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A Standard Ionosphere for the Study of Electromagnetic Pulse Propagation

Capt. Michael A. Messier
Air Force Weapons Laboratory

Abstract

Two ionosphere models are introduced for the purpose of providing standards for EMP propagation research and exoatmospheric environment simulator studies. The two models are realistic in their structure and represent extremes which bound the ionospheres one would normally expect to find on a given twenty four hour day. Both electron density and electron collision frequency profiles are shown. In addition, a number of simple propagation formulae are included to aid in the evaluation of the effects of these ionospheres on wave propagation.

1. Introduction

This note introduces two model ionospheres which the author proposes as standards for the study of electromagnetic pulse (EMP) propagation. This note was inspired by the need for a standard model for studying the variation in fields that an exoatmospheric environment simulator would have to reproduce. Such a model serves just as well for other EMP propagation studies.

The two profiles chosen represent a daytime sunspot maximum ionosphere and a nighttime sunspot minimum ionosphere. They are based on ionospheres published by Johnson[1] and consist of characteristic points in tabular form, assuming exponential variation between these points. This form is simple and allows easy standardization. Its biggest drawback occurs when a wave treatment of propagation is attempted, since the gradient discontinuities introduce unrealistic reflections and the exponential variation causes mathematical difficulties in analytical calculations. When these problems are encountered, the reader can revert back to Johnson's profiles or use a more suitable representation, such as a spline fit.

The final section of this note includes some simple formula for the study of high frequency propagation. Also included are a number of graphs showing the variation of the ionospheric cutoff frequency with altitude and launch angle.

2. Ionosphere Profiles and Characteristics

Table 1 lists the characteristic points of both the daytime and nighttime ionospheres, including both electron density and collision frequency. These are plotted in figure 1 showing the exponential variation between points. Figure 2 is a plot of the integral of the electron density along vertical and earth tangential paths, for both ionospheres, as a function of altitude. These integrals are useful in calculating signal phase, delay, and dispersion as indicated in section 3. The total

Nighttime, Sunspot Minimum

Altitude (km)	Electron Density (m^{-3})	Electron Collision Frequency (Hz)
80	1(8)	4.7(5)
90	1(9)	2.2(5)
100	2(9)	1.0(5)
150	2(8)	5(2)
200	9(8)	4.5(1)
225	1(10)	2(1)
250	7(10)	1(2)
275	1.9(11)	4(2)
300	2(11)	5(2)
350	1.8(11)	4(2)
500	3(10)	1(2)
700	1.6(10)	4(1)
1000	8.5(9)	2.5(1)

Daytime, Sunspot Maximum

Altitude (km)	Electron Density (m^{-3})	Electron Collision Frequency (Hz)
60	1(8)	6(5)
100	4(8)	1.1(5)
130	2(11)	6(3)
160	3(11)	1.5(3)
230	6(11)	6(2)
280	2(12)	1.4(3)
1000	1(11)	8(1)

