Theoretical Notes

Note 162

POTENTIAL INDUCED ON A SPHERE DUE TO TRANSIENT RADIATION

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ABSTRACT

This paper documents an application of some work by Dr. Conrad Longmire concerning a first-order approximation to the potential developed on a sphere in free space due to a nuclear burst. The burst is assumed exoatmospheric and charge is assumed to be emitted from the sphere instantaneously. A study is done to compare the effect of several different X-ray spectra, yields, and photoelectric conversion efficiency. Results are given graphically as potential with respect to infinity versus range, and sensitivity of potential to changes in yield or conversion efficiency.

SECTION I

INTRODUCTION

Recent interest in system-generated electromagnetic pulse, often called IEMP, has spurred efforts to determine the fields and currents produced in an object due to X rays and γ rays from a nuclear burst. This paper documents an application of a first-order approximation proposed by Dr. Conrad Longmire (Ref. 1). Specifically, this paper deals with the potential with respect to infinity developed on a sphere as a function of X-ray spectra, weapon yield, and distance.

Special acknowledgement is given to Capt Clovis R. Hale for proposing the problem, and for his continued guidance and suggestions.

The following assumptions are made:

1. The model is a 3-meter diameter aluminum sphere.

2. For purposes of computing charge removal, the area from which charge is emitted is a 3-meter diameter disc perpendicular to the incident radiation.

3. The nuclear burst is exoatmospheric; i.e., there is no absorption of radiation between the burst and target.

4. All charge is emitted instantaneously; i.e., as a delta function in time.

5. All bursts are assumed to yield 4 megaton (MT) X rays in a Planckian distribution and .015 MT gammas (γ) at 1 MeV.

Results will be given as numerical solutions of two simultaneous equations.

SECTION II

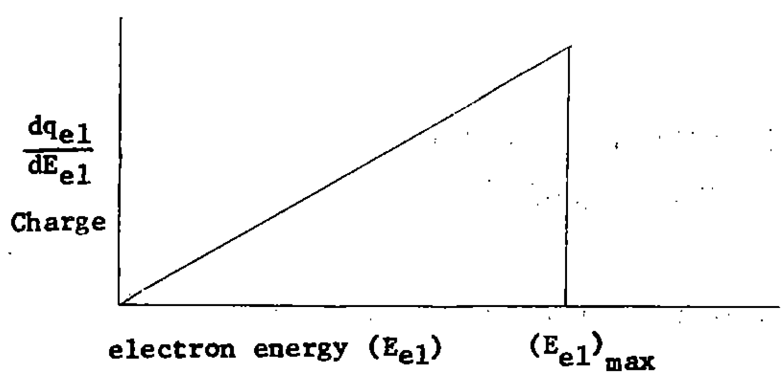
THEORY

1. Assume a discrete energy photon. The charge emitted from a surface, q, is just the number of incident photons per photon energy increment, $\frac{dN}{dE_\gamma}$, multiplied by some conversion efficiency as a function of photon energy, $K(E_\gamma)$. Discretely then, the charge emitted is

$$\Delta q = \frac{dN}{dE_\gamma} \left(K(E_\gamma) \right) \Delta E_\gamma \tag{1}$$

The conversion efficiency used will be that found by Kusnezov (Ref. 2) where the following qualifications were made: (a) the charge is moving either in or opposite to the direction of the beam; (b) electrons and photons are in equilibrium; i.e., conversion occurs in a sheet that is thin compared to a photon range and thick compared to an electron range.

2. From the work of Chadsey (Ref. 3) and Hale (Ref. 4), it is seen that a triangular function is the simplest function which approximates a wide range of measured and theoretically computed energy spectra. Use of this triangular function omits the Compton and Auger electrons from the spectrum but does include their number.



