengths.

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# COMPUTED ELECTRON DRIFT VELOCITY IN MOIST AIR

## I. INTRODUCTION

1. Subject. This report covers an investigation of the drift velocity of free electrons under impressed electric fields in moist air. The electric field strengths vary from zero to about 450,000 volts per meter (for one atmosphere pressure). The moistness of the air varies from 0 percent water vapor content to 6 percent water vapor content.

2. Motivation. Impurity gases have been shown to have a strong effect on the mobility of free electrons in certain gases, such as argon and other noble gases according to McDaniel (1), page 539; and nitrogen, methane, hydrogen, carbon dioxide, ethylene, cyclopropane (2), and many others. Water vapor has been shown to have such a strong effect as an impurity gas according to Hurst, Stockdale, and O'Kelly (2).

The effect of various concentrations of water vapor on free electron mobility in air is of interest because this effect apparently has not been investigated experimentally. Furthermore, the conductivity of air is important to an effect such as the "nuclear electromagnetic pulse" (NEMP). (Sollfrey (3) indicates this in an unclassified way.)

3. Previous Investigation. According to Baum (4)(5), earlier theoretical efforts to evaluate the effect of water vapor on air conductivity have suggested that the free electron mobility would be lowered significantly by the addition of a small percentage of water vapor throughout ranges of the electric field strength from zero to about 100,000 volts per meter at atmospheric pressure. Different results were obtained in this report.

Experimental work of adding water vapor to pure nitrogen shows that the free electron mobility is markedly increased over wide ranges of the electric field strength (2). The probable cause of this increase in free electron mobility, as suggested by the investigators, is the decrease of the electron agitation energy (the average kinetic energy of a free electron) in the nitrogen caused by the extremely low electron agitation energy in the pure water vapor. (The effect may be regarded as a consequence of the large electron collision cross section in nitrogen.) This decrease overrules the effect of the low electron mobility in pure water vapor, so that the electron mobility increases in the mixture.

4. A New Consideration. This result suggests that the same effect might be observed in air. The effect requires that the electron energy factor (which is the electron agitation energy divided by the molecular agitation energy, or the electron temperature divided by the molecular temperature) be high for dry air just as it is high for dry nitrogen. Crompton, Huxley, and Sutton (6) or Brown (7), page 83, have determined the electron energy factor for air for various electric field strengths. This requirement seems to be satisfied.

## II. INVESTIGATION

5. The "Average Electron" Method. Klema and Allen (8) summarize a method, developed by others, which can be used to calculate the drift velocity of free electrons, in a mixture of gases; graphs or tabulations of the free electron drift velocity and energy factor are used in the calculations as functions of the electric field strength for each component of the gas mixture. Their approach has been called the average electron method. The method is limited in that it examines the behavior of only the average electron and does not take into account the actual velocity distribution of the electrons, according to Uman (9); however, this limitation is not significant for the purposes of this report. The equations of the average electron method are derived in the appendix of this report. The basic equations developed are

$$\left(\frac{E}{p}\right)W = \sum_i \left(\frac{p_i}{p}\right)\left(\frac{E_i}{p}\right) W_i \tag{1}$$

$$\left(\frac{E}{p}\right)\frac{1}{W} = \sum_i \left(\frac{p_i}{p}\right)\left(\frac{E_i}{p}\right) \frac{1}{W_i} \tag{2}$$

$$W = \left\{\left[\left(\frac{E}{p}\right)W\right] \Big/ \left[\left(\frac{E}{p}\right)\frac{1}{W}\right]\right\}^{\frac{1}{2}} \tag{3}$$

and

$$\frac{E}{p} = \left\{\left[\left(\frac{E}{p}\right)W\right]\left[\left(\frac{E}{p}\right)\frac{1}{W}\right]\right\}^{\frac{1}{2}} \tag{4}$$

Here, $W$ = free electron drift velocity
$E/p$ = electric field strength per unit pressure for the gas mixture
$p_i/p$ = fraction of the mixture which is the i-th gas component
$E_i/p$ = electric field strength per unit pressure
$W_i$ = electron drift velocity as functions of the energy factor, as determined from experimental data, for the i-th gas component. These equations are based on momentum loss rate and energy loss rate considerations.

6. Input Data. Crompton, Huxley, and Sutton (6) obtain for air, where K is the energy factor

$$K = 1 + 33 \frac{E}{p} \text{ for } K < 2$$

$$W = 1.59 \times 10^7 \left(\frac{E}{p}\right) \div \left(1 + 33 \frac{E}{p}\right)$$

in centimeters per second for this same range of K. For air, for K between 4 and 9, these authors obtain $K = 13.3\ (E/p)^{\frac{1}{2}}$ and $W = 1.23 \times 10^6\ (E/p)^{\frac{1}{2}}$ in centimeters per second. Their results are used as input data for the calculations performed for this report. For K greater than 9, the energy factor data used are those obtained by Crompton, Huxley, and Sutton, and the free electron drift velocity data used are those of Nielson and Bradbury (10), or McDaniel (1), page 546, or Brown (7), page 58.