Table 1

Ionosphere Profiles

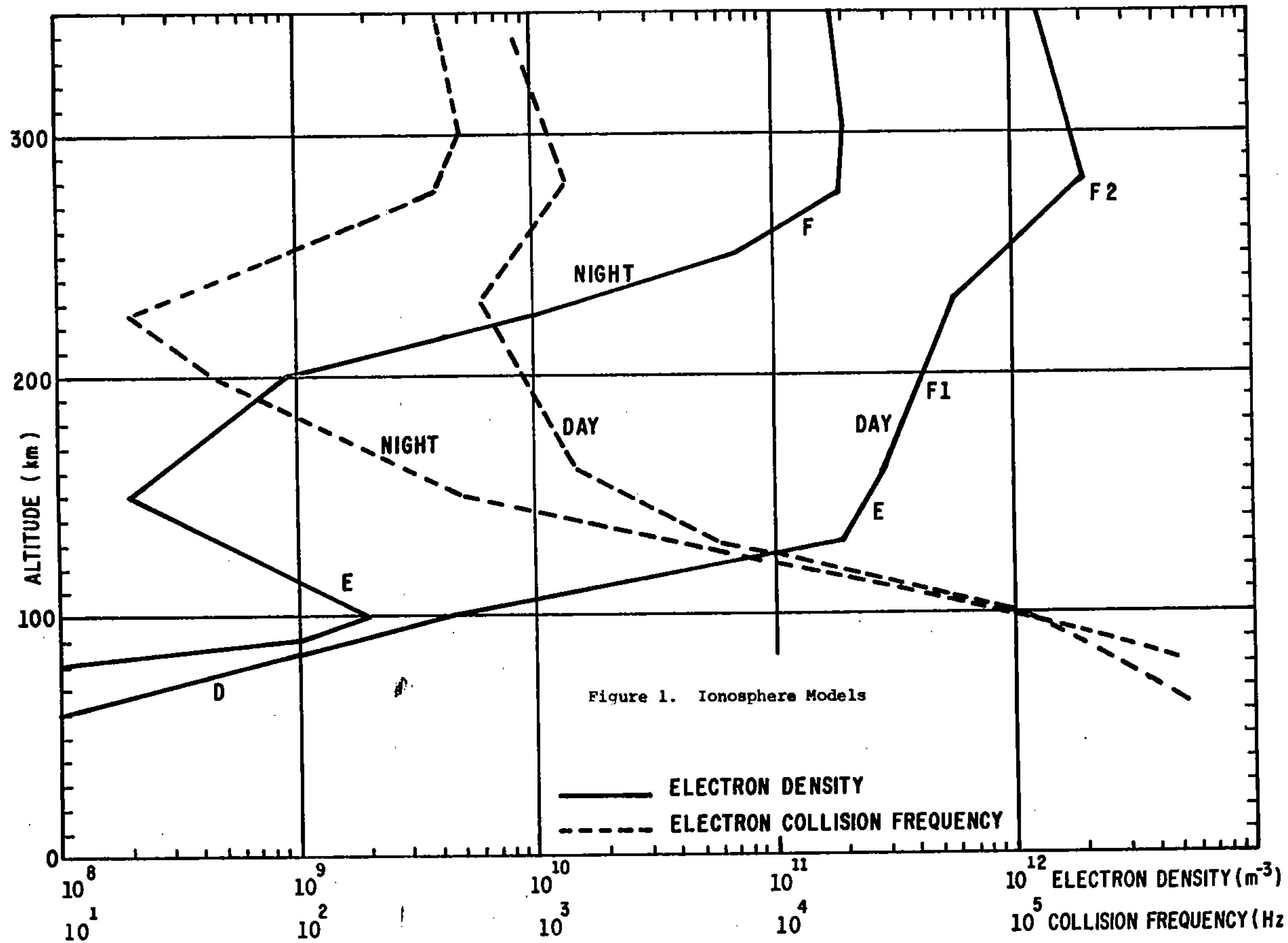
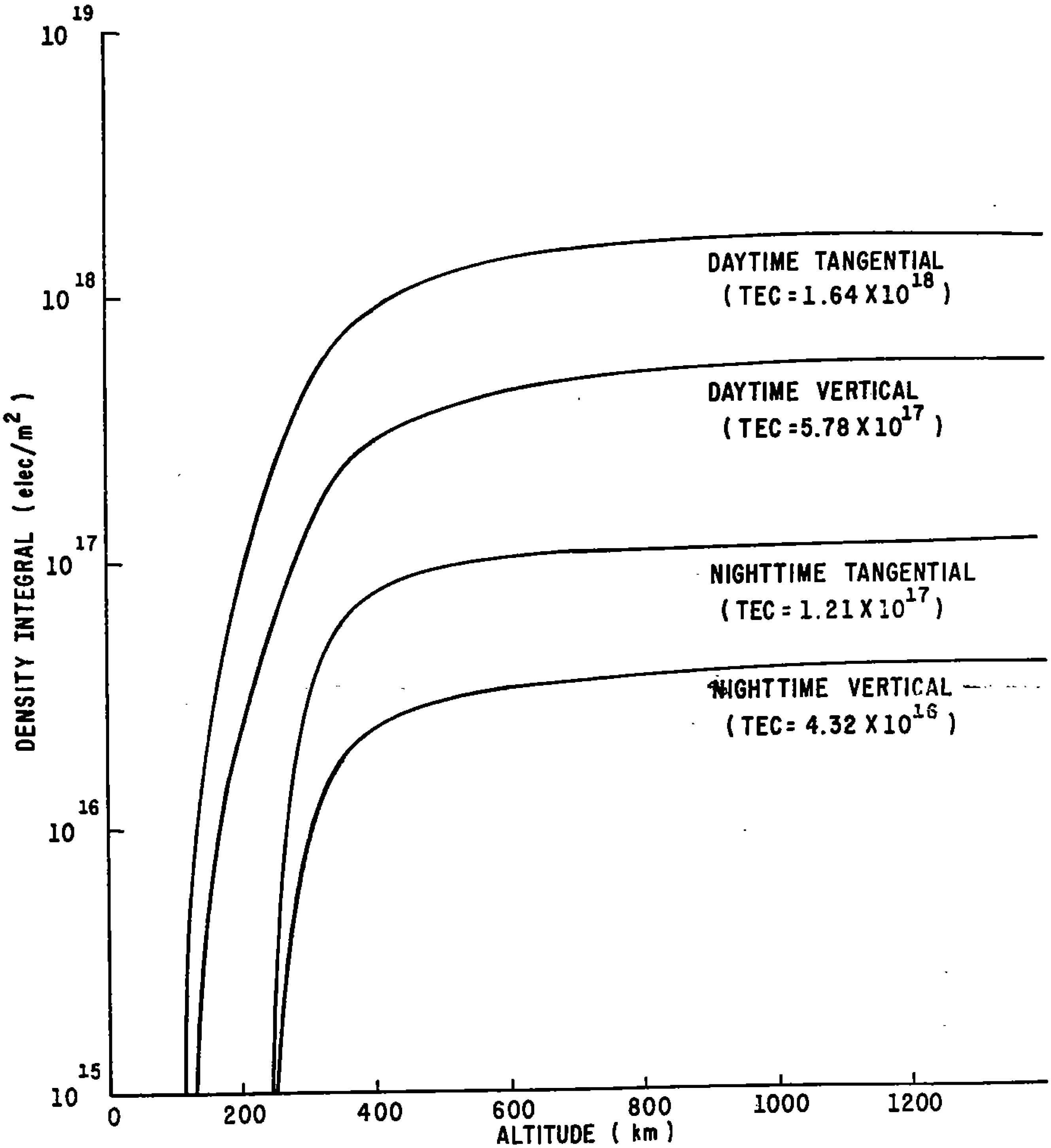


Figure 1. Ionosphere Models

Figure 2. Electron Density Integral vs. Altitude



electron content (TEC), ie, the integral to infinity, is shown alongside each curve.

