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Radio Flash Arising from Differential Scattering

by

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ABSTRACT

Owing to the atmospheric density gradient, scattered gamma ray photons which arrive at a point distant from a nuclear burst have a distribution in the transverse direction which is not isotropic. This leads to a net Compton electron current in the ϑ-direction which in turn emits an electromagnetic signal. In this note we estimate this signal.

Owing to scattering, a gamma ray from an air burst will arrive at a distant point R making some angle ψ with the radius vector. Thus its motion, while generally in the radial direction, will also have a transverse component. If we consider now all the gammas arriving at R they will have a distribution of transverse components which, owing to the atmospheric density gradient, will not be isotropic but will be directed preferentially downward. Thus in addition to the radial component J_r, the Compton electron current density will also have a transverse component J_θ which has its origin in differential scattering. Being a transverse current, J_θ is very efficient in producing a radiated signal, and thus may be important even though it is very small.

In reference 1 a simple prescription is given for calculating the signal radiated once J_θ is known; thus our problem reduces to calculating this quantity. To calculate J_θ properly would require a time dependent solution of the gamma ray transport problem in the nonuniform atmosphere. In the absence of such a solution we shall here make an estimate based on consideration of first scatterings only. Note that we are here only calculating a differential effect, not the total number of scatterings, thus our results will be wrong only insofar as a "typical" multiple scattering path may differ from a single scattering path.

Consider a source at point S and a gamma ray which leaves the source, scatters at point P, and arrives at point R_1 as in Fig. 1.

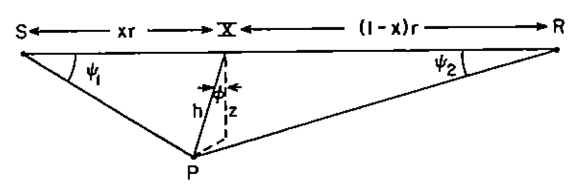

Fig. I. Scattering Geometry.

The point P is a distance h from the line SR and the plane SPR makes an angle φ with the vertical. Thus the vertical component z of h is

$$(1)\quad z = h \cos \varphi .$$

The line SR is of length r and the plane through P perpendicular to SR meets SR at X and divides this length into segments xr and $(1 - x)r$. The angles PSR and PRS are respectively labeled ψ_1 and ψ_2.

The photon travelling over the scattering path SPR rather than directly is delayed by a time* τ given by

$$(2) \quad c\tau + r = \sqrt{(xr)^2 + h^2} + \sqrt{(1 - x)^2 r^2 + h^2} .$$

As gamma ray scattering is strongly peaked in the forward direction only those scatterings for which h is appreciably less than xr are important. We may therefore expand the square roots obtaining

$$(3) \quad c\tau = \frac{h^2}{2rx(1 - x)} .$$

The two angles ψ_1, ψ_2 are given by

$$(4) \quad \tan \psi_1 = \frac{h}{rx} , \quad \tan \psi_2 = \frac{h}{r(1 - x)} .$$

Note that for every scattering at P at the distance xr from S, there is an exactly symmetric one at xr from R. To describe this symmetric scattering we simply interchange x with (1 - x) in our formulas, leaving Eq. (3) invariant, and interchanging ψ_1 and ψ_2 in Eq. (4). If we consider this symmetric pair of scatterings together, they have the time delay of Eq. (3) and produce the average tangent

$$(5) \quad \tan \psi \equiv \frac{1}{2} (\tan \psi_1 + \tan \psi_2) = \frac{h}{2rx(1 - x)} .$$

We shall now assume that the solution to the problem of scattering in a homogeneous medium is known and is described in terms of two functions, a build-up factor B(r) which is the ratio of scattered to direct radiation at r, and a response function $u_0(r,\tau)$ which describes the time history of arrival at the point r of photons from a δ-function source. Thus if the gamma rays at the source have the time history $f(\tau)$, the Compton current in a uniform atmosphere is given by

$$(6) \quad J = -\mathcal{G}(r)\left\{f(\tau) + B(r)\int_0^{\tau} f(\tau')u_0(r,\tau - \tau')d\tau'\right\},$$

where $\mathcal{G}(r)$ is the spacial factor including source strength, gamma ray absorption, and inverse r^2 attenuation. To be specific we shall write u_0 in the form

$$(7) \quad u_0(\tau) = \kappa e^{-\mu}, \quad \mu \overset{\text{def}}{=} \int_0^{\tau} \kappa(\tau')d\tau' ,$$

where κ is a slowly varying function of τ but otherwise unspecified.