The drift velocity of electrons in water vapor has been measured to low values of E/p by Pack, Voshall, and Phelps (11) and by Lowke and Rees (12), whose results affirm that the electron mobility is constant for an E/p less than about 8 volts per centimeter per millimeter of Mercury. In fact, $W = 7.1 \times 10^4\ E/p$ in centimeters per second over this range. Because variations in the energy factor-electric field strength curve usually correspond to variations in the drift velocity-electric field strength curve, a similar linearity for the energy factor as a function of E/p--$K = 1 + 0.15\ E/p$, for E/p less than about 8 volts per centimeter per millimeter of Mercury--will be assumed.

To avoid reading values directly (and inaccurately) from the graph for water vapor obtained by Pack, Voshall, and Phelps (11) or the graph shown by McDaniel (1), page 543, the graph was approximated by simple power laws:

$$W = 8 \times 10^4 \frac{E}{p} \text{ for } \frac{E}{p} < 10$$

$$W = 8 \times 10^5 \left(\frac{E}{p} \div 10\right)^{1.5} \text{ for } \frac{E}{p} \text{ between 10 and 13}$$

$$W = 1.186 \times 10^6 \left(\frac{E}{p} \div 13\right)^4 \text{ for } \frac{E}{p} \text{ between 13 and 20}$$

The first expression does not agree perfectly with the expression given previously that $W = 7.1 \times 10^4$ E/p for E/p less than 8, but the disparity is considered unimportant for the purposes of this report. The water vapor data given by Healy and Reed (13), pages 81 and 86, or data cited by McDaniel (1), pages 537 and 547, are used to supplement the drift velocity data of Pack, Voshall, and Phelps for an E/p greater than 20, and are used as the free electron energy factor data for water vapor. These energy factor data of Healy and Reed were approximated by $K = 2.6$ $(E/p \div 10)^{2.5}$ for E/p between 10 and 17. For E/p greater than 17, K was read directly from the graph.

7. Results of Computations. For air and water vapor, the free electron drift velocity and corresponding electric field strength are presented in Table I as functions of the energy factor (or of the ratio of electron temperature to molecular temperature). The free electron drift velocity and corresponding electric field strength were computed as functions of the energy factor by means of the average electron method for several air-water vapor mixtures; the results are presented in Tables II, III, and IV for water vapor fractions of 0.5 percent, 1 percent, 1.6 percent, 2 percent, 3 percent, and 6 percent. These results are also portrayed in Figs. 1, 2, and 3 for water vapor fractions of 1 percent, 2 percent, 3 percent, and 6 percent.

Water vapor concentrations of 1 percent, 2 percent, 3 percent, and 6 percent in air represent saturation at $45^\circ$, $64^\circ$, $76^\circ$, and $98^\circ$ Fahrenheit, respectively.

8. An Analytic Approximation for Weak Electric Fields. An analytical approximation to the free electron mobility is obtainable for low electric field strengths, for air-water vapor mixtures.

Expand E/p in a Maclaurin series in W:

Table I. Data for Free Electron Drift Velocity and Corresponding Electric Field Strength as Functions of the Energy Factor for Air and Water Vapor

| K | E/p ($H_2O$) (v/cm/mm Hg) | W ($H_2O$) (x $10^6$ cm/sec) | E/p (air) (v/cm/mm Hg) | W (air) (x $10^6$ cm/sec) |
|---|---|---|---|---|
| 1.1 | 0.67 | 0.0473 | 0.00303 | 0.0438 |
| 1.2 | 1.33 | 0.0947 | 0.00606 | 0.0803 |
| 1.3 | 2.00 | 0.1420 | 0.00909 | 0.1110 |
| 1.4 | 2.67 | 0.1890 | 0.01212 | 0.1380 |
| 1.5 | 3.33 | 0.2370 | 0.01515 | 0.1610 |
| 1.6 | 4.00 | 0.2840 | 0.01818 | 0.1810 |
| 1.7 | 4.67 | 0.3310 | 0.02121 | 0.1980 |
| 1.8 | 5.33 | 0.3790 | 0.02424 | 0.2140 |
| 1.9 | 6.00 | 0.4460 | 0.02727 | 0.2280 |
| 2.0 | 6.67 | 0.4730 | 0.0303 | 0.2410 |
| 4.0 | 11.88 | 1.0370 | 0.0905 | 0.3870 |
| 4.5 | 12.45 | 1.1110 | 0.1145 | 0.4260 |
| 5.0 | 12.98 | 1.1830 | 0.1413 | 0.4740 |
| 5.5 | 13.49 | 1.3750 | 0.1710 | 0.5220 |
| 6.0 | 13.97 | 1.5810 | 0.2035 | 0.5680 |
| 6.5 | 14.43 | 1.8000 | 0.2388 | 1.6160 |
| 7.0 | 14.86 | 2.0250 | 0.2770 | 0.6630 |
| 7.5 | 15.28 | 2.2620 | 0.3180 | 0.7110 |
| 8.0 | 15.68 | 2.5070 | 0.3618 | 0.7580 |
| 8.5 | 16.06 | 2.7570 | 0.4084 | 0.8040 |
| 9.0 | 16.44 | 3.0370 | 0.4579 | 0.8530 |
| 10.0 | 17.00 | 3.5500 | 0.54 | 0.8800 |
| 15.0 | 19.00 | 5.4300 | 0.93 | 1.1200 |
| 20.0 | 20.80 | 6.6000 | 1.35 | 1.3500 |
| 30.0 | 22.70 | 8.0000 | 2.5 | 1.8300 |
| 40.0 | 25.20 | 8.8000 | 5.4 | 3.2000 |