For purposes of discussion, let the exponential variation of electron density and collision frequency be represented by the following equations:

$$N = N_o \exp(a_N h)$$

$$\nu = \nu_o \exp(a_\nu h) \quad ,$$

where

h is the altitude

N is the electron density

ν is the electron collision frequency

and N_o, a_N, ν_o, and a_ν are altitude dependent parameters. Table 2 lists these parameters for the intervals between characteristic points for the nighttime ionosphere. Table 3 lists these parameters for the daytime ionosphere. When extrapolating beyond the endpoints of the model, use parameters for the end intervals. These parameters assume that the altitude is in kilometers.

3. Propagation Formulas

The propagation of a wave through the ionosphere, for the frequencies of interest, can be approximated by considering that of a ray with the geometric attenuation (r_o/r) of a spherical wave superimposed. The transfer function of the ionosphere can be calculated by tracing single frequencies and determining the relative phase and amplitude.

Let ℓ by the distance along the propagation path. A plane wave is described by

Altitude Interval (km)	N_o (m^{-3})	a_N (km^{-1})	ν_o (sec)	a_ν (km^{-1})
80-90	1.000(0)	2.303(-1)	2.039(8)	-7.591(-2)
90-100	1.953(6)	6.931(-2)	2.656(8)	-7.885(-2)
100-150	2.000(11)	-4.605(-2)	4.000(9)	-1.060(-1)
150-200	2.195(6)	3.008(-2)	6.859(5)	-4.816(-2)
200-225	3.874(0)	9.632(-2)	2.956(4)	-3.244(-2)
225-250	2.478(2)	7.784(-2)	1.024(-5)	6.438(-2)
250-275	3.225(6)	3.994(-2)	9.537(-5)	5.545(-2)
275-300	1.081(11)	2.052(-3)	3.436(1)	8.926(-3)
300-350	3.763(11)	-2.107(-3)	1.907(3)	-4.463(-3)
350-500	1.177(13)	-1.195(-2)	1.016(4)	-9.242(-3)
500-700	1.444(11)	-3.143(-3)	9.882(2)	-4.581(-3)
700-1000	7.000(10)	-2.108(-3)	1.198(2)	-1.567(-3)

Table 2

Ionosphere Exponential Fit Parameters
Nighttime, Sunspot Minimum

Altitude Interval (km)	N_o (m^{-3})	a_N (km^{-1})	ν_o (sec^{-1})	a_ν (km^{-1})
60-100	1.250(7)	3.466(-2)	7.643(6)	-4.241(-2)
100-130	4.032(-1)	2.072(-1)	1.787(9)	-9.696(-2)
130-160	3.451(10)	1.351(-2)	2.438(6)	-4.621(-2)
160-230	6.153(10)	9.902(-3)	1.218(4)	-1.309(-2)
230-280	2.360(9)	2.408(-2)	1.217(1)	1.694(-2)
280-1000	6.412(12)	-4.161(-3)	4.261(3)	-3.975(-3)

Table 3

Ionosphere Exponential Fit Parameters
Daytime, Sunspot Maximum

$$E = E_o \exp\left[i\left(\omega t - \int_o^L k d\ell\right)\right], \qquad (1)$$

where

$$k = k_r - ik_i \ . \qquad (2)$$

Substitution in the wave equation,

$$\nabla^2 E = \varepsilon\mu_o \frac{\partial E^2}{\partial t^2} + \sigma\mu_o \frac{\partial E}{\partial t} , \qquad (3)$$

yields

$$k^2 = \frac{1}{\lambdabar_o^2}\left[1 - i \frac{\sigma}{\varepsilon_o \omega}\right] , \qquad (4)$$

where

$\lambdabar_o = c/\omega$

ω = signal frequency (radians/sec)

σ = conductivity

ε_o = permittivity of free space.

Either ε or σ can be considered complex quantities for propagation in a conducting medium. Let

$$\sigma = \sigma_r - i\sigma_i \ , \qquad (5)$$

and

$$\sigma' = \frac{\sigma}{\varepsilon_o \omega} \ . \qquad (6)$$

From Johnson,

$$\sigma = \frac{Nq^2}{m}\left[\frac{1}{\nu+i\omega}\right] \tag{7}$$

$$= \varepsilon_o \omega_p^2\left[\frac{\nu}{\nu^2+\omega^2} - i\,\frac{\omega}{\nu^2+\omega^2}\right], \tag{8}$$

where

ν = electron collision frequency (radians/sec)

q = electron charge (1.602×10^{-19} coulomb)

N = electron density (m^{-3})

m = electron mass (9.108×10^{-31} kg)

ω_p = plasma frequency (radians/sec).