If the value of the most energetic electrons emitted $(E_{el})_{max}$ is y, the total electron charge dq_{el} is the area under the triangle and is equal to $\frac{1}{2}y(E_{el})_{max}$. As the predominant production method at the energies of interest is photoelectric, the energy of the most energetic electron emitted $(E_{el})_{max}$ is the photon energy E_γ less the 1.56 keV K edge of aluminum. Then

$$\Delta q = \frac{1}{2}y(E_\gamma - 1.56 \text{ keV}) \quad (2)$$

and

$$y = \left(\frac{dq_{el}}{dE_{el}}\right)\Bigg|_{(E_{el})_{max}} = \frac{2\Delta q}{(E_\gamma - 1.56 \text{ keV})} \quad (3)$$

Then for any electron energy E_{el}, the value of the differential charge emission is

$$\frac{dq_{el}}{dE_{el}} = \frac{2\Delta q \; E_{el}}{(E_\gamma - 1.56 \text{ keV})^2} \qquad \text{for } E_{el} \leq E_\gamma - 1.56 \quad (4)$$

and $\frac{dq_{el}}{dE_{el}} = 0 \qquad \text{for } E_{el} > E_\gamma - 1.56$

Then

$$dq_{el}(E_{el}) = \frac{2\frac{dN}{dE_\gamma}\left(K(E_\gamma)\right) E_{el} \; dE_\gamma dE_{el}}{(E_\gamma - 1.56 \text{ keV})^2} \quad (5)$$

and for total charge emitted having energy greater than Ve.

$$Q(Ve) = \int_{(Ve)}^{(E_{\gamma max}-1.56\ keV)} E_{el} \left\{ \int_{(Ve\ +\ 1.56\ keV)}^{(E_{\gamma max})} 2\ \frac{dN}{dE} \frac{K(E\ )}{(E_{\gamma}-1.56\ keV)^2} dE \right\} dE_{el} \tag{6}$$

3. The development so far has considered only the photoelectric effect. As the photon energy leaves the X-ray region, the Compton process begins to dominate. Instead of the maximum energy of the electron being just $E_{\gamma max}$-1.56 keV, the maximum electron energy is the electron energy at a 180 degree recoil angle. As 1 MeV gamma energy was assumed, the integration of equation (6) will be performed only to an electron energy of 800 keV, the maximum recoil energy.

4. If the incident photon spectrum is in calories per square meter as determined by energy yield Y and distance r, Q(Ve) is the total amount of charge emitted with energy greater than Ve. If A is the area of the disc perpendicular to the photon flux, the total charge emitted is controlled by the factor $\frac{YA}{4\pi r^2}$ which considers the spherical attenuation of energy from the burst.

5. If equation (6) is divided by a potential V, we get a new function f(V) such that

$$\frac{Q(Ve)}{V} \quad \frac{YA}{4\pi r^2} f(V) \tag{7}$$

which relates charge Q(Ve) to potential V.

6. Another relation between charge and potential is $Q(Ve) = CV$ where C is the capacitance of the sphere. From these two relationships we would like to find the value V_o which satisfies

$$f(V_o) = \frac{4 r^2 C}{YA} \qquad (8)$$

As f(V) is a function of V, but the right side of equation (8) is constant, V_o may be determined numerically. V_o is then the kinetic energy of the least energetic electron which can escape to infinity, and also the potential to which the sphere will charge with respect to infinity.

SECTION III

SAMPLE CALCULATION

Consider a 5.0 keV spectrum. Using

$$C=4\pi\epsilon_0 R \qquad (9)$$

where C is the capacitance of a sphere of radius R, for 10^2 kilometers distance $f(V)=7.42\times10^{-13}$. This value of f(V) corresponds to approximately 100 kilovolts potential on the Q(Ve)/V curve, Figure 1. Plotting volts potential versus radial distance from burst gives Figure 2.

SECTION IV

ANALYSIS

1. Figure 2 shows that it is possible to obtain an extremely high potential on a sphere. The maximum potential on the sphere is equal to the kinetic energy in electron volts of the most energetic electron emitted. At very close ranges such as 1 to 10 Km, the potential is constant, equal to the kinetic energy of the most energetic electron released from the 1-MeV gamma Compton interaction. However, although the potential is constant, the incident fluence is falling off as r^{-2}. As the distance from the burst increases to 10^2 Km, the charge on the sphere becomes insensitive to the gamma-induced charge and the potential decreases as r^{-2}. At ranges greater than 10 Km, the sphere sees the effect of the less energetic but much more numerous X rays. If instead of .015 MT energy for the gamma rays,the energy were closer to .005 MT the constant potential line from the gamma contribution would begin to fall off sooner so that the effect of the X-ray spectrum would be seen at shorter ranges. Figure 3 shows the effects of a change in the X-ray spectra. For this study four black-body spectra were used, each with a common .015 MT of gamma energy and 4 MT of X-ray energy. At any given range there is one black-body which produces the greatest potential on a sphere. For example, at a range of 40,000 Km, worst case is a 2-keV black-body.