In the nonuniform atmosphere the scattering centers have the density distribution

$$(8) \quad \rho = \rho_0 e^{-z/H} \doteq \rho_0\left[1 - \frac{h \cos \varphi}{H}\right] ,$$

where H is the atmospheric relaxation height. The number of scatterings per incident photon is clearly proportional to $\rho/\rho_0 = 1 - h \cos \varphi/H$. But owing to the density gradient, the attentuation of the primary and of the scattered beams also depend on z. Clearly, attenuation over the path SX is

$$\exp [-rx/\lambda]$$

whereas over SP it is

$$\exp \left\{- \frac{rx}{\lambda}\left(1 - \frac{rx \cos \theta}{H}\right)\right\} ,$$

where θ is the angle SP makes with the vertical. But $\cos \vartheta = - \sin (\pi/2 - \theta) = z/rx$, whence we see that the differential absorption between SX and SP is simply

$$\exp \left\{\frac{xrh \cos \varphi}{2\lambda H}\right\} .$$

Similarly differential absorption over PR is the same with x replaced by (1 - x) and the total differential absorption over SPR is

$$(9) \quad dA = \exp\left\{\frac{rh \cos \varphi}{2\lambda H}\right\} \doteq 1 + \frac{rh \cos \varphi}{2\lambda H} .$$

We can now construct the differential response function $u(\varphi,\tau)$, defined such that $u d\tau d\varphi$ is the fraction of total photons which arrive in the interval $(\tau,\tau + d\tau)$ and were scattered in the volume between the two half-planes φ and $\varphi + d\varphi$. We have

*Throughout this note we work in retarded time $\tau = t - r/c$ so that delays are relative to the direct path flight time. We call the retarded time τ simply "time".

simply to multiply u_0 by the density correction factor, Eq. (8) and the differential absorption factor, Eq. (9), obtaining

$$(10) \quad u = u_0 \exp\left\{\frac{h \cos\varphi}{H}\left(\frac{r}{2\lambda} - 1\right)\right\} .$$

To calculate the vertical component of the current we need the vertical response function $u_\perp$, which is the average value of the vertical projection of $\tan\psi$, that is the average value of $\tan\psi \cos\varphi$. Thus

$$(11) \quad u_\perp \equiv \langle \cos\varphi \tan\psi \rangle = \int_0^{2\pi} \cos\varphi \tan\psi \; u(\varphi,\tau) d\varphi .$$

Using Eqs. (5), (7) and (10) and expanding the exponentials we obtain

$$(12) \quad u_\perp(r,\tau) = \frac{\kappa e^{-\mu}}{2H} \frac{h^2}{2rx(1 - x)}\left(\frac{r}{2\lambda} - 1\right)$$

$$= \frac{\kappa c\tau}{2H} e^{-\mu}\left(\frac{r}{2\lambda} - 1\right),$$

where we have used Eq. (3) to eliminate h and x.

The radial current density is given by Eq. (6) and the transverse current density by

$$(13) \quad J_\theta = -\mathcal{J}(r) \cdot B(r)\int_0^\tau f(\tau')u_\perp(r,\tau - \tau')d\tau' .$$

At the great distances where the radiated signal is generated $B(r)$ is normally $\gg 1$ and $f(\tau)$ a very short pulse compared with the response functions u_0, $u_\perp$. Thus we obtain to a very good approximation

$$(14) \quad \begin{cases} J_r = -\mathcal{J}(r)B(r)\kappa e^{-\mu} \\ \\ J_\theta = -\mathcal{J}(r)B(r) \dfrac{c\tau}{2H}\left(\dfrac{r}{2\lambda} - 1\right)\kappa e^{-\mu} , \end{cases}$$

that is to say

$$(15) \quad J_\theta = \frac{c\tau}{2H}\left(\frac{r}{2\lambda} - 1\right)J_r .$$

If we put this source into the equation for (rE_0) and proceed as for the geomagnetic signal (Ref. 1) we find that J_θ produces a radiated signal given by

$$(16) \quad rE_\theta = \frac{c\tau}{2H}\left(\frac{\overline{R}_s}{2\lambda} - 1\right)\overline{R}_s E_s ,$$

where $\overline{R}_s$, E_s are defined, as in reference (1), by

$$(17) \quad 2\pi\sigma(\overline{R}_s) = 1/\lambda ,$$

and

$$(18) \quad E_s = -J_r/\sigma .$$

As $\overline{R}_s$ is normally several (5 to 10) times λ, we see that differential absorption of the scattered beams overrides differential scattering, leading to a net downward current of Compton electrons, i.e. a negative J_θ and a positive radiated signal.

As an example we have chosen $\overline{R}_s$ to be initially (at the end of the α-phase) 10λ and the time history of σ to be the same as for Fig. 1 of reference 1, which we reproduce here as Fig. II.