Table II. Data for Free Electron Drift Velocity and Corresponding Electric Field Strength as Functions of the Energy Factor for Water Vapor Fractions of 1 and 2 Percent

| K | 99% Air, 1% $H_2O$ | | 98% Air, 2% $H_2O$ | |
|---|---|---|---|---|
| | E/p (v/cm/mm Hg) | W (x $10^6$ cm/sec) | E/p (v/cm/mm Hg) | W (x $10^6$ cm/sec) |
| 1.1 | 0.00968 | 0.0462 | 0.0163 | 0.0467 |
| 1.2 | 0.01940 | 0.0900 | 0.0327 | 0.0919 |
| 1.3 | 0.02920 | 0.13200 | 0.0492 | 0.1360 |
| 1.4 | 0.03910 | 0.17100 | 0.0658 | 0.1790 |
| 1.5 | 0.04920 | 0.21000 | 0.0825 | 0.2200 |
| 1.6 | 0.05930 | 0.24700 | 0.0984 | 0.2610 |
| 1.7 | 0.06970 | 0.28200 | 0.1160 | 0.3010 |
| 1.8 | 0.08010 | 0.31700 | 0.1340 | 0.3400 |
| 1.9 | 0.09070 | 0.35000 | 0.1510 | 0.3790 |
| 2.0 | 0.10200 | 0.38200 | 0.1690 | 0.4160 |
| 4.0 | 0.23400 | 0.67500 | 0.3590 | 0.7830 |
| 4.5 | 0.26600 | 0.70200 | 0.3980 | 0.7830 |
| 5.0 | 0.29800 | 0.73700 | 0.4370 | 0.8540 |
| 5.5 | 0.33900 | 0.80200 | 0.4870 | 0.9410 |
| 6.0 | 0.38500 | 0.87000 | 0.5410 | 1.0300 |
| 6.5 | 0.43400 | 0.93500 | 0.5990 | 1.1100 |
| 7.0 | 0.48500 | 0.99600 | 0.6590 | 1.1900 |
| 7.5 | 0.53900 | 1.0600 | 0.7230 | 1.2600 |
| 8.0 | 0.59600 | 1.1100 | 0.7910 | 1.3300 |
| 8.5 | 0.65600 | 1.1600 | 0.8610 | 1.4000 |
| 9.0 | 0.72000 | 1.2300 | 0.9360 | 1.4800 |
| 10.0 | 0.83900 | 1.2800 | 1.0800 | 1.5500 |
| 15.0 | 1.33000 | 1.5500 | 1.6500 | 1.8700 |
| 20.0 | 1.80000 | 1.7600 | 2.1700 | 2.0800 |
| 30.0 | 2.96000 | 2.1400 | 3.3700 | 2.4100 |
| 40.0 | 5.73000 | 3.3700 | 6.2100 | 3.5300 |

Table III. Data for Free Electron Drift Velocity and Corresponding Electric Field Strength as Functions of the Energy Factor for Water Vapor Fractions of 3 and 6 Percent

| K | 97% Air, 3% $H_2O$ | | 94% Air, 6% $H_2O$ | |
|---|---|---|---|---|
| | E/p (v/cm/mm Hg) | W (x $10^6$ cm/sec) | E/p (v/cm/mm Hg) | W (x $10^6$ cm/sec) |
| 1.1 | 0.0230 | 0.0469 | 0.0429 | 0.0471 |
| 1.2 | 0.0460 | 0.0927 | 0.0858 | 0.0937 |
| 1.3 | 0.0691 | 0.1380 | 0.1290 | 0.1400 |
| 1.4 | 0.0924 | 0.1820 | 0.1720 | 0.1850 |
| 1.5 | 0.1160 | 0.2240 | 0.2150 | 0.2310 |
| 1.6 | 0.1390 | 0.2680 | 0.2570 | 0.2760 |
| 1.7 | 0.1630 | 0.3100 | 0.3030 | 0.3200 |
| 1.8 | 0.1870 | 0.3510 | 0.3470 | 0.3640 |
| 1.9 | 0.2110 | 0.3920 | 0.3910 | 0.4080 |
| 2.0 | 0.2360 | 0.4320 | 0.4340 | 0.4510 |
| 4.0 | 0.4800 | 0.8410 | 0.8370 | 0.9220 |
| 4.5 | 0.5250 | 0.8800 | 0.9000 | 0.9730 |
| 5.0 | 0.5700 | 0.9220 | 0.9610 | 1.020 |
| 5.5 | 0.6270 | 1.0200 | 1.0400 | 1.160 |
| 6.0 | 0.6890 | 1.1200 | 1.1200 | 1.290 |
| 6.5 | 0.7540 | 1.2200 | 1.2000 | 1.420 |
| 7.0 | 0.8220 | 1.3100 | 1.2800 | 1.540 |
| 7.5 | 0.8940 | 1.4000 | 1.3700 | 1.660 |
| 8.0 | 0.9700 | 1.4900 | 1.4700 | 1.780 |
| 8.5 | 1.0500 | 1.5700 | 1.5700 | 1.890 |
| 9.0 | 1.1300 | 1.6600 | 1.6700 | 2.0100 |
| 10.0 | 1.3000 | 1.7500 | 1.8800 | 2.1700 |
| 15.0 | 1.9300 | 2.1200 | 2.6600 | 2.6900 |
| 20.0 | 2.5000 | 2.3500 | 3.3500 | 2.9700 |
| 30.0 | 3.7400 | 2.6400 | 4.7100 | 3.2300 |
| 40.0 | 6.3600 | 3.6800 | 7.2100 | 4.1000 |

