Theoretical Notes
Note 109

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The Transmission of Electromagnetic Waves and Pulses into the Earth*

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The transmission into the earth of an electromagnetic transient, such as might be generated by a nuclear explosion, is of interest from the point of view of determining the shielding properties of the earth. A plane electromagnetic pulse normally incident on a plane earth is studied as a convenient model. The steady-state transfer function is first evaluated and its properties analyzed for dry, moist, and wet earth over the complete frequency spectrum. The transmission of an incident pulse of Gaussian shape in air is formulated and its shape determined as it enters and advances into the earth. The numerical results determined by a high-speed computer are analyzed and discussed.

INTRODUCTION

One of the simpler boundary-value problems in electromagnetic theory is the transmission and reflection of a plane wave normally incident in air on an infinite imperfectly conducting half-space. The determination of the reflection and transmission coefficients and the analysis of the standing-wave pattern in the air and the traveling-wave disturbance in the conducting medium are familiar elementary exercises in introductory courses and textbooks. The well-known simple formulas are perfectly general in the sense that they apply at all frequencies. However, a description of the actual reflection and transmission for frequencies that extend from zero to infinity does not appear to have been given. Such a spectrum of frequencies is involved in the transmission and reflection of an incident plane electromagnetic disturbance in the form of a pulse.

Transient fields in a dissipative medium have been the subject of a number of investigations. Wait,[1] Richards,[2] Zisk,[3] and Andersen and Moore[4] have determined the electromagnetic fields generated by pulsed electric and magnetic dipoles in an infinite conducting medium with special emphasis on the behavior of the disturbance as a function of the distance from the point source. Wait,[5,6] Levy and Keller,[7] and Keilson and Rowe[8] investigated the propagation of transient ground or surface waves over curved and plane earths including the diffusion of the disturbance into the earth. Harrison,[9] Harrison *et al.*,[10] and Harrison and Papas[11] have analyzed the propagation of transient electromagnetic fields through conducting walls into plane and spherical cavities. A study of the shielding by a metal cylinder of small cross section and finite length was made by King and Harrison.[12] Grumet[13] has reported on the transient electromagnetic fields produced in a semi-infinite conductor when a uniform electric field of infinite extent is abruptly applied at the surface in the form of a step function.

All of these previous investigations are related to the present one in the sense that propagation in a dissipative medium is involved. However, none of them is concerned with the propagation of a pulsed disturbance traveling from air into a dissipative half-space in a manner that involves both the reflection at the surface and the subsequent propagation into the medium. Interest in the earlier work has centered, on the one hand, on the problem of communication by means of transient signals through a dissipative medium; on the other hand, on the shielding properties of thin metal walls enclosing cavities. The primary purpose of the present study is the determination of the shielding properties of the earth against transient electromagnetic pulses originating in the air above the surface. This involves both reflection at the boundary and attenuated propagation into the medium. Since only the latter has been treated in previous work, a new and more complete investigation of the entire problem is required. The use of a pulse of finite width without sharp discontinuities would appear to offer a somewhat more realistic representation of physically available disturbances than a step function.

When a nuclear explosion occurs above the surface of the earth, an intense electromagnetic field is generated that impinges on the earth in the form of a rising and then decaying amplitude in a short period of time. Depending on the distribution of frequencies in this incident pulse and the electrical properties of the earth, it is partly reflected, partly transmitted. A description of the transmitted pulse at various depths is of interest

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[1] J. R. Wait, J. Appl. Phys. **24**, 341 (1953).
[2] P. I. Richards, Trans. IEEE **AP-6**, 178 (1958).
[3] S. H. Zisk, Trans. IEEE **AP-8**, 229 (1960).
[4] W. L. Anderson and R. K. Moore, Trans. IEEE. **AP-8**, 603 (1960).
[5] J. R. Wait, Can. J. Phys. **34**, 27 (1956).
[6] J. R. Wait, Can. J. Phys. **35**, 1146 (1957).
[7] B. R. Levy and J. B. Keller, Trans. IEEE **AP-6**, 56 (1958)
[8] J. Keilson and R. V. Row, J. Appl. Phys. **30**, 1595 (1959).
[9] C. W. Harrison, Jr., Trans. IEEE **AP-12**, 319 (1964).
[10] C. W. Harrison, Jr., M. L. Houston, R. W. P. King, and T. T. Wu, Trans. IEEE, **AP-13**, 149 (1965).
[11] C. W. Harrison, Jr. and C. H. Papas, Trans. IEEE **AP-13**, 960 (1965).
[12] R. W. P. King and C. W. Harrison, Jr., Trans. IEEE, **AP-9**, 166 (1961).
[13] A. Grumet, J. Appl. Phys. **30**, 682 (1959).

in conjunction with the shielding characteristics of the earth. In broad outline, these shielding properties are of two kinds: (1) reflection at the surface so that only a fraction of the incident field enters the earth, and (2) attenuation with depth of the part that is transmitted. Since the frequency characteristics of reflection at the surface and attenuation in the course of transmission are quite different, it may be anticipated that the pulse transmitted into the earth must differ greatly from the incident pulse.

