EMP Theoretical Notes

Note 124

24 August 1971

EXTERNAL SYSTEM GENERATED EMP ON SOME TYPES OF SATELLITE STRUCTURE

Conrad L. Longmire
Mission Research Corporation

Work Supported by Air Force Weapons Laboratory

ABSTRACT

A model is developed for estimating currents and voltages induced by short pulses of x rays and γ rays on the external structure of satellites. A condition for the validity of the model is that the ejected electrons travel distances small compared with the satellite dimensions in the duration of the radiation pulse.

SECTION 1

INTRODUCTION

In this note we discuss some simple estimates of the electrical response of satellite structures to the x-ray and γ-ray pulses emitted by nuclear explosions.

We use the approximation that the x-ray and γ-ray pulses are very short in duration, which is valid for large structures. In the following sections of this report we discuss:

2 - x-ray and γ-ray sources;
3 - photon induced electron emission;
4 - the spectrum of electrons ejected by a spectrum of photons;
5 - essentials of satellite geometry in the large;
6 - the delta function approximation

An example is discussed in a classified addendum to this report.

The author is grateful to Dr. John Darrah for suggesting a problem to which an approximate solution could be found by analytical techniques. He also acknowledges helpful discussions with Capt. Clovis Hale (AFWL) and Dr. Jack Davis (EG&G), and is grateful to EG&G (Albuquerque) for quickly establishing suitable working arrangements.

SECTION 2

THE X-RAY AND Γ-RAY SOURCE

While both x rays and γ rays are electromagnetic quanta, and therefore identical in nature, it is conventional and useful to separate them according to their origin in the nuclear explosion.

From the widely known (unclassified) properties of fission of uranium and plutonium, one can easily deduce that, in the center of an efficient nuclear explosion, the temperature must be of the order of 10 kev and most of the energy must be in the form of electromagnetic radiation. This radiation is emitted by electrons (both free and bound), and is therefore called the thermal x-radiation. Some fraction of the x rays escape from the device. The emerging spectrum cannot be estimated easily, but must be determined by either detailed calculations or experiment. Actual spectra are classified. The major fraction of the device yield typically escapes in x rays.

The γ rays are emitted by nuclei which have been left in excited states by fission or by neutron reaction. The energy of the γ rays ranges from hundreds of kev to Mev. Only a fraction of the γ rays generated in the device escape to the outside. Typically, of the order of 0.1% of the device yield escapes in γ rays, and an average gamma energy of 1 Mev is often used. To find the actual gamma spectrum, again detailed calculations or measurements must be made.

Without seriously violating the definition in terms of origin, we may call the energy range below 100 kev the x-ray range, and the energy range above 100 kev the γ-ray range.

Let

$$x = \text{quantum energy in kev} \quad . \qquad (1)$$

We shall assume that we have been given an energy spectrum S(x), ergs/kev, so that

$$S(x)\ dx(\text{ergs}) = \text{energy in } dx \quad . \qquad (2)$$

The spectrum G(x) of photons is then

$$G(x)\ (\text{photons/kev}) = 6.25\times10^{8}\ \frac{S(x)}{x} \quad . \qquad (3)$$

Examples of these spectra are given in an addendum to this report (Ref. 1).

The x rays and γ rays are emitted by the nuclear device in a pulse with duration of the order of 10^{-8} second. Actual pulse shapes are classified; more information is given in Ref. 1.

If the radiation passes through air on its way from source to target, the amount, spectrum, and pulse shape of the radiation will be altered. For altitudes above 50 km γ rays are little affected, and above 100 km x rays are little affected. We shall assume that the intervening air is negligible. Then the spectral flux of photons F(x) at the target is

$$F(x) = \frac{G(x)}{4\pi R^2} \text{ photons/kev-unit area} \quad , \qquad (4)$$

where R is the distance from source to target. The effect of intervening air could be taken into account by modifying G(x) , if calculations of the transmission of the photons are available.

The quantities S(x) , G(x) , and F(x) as introduced above are time integrated. In some cases we shall need the corresponding time dependent quantities $s(x,t)$, $g(x,t)$, and $f(x,t)$, which are related to the former quantities by

$$F(x) = \int_{-\infty}^{\infty} f(x,t)\, dt , \quad \text{etc.} \tag{5}$$

SECTION 3

PHOTON-INDUCED ELECTRON EMISSION

When x rays or γ rays strike a material surface, electrons are ejected from the surface. If the object is thin enough to transmit some of the photons, electrons are ejected from both front and back surfaces.