Table IV. Data for Free Electron Drift Velocity and Corresponding Electric Field Strength as Functions of the Energy Factor for Water Vapor Fractions of 0.5 and 1.6 Percent

| K | 99.5% Air, 0.5% $H_2O$ |  | 98.4% Air, 1.6% $H_2O$ |  |
|---|---|---|---|---|
|  | E/p (v/cm/mm Hg) | W (x $10^6$ cm/sec) | E/p (v/cm/mm Hg) | W (x $10^6$ cm/sec) |
| 1.1 | .00636 | 0.0456 | 0.0137 | .0465 |
| 1.2 | .01270 | 0.9876 | 0.0274 | 0.0913 |
| 1.3 | .01920 | 0.0260 | 0.0412 | 0.1350 |
| 1.4 | .02570 | 0.1630 | 0.0551 | 0.1770 |
| 1.5 | .03240 | 0.1970 | 0.0692 | 0.2090 |
| 1.6 | .03910 | 0.2290 | 0.0834 | 0.2570 |
| 1.7 | .04590 | 0.2600 | 0.0978 | 0.2960 |
| 1.8 | .05290 | 0.2890 | 0.1120 | 0.3330 |
| 1.9 | .06000 | 0.3170 | 0.1270 | 0.3700 |
| 2.0 | .06620 | 0.3380 | 0.1420 | 0.4070 |
| 4.0 | .16700 | 0.5770 | 0.3090 | 0.7480 |
| 4.5 | .19500 | 0.6030 | 0.3460 | 0.7790 |
| 5.0 | .22500 | 0.6390 | 0.3820 | 0.815 |
| 5.5 | .26100 | 0.6960 | 0.4290 | 0.896 |
| 6.0 | .30100 | 0.7500 | 0.4800 | 0.973 |
| 6.5 | .34300 | 0.8050 | 0.5340 | 1.05 |
| 7.0 | .38800 | 0.8580 | 0.5920 | 1.12 |
| 7.5 | .43600 | 0.9110 | 0.6520 | 1.18 |
| 8.0 | .48700 | 0.9630 | 0.7160 | 1.26 |
| 8.5 | .54100 | 1.0100 | 0.7820 | 1.32 |
| 9.0 | .59800 | 1.0700 | 0.8530 | 1.39 |