THEORY OF STEADY-STATE TRANSMISSION AS A FUNCTION OF THE FREQUENCY

Let the upper half space (region 0, air) be characterized by the wavenumber $k_0=\omega/c$, where $c=1/(\mu_0\epsilon_0)^{1/2}\doteq 3\times 10^8$ m/sec is the velocity of light, and the characteristic resistance $\zeta_0=(\mu_0/\epsilon_0)^{1/2}\doteq 120\pi\ \Omega$. The lower half-space (region 1, earth) has the complex wavenumber

$$k_1=\beta_1-j\alpha_1=k_0(\epsilon_{1r})^{1/2}(1-jp_1)^{1/2}, \tag{1}$$

where $\epsilon_1=\epsilon_0\epsilon_{1r}$ is the real effective permittivity, σ_1 is the real effective conductivity in mho/m, and $p_1=\sigma_1/\omega\epsilon_1$ is the loss tangent. It is assumed that $\mu_1=\mu_0$; as usual, $\omega=2\pi f$. The real phase constant β_1 and attenuation constant α_1 can be expressed in terms of the convenient tabulated functions[14,15]

$$f(p)=\cosh(\tfrac{1}{2}\sinh^{-1}p)$$
$$=\{\tfrac{1}{2}[(1+p^2)^{1/2}+1]\}^{1/2}$$
$$g(p)=\sinh(\tfrac{1}{2}\sinh^{-1}p)$$
$$=\{\tfrac{1}{2}[(1+p^2)^{1/2}-1]\}^{1/2}=p/2f(p). \tag{2}$$

Since

$$(1-jp)^{1/2}=f(p)-jg(p), \tag{3}$$

it follows that

$$\beta_1=k_0(\epsilon_{1r})^{1/2}f(p_1)=[\sigma_1\zeta_0/(\epsilon_{1r})^{1/2}][f(p_1)/p_1] \tag{4a}$$

$$\alpha_1=k_0(\epsilon_{1r})^{1/2}g(p_1)=[\sigma_1\zeta_0/(\epsilon_{1r})^{1/2}][g(p_1)/p_1]. \tag{4b}$$

Graphs of $f(p)$, $g(p)$, $f(p)/p$, and $g(p)/p$ as functions of p are given in Fig. 1. The characteristic impedance of the medium is

$$\zeta_1=\zeta_0/(\epsilon_{1r})^{1/2}(1-jp_1)^{1/2}$$
$$=\zeta_0[f(p_1)+jg(p_1)]/(\epsilon_{1r})^{1/2}(1+p_1^2)^{1/2}. \tag{5}$$

The plane bounding the two regions is defined by $z=0$, with the positive z axis directed downward into the earth. A plane electromagnetic wave normally incident in air on the boundary surface is represented by

$$e_x^i(z,t)=\mathrm{Re}E_x^i(z,\omega)e^{j\omega t}$$
$$=\mathrm{Re}E_x^i(0,\omega)\exp[j\omega(t-z/c)]. \tag{6a}$$

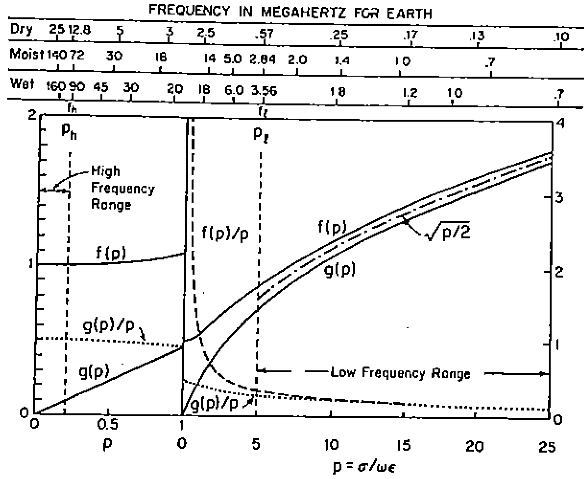

FIG. 1. The functions $f(p)$ and $g(p)$ and related quantities.

The complex amplitude is

$$E_x^i(z,\omega)=E_x^i(0,\omega)\exp(-jk_0z). \tag{6b}$$

The associated incident magnetic field has the complex amplitude $B_y^i(z,\omega)=E_x^i(z,\omega)/c$. The transmitted wave has the complex amplitude

$$E_x^t(z,\omega)=E_x^i(0,\omega)\Gamma_t\exp(-jk_1z) \tag{7a}$$

and the real instantaneous value

$$e_x^t(z,t)=E_x^i(0,\omega)\,|\Gamma_t|\exp(-\alpha_1z)\cos(\omega t-\beta_1z+\Psi_t), \tag{7b}$$

where the complex transmission coefficient is

$$\Gamma_t=2k_0/(k_0+k_1)=|\Gamma_t|\exp(j\Psi_t). \tag{8}$$

Evidently, (7b) represents a plane wave traveling downward into the earth with the phase velocity $v_p=\omega/\beta_1$ and an amplitude that is exponentially attenuated. The ratio of the field at depth $z=d$ to the *transmitted* field at the surface is simply

$$E_x^t(d,\omega)/E_x^t(0,\omega)=\exp(-jk_1d)$$
$$=\exp(-\alpha_1d)\exp(-j\beta_1d). \tag{9}$$

The magnitude of this ratio—which is simply $\exp(-\alpha_1d)$—is shown in Fig. 2 as a function of frequency in megahertz with the depth d in meters as the parameter for three different types of earth. Note that at a sufficiently high frequency, the magnitude of (9) becomes constant. The reflected wave at the boundary is the difference between the transmitted field and the incident field; that is, $E_x^r(0,\omega)=E_x^t(0,\omega)-E_x^i(0,\omega)$. By the means of (1)–(4) it is readily shown that in general

$$|\Gamma_t|=2\{[1+(\epsilon_{1r})^{1/2}f(p_1)]^2+\epsilon_{1r}g^2(p_1)\}^{-1/2}$$
$$\Psi_t=\tan^{-1}\{(\epsilon_{1r})^{1/2}g(p_1)/[1+(\epsilon_{1r})^{1/2}f(p_1)]\}. \tag{10}$$

The quantity of primary interest is the ratio of the transmitted electric field at an arbitrary depth $z=d$ to

[14] R. W. P. King, *Fundamental Electromagnetic Theory*, Appendix II (Dover Publications, Inc., New York, 1963).

[15] C. W. Harrison, Jr. and E. A. Aronson, Sandia Corp. Monograph, SC-R-67-1052, Jan. 1967.