2. Sensitivity to changes in yield or photoelectric conversion efficiency is shown in figure 4 using a section of the 5-keV black body-produced

potential. The change in potential is a function of slope, which in turn is a function of spectrum. The potential curves shift to the right by a factor of $\sqrt{Y_2/Y_1}$ where Y_2 and Y_1 are the respective yields. For example, an increase of a factor of 10 in either yield or photoelectric conversion efficiency gives less than a factor of 2 increase in potential at $4X10^3$ kilometers.

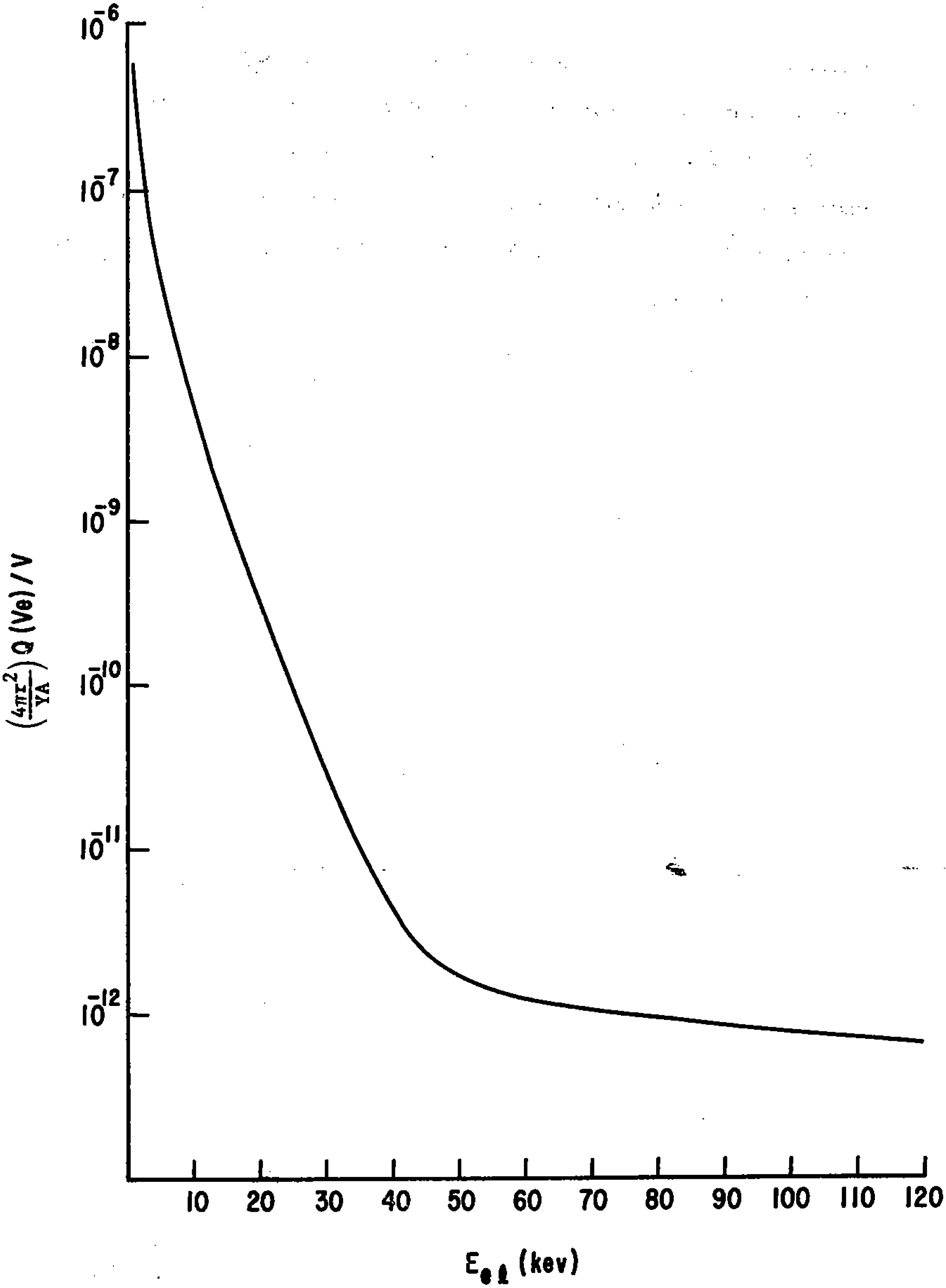


Figure 1. Charge greater than Energy Ve divided by Potential versus Electron energy.