The plasma frequency is related to the electron density by

$$\omega_p^2 = \frac{q^2 N}{\varepsilon_o m} = 3178N \tag{9}$$

Solving for the real and imaginary parts of the wave number, k, we find

$$k_i = \frac{\sqrt{1 - \sigma_i'}}{\sqrt{2}\lambda_o}\left\{\sqrt{1 + \left(\frac{\nu}{\omega}\right)^2\left(\frac{\sigma_i'}{1-\sigma_i'}\right)^2} - 1\right\}^{1/2} \tag{10}$$

$$k_r = \frac{\sqrt{1 - \sigma_i'}}{\sqrt{2}\lambda_o}\left\{\sqrt{1 + \left(\frac{\nu}{\omega}\right)^2\left(\frac{\sigma_i'}{1-\sigma_i'}\right)^2} + 1\right\}^{1/2}, \tag{11}$$

where

$$\sigma_i' = \frac{\omega_p^2}{\nu^2+\omega^2} . \tag{12}$$

It can be seen that k_i is the attenuation index and k_r is the phase index. k_r is related to the index of refraction by

$$n = \lambda_o k_r . \tag{13}$$

For small collision frequencies and $\omega >> \omega_p$,

$$n = \sqrt{1 - \sigma_i'} = \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2} \tag{14}$$

and

$$k_i = \frac{\omega_p^2}{2c} \frac{\nu}{\nu^2+\omega^2} \tag{15}$$

or

$$k_i = \frac{q^2}{2c\varepsilon_o m} \frac{N\nu}{\nu^2+\omega^2} . \tag{16}$$

For an isotropic lossless medium, the phase or wave velocity is given by

$$v_p = \frac{c}{n} \tag{17}$$

and the group velocity of a small band of frequencies is given by

$$v_g = nc . \tag{18}$$

The fact that these velocities are frequency dependent leads to the dispersion of a pulse as it propagates through the ionosphere. For frequencies much greater than the plasma frequency,

$$v_p \approx c\left[1 + \frac{1}{2}\left(\frac{\omega_p}{\omega}\right)^2\right] \tag{19}$$

$$v_g \approx c\left[1 - \frac{1}{2}\left(\frac{\omega_p}{\omega}\right)^2\right] . \tag{20}$$

The wave phase is given by

$$\phi_p = \omega\int_o^L \frac{d\ell}{v_p} \tag{21}$$

and the dispersed wave phase is given by

$$\phi_{pd} = \phi_p - \frac{\omega L}{c} ,$$

where $d\ell$ is an element of the propagation path and L is the total path length. For high frequencies, then

$$\phi_{pd} = \frac{1}{2c\omega}\int_o^L \omega_p^2 d\ell = \frac{q^2}{2c\omega\varepsilon_o m}\int_o^L N d\ell . \tag{22}$$

This last formula shows the importance of the density integral curve. Similarly, the group phase is given by

$$\phi_g = \omega\int_o^L \frac{d\ell}{v_g} . \tag{23}$$

For high frequencies, then, the dispersed group phase is given by

$$\phi_{gd} = -\phi_{pd} \quad (24)$$

When an isotropic lossless plasma is considered, the dispersed group delay is given by

$$\tau_g = \frac{\phi_{gd}}{\omega} = \frac{1}{c}\int_o^L \frac{d\ell}{n} , \quad (25)$$

or for high frequencies,

$$\tau_g \approx \frac{1}{2c\omega^2}\int_o^L \omega_p^2 d\ell . \quad (26)$$

Because the index of refraction varies with altitude, the path direction is curved. Let θ denote the angle from the vertical and let θ_L denote the launch angle. θ is determined at any point along the path by Snell's law. In a planar ionosphere, Snell's law has the familiar form

$$n_o \sin\theta_L = n \sin\theta , \quad (27)$$

where n_o is the index of refraction at the launch point, usually equal to unity. When a spherical ionosphere is considered, Snell's law takes the form

$$n_o R_o \sin\theta_L = nR \sin\theta , \quad (28)$$

where R and R_o are the radial distances to the points of interest. Snell's law can be used to change the variable of integration from ℓ to the vertical distances Z (planar ionosphere) or R (spherical ionosphere). Thus, with $n_o = 1$,

$$d\ell = \frac{dZ}{\cos\theta} = \frac{ndZ}{\sqrt{n^2 - \sin^2\theta_L}} \quad (29)$$

or

$$d\ell = \frac{dR}{\cos\theta} = \frac{n dR}{\sqrt{n^2 - \left(\frac{R_o}{R}\right)^2 \sin^2\theta_L}} . \quad (30)$$

Consider a wave launched at some angle, θ_L, into a planar ionosphere. The group delay is given by

$$\tau_g = \frac{1}{c}\int_o^Z \frac{dZ}{d \cos\theta} = \frac{1}{c}\int_o^Z \frac{dZ}{\sqrt{n^2 - \sin^2\theta_L}} .$$

The delay will be infinite at altitudes for which $n^2 \leq \sin^2\theta_L$, or

$$\omega_p \geq \omega \cos\theta_L .$$

Then, the cutoff frequency is given by

$$\omega_c = \frac{\omega_p}{\cos\theta_L} . \quad (31)$$

Similarly, for a spherical ionosphere,

$$\omega_c = \frac{\omega_p}{\sqrt{1 - \left(\frac{R_o}{R_m}\right)^2 \sin^2\theta_L}} , \quad (32)$$

where R_m is the radial distance to the ω_p considered. Figures 3 and 4 show plots of altitude versus cutoff frequency for the two spherical ionospheres using several launch angles. Figure 5 shows minimum ionosphere penetration frequency versus launch angle for the two spherical ionospheres. Also included is a curve showing the same calculation for a nighttime planar ionosphere to show where the two models diverge.