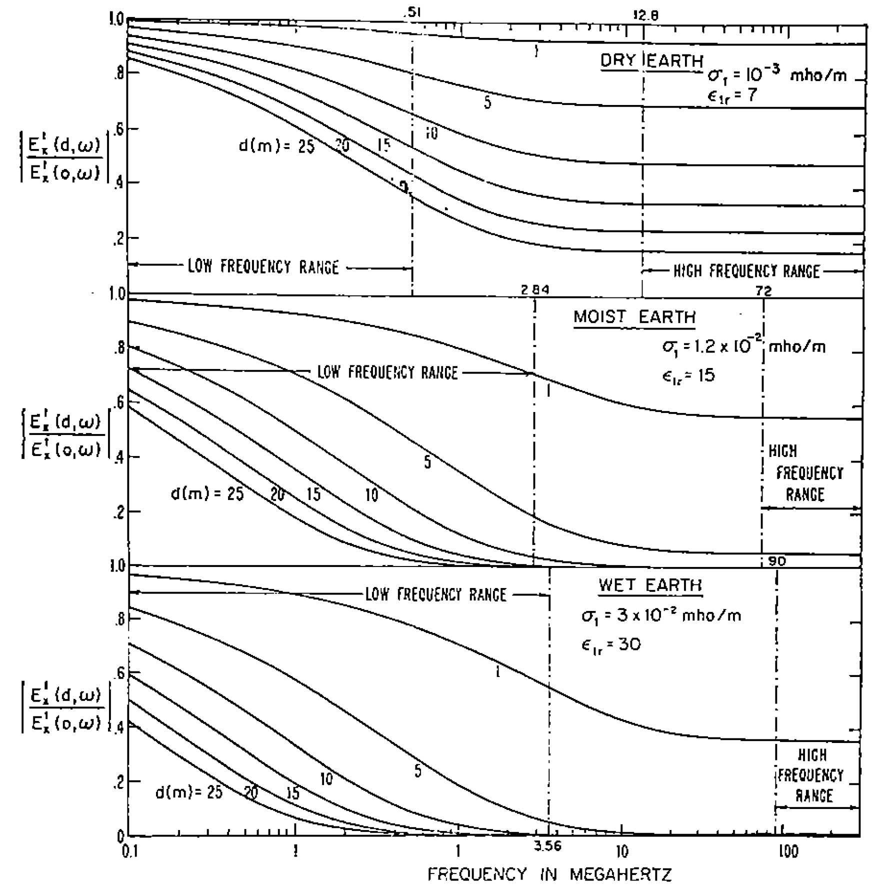

FIG. 2. The ratio $E_x^t(d,\omega)/E_x^t(0,\omega)=e^{-\alpha_1 d}$ as a function of frequency.

the incident field at $z=0$. This ratio is known as the steady-state transfer function and is denoted by

$$G(d,\omega)=G_R(d,\omega)+jG_I(d,\omega)$$
$$=|G(d,\omega)|\exp[j\Phi(d,\omega)]. \quad (11)$$

Thus,

$$G(d,\omega)=E_x^t(d,\omega)/E_x^i(0,\omega)=|\Gamma_t|\exp[j(\Psi_t-k_1d)]. \quad (12)$$

It follows that

$$|G(d,\omega)|=|\Gamma_t|\exp(-\alpha_1 d); \qquad \Phi(d,\omega)=\Psi_t-\beta_1 d. \quad (13a)$$

In particular, at $d=0$,

$$|G(0,\omega)|=|\Gamma_t|; \qquad \Phi(0,\omega)=\Psi_t. \quad (13b)$$

The steady-state properties of the wave transmitted into the earth are completely determined by the complex transfer function $G(d,\omega)$. It is instructive to study its properties first at both ends of the spectrum, that is, in the low-frequency and high-frequency ranges for which useful simplifications occur.

The low-frequency range is defined by the inequality

$$p_1^2 \gg 1 \quad \text{or} \quad p_1^2 \geq 25 \quad (14a)$$

which is equivalent to

$$\omega \leq 0.2\sigma_1/\epsilon_1 \quad \text{or} \quad f \leq f_l, \quad (14b)$$

where

$$f_l = 3560(\sigma_1/\epsilon_{1r}) \text{ MHz} \quad (14c)$$

is the upper limit of the low-frequency range. In this range it is readily shown that $f(p_1) \doteq g(p_1) \doteq (p_1/2)^{1/2}$ (see Fig. 1) so that

$$\Gamma_t \doteq 2(\omega\epsilon_0/\sigma_1)^{1/2},$$
$$\Psi_t \doteq \tan^{-1}\{(\sigma_1/2\omega\epsilon_0)^{1/2}/[1+(\sigma_1/2\omega\epsilon_0)^{1/2}]\} \doteq \pi/4 \quad (15)$$
$$\beta_1 \doteq \alpha_1 \doteq \beta_0(\epsilon_{1r}p_1/2)^{1/2} \doteq (\omega\mu_0\sigma_1/2)^{1/2} = 1/d_s, \quad (16)$$

where d_s is the skin depth. It follows that in the low-frequency range

$$|G(d,\omega)| \doteq 2(\omega\epsilon_0/\sigma_1)^{1/2}\exp[-d(\omega\mu_0\sigma_1/2)^{1/2}] \quad (17a)$$
$$\Phi(d,\omega)=\Psi_t-\beta_1 d \doteq (\pi/4)-d(\omega\mu_0\sigma_1/2)^{1/2}. \quad (17b)$$

It is seen from (17a) that $|G(d,\omega)|$ increases with frequency owing to the term $\omega^{1/2}$ and simultaneously decreases due to the exponential attenuation. The maximum of $|G(d,\omega)|$ (provided it exists in the specified low-frequency range) occurs when

$$d(\omega\mu_0\sigma_1/2)^{1/2}=1 \quad \text{or} \quad f=1/\pi d^2\mu_0\sigma_1. \quad (18)$$

The associated maximum value is

$$|G(d,\omega)|_{max}=(2/d\sigma_1)(2\epsilon_0/\mu_0)^{1/2}e^{-1}=(2.76/d\sigma_1)\times10^{-3}. \quad (19)$$

It is clear from (17a, b) that at sufficiently low frequencies, i.e., $0\leq f<f_l$, $G(d,\omega)$ increases as $f^{1/2}$ along the 45° line in the complex $G(d,\omega)$ plane. When $d>0$, $G(d,\omega)$, as given in (17a, b), initially increases as $f^{1/2}$ along the 45° line as f is raised from zero, but then reaches a maximum and decreases exponentially. At the same time, the angle $\Phi(d,\omega)$ increases as $f^{1/2}$ in the clockwise direction so that the endpoint of the phasor $G(d,\omega)$ traces a spiral about the origin that expands at low frequencies, reaches a maximum and then contracts. Note, however, that this description is valid only until $f=f_l$. Depending upon the properties of the conducting medium and the depth d, this frequency may be reached before or after $G(d,\omega)$ has attained a maximum.

