Electromagnetic Pulse Theoretical Notes
Note 63
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Radial Current and Dose Rate from Neutron Induced Gamma Quanta in Uniform Air by Means of the Stationary Source Approximation

Capt Clovis R. Hale
Air Force Weapons Laboratory

Abstract

The gamma source induced by inelastic scatter and fast capture of neutrons in air may be approximated by a stationary point source over an appreciable time span, depending upon observer distance. This note describes the required expressions for this approximation, and then presents calculations of dose and radial current for pulsed neutron sources in narrow energy bands.

I. Introduction

A very good approximation for neutron-induced electromagentic pulse (EMP) sources over a particular domain of time-range is the so-called "Rattle" model.* The approximation makes use of the fact that the highest energy neutrons from a nuclear burst (14.1MEV) are travelling at a speed of approximately one-sixth the velocity of the induced gamma rays, so that a distant observer sees essentially a stationary point source of gammas over an appreciable time span. Thus, the model requires no neutron transport, but only that the neutron spectrum as a function of time be computed, with the resulting gamma source being lumped at the origin. One may then use a set of empirical Green's functions for point gamma sources, such as those obtained by Schaefer[1], and perform a convolution integral to obtain the dose and radial current due to neutron-induced gammas.

II. Expressions for Dose Rate and Radial Current

For a homogeneous medium, the Boltzmann transport equation may be integrated over velocity angle and volume to obtain an integro-differential equation for the neutron spectrum as a function of time. Upon application of the multi-group approximation to this expression, there results a coupled system of first order linear differential equations in time. The series of steps leading from the general Boltzmann equation to this coupled set is given in Appendix A. A solution is straightforward by matrix methods; however, the absence of up-scatter (neutron gaining energy in a collision) makes possible a relatively simple solution that is given in Appendix B. The differential equations and solution taken from the Appendices are given below:

$$\frac{dn_i(t)}{dt} + \sigma_{ti} v_i n_i(t) = \sum_{j=1}^{NN} \sigma_{ji} v_j n_j(t) + s_i(t) \qquad i=1, NN \quad (1)$$

$$n_i(t) = \sum_{j=1}^{i} A_{ji} \exp(-a_j t) \qquad i=1, NN \quad (2)$$

$$A_{ki} = \begin{cases} N_i + \sum_{j=1}^{i-1} b_{ji} \sum_{\ell=1}^{j} \frac{A_{\ell j}}{a_\ell - a_i}, & k=i \\ \frac{1}{a_i - a_k} \sum_{j=k}^{i-1} b_{ji} A_{kj}, & k<i \end{cases} \quad (3)$$

* To my knowledge, both the idea and term "Rattle" model originated with W.R. Graham, Rand Corporation

Where $n_i(t)$ = number of neutrons of average energy E_i as function of time,

σ_{ti} = total reaction cross section in group i,

v_i = average neutron velocity in group i,

σ_{ji} = average transfer cross section from group j to group i,

NN = number of neutron energy groups,

NG = number of gamma energy groups,

a_j = $(\sigma_{tj} - \sigma_{jj})\, v_j$,

b_{ji} = $\sigma_{ji}\, v_j$, and

N_i = initial number of neutrons in group i.

To obtain the gamma source, we assume the gamma energy interval to be segmented in the same manner as the neutron energy interval, and that there exists a matrix whose elements B_{ji} are physically the cross-sections for production of gammas of energy E_i by neutrons of energy E_j. Then the rate of production of photons of energy E_i is given by

$$z_i(t) = \sum_{j=1}^{NN} v_j\, B_{ji}\, n_j(t) \qquad i=1,\ NG \tag{4}$$

If $n_j(t)$ is replaced by (2), the result is

$$z_i(t) = \sum_{j=1}^{NN} v_j B_{ji} \sum_{k=1}^{j} A_{kj}\, e^{-a_k t}$$

$$= \sum_{k=1}^{NN} e^{-a_k t} \sum_{j=k}^{NN} v_j\, B_{ji}\, A_{kj} \tag{5}$$

Let $$C_{ki} = \sum_{j=k}^{NN} v_j\, B_{ji}\, A_{kj} \tag{6}$$

Then $$z_i(t) = \sum_{k=1}^{NN} C_{ki}\, e^{-a_k t} \tag{7}$$

Now let $q(r,E,t)$ be the Green's function for dose rate, where

r = distance from origin,
E = photon energy, and
t = time since direct beam arrival at r.