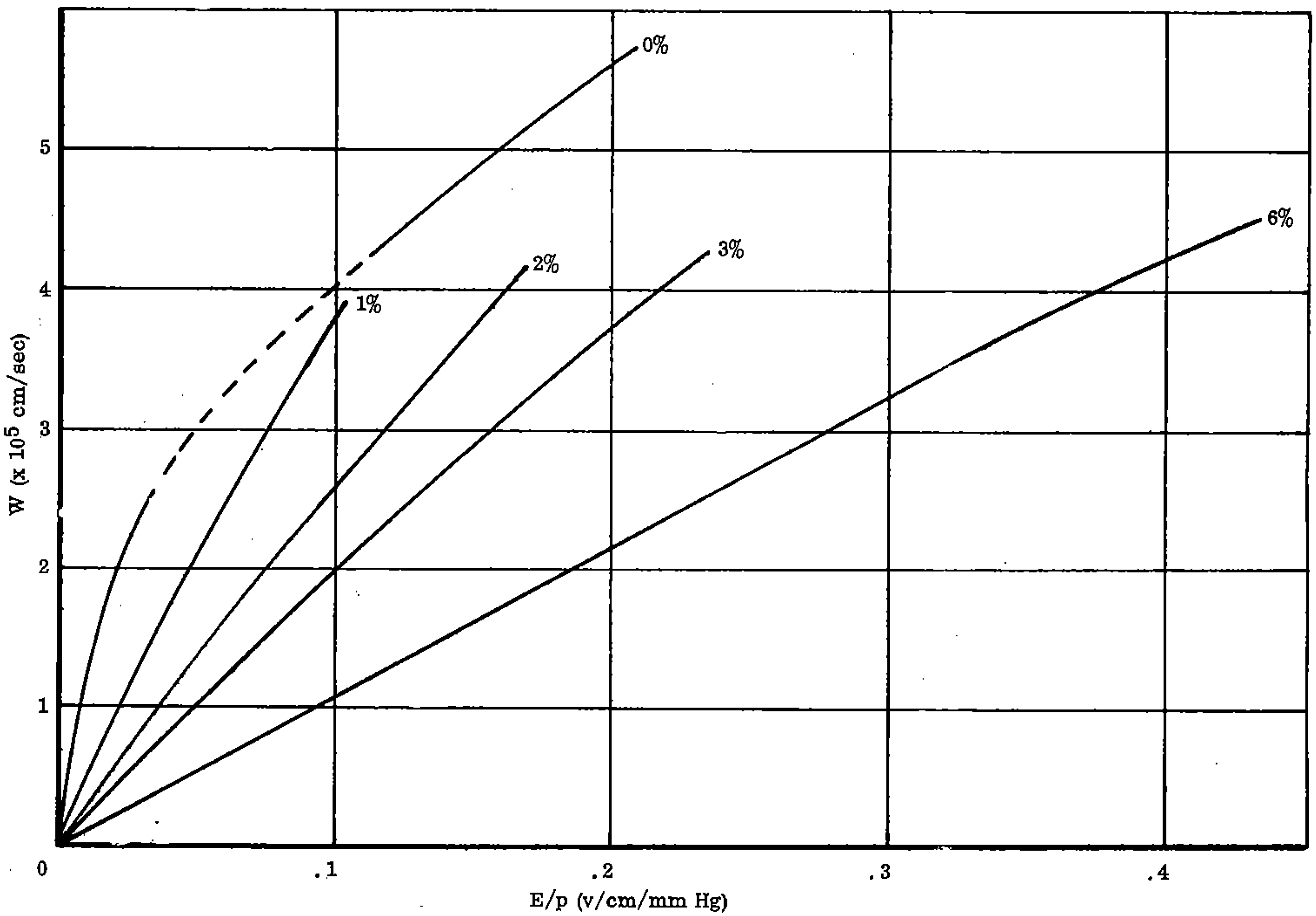

Fig. 1. Electron drift velocity for low electric field strengths.