Note in particular that as $f\rightarrow0$, $|G(d,\omega)|\rightarrow0$. This means that the incident wave approaches complete reflection as $f\rightarrow0$ so that nothing is transmitted. This fact is of major importance in the study of the transmission of a pulse into the earth.

The high-frequency range is defined by the inequality

$$p_1^2\ll1 \quad \text{or} \quad p_1^2\leq0.04, \quad (20a)$$

which is equivalent to

$$\omega\geq5\sigma_1/\epsilon_1 \quad \text{or} \quad f\geq f_h, \quad (20b)$$

where

$$f_h=9\times10^4(\sigma_1/\epsilon_{1r})\text{ MHz} \quad (20c)$$

is the lower limit of the high-frequency range. In this range, $f(p_1)\doteq1$, $g(p_1)\doteq p_1/2=\sigma_1/2\omega\epsilon_1$ (see Fig. 1) so that

$$\Gamma_t\doteq2/[1+(\epsilon_{1r})^{1/2}]; \qquad \Psi_t\doteq\sigma_1/2\omega\epsilon_0[(\epsilon_{1r})^{1/2}+\epsilon_{1r}]\ll1 \quad (21)$$

$$\beta_1\doteq\omega(\mu_0\epsilon_1)^{1/2}, \qquad \alpha_1\doteq\sigma_1(\mu_0/\epsilon_1)^{1/2}/2. \quad (22)$$

Note in particular that in this range the attenuation constant, α_1, is *independent of the frequency* and the wavenumber, β_1, is linear in the frequency. It follows that the transfer function is given by

$$|G(d,\omega)|\doteq\{2/[1+(\epsilon_{1r})^{1/2}]\}\exp[-\sigma_1d(\mu_0/\epsilon_1)^{1/2}/2]. \quad (23a)$$

$$\Phi(d,\omega)\doteq\sigma_1/2\omega\epsilon_0[(\epsilon_{1r})^{1/2}-\epsilon_{1r}]-\omega d(\mu_0\epsilon_1)^{1/2}$$
$$\doteq-\omega d(\mu_0\epsilon_1)^{1/2}. \quad (23b)$$

Thus, at sufficiently high frequencies, $f>f_h$, and at any given depth d, $|G(d,\omega)|$ is a constant independent of the frequency. When $d=0$, $\Phi(0,\omega)\rightarrow0$ as $f\rightarrow\infty$; when $d>0$, the leading part of $\Phi(d,\omega)$ increases linearly with ω. This means that the endpoint of the phasor $G(d,\omega)$ traces a circle in the clockwise (negative Φ) direction when $f>f_h$.

DISCUSSION AND NUMERICAL EXAMPLES OF STEADY-STATE TRANSMISSION

In order to illustrate the behavior of the steady-state transfer function, $G(d,\omega)=|G(d,\omega)|\exp[j\Phi(d,\omega)]=G_R(d,\omega)+jG_I(d,\omega)$, it is useful to consider three practically important numerical cases. These are

Dry earth:

$$\epsilon_{1r}=7, \quad \sigma_1=10^{-3}\text{ mho/m}, \quad p_1=2.58\times10^6/f,$$
$$f_l=0.51\text{ MHz}, \quad f_h=12.8\text{ MHz}.$$

Moist earth:

$$\epsilon_{1r}=15, \quad \sigma_1=1.2\times10^{-2}\text{ mho/m}, \quad p_1=14.4\times10^6/f,$$
$$f_l=2.84\text{ MHz}, \quad f_h=72\text{ MHz}.$$

Wet earth:

$$\epsilon_{1r}=30, \quad \sigma_1=3\times10^{-2}\text{ mho/m}, \quad p_1=18.0\times10^6/f,$$
$$f_l=3.56\text{ MHz}, \quad f_h=90\text{ MHz},$$

where f_l is the upper limit of the low-frequency range with an associated $p_l=5.05$, f_h is the lower limit of the high-frequency range with an associated $p_h=0.20$. For the intermediate range the general formulas must be used. The relationship between the loss tangent p_1, the function $\beta_1\sim f(p_1)/p_1$ and $\alpha_1\sim g(p_1)/p_1$ and the frequency in the three types of earth is indicated in Fig. 1. The high- and low-frequency ranges are also indicated in terms of p and f.

DRY EARTH

The transfer function $G(d,\omega)$ is shown in Fig. 3 in the complex plane with the frequency in megahertz as the parameter. When $d=0$, $G(0,\omega)$ is seen to increase as $\omega^{1/2}$ quite closely along $\Phi(0,\omega)=45°$ up to the limiting frequency $f_l=0.505$ MHz of the low-frequency range. The high-frequency range begins at $f=12.8$ MHz and extends to $f=\infty$. Note that $G(0,\infty)=2/[1+(\epsilon_{1r})^{1/2}]=0.55$ at $\Phi(0,\infty)=0$.