Then the total dose rate from the neutron induced gammas is given by

$$Q(r,t) = \sum_{i=1}^{NG} \int_{-\infty}^{\infty} q(r,E_i,\tau) \quad z_i(t-\tau)d\tau \tag{8}$$

Where the convolution on energy has been replaced by a sum.

The infinite limits on τ may be replaced by finite limits, since

$$q(r,E,t) = 0, \ t<0$$
$$z_i(t) = 0, \ t<0 \tag{9}$$

After replacing $z_i(t)$ by expression (7) and rearranging, the expression for total dose rate becomes

$$Q(r,t) = \sum_{i=1}^{NG} \sum_{k=1}^{NN} C_{ki} \int_{o}^{t} q(r,E_i,\tau)e^{-a_k (t-\tau)}d\tau \tag{10}$$

The expression for radial electron current is identical to (10) if q is a Green's function for radial current.

Schaefer has obtained an empirical fit for $q(r,E,t)$ as well as for the radial current Green's function[1], based upon the Monte Carlo gamma transport calculation of LeLevier[2]. For ease of application, the expressions used in this note are related to those of Schaefer below.

$$q(r,E,t) = Q_d[\delta \ (t) + f(E,r,t)] \tag{11}$$

where

Q_d = energy deposited by the direct or unscattered gamma beam

$\delta(t)$ = Dirac delta function

$f(E,r,t)$ = the empirical fits obtained by Schaefer

The expression for the radial current Green's function is similarly defined.

III. Numerical Results

Numerical results are presented for the dose rate and radial current obtained from the application of equation (10), using the emperical formulas of Schaefer[1] for the Green's function. The neutron cross-sections are basically from the ENDF-B file[7], and gamma production cross-sections were obtained from many sources. The cross-sections consisted in a matched set of 22 neutron groups and 18 gamma groups, with the neutron groups distributed over the energy range thermal to 15 MEV, and the gamma energies distributed over the range .02 to 10 MEV. A more complete description is given in reference 4. All results are for STP air of density 1.2929 mg/cc.

The direct beam dose and current for equation (11) were computed as follows:

$$Q_d\,(r,E) = \frac{U\,(E)\,e^{-r/\lambda(E)}}{4\pi r^2} \tag{12}$$

$$J_d\,(r,E) = \frac{W\,(E)\,e^{-r/\lambda(E)}}{4\pi r^2} \tag{13}$$

$$U(E) = E\,\sigma_d\,(E) \tag{14}$$

$$W(E) = eR\,(E)/\lambda_c\,(E) \tag{15}$$

Where

E = gamma energy

$\lambda(E)$ = gamma mean free path (all reactions)

$\sigma_d(E)$ = gamma cross-section for ionizing energy loss

e = electronic charge

$R(E)$ = average electron range in direction of incident gamma[5]

$\lambda_c(E)$ = mean free path for Compton collisions

The cross-section $\sigma_d(E)$ includes gamma losses due to photoelectric effect, Compton scattering, and pair production. It was assumed that electron currents were produced only from Compton scattering, and that the production of current and deposition of energy were local processes.

The first two sets of data display r^2 times dose rate and r^2 times radial current as a function of retarded time for various ranges. Figs. 1-3 show the detailed structure of the wavefront for the first microsecond, and Figs. 4-17 give the "rattle model" prediction for dose rate and radial current out to arrival time of the neutron