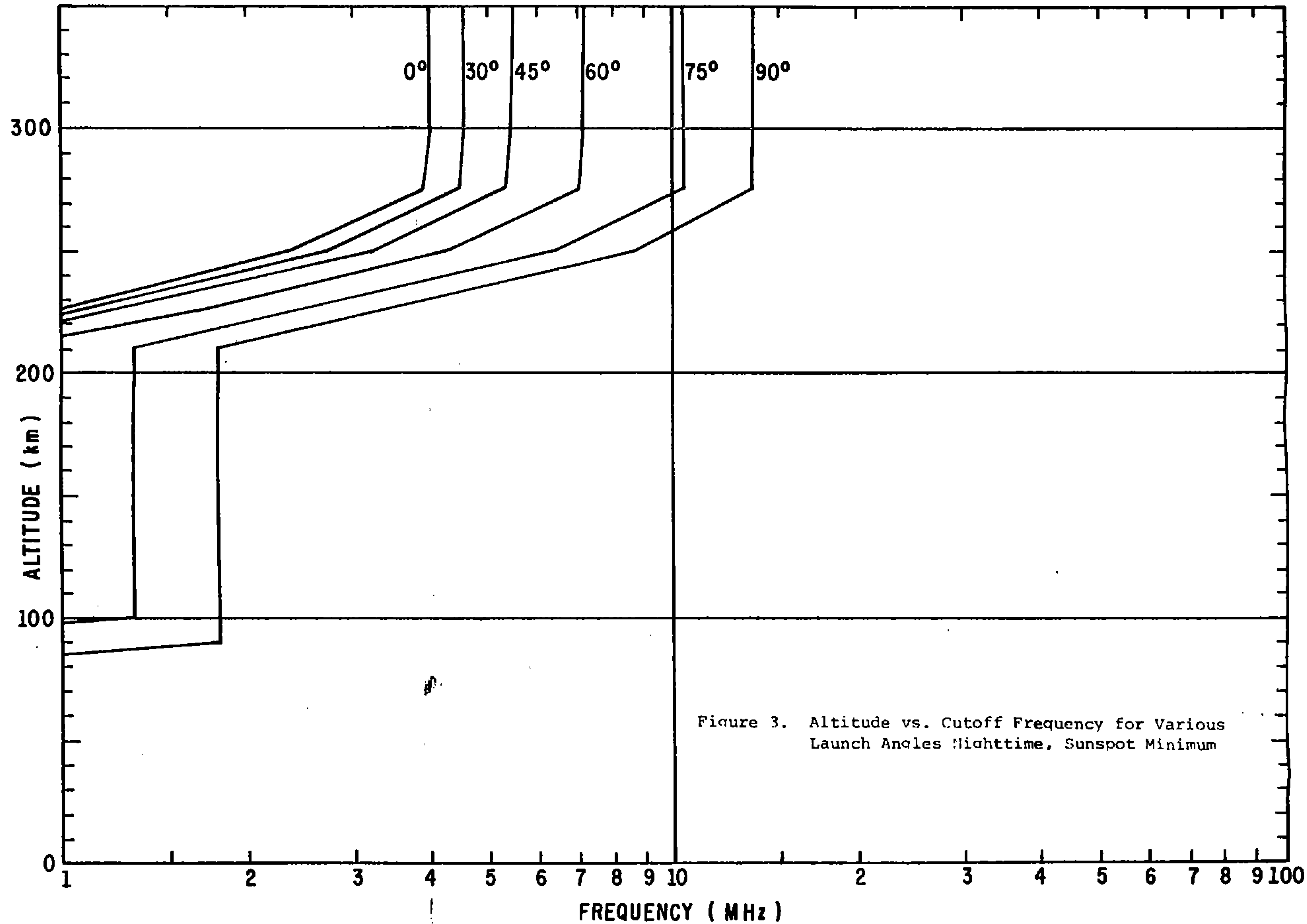


Figure 3. Altitude vs. Cutoff Frequency for Various Launch Angles Nighttime, Sunspot Minimum