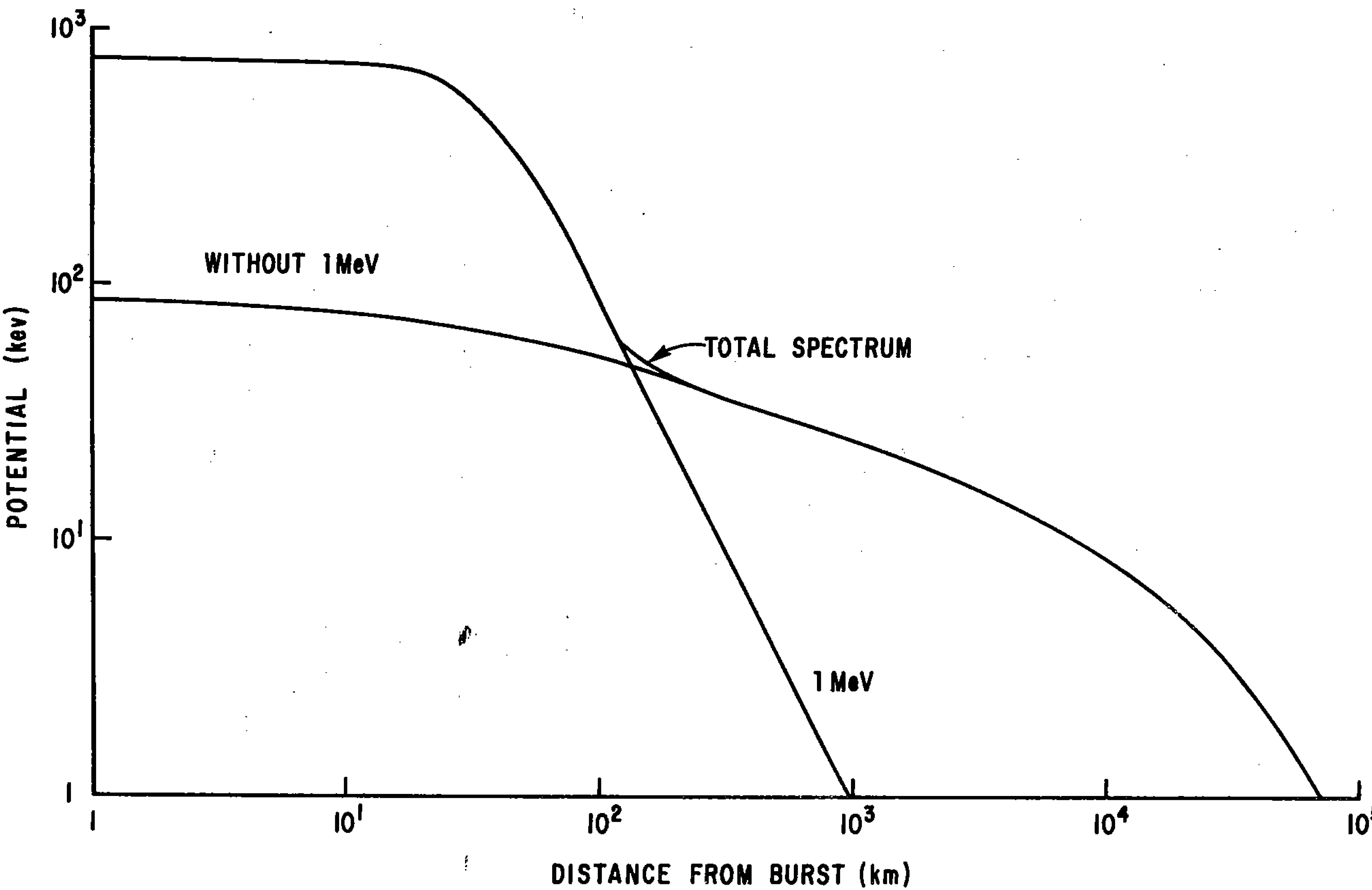


Figure 2. Potential versus Range for a 5 keV Black Body and a 1 MeV γ

