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An Electron Dynamics Scheme for Close-in Self-Consistent EMP Applications

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ABSTRACT

A scheme for representing and evaluating the interaction of driving (Compton) electrons with their collectively generated, time and space varying fields in the vicinity of a medium interface is presented. The collisional influence on the electron motion is approximated according to a stopping power model. The scheme is particularly designed to allow representation of source electron energy and angular distributions on available computers.

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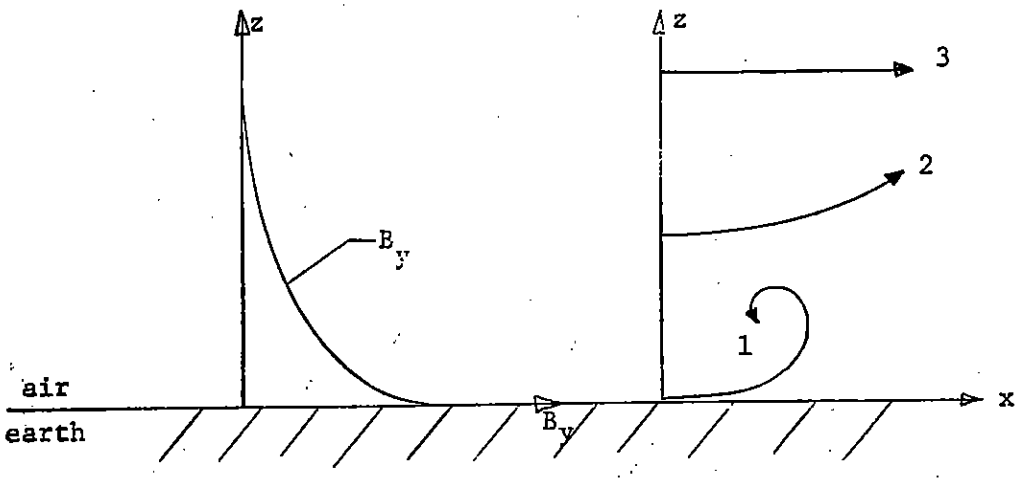
I. Introduction.

This note presents a scheme (model and procedure) for treating the turning of driving (Compton recoil) electrons in their collectively generated fields. The incorporation of this turning into a field generation code results in a self-consistent EMP code (i.e., the fields and electron motions are consistent).

The scheme presented here is designed for incorporation into the one-dimensional code developed by W. R. Graham, It is especially designed to apply to early times when the fields vary rapidly with position (altitude) and time. At these times the Compton recoil electrons can be expected to experience a variation in fields throughout their lifetime. At later times the electrons will see a constant field and are amenable to simpler treatments.

It may prove convenient to interpret the proposed scheme with reference to the following illustration.



The magnetic field B_y is directed into the paper and is assumed to be varying significantly within an electron range near the surface. At some altitude (z) the magnetic field will approach zero. The x direction is chosen pointing away from the projection of the photon source (which drives the electrons) on the earth's surface.

The electrons (labeled (1), (2), and (3)) at different heights experience different turning influences depending upon the field distribution, and, fall behind the field wavefront-which travels in the x direction at approximately the speed of light-at different rates. The illustration unfortunately implies that the magnetic field is monotonically decreasing away from the media interface and that the electrons travel in the xz plane. Neither of these features is necessarily present in the actual phenomenon and neither is assumed in the following scheme.

The effect of the electron turning phenomenon is to distribute the EMP driver in z and hence to effect the distribution of EMP fields. In general, field peaks are lowered and broadened, both in space and time, by the turning process.

II. The Model

The model used to represent electron turning in the proposed scheme is defined as follows. The electron source which drives the field equations is assumed to be independent of x at a constant retarded time, τ.* The fields driven by the electrons are also assumed to be independent of x at τ. Thus, the turning and energy loss histories experienced at τ are independent of x. This fact allows us to ignore the x position of the electrons because any electrons lost by flow out of Δx at z, τ are exactly replenished by flow into Δx.

The distribution of driving electrons is represented by a finite number of charge aggregates characterized by their charge density, velocity components (or energy and direction), and z position at each time, τ. As the local retarded time in the field equations is advanced the charge aggregate parameters (velocity, position, etc.) are updated through application of the electron dynamics equations.