Curves of $G(d,\omega)$ for $d=1$, 5, 15, and 25 m are also shown in Fig. 3. These are readily understood in terms of the known behaviors in the low- and high-frequency ranges. Since the former extends only from $f=0$ to $f=f_l=0.505$ MHz, only relatively small sections of the curves in Fig. 3 are involved. All start from zero along the 45° line but bend away from this owing to the term $d(\omega\mu_0\sigma_1/2)^{1/2}$ in (17b). The high-frequency range starts at $f=f_h=12.8$ MHz. It is seen that when $d=1$ and 5 and practically when $d=15$, this range is actually reached. The curves are seen to be circles with radii given by (23a). They are $G(1,\omega)=0.55\exp(-0.0714)=0.51$, $G(5,\omega)=0.55\exp(-0.357)=0.39$, $G(15,\omega)=0.55\exp(-1.08)=0.187$. On the other hand, when $d=25$, $G(25,\omega)=0.55\exp(-1.78)=0.093$. It is seen in Fig. 3 that at $f=5$ MHz the curve has already spiraled inward quite close to the ultimate circle of radius 0.093. Note that when d is small and ω not too large,

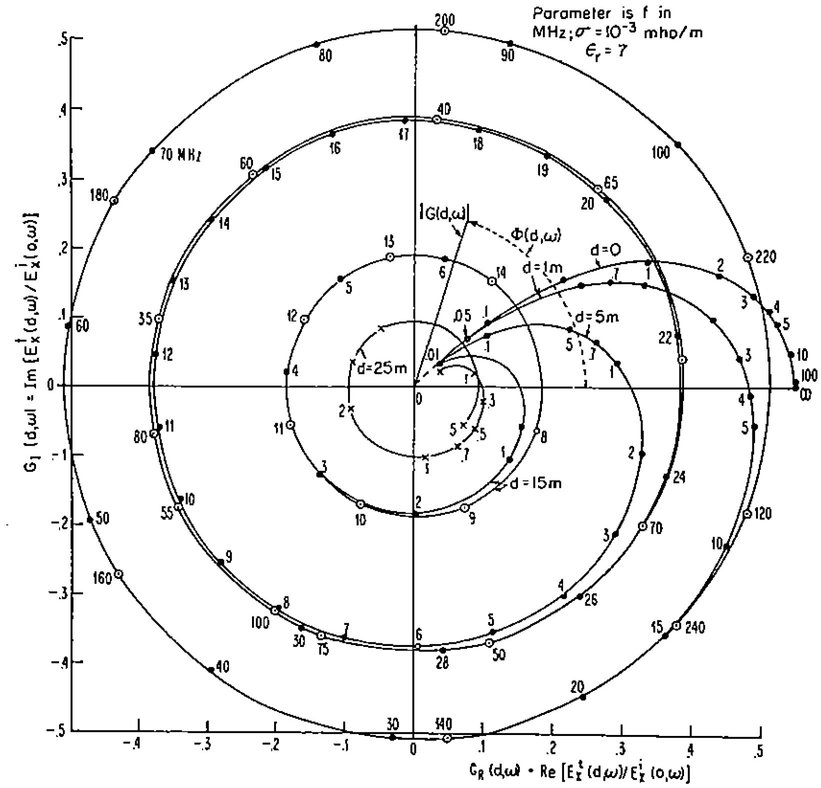

FIG. 3. The steady-state transfer function for dry earth ($\sigma_1=10^{-3}$ mho/m, $\epsilon_{1r}=7$) in the complex plane. The parameters are the frequency f in megahertz and the depth d in meters.

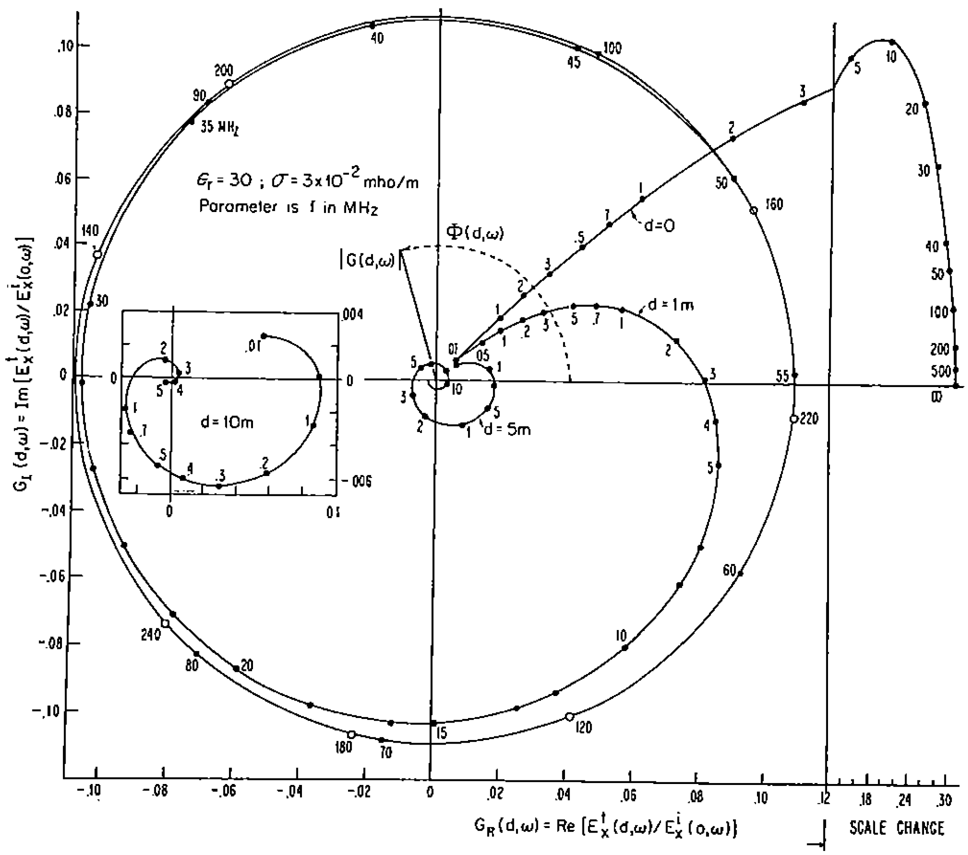

FIG. 4. Like Fig. 3 for wet earth ($\sigma_1=3\times10^{-2}$ mho/m, $\epsilon_{1r}=30$).