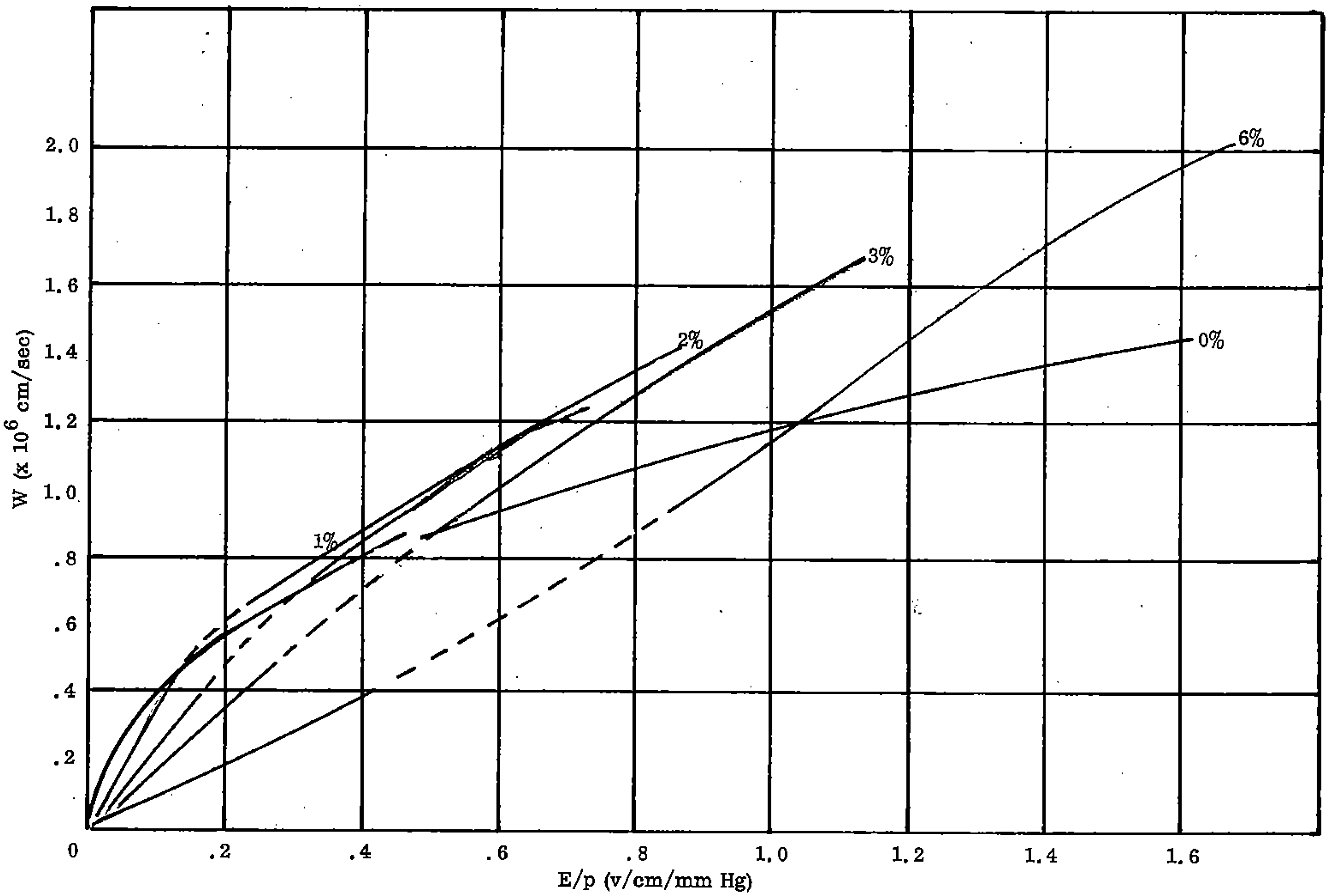

Fig. 2. Electron drift velocity for medium electric field strengths.

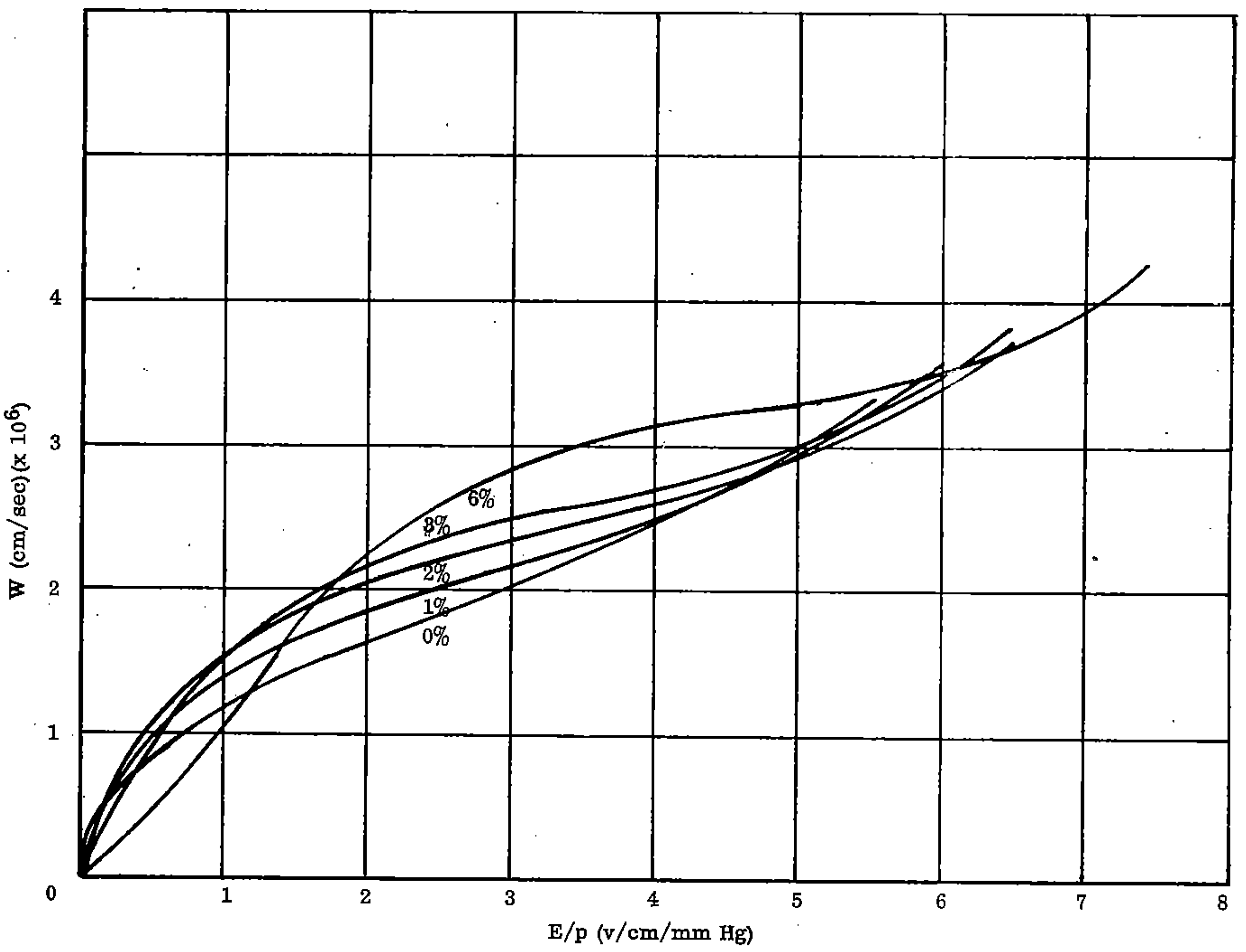

Fig. 3. Electron drift velocity for high electric field strengths.

$$\frac{E}{p}(W) = \frac{E}{p}(O) + \left(\frac{E}{p}\right)'(O) \cdot W + \ldots$$

Now, $\frac{E}{p}(K) = \sqrt{a(K) \cdot b(K)}$ and $W(K) = \sqrt{a(K)/b(K)}$,

where $a(K) = \left(\frac{E}{p}\right)W$ and $b(K) = \left(\frac{E}{p}\right) \div W$.