The ejected electrons originate in several kinds of processes. The more important of these are the following:

(a) photoelectrons, ejected from atoms with simultaneous absorption of a photon;

(b) Compton recoil electrons, produced by scattering of a photon by an electron;

(c) Auger electrons, emitted by atoms in filling an inner shell vacancy previously created by photo effect or Compton recoil;

(d) knock-on electrons produced by an electron of type (a), (b), (c), or (d).

3.1 Γ-Ray Induced Emission

For photons in the γ-ray range, the Compton recoil process is dominant, except in materials with high atomic number. For a steady flux f_γ of γ rays passing through a material medium, there is an accompanying flux of Compton recoil electrons f_e , given by

$$f_e = 0.007\, f_\gamma \quad , \tag{6}$$

and this relation holds almost independently of the type of material and over a wide range of gamma energy. The flux of electrons makes an electric current density j ,

$$j(\text{amps/unit area}) = -1.1\times10^{-21} f_\gamma(\text{photons/unit area-sec}) \quad . \tag{7}$$

Integrating over time, we obtain the total charge q dislodged per unit area

$$q(\text{Coulombs/unit area}) = -1.1\times10^{-21}\ F_\gamma(\text{photons/unit area}) \quad . \tag{8}$$

Since the Compton recoil electrons are concentrated in angle near the forward direction of the original γ ray, the Compton current flows out the back surface, and not out of the front surface.

The stopping range of Compton recoil electrons in matter is much smaller (of the order of 1 gm/cm^2) than the mean free path of the γ rays (of the order of 30 gm/cm^2) that produce them. Hence the Compton recoil electrons that escape from the back surface are produced uniformly in depth near the back surface. Since Mev electrons lose energy at a rate which as approximately constant along their track, we expect the spectrum of emitted Compton electrons to be flat, in a crude approximation.

3.2 X-Ray Induced Emission

For photons in the x-ray range, the photoelectric effect (including subsequent Auger effect) is dominant. In this energy range, the maximum Compton recoil energy is only a small fraction of the photon energy. Data and calculations on emitted electrons are surveyed in a report edited by Clovis Hale (Ref. 2).

We shall discuss here the x-ray induced emission from aluminum, a typical satellite material. We consider x rays with quantum energy large compared with the K-edge (1.56 kev). The ejected electrons will then have energy near that of the incident x-ray, and the Auger energy is small and may be neglected.

The photoelectrons are ejected from the atom predominantly at angles near the plane perpendicular to the initial photon path. However, multiple scattering is a very strong effect for electrons in the energy range we are considering. The electron soon forgets its initial direction.

Let us now estimate the number of photoelectrons emitted per incident photon for a thick target. Consider first photoelectrons born just at the surface. About 1/2 of these electrons, those that are directed outwards, will leave the surface immediately. Of the other 1/2, which were initially directed into the target, a fraction of the order of 1/2 will leave the surface after multiple scattering. Thus we estimate 3/4 of photoelectrons born just inside the surface will escape.

Electrons born deeper in the target than one electron range will not escape. The escape probability will fall approximately linearly with depth of birth in the target.

The x-ray absorption length is long compared with the electron range. We therefore conclude that about 3/8 of the electrons born in the first electron range should escape. If $\mu(\mathrm{cm}^2/\mathrm{gm})$ is the x-ray mass absorption coefficient and $R_e(\mathrm{gm}/\mathrm{cm}^2)$ is the mean range of the resulting photoelectrons, then the photoelectric yield Y_{pe} is

$$Y_{pe}\left(\frac{\text{electrons}}{\text{incident photon}}\right) \approx \frac{3}{8}\mu R_e \quad . \qquad (9)$$

As a numerical example, consider 30-kev x rays. The absorption coefficient of Al is then $\mu = 1.25\ \mathrm{cm}^2/\mathrm{gm}$. The mean range of 30-kev electrons in Al is $R_e = 0.0011\ \mathrm{gm}/\mathrm{cm}^2$. We therefore estimate

$$Y_{pe} \approx \frac{3}{8}(1.25)(0.0011) \approx 0.0005\ \frac{\text{electrons}}{\text{incident photon}}$$

$$\approx 0.05\% \quad \text{(at 30 kev)} \quad . \qquad (10)$$

Russian measurements are quoted in Ref. 2, giving Y_{pe} = 0.4% at 30 keV on Al , with an angle of 30° between the surface plane of the target and the x-ray direction. Eq. (9) is for normal incidence. Since the photoelectrons are made twice as far from the surface at 90° incidence relative to 30° incidence, we may divide the Russian result by 2 to compare with the result (10). Still the Russian experimental result is four times larger than our estimate.