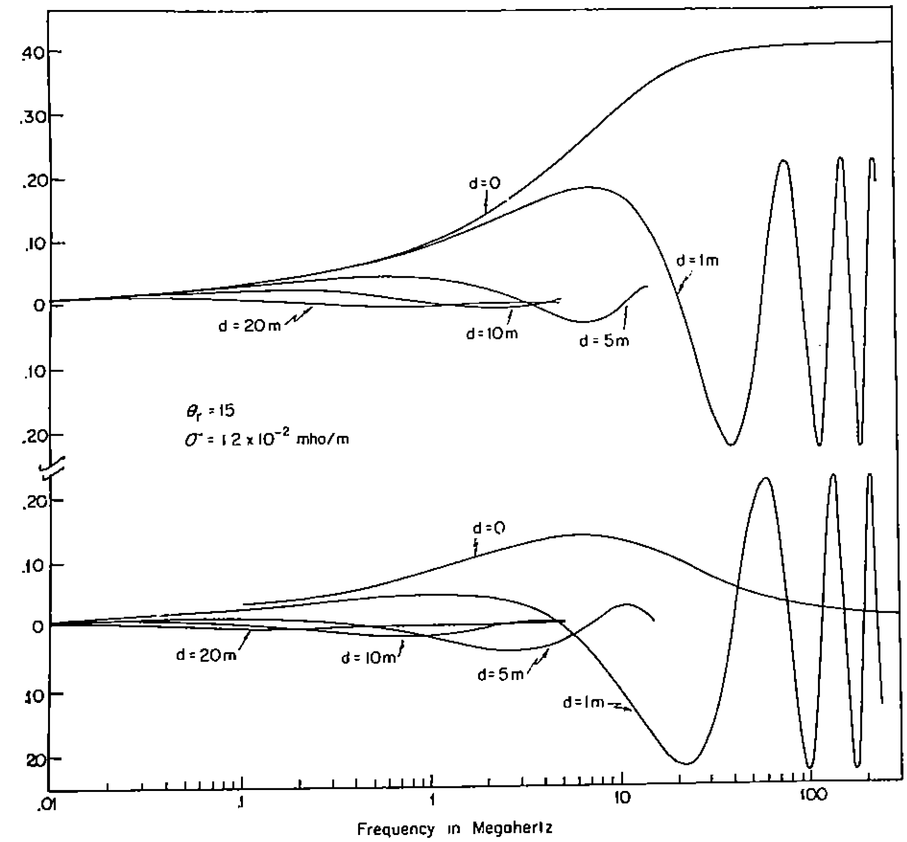

FIG. 5. Real and imaginary parts of the steady-state transfer function for moist earth ($\sigma_1=1.2\times10^{-2}$ mho/m, $\epsilon_{1r}=15$) as functions of the frequency in megahertz. The parameter is the depth d in meters.

the term $|\Gamma_t|$ in (12) dominates. Since it increases with frequency, the curves spiral outward to the final high-frequency circle. When d and ω are sufficiently large, the exponential term in (12) dominates and the curve spirals inward to the final high-frequency circle, which is independent of the frequency. For the dry earth represented in Fig. 3, a slight turning inward can be observed in the curve for $d=25$ m.

WET EARTH

The transfer function $G(d,\omega)$ is shown in Fig. 4 in the complex plane. When $d=0$, the curve starts along the 45° line as the frequency is increased from zero. In this case, the upper limit of the low-frequency range is $f_l=3.56$ MHz, which is large enough to include a significant part of the curve. The high-frequency range begins at $f=90$ MHz. As $f\rightarrow\infty$, $G(0,\infty)\rightarrow 2/[1+(30)^{1/2}]=0.309$, $\Phi(0,\infty)=0$.

When $d=1$ m the behavior of $G(d,\omega)$ is similar to that in Fig. 3 for dry earth except that α_1 is greater so that the amplitude in the limiting high-frequency circle is much smaller. Note that the high-frequency range begins at 90 MHz or after about $1\frac{1}{2}$ turns around the origin. When $d=5$ m the attenuation is large and becomes significant in the low-frequency range. As a result, the maximum value of $|G(5,\omega)|_{max}=0.0184$ occurs at $f=0.338$ MHz as obtained from (18) and (19). $G(5,\omega)$ continues to spiral inward as f is increased beyond this value. It should reach the high-frequency range at $f=90$ MHz, i.e., after very many turns around the spiral, and a circle with the small radius $|G(5,\omega)|=0.0018$. When $d=10$, α_1 is still greater, $|G(10,\omega)|_{max}=0.0092$ at $f=0.0846$ MHz, well within the low-frequency range. $G(10,\omega)$ then spirals inward and after many rotations should reach the limiting high-frequency circle with radius, $|G(10,\omega)|=10^{-5}$ at $f=90$ MHz.

Curves of $G_R(d,\omega)$ and $G_I(d,\omega)$ as functions of frequency are shown in Fig. 5 for the intermediate case of moist earth with $\sigma_1=1.2\times10^{-2}$ mho/m and $\epsilon_{1r}=15$.

The graphs in Figs. 3 and 4 provide the entire frequency behavior of the transfer function $G(d,\omega)$. Note that with $d=0$, $G(0,\omega)=\Gamma_t$ is the complex transmission coefficient that characterizes the complete transmitted wave as it enters the earth. The associated reflected wave is given by the reflection coefficient, $\Gamma_r=\Gamma_t-1$. The attenuating and phase-changing effects of d meters of earth are given by the difference between $G(0,\omega)$ and $G(d,\omega)$.