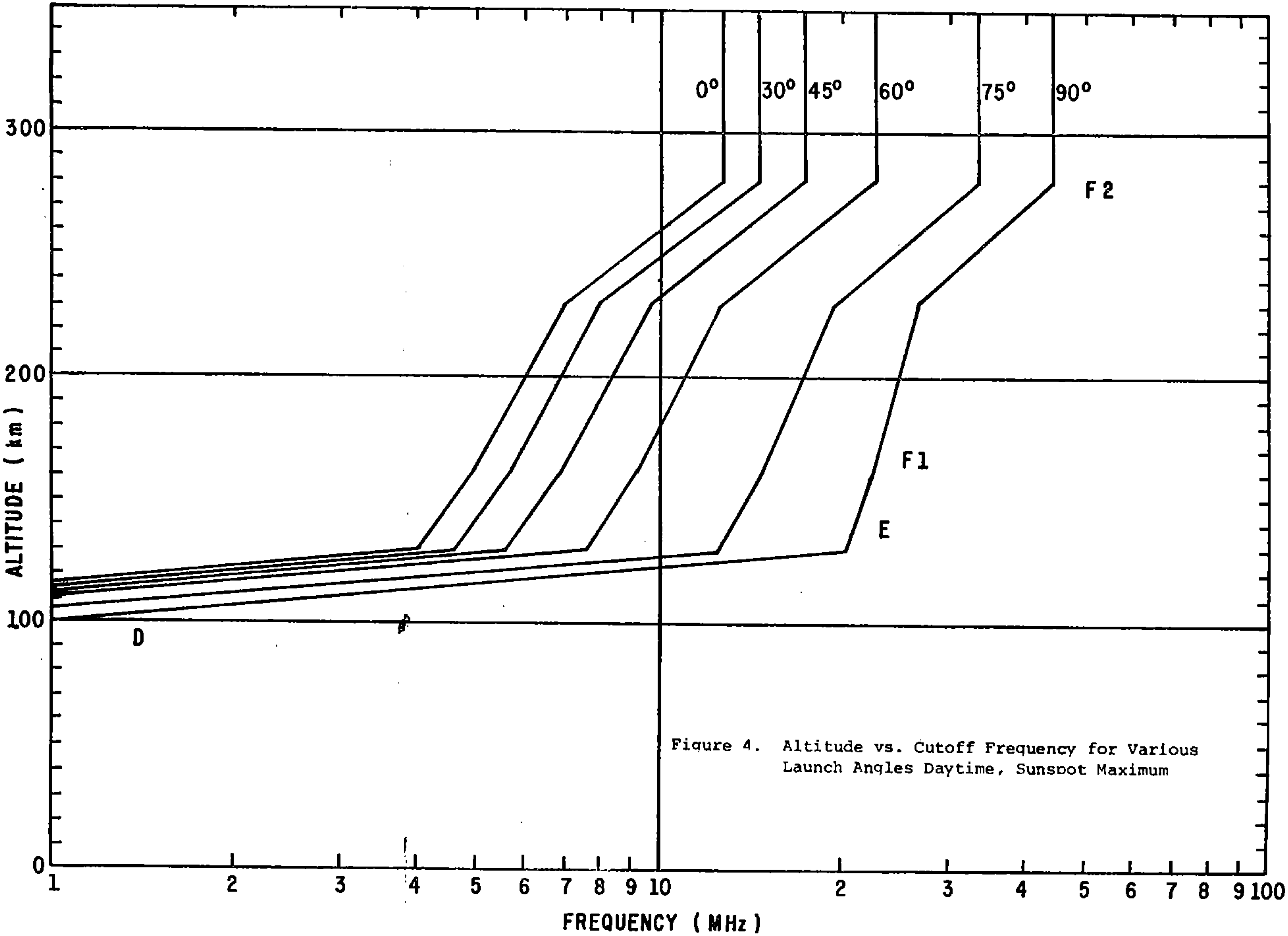
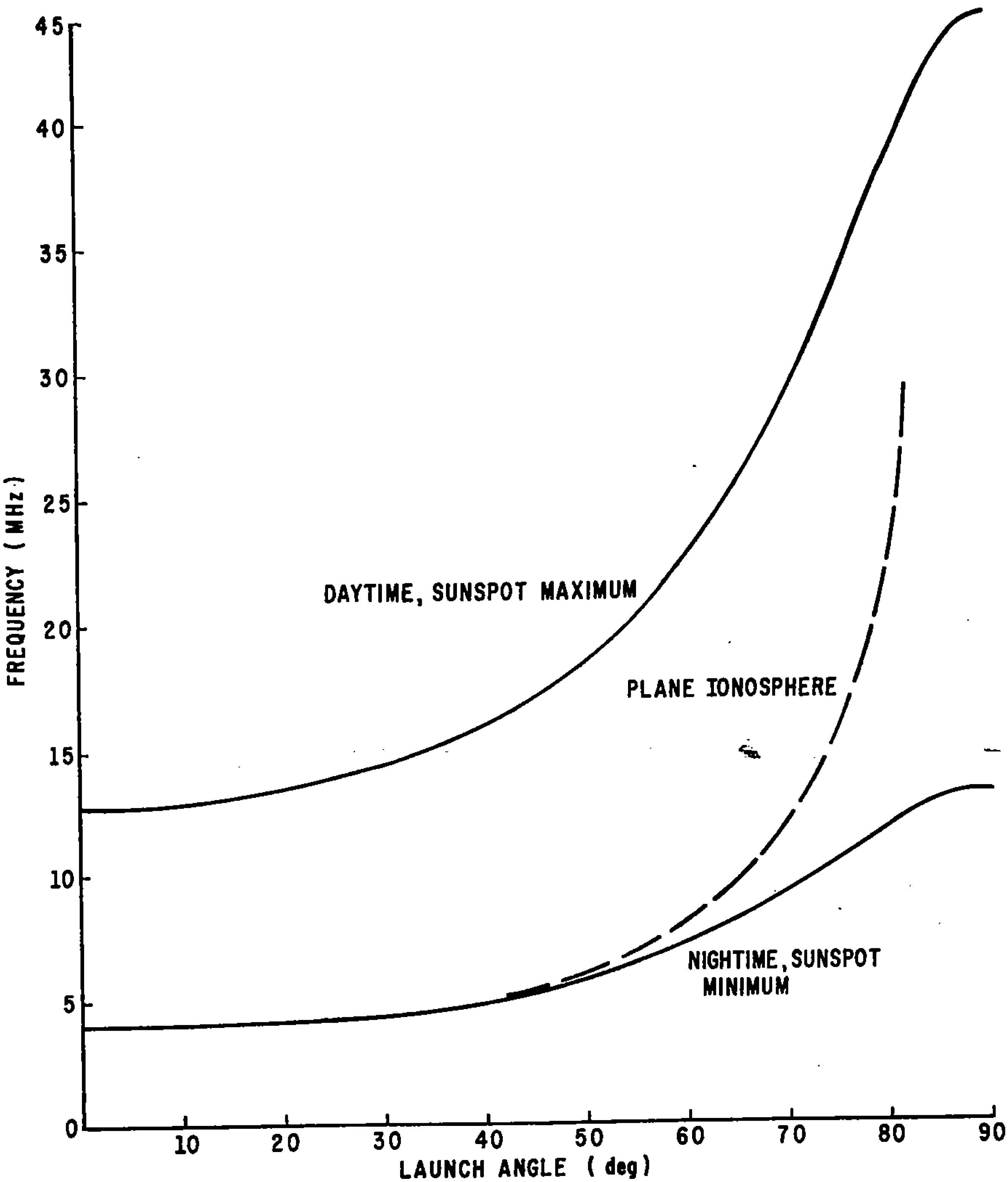