It is hard to believe that there could be many more <u>energetic</u> electrons escaping than our estimate (10). It is possible that the remainder of the observed electrons may be very low energy knock-on electrons. The experiments indicate that there may be many low energy electrons.

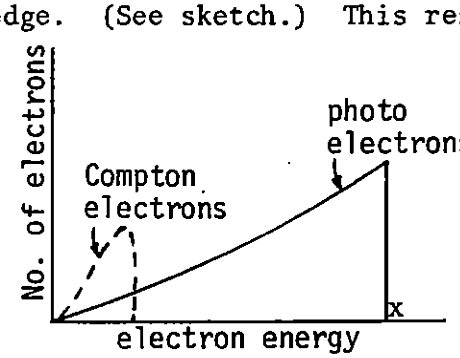
It is important for our purposes to know the number and spectrum of ejected electrons. While I have not made a thorough study of the existing data, I am bothered by the discrepancy noted above and believe it deserves some investigation. For the present, we shall use our estimate (10).

The dependence of Y_{pe} on the x-ray energy x can be understood from Eq. (9). Since $\mu \sim x^{-3}$ and $R_e \sim x^2$, we have (far from absorption edges)

$$Y_{pe} \sim \frac{1}{x} \quad . \tag{11}$$

The data quoted in Ref. 2 indeed have this dependence on x .

For the energy spectrum of the ejected electrons, the measurements, and Monte Carlo calculations by Chadsey (reviewed in Ref. 2), indicate a triangular spectrum, when the x-ray energy is large compared with the K-edge. (See sketch.) This result is also plausible in terms of the simple model discussed above. The Compton electrons form an additional triangle, at lower energy than the photoelectrons. We shall ignore the Compton electrons (from x rays) in this report, although this can not always be done with safety.



Following the discussion above, we take the emission function $\psi(w,x)$, the number of electrons emitted per unit energy w (kev) per incident photon of energy x, to be

$$\psi(w,x) = \eta \frac{w}{x^3} \quad (0 < w < x) \quad , \tag{12}$$

with

$$\eta = 0.03 \quad \text{for Al , one surface} \quad . \tag{13}$$

Then, as required

$$\int_0^x \psi(w,x)\, dw = 0.0005 \left(\frac{30}{x}\right) \quad . \tag{14}$$

If the target is thin for x-rays, but not for the photoelectrons, the result (12) should be doubled to account for emission from both front and back surfaces. In this connection, it may be useful to record the following formula for the mean range of electrons of energy E in aluminum:

$$R_e \ (\text{gm/cm}^2) \approx \frac{0.42\ E^2}{0.30+E} \quad (\text{E in Mev}) \quad . \tag{15}$$

This formula is valid over the energy range from a few kev to a few Mev.

For x rays approaching 100 kev, the photoelectric angular distribution is shifted appreciably in the forward direction of the incident x ray. This effect will make the front surface emission decrease a little, and the back surface emission increase a little. The effect is mollified by multiple scattering, and we shall ignore it for x rays.

SECTION 4

THE FOLDED ELECTRON DISTRIBUTION

In section 2 we introduced the time dependent flux $f(x,t)$ of x-rays at the target, x-rays per unit area per unit time. Folding this with the emission function $\psi(w,x)$, we obtain the net emission rate $\phi(w,t)$ of electrons of energy w per unit area of the target

$$\phi(w,t) = \int_{w}^{\infty} f(x,t)\psi\ (w,x)dx\ . \tag{16}$$

The corresponding time integrated energy distribution is:

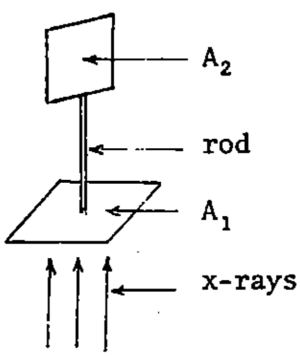
$$\phi(w) = \int_{w}^{\infty} F(x)\ \psi(w,x)\ dx\ . \tag{17}$$

In writing Eq.(16), we have assumed that the electrons leave the surface instantaneously after absorption of the x-ray. Since the electrons travel distances in the metal of the order of 10^{-2}cm at velocities greater than 10^{9}cm/sec, this is a good approximation for photo electrons from x-rays. It is also a good approximation for Compton recoil electrons from γ-rays.