PULSE TRANSMISSION

Instead of the periodic traveling plane wave defined in (6a), the incident electric field may consist of an arbitrary pulse in the z coordinate and in time. A

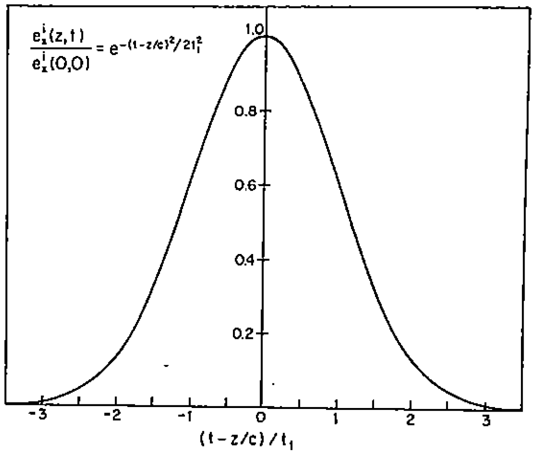

FIG. 6. Incident Gaussian pulse.

generally useful and simple distribution is the Gaussian pulse in the form

$$e_x(z, t) = e_x(0, 0) \exp[-(t-z/c)^2/2t_1^2], \quad (24)$$

where t_1 is a constant parameter that is a measure of the pulsewidth in time and $z_1 = ct_1$ is the corresponding pulsewidth in space. The nature of the distribution (24) is illustrated in Fig. 6. At $z=0$ this gives $e_x(0, t)/e_x(0, 0)$ as a function of the time; and at $t=0$ it gives $e_x(z, 0)/e_x(0, 0)$ as a function of the normalized distance, z/c. Note that $t=t_1$ is the time in which the pulse is reduced to $e^{-1/2}=0.606$ of its maximum at $t=0$. (The pulse is reduced to half-amplitude in the time $1.175t_1$.)

The spectrum contained in (24) is given by its Fourier transform. This is

$$E_x^i(z, \omega) = \int_{-\infty}^{\infty} e_x^i(z, t) e^{-j\omega t} dt$$

$$= e_x^i(0, 0) \int_{-\infty}^{\infty} \exp[-(t-z/c)^2/2t_1] e^{-j\omega t} dt. \quad (25a)$$

This is readily integrated to give[16]

$$E_x^i(z, \omega) = e_x^i(0, 0) \exp(-j\omega z/c) t_1 (2\pi)^{1/2} \times \exp(-\omega^2 t_1^2/2). \quad (25b)$$

At $z=0$, i.e., the boundary between the air and the dissipative medium (earth),

$$E_x^i(0, \omega) = e_x^i(0, 0) t_1 (2\pi)^{1/2} \exp(-\omega^2 t_1^2/2). \quad (25c)$$

The field transmitted into the earth (region 1) to a distance $z=d$ at the frequency ω is

$$E_x^t(d, \omega) = E_x^i(0, \omega) G(d, \omega), \quad (26)$$

where $G(d, \omega)$ is the steady-state transfer function defined in (11). The instantaneous field at the depth d due to the entire Gaussian pulse is obtained from the following Fourier inversion integral:

$$e_x^t(d, t) = (2\pi)^{-1} \int_{-\infty}^{\infty} E_x^t(d, \omega) e^{j\omega t} d\omega$$

$$= (2\pi)^{-1} \int_{-\infty}^{\infty} E_x^i(0, \omega) G(d, \omega) e^{j\omega t} d\omega. \quad (27)$$

With (10) this becomes

$$e_x^t(d, t) = (2\pi)^{-1} \int_{-\infty}^{\infty} E_x^i(0, \omega) \mid G(d, \omega) \mid \times \exp\{ j[\omega t + \Phi(d, \omega)]\} d\omega. \quad (28)$$

However, since $e_x^t(d, t)$ is real, as is $E_x^i(0, \omega)$, it follows that

$$e_x^t(d, t) = (2\pi)^{-1} \int_{-\infty}^{\infty} E_x^i(0, \omega) \mid G(d, \omega) \mid \times \cos[\omega t + \Phi(d, \omega)] d\omega \quad (29)$$

and, since the integrand is even in ω

$$e_x^t(d, t) = \pi^{-1} \int_0^{\infty} E_x^i(0, \omega) \mid G(d, \omega) \mid \cos\Phi(d, \omega) \times \cos\omega t d\omega - \pi^{-1} \int_0^{\infty} E_x^i(0, \omega) \mid G(d, \omega) \mid \times \sin\Phi(d, \omega) \sin\omega t d\omega. \quad (30)$$

With (25c) this becomes

$$e_x^t(d, t) = \frac{2e_x^i(0, 0) t_1}{(2\pi)^{1/2}} \left[\int_0^{\infty} \mid G(d, \omega) \mid \cos\Phi(d, \omega) \times \exp(-\omega^2 t_1^2/2) \cos\omega t d\omega - \int_0^{\infty} \mid G(d, \omega) \mid \sin\Phi(d, \omega) \times \exp(-\omega^2 t_1^2/2) \sin\omega t d\omega \right]. \quad (31)$$

Here the first integral is an even function of t, the second integral an odd function of t. Note that the incident field at $z=0$ is given by

$$e_x^i(0, t) = \frac{e_x^i(0, 0) t_1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \exp(-\omega^2 t_1^2/2) \times \cos\omega t d\omega = e_x^i(0, 0) \exp(-t^2/2t_1^2), \quad (32)$$

which is an even function of t. Thus, the transmitted pulse has both positive and negative parts even though the incident pulse is entirely positive.

The fact that the incident field is entirely positive means that its spectrum contains a significant component at zero frequency. However, it has been shown that the steady-state transfer function is zero for zero frequency. Hence, the zero-frequency component is totally reflected so that the transmitted pulse must

[16] See, for example, A. Papoulis, *The Fourier Integral and Its Applications* (McGraw-Hill Book Co., New York, 1962), p. 24.