Figure 4. Altitude vs. Cutoff Frequency for Various Launch Angles Daytime, Sunspot Maximum

Figure 5. Minimum Ionosphere Penetrating Frequency vs. Angle of Launch



The presence of a real conductivity in the ionosphere (due to electron collisions and resulting energy loss) means that absorption will occur. The attenuation is given by

$$A = \exp\left[-\int_o^L k_i d\ell\right] , \tag{33}$$

or, in decibels by

$$A(db) = 1.44 \times 10^{-8} \int_o^L \frac{\omega_p^2 \nu}{\nu^2+\omega^2} d\ell \tag{34}$$

and

$$A(db) = 4.85 \times 10^{-5} \int_o^L \frac{N\nu}{\nu^2+\omega^2} d\ell . \tag{35}$$

The last equation, in terms of Hertz instead of radians per second (ω) is

$$A(db) = 7.28 \times 10^{-6} \int_o^L \frac{N\nu}{\nu^2+f^2} d\ell , \tag{36}$$

where it is assumed that ν is now in units of Hertz. Using this equation, the attenuation rate, in decibels/meter, is given by

$$A(db/m) = 7.28 \times 10^{-6} \left[\frac{N\nu}{\nu^2+f^2}\right] . \tag{37}$$

From the electron collision frequency profiles it can be seen that $\nu^2 << f^2$ in most of the ionosphere so that

$$A(db/m) \approx 7.28 \times 10^{-6} \frac{N\nu}{f^2} . \tag{38}$$

Figure 6 shows the quantity $7.28 \times 10^6 N\nu$ or f^2*A(db/m) plotted as a function of altitude. Figure 7 shows the integral of this quantity as a function of altitude, or f^2*A(db). In both cases, only the vertical path is considered.