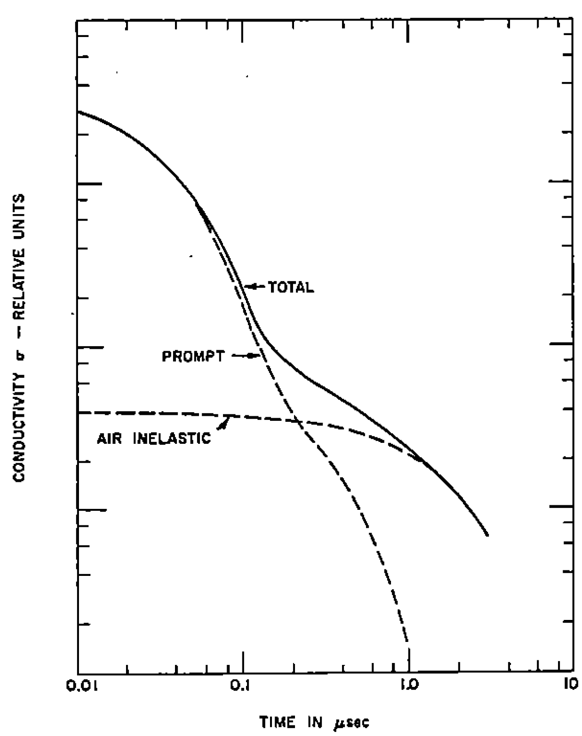

Fig. II. Typical σ vs. τ History.

In Fig. III we show the differential scattering signal thus calculated from Eq. (16). Two curves are shown, one including and the other excluding the long tail of gamma radiation arising from air inelastic gammas. A dotted line in the case with no long tail shows the effect of ionic conductivity, which was omitted in calculating the solid curves. To put this signal into proper perspective we have reproduced in Fig. IV the air asymmetry signal given in reference 1, together with a second curve showing the combined effects of air asymmetry and differential scattering.

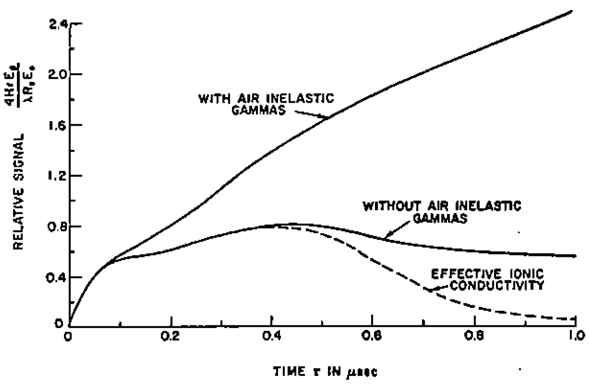

Fig. III. The Differential Scattering Signal. Variable κ.

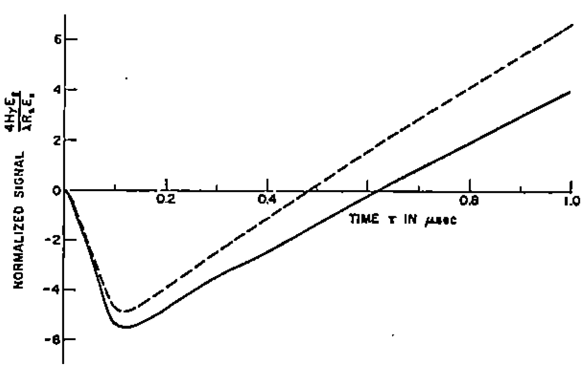

Fig. IV. Air Asymmetry With (Dashed) and Without (Solid) Differential Scattering (Equatorial Plane). Variable κ and Air Inelastic γ's.

The magnitude of the differential scattering signal is roughly proportional to R_s^2, and so is the atmospheric asymmetry signal. One would therefore not expect their relative magnitudes to depend very much on yield. Because of the factor $(\overline{R}_s/2\lambda - 1)$, the shape of the differential scattering signal is yield dependent, especially for small yields.

The above calculation is somewhat crude. It probably gives about the right shape for the differential scattering signal but not necessarily the correct amplitude. By treating <u>all</u> scatterings with a given time delay as single scatterings we have placed the scattering point P at its maximum possible distance from the line SR and thus overestimated the differential effects; i.e. the amplitude of our function u, Eq. (10), is too large. On the other hand, replacing multiple scatterings by single scatterings underestimates the angle ψ_2 that they make with the radius vector at R. It follows, then, that in Eq. (11) we have used a value of tan ψ which is too small and a value of u which is too large and there is some compensation of errors. The signal seems rather small but not completely negligible and may well deserve proper treatment, starting with a numerical solution of the gamma ray transport problem.

REFERENCES

1. B. R. Suydam, Radio Flash from a Low Altitude Air Burst, Los Alamos Scientific Laboratory Report LA-4245-MS.