SECTION 5

GEOMETRICAL CONSIDERATIONS

A satellite is likely to have appended to it several structures of large area. For example, there may be a microwave dish facing down toward the earth, and solar cell paddles facing the sun or other directions. These structures may be connected by support members of small diameter.



For our purposes, we may epitomize the structure by the simple arrangement shown in the sketch. The area A_1 is broadside to the incident radiation, while the area A_2 is nearly parallel to the radiation. Therefore, more electrons are ejected from A_1 than from A_2.

Some of the ejected electrons escape to large distances, leaving the satellite charged and at a potential with respect to space. The positive charge on the satellite redistributes itself, sharing the charge between A_1 and A_2. A current flows in the rod while the redistribution is occurring.

The time dependence of the current pulse is controlled by several factors, including:

(a) the duration of the x-ray and γ-ray pulse;

(b) the time required for the escaping electrons to reach a distance somewhat greater than the satellite length;

(c) the natural ringing period of the structure, as determined by the inductance of the rod and the capacitance between A_1 and A_2.

[illegible] simultaneously [illegible], in [illegible], require a computer program. However, for large structures we can at least separate the factor (a) from the other two.

For present purposes, a structure will be called large if the ejected electrons move a distance that is only a small fraction of the structure dimension in the duration of the x-ray pulse. Then, for currents and voltages on the structure in the large, we may assume that the x-ray pulse is a delta function. This assumption allows a simple treatment of subsequent motion of the ejected electrons, as will be discussed in the following section.

For structures that are small, there are other approximations one can use to get estimates of currents and voltages. We hope to take up this problem in subsequent work.

We note that, for short pulses, there will be a redistribution current even if A_2, as well as A_1, is normal to the incident x-rays, when one of the areas is struck first by the x-rays.

SECTION 6

THE DELTA FUNCTION APPROXIMATION

Imagine that all of the ejected electrons leave the surface of the area A_1 simultaneously. Then, if the surface is not too convoluted, the electrons will be ordered (approximately) in space, at any later time, according to their initial kinetic energy. Electrons with larger initial kinetic energy will be farther from the surface. Thus a given electron moves in the electric field produced by all the electrons that have higher energy than the given electron. From this fact we can easily estimate the amount of electron charge that escapes to infinity.

Equation (17) gives the energy spectrum $\Phi(w)$ of the electrons ejected per unit area. From $\Phi(w)$ we can calculate the total charge $q(w)$ emitted per unit energy; except for a minus sign,

$$q(w) \text{ (Coulombs/kev)} = 1.6 \times 10^{-19} A_1 \Phi(w), \tag{18}$$

or, from Eq. (4) .

$$q(w) = \frac{A_1}{4\pi R^2} Q(w), \tag{19}$$

where

$$Q(w) \text{ (Coulombs/kev)} = 1.6 \times 10^{-19} \int_w^\infty G(x) \psi(w,x)\, dx . \tag{20}$$

Note that $Q(w)$ is the charge spectrum that would be produced if all of the x-rays from the device fell upon the target.

According to the discussion at the beginning of this section, we also need the charge q(>w) associated with electrons having energy greater than w,

$$q(>w) \text{ (Coulombs)} = \frac{A_1}{4\pi R^2} \; Q(>w) \tag{21}$$

where

$$Q(>w) \text{ (Coulombs)} = \int_w^\infty Q(w) \, dw \; . \tag{22}$$

Now let C be the capacitance (farads) between the area A_1 and infinity. Then the work done against the electric field by an electron of energy w in escaping to infinity is

$$\text{work(ev)} = \frac{q(>w)}{C} \; . \tag{23}$$

The last electron to escape will be that one for which the work done is just equal to its initial kinetic energy. Let

$$V\text{(volts)} \equiv 10^3 \; w\text{(kev)} \; , \tag{24}$$

so that V is the voltage that an electron has to fall through to obtain w kev of energy. Then the condition on the last electron to escape is