Knowledge of the aggregate parameter allows one to calculate the contribution to current components and the ionization rate. This can be done by summing the contributions from each aggregate to each altitude interval, Δz, and assigning the value of the contribution to the midpoint of the interval. Note that the z grids associated with electron dynamics, EMP source scoring, and field calculation can be independent from one another.

Thus, the major features of the model are one-dimensionality, (radial variations in particle densities and field intensities are ignored at constant retarded time), and representation of the electrons, which drive the EMP field and charged particle density equations, by a manageable, finite number of electron aggregates. The operations on the aggregates consist of updating aggregate parameters at each time step and interpreting these parameters in terms of current and ionization rate.

III. The Procedure

The procedure of evaluating the electron dynamics in a field prediction code consists of initiation of the electron aggregates, updating of the aggregate parameters (position and velocity components), and evaluation of the electron currents and ionization rates.

a. Aggregate Initiation.

Upon stepping $\Delta\tau$ a set of electron aggregates is initiated.

*The retarded time τ is defined as the local time since initial arrival of the driving photons from the source. We assume that the photon source is turned on at $x = 0$ and $t = 0$, hence $\tau = t - x/c$ where t is the ordinary time.

Each of these aggregates is identified, for example, with indices (I, J, K, L, M) corresponding to their initial categories (energy, polar angle with respect to the radial*, azimuthal angle with respect to the radial altitude, and initiation time) as obtained from a radiation transport calculation. For each of these aggregates the initial charge density - $n(I,J,K,L,M)$, velocity components - $v_{x,y,z}(I,J,K,L,M)$, and altitude - $z(I,J,K,L,M)$ are established. These parameters (with the exception of charge density) will be updated at each subsequent time step. The initial aggregate parameters, v_x, v_y, v_z, and z are easily obtained on the basis of the initial category definitions. The aggregate parameter charge density, however, depends upon the radiation source intensity and position and time of observation (field calculation) with respect to the nuclear burst as well as the x component of aggregate velocity.

The x component of aggregate velocity is relevant for the following reasons. We can compute the number of electrons, ΔN, crossing a unit area perpendicular to the x axis in $\Delta\tau$. This quantity is also expressible as a number density times the x-velocity component times $\Delta\tau$. Equating these two quantities yields the initial aggregate electron density in terms of electron source strength, the x component of velocity and $\Delta\tau$.

Mathematically,

$$n(I,J,K,L,M) = \frac{\Delta N}{v_x(I,J,K,L,M) \cdot \Delta\tau}$$

The number of electrons crossing a unit area perpendicular to x, ΔN, can be expressed as

$$\Delta N = Se\ (I,J,K,L,M,) \cdot \Delta x \cdot \Delta\tau \cdot v_x/|v_x|$$

where Se (I,J,K,L,M) is the electron source intensity in terms of electrons per unit volume per unit time (driven by photons) in the relevant category and at x

Δx is the distance to be traveled by the electrons in

$\Delta\tau$ is the retarded time step

Since Se is supplied by an EMP source code or subroutine and $\Delta\tau$ is given we have only to calculate Δx in order to obtain ΔN and thereby the $n(I,J,K,L,M)$.

For an electron with positive x velocity starting at $x - \Delta x$ to arrive at x in $\Delta\tau$

$$\Delta\tau = \Delta x\ (1/v_x - 1/c)$$

where c is the speed of light

*The radial from the normal axis through the burst.

This allows us to express Δx in terms of $\Delta\tau$ and $v_x(I,J,K,L,M)$

$$\Delta x = \Delta\tau/(1/v_x(I,J,K,L,M) - 1/c)$$

For electrons with negative x velocities

$$\Delta x = \Delta\tau/(1/|v_x| + 1/c)$$

Thus, for any case

$$\Delta x = \Delta\tau/(1/|v_x| - v_x/(|v_x| \cdot c))$$

and, finally, the charge density can be set at

$$n(I,J,K,L,M) = \frac{Se(I,J,K,L,M) \cdot \Delta\tau}{(1 - v_x/c)}$$

Generally this charge density will be set for all time throughout the subsequent calculations. However, it may be desirable to manipulate this variable to represent fractional transmission of the electrons.