References

1. Johnson, Francis S., 1965. "Satellite Environment Handbook (Second Edition)," Stanford University Press, California.

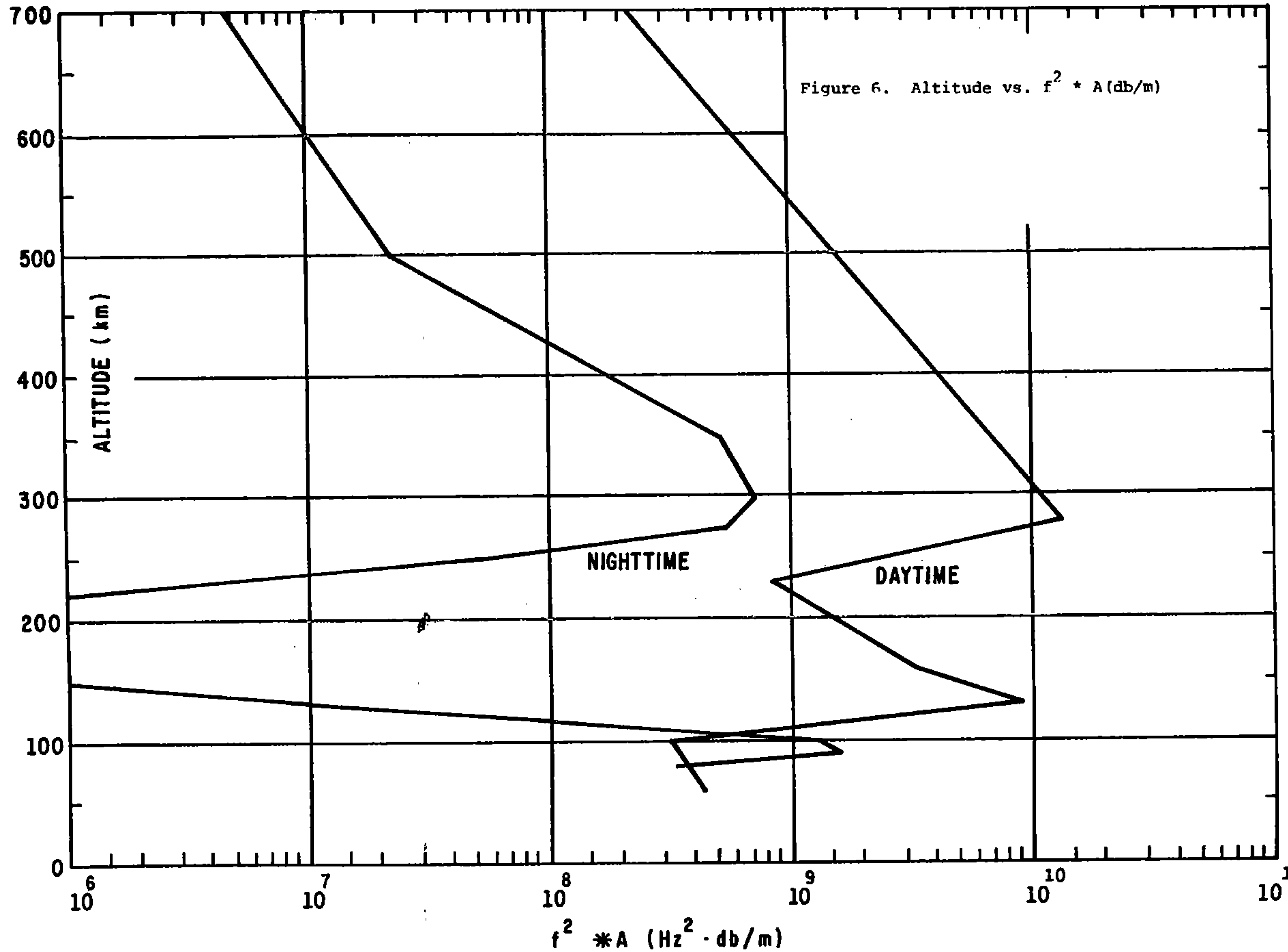


Figure 6. Altitude vs. $f^2 * A$(db/m)

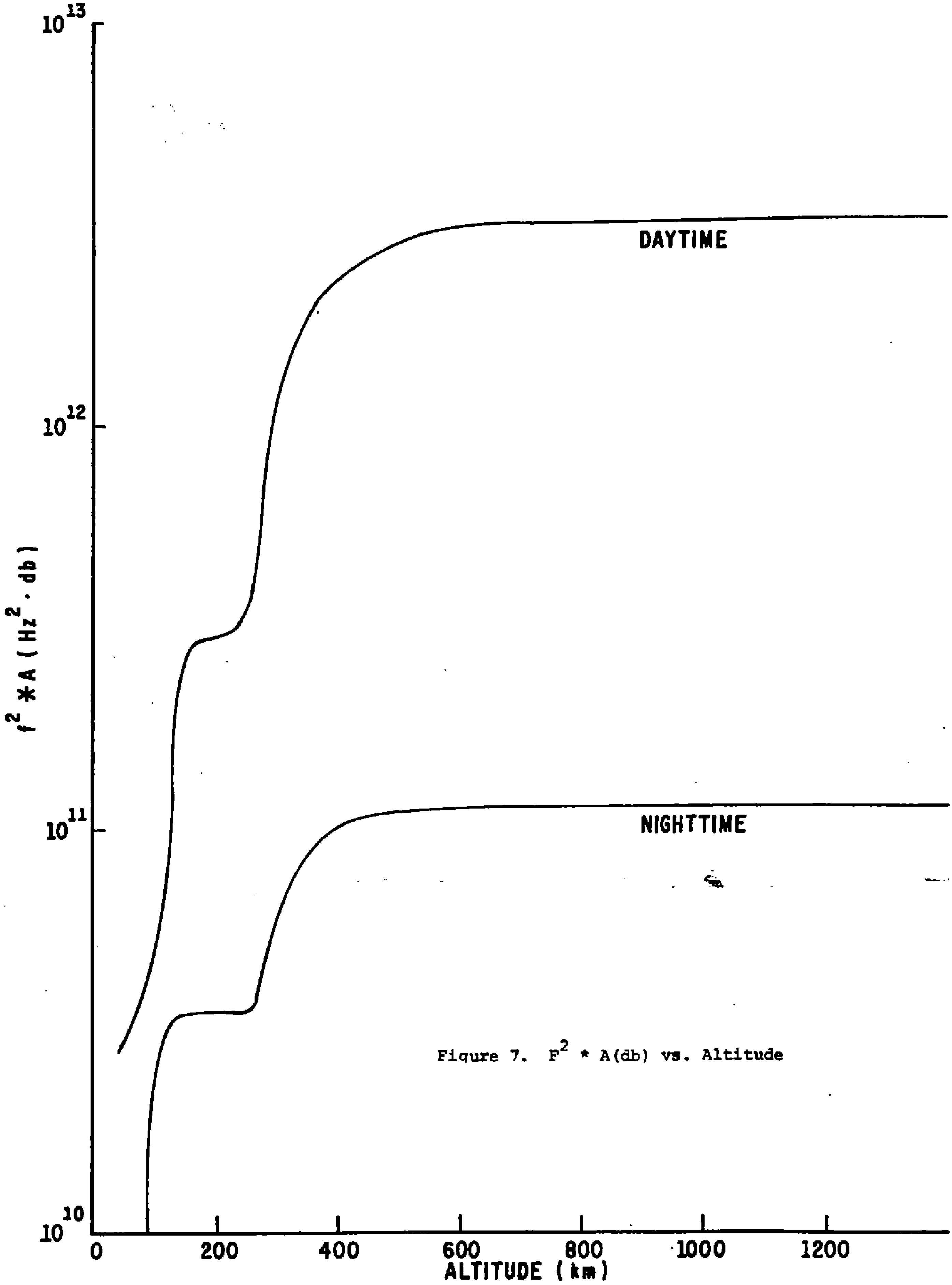


Figure 7. $F^2 * A(db)$ vs. Altitude