Clearly, $\left(\frac{E}{p}\right)'(W) = d\left(\frac{E}{p}\right) \div dK \cdot \frac{dK}{dW}$.

The notation (E/p)'(W) means the first derivative of E/p with respect to W, E/p being considered a function of W. E/p(O) and E/p)'(O) are the functions E/p and (E/p)' evaluated at W = 0.

By the chain rule,

$$\frac{d(E/p)}{dK} = \frac{\partial (E/p)}{\partial a} \cdot \frac{da}{dK} + \frac{\partial (E/p)}{\partial b} \cdot \frac{db}{dK},$$

and

$$\frac{dW}{dK} = \frac{\partial W}{\partial a} \cdot \frac{da}{dK} + \frac{\partial W}{\partial b} \cdot \frac{db}{dK}.$$

Thus,

$$\frac{d(E/p)}{dK} = \frac{1}{2}\,\frac{E/p}{a} \cdot \frac{da}{dK} - \frac{1}{2}\,\frac{E/p}{b} \cdot \frac{db}{dK},$$

and

$$\frac{dW}{dK} = \frac{1}{2}\,\frac{W}{a} \cdot \frac{da}{dK} - \frac{1}{2}\,\frac{W}{b} \cdot \frac{db}{dK}.$$

Therefore,

$$\frac{d(E/p)}{dK} \cdot \frac{dK}{dW} = \frac{(E/p)}{(W)} \cdot \frac{\frac{1}{a}\frac{da}{dK} + \frac{1}{b}\frac{db}{dK}}{\frac{1}{a}\frac{da}{dK} - \frac{1}{b}\frac{db}{dK}} ,$$

$$= b \cdot \frac{1 + \frac{a}{b} \cdot \frac{db}{dK} \cdot \frac{dK}{da}}{1 - \frac{a}{b} \cdot \frac{db}{dK} \cdot \frac{dK}{da}} ,$$

$$= b \quad \frac{1 + \frac{a}{b}\frac{db}{da}}{1 - \frac{a}{b}\frac{db}{da}}$$

By the chain rule,

$$\frac{db}{da} = \frac{\partial b}{\partial (E/p)} \cdot \frac{\partial (E/p)}{\partial a} + \frac{\partial b}{\partial W} \cdot \frac{\partial W}{\partial a} =$$

$$\frac{1}{W} \cdot \frac{1}{W} + \left( \frac{-E/p}{W^2} \right) \cdot \frac{1}{E/p} = 0 .$$

This result is also clear from the fact that a and b can be considered independent variables; hence,

$$\frac{db}{da} = 0 .$$

Consequently, $$\frac{d(E/p)}{dK} \cdot \frac{dK}{dW} = b .$$

The Maclaurin series then becomes

$$E/p(W) = E/p(O) + b\Big|_{W=0} \cdot W + \ldots \quad .$$

$$\text{Now } b\Big|_{W=0} = b\Big|_{K=1} = \frac{(E/p)}{W}\Big|_{K=1} = 6.29 \times 10^{-8} f_1 + 1.41 \times 10^{-5} f_2$$

where $f_1$ is the air fraction and $f_2$ is the water vapor fraction.

Therefore,

$$E/p = (6.29 \times 10^{-8} f_1 + 1.41. \times 10^{-5} f_2)\, W,$$

or

$$W = \frac{1.59 \times 10^7}{1+224\,\frac{f_2}{f_1}} \cdot E/p\,.$$

A similar result was obtained by Baran and Baum (14) who used a different theory.

## III. DISCUSSION

9. Evaluation. The method used in this report gives the same sort of answer as the theory set forth in Baum (4)(5) for low electric field strengths but markedly different results for high electric field strengths. Baum's theory apparently predicts that the free electron mobility in air is always decreased by the addition of water vapor (5). However, the method used in this report indicates that the free electron mobility will be decreased for low electric field strengths but will be increased for high electric field strengths. The reader is advised to compare experimental results for nitrogen and water vapor, shown in the Hurst, Stockdale, and O'Kelly (2) graph with Fig. 3 in this report.

# IV. CONCLUSIONS

10. <u>Conclusions.</u> It is concluded that:

a. The electron mobility in air is decreased by addition of water vapor for low electric fields, it is increased by addition of water vapor for high electric fields, and the change caused by addition of water vapor dwindles for still higher electric fields.

b. The electron mobility increases by 19 percent for 1 percent water vapor content which occurs for an E/p from 0.75 to 1.0; the crossover is at an E/p = 0.12 for 1 percent water vapor content.

c. The electron mobility increases by 32 percent for 2 percent water vapor content which occurs for an E/p near 1.0; the crossover is at an E/p = 0.33 for 2 percent water vapor content.

d. The electron mobility increases by 32 percent for 3 percent water vapor content which occurs for an E/p near 1.3; the crossover is at an E/p = 0.51 for 3 percent water vapor content.

e. The electron mobility increases by as much as 43 percent for 6 percent water vapor content which occurs for an E/p near 2.5; the crossover is at an E/p = 1.04 for 6 percent water vapor content.

f. The maximum increase is less for smaller water vapor contents and then occurs at lower electric field strengths.

g. Direct experimental measurements of the electron drift velocity in moist air are needed.