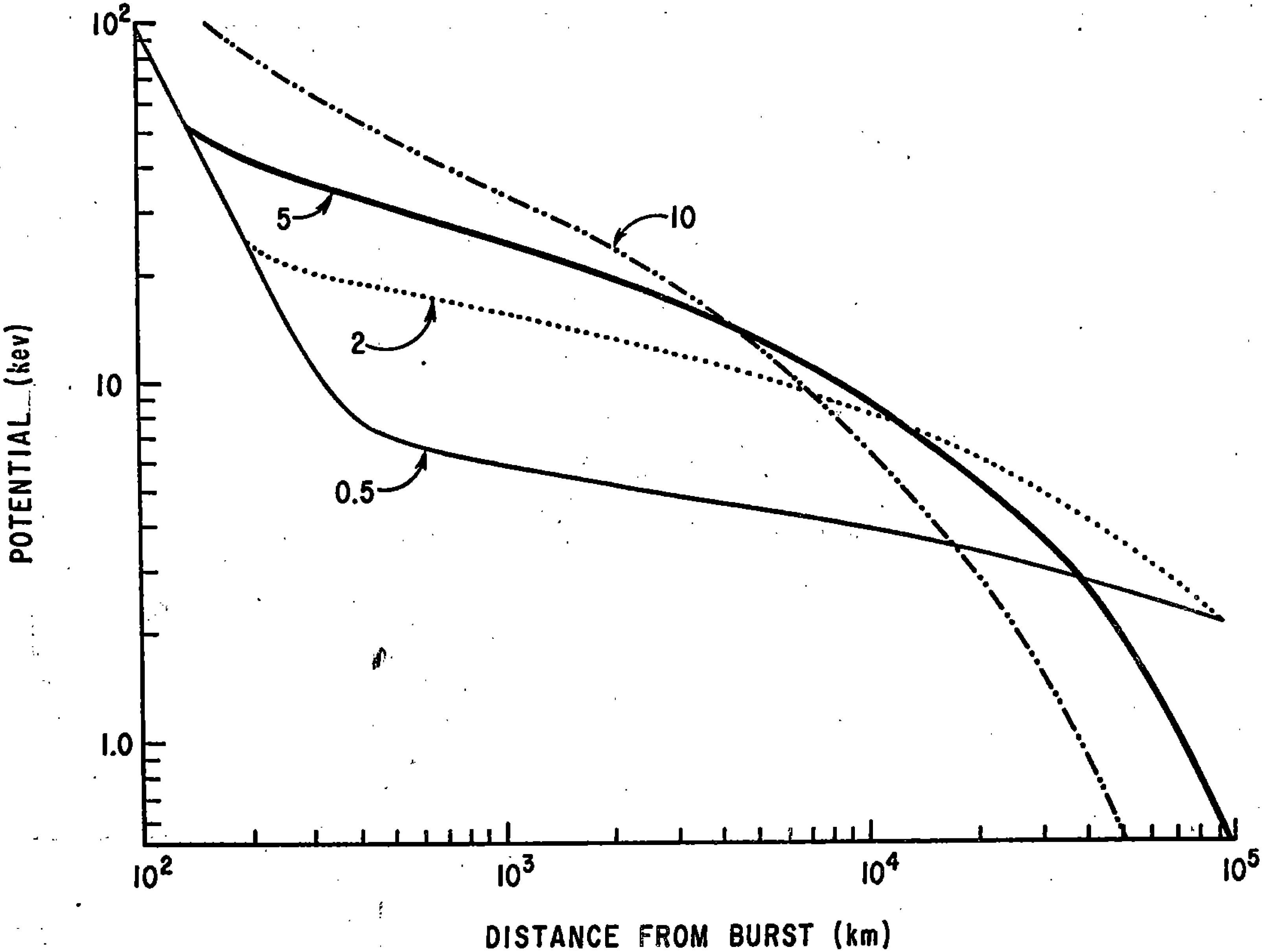


Figure 3. Potential versus Range for various spectra.