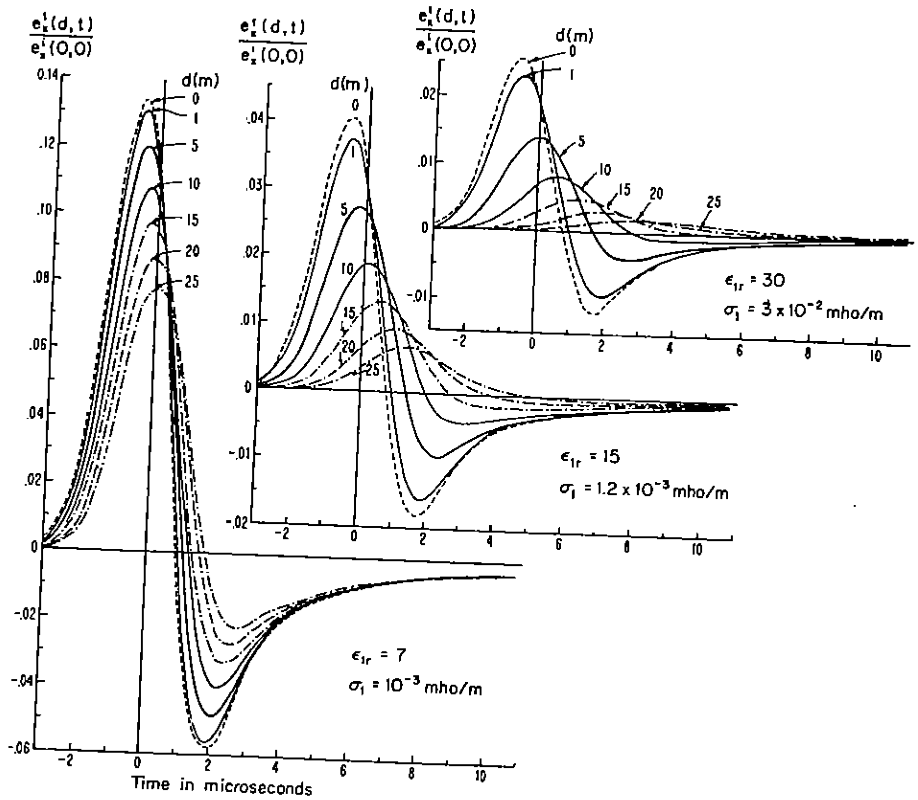

FIG. 7. Transmitted part of a Gaussian pulse at time t (microseconds) and depth d (meters) in dry earth ($\sigma_1=10^{-3}$ mho/m, $\epsilon_{1r}=7$), moist earth ($\sigma_1=1.2\times10^{-2}$ mho/m, $\epsilon_{1r}=15$), and wet earth ($\sigma_1=3\times10^{-2}$ mho/m, $\epsilon_{1r}=30$).

have equal positive and negative parts in the sense that the average is zero. This means that the shape of the transmitted pulse for all $z\geq0$ must differ greatly from the incident pulse at $z\leq0$.

The expression (31) cannot be evaluated numerically with the infinite upper limit. However, it has been shown [3] that for the Gaussian pulse a negligible error is made if the upper frequency limit is taken at $\omega_c=2.6/t_1$ instead of at infinity. The integral to be evaluated numerically is, therefore,

$$e_x^t(d,t)=\frac{2e_x^i(0,0)t_1}{(2\pi)^{1/2}}\int_0^{2.6/t_1}\left[G_R(d,\omega)\cos\omega t - G_I(d,\omega)\sin\omega t\right]\exp(-\omega^2t_1^2/2)d\omega. \quad (33)$$

This formula has been evaluated by high-speed computer and the quantity $e_x^t(d,t)/e_x^i(0,0)$ determined for the following conditions: dry earth ($\sigma_1=10^{-3}$, $\epsilon_{1r}=7$), moist earth ($\sigma_1=1.2\times10^{-2}$, $\epsilon_{1r}=15$), and wet earth ($\sigma_1=3\times10^{-2}$, $\epsilon_{1r}=30$) with a pulsewidth of $t_1=1$ μsec at the depths $d=0$, 1, 5, 10, 15, 20, and 25 m. Curves of this quantity as functions of the time are given in Fig. 7. Note that as predicted each curve is a superposition of an even and an odd function of the time. The positive lobe is higher but shorter than the negative one.

In dry earth the amplitude of the pulse (on the left and bottom in Fig. 7) decreases quite slowly with increasing depth d and its shape broadens and flattens only gradually. In moist earth (in the center of Fig. 7) the decay with depth is much more rapid and the curves quickly become broad and flat as the depth is increased. This is due to the exponential attenuation of the higher frequencies which are the principal components of the transmitted pulse since the very low frequencies are largely reflected at the surface. In wet earth (on the right and top in Fig. 7) the decay and broadening of the curves are still more rapid. Even so, a significant signal still exists under 25 m of wet earth!

CONCLUSION

The interesting properties of a transient electromagnetic disturbance in the form of a plane Gaussian pulse normally incident on the plane surface of the earth have been determined for various depths. The earth acts as a shield that reflects a major part of the incident field and then attenuates the other part that is transmitted. Since the low frequencies in the incident unidirectional pulse are almost totally reflected, the field that enters the earth includes largely higher frequencies that form a pulse with both positive and negative parts. As the transmitted pulse proceeds into the earth, the amplitudes of the individual frequencies decay exponentially with distance. However, for given values of σ_1 and ϵ_{1r}, the attenuation constant in (4b) is proportional to $g(p_1)/p_1$ and this increases with increasing frequency or decreasing p_1 (See Fig. 1). Thus, whereas the low frequencies are largely excluded from the transmitted field by reflection, the high frequencies are attenuated most rapidly with depth. This means that at any given depth in an earth with specified ϵ_{1r} and σ_1 there may be an optimum frequency for which the amplitude is a maximum. However, if σ_1 and ϵ_{1r} have values such that this maximum amplitude occurs in the high-frequency range defined in (20a, b, c), the attenuation and, hence, the amplitude for all higher frequencies is essentially independent of frequency.

In general, the shielding and absorbing properties of the earth as a function of frequency are not entirely simple owing to the greatly different frequency dependencies of the reflection at the air-earth surface and attenuation along the path of transmission in the earth.

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