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# APPENDIX

## DERIVATION OF THE EQUATIONS USED IN THE AVERAGE ELECTRON METHOD

It may be helpful to derive the equations (1) and (2) used in the text. It is assumed that the free electron momentum loss rate resulting from collisions with a component (or species) of the gas is proportional to the partial pressure of that component of the gas. It is also assumed that the free electron energy loss rate resulting from a component of the gas is proportional to the partial pressure of that component. The total electron momentum or energy loss rate in a gas mixture is the sum of that resulting from the components of the gas. The free electrons will here be assumed to be essentially monoenergetic (the average electron model).

For equilibrium, the force applied by an electric field to a drifting free electron is e E. The momentum loss rate is the product of the collision frequency and the drift component of the electron momentum, as derived by Rossi and Staub (15). Their argument is strengthened by examination of the definition of the "momentum transfer collision frequency."

$$eE = \left(\frac{up}{\lambda}\right) mW$$

where m = electron mass

W = drift velocity

u = electron speed

p = gas pressure in atmospheres

and λ = electron mean free pathlength at 1 atmosphere pressure

$$eE = \sum_i \left(\frac{up_i}{\lambda_i}\right) mW .$$

The quantity $\frac{up_i}{\lambda_i}$ is the frequency of collision with the component labelled "i" in the gas mixture.

$$\frac{eE}{umW} = \sum_i \frac{p_i}{\lambda_i}$$

Now, $p_i$ is given by the proportion of the gases in the gas mixture. The $\lambda_i$ is the pathlength of an electron in 1 atmosphere pressure of the component labelled "i," for the electron speed u. Suppose experimental data exist for the drift velocity and electric field strength for that same electron speed u in 1 atmosphere pressure of the component labelled "i." Thus,

$$eE_i = \left(\frac{up'}{\lambda_i}\right) mW_i$$

where p' is 1 atmosphere, for these experimental data.

Clearly,

$$\frac{1}{\lambda_i} = \frac{eE_i}{up'mW_i}$$

So

$$\frac{eE}{umW} = \sum_i p_i \frac{eE_i}{up'mW_i} ,$$

$$\frac{E}{W} = \sum_i p_i \frac{E_i}{p'W_i} ,$$

$$\frac{1}{W}\left(\frac{E}{p}\right) = \sum_i \left(\frac{p_i}{p}\right)\left(\frac{E_i}{p'}\right)\frac{1}{W_i}$$

The variables $E_i/p'$ and $W_i$ are functions of the electron speed u and thus, functions of the Townsend energy factor K.

For equilibrium, the power applied by an electric field to a drifting free electron is $eEW$. The energy loss rate is the product of the collision frequency and the energy lost per collision. The energy lost per collision can be written as the production of the average fractional energy loss per collision and the excess of the electron energy over the thermal energy.

$$eEW = \left(\frac{up}{\lambda}\right)(h)\left(\mathfrak{D}/\mathcal{K} - kT\right),$$

where h is the electron fractional energy loss per collision, $\mathfrak{D}/\mathcal{K}$ is the ratio of the diffusion constant to the mobility and is equal to the electron energy, and kT is the thermal energy.

For a mixture of gases,

$$eEW = \sum_i \frac{up_i}{\lambda_i} h_i \left(\mathfrak{D}/\mathcal{K} - kT\right)$$

As before, $p_i$ is given by the proportion of the gases in the mixture. The $\lambda_i$ is the pathlength of an electron in 1 atmosphere pressure of the component labelled "i," for the electron energy $\mathfrak{D}/\mathcal{K}$. Suppose experimental data exist for the drift velocity and electric field strength for that same electron energy $\mathfrak{D}/\mathcal{K}$ in 1 atmosphere pressure of the component labelled "i." Thus,

$$eE_iW_i = \frac{up'}{\lambda_i} h_i \left(\mathfrak{D}/\mathcal{K} - kT\right),$$

where p' is 1 atmosphere, for these experimental data. Clearly,

$$\frac{h_i}{\lambda_i} = \frac{eE_iW_i}{up'\left(\mathfrak{D}/\mathcal{K} - kT\right)}$$

So

$$eEW = \sum_i up_i \left(\mathfrak{D}/\mathcal{K} - kT\right) \frac{eE_iW_i}{up'\left(\mathfrak{D}/\mathcal{K} - kT\right)},$$

$$e\,EW = \sum_i e\,p_i\,W_i\,(E_i/p'),$$

$$W\left(\frac{E}{p}\right) = \sum_i \left(\frac{p_i}{p}\right)\left(\frac{E_i}{p'}\right) W_i \quad .$$