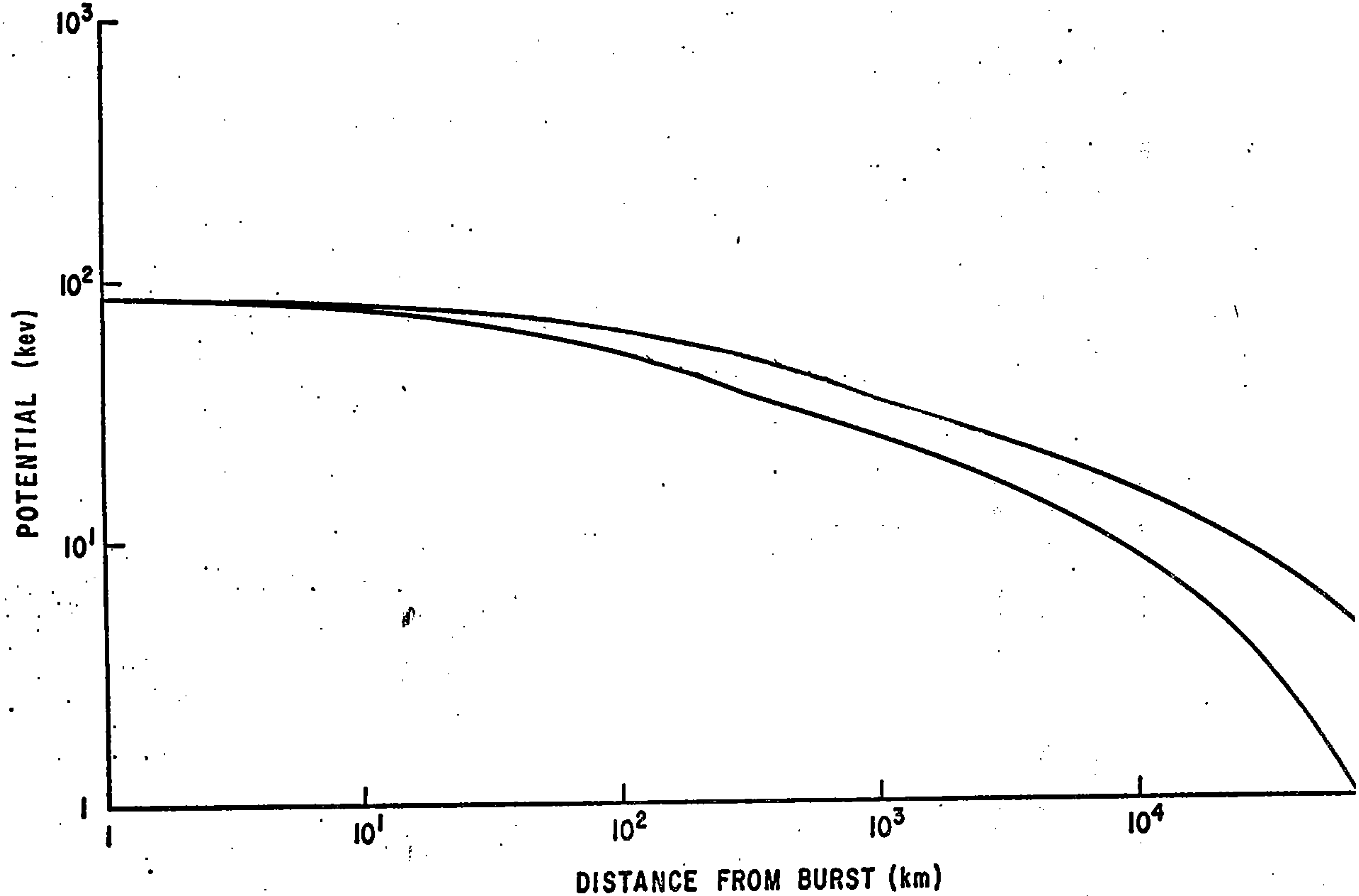


Figure 4. Effect of a factor of ten (10) increase in yield of photoelectric conversion efficiency for a 5 keV Black Body.

SECTION V

SUMMARY

A high potential can be induced on a sphere due to a nuclear burst. Four different spectra have shown a high dependence on the X-ray portion of the spectra at fluxes of 1 calorie/cm^2 or less. Potential is fairly insensitive to changes in yield or photoelectric conversion efficiency.

REFERENCES

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