The choice of $\Delta\tau$ should be made so that the maximum Δx experienced by an electron aggregate is a small fraction of an electron range. To insure this one might choose a maximum acceptable fraction of an electron range, K, and, using the maximum v_x for an aggregate, compute $\Delta\tau$.

$$\Delta\tau = \frac{KRe}{v_x} \quad (1 - v_x/c)$$

This $\Delta\tau$ corresponds to an electron dynamics time step and can be different from the field evaluation or aggregate initiation time step. If v_x is very nearly the speed of light then the $\Delta\tau$ becomes very small. This is consistent with our early models in which the electrons were assumed to travel with $v_x = c$ and hence the electron current was coincident with the photon current.

b. Updating the Aggregate Parameters

The aggregate parameters; z, v_x, v_y, and v_z (I,J,K,L,M), are to be updated at each $\Delta\tau$ in the dynamics tracking. For any aggregate the following dynamic equation (including a stopping power representation for the collisional energy losses) applies.

$$\frac{d(\gamma\vec{v})}{dt} = \frac{q}{mo} \left[\vec{E} + \vec{v} \times \vec{B} - \frac{\vec{v}}{q|\vec{v}|} \frac{dT}{ds}\right]$$

where q is the electronic charge

mo is the electronic mass

γ equals $1/\sqrt{1-v^2/c^2}$

and dT/ds is the ordinary electron stopping power (a function of energy).

This vector dynamics equation can be separated into three coupled scalar equations expressing the change in v_x, v_y, and v_z in Δt. In order to increment each $\Delta\vec{v}$ component for each aggregate then we need to establish the Δt corresponding to $\Delta\tau$ for each value of $v_x(I,J,K,L,M)$.

$$\Delta t = \Delta x/|v_x|$$

$$= \Delta\tau/(1-v_x/c)$$

Thus, for positive v_x, Δt is larger than $\Delta\tau$ and for negative v_x, Δt is smaller than $\Delta\tau$. This value of Δt should be used in the dynamics equations. Since we have chosen $\Delta\tau$ to insure that no electron travels an unacceptably large fraction of its range the Δt's should all be acceptable for the dynamics equation from the standpoint of small collisional energy losses. However, extreme field influences may require further reductions in the $\Delta\tau$ or Δt's. Perhaps the field influences as well as the collision influence could be considered in establishing $\Delta\tau$ in the first place.

c. Current and Ionization Scoring

Occasionally it will be advisable to update the field values. This will require interpretation of the electron aggregate parameters in terms of currents and ionization rates. Many approaches to this interpretation are possible. A possibly suitable approach is suggested here. However, experience with implementation may result in the alteration or rejection of these suggestions.

One could establish a z grid corresponding to the centers of each charge aggregate. That is, each aggregate could apply to a Δz interval with the initial $z(I,J,K,L,M)$ being the centers of this interval. Likewise an interval could be assigned to each current and ionization rate scoring (interpretation) point. The contributions to current or ionization would be weighted by the fraction of the aggregate interval falling within the scoring interval.

That is

$$J_i(z_j,I,J,K,L,M) = n(I,J,K,L,M) \cdot q \cdot v_i(I,J,K,L,M) \cdot F$$

where $J_i(z_j,I,J,K,L,M)$ is the contribution to i'th component of current at z_j due to aggregate (I,J,K,L,M)

F is the fraction of aggregate (I,J,K,L,M) falling within the Δz_j interval about z_j.

The total i'th current component at z_j can be expressed thus as

$$J_{ij} = \sum_{I,J,K,L,M} J_i(z_j,I,J,K,L,M)$$

The ionization rate is more complex since what is required is not the instantaneous value at z_j as with the current but the total charged particle number density. This involves the results of continual insertion of the ionization rate into some species density (or air chemistry) equations. It is suggested that the ionization rate be computed and fed into the species density equations at each electron dynamics step as

$$Q\ (z_j,I,J,K,L,M) = \frac{n(I,J,K,L,M)}{E_{ip}} \cdot \left|\vec{v}(I,J,K,L,M)\right| \cdot \frac{dT}{ds}$$

$$Q_j = \sum_{I,J,K,L,M} Q(z_j,I,J,K,L,M)$$

where $Q(z_j,I,J,K,L,M)$

is the ionization rate contribution to z_j due to aggregate (I,J,K,L,M)

E_{ip} is the average energy required per ion pair creation

and Q_j is the total ionization rate from all aggregates.