$$V\text{(volts)} = \frac{q(>V)}{C} \tag{25}$$

From Eq.(21), this criterion can be written

$$\frac{Q(>V)}{V} = \frac{4\pi R^2}{A_1} \; C \; . \tag{26}$$

For a given x-ray wource spectrum, the function Q(>V)/V can be calculated and graphed as a function of V. Then for a given distance R, area A_1, and capacitance C, the value of V which satisfies Eq.(26) can be read from the graph. The value of V so determined will equal the energy (in ev) of the last electron to escape to infinity. It will also equal the voltage of the area A_1 with respect to infinity, after all electrons with energy less than V have returned to A_1. Furthermore, the charge that escapes to infinity is just

$$q(>V) = C\ V\ . \tag{27}$$

The capacitance C can usually be estimated with sufficient accuracy for our purposes. The capacitances to infinity of a sphere and of a disk, both of radius r cm, are

$$C\ (\text{picofarads}) = 1.11\quad r(\text{cm})\quad (\text{sphere}) \tag{28}$$

$$C\ (\text{picofarads}) = 0.71\quad r(\text{cm})\quad (\text{disk}) \tag{29}$$

The capacitances of objects with the same surface area tend to be the same; the capacitances of a sphere and a disk with the same surface area are in the ratio

$$\frac{C(\text{sphere})}{C\ (\text{disk})} = \frac{\pi}{2\sqrt{2}} \approx 1.11\ (\text{same area}) \tag{30}$$

As stated in section 5, the positive charge left on A_1 will distribute itself over the rest of the structure. For example, if A_1 and A_2 are approximately equal, half the charge will eventually transfer to A_2. In fact, the charge may oscillate for a while, damped by resistance in the structure and by radiation. The ringing frequency can be estimated in terms of the capacitance between A_1 and A_2 and the inductance of the structure coupling them.

The charge cannot transfer earlier than the time required for most of the escaping electrons to move beyond the distance between A_1 and A_2. This time can be estimated by using the velocity going with the energy V(ev) of the last electron to escape. If this time is longer than the ringing time, the ringing will be suppressed.

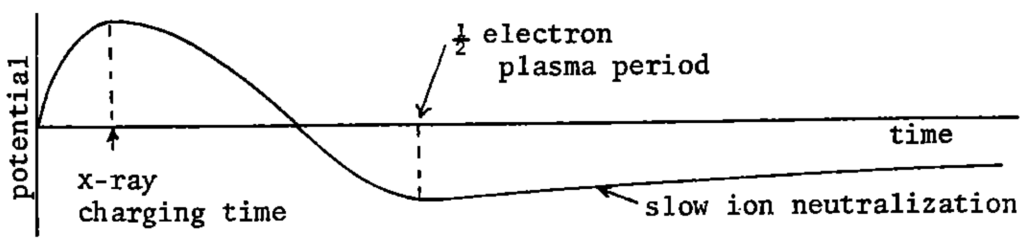
The positive charge left on the satellite will be neutralized eventually by electrons drawn from the surrounding ambient plasma. Neutralization may be expected to occur at the plasma frequency

$$\omega_p \text{ (radians/sec)} \approx 0.6 \times 10^5 \sqrt{N_e}$$

where N_e is the ambient electron density, electrons/cm^3.
At synchronous satellite altitudes, N_e ranges between 10 and 100 electrons/cm^3, so that

$$\omega_p \approx (2 \text{ to } 6) \times 10^5 \text{ radians/sec} \qquad (32)$$

The time dependence of the satellite potential may show the following interesting effect. The potential starts out positive, after ejected electrons escape. Ambient electrons are drawn into the satellite, reducing its potential. At the instant the potential becomes zero, some electrons are still moving inwards, and continue to hit and become stuck in the satellite. Thus the potential becomes negative, such as to repel further electrons. Since the satellite cannot emit electrons freely, the potential then remains negative until positive ions neutralize it, on a longer time scale. This behaviour is illustrated in the sketch below.



It should be noted that satellites normally assume a non-zero potential due to impacts of charged particles from the surrounding plasma.

REFERENCES

2. EMP Theoretical Note 121, REVIEW OF INTERNAL EMP TECHNOLOGY, edited by: Capt. C. R. Hale, Air Force Weapons Laboratory, April 1971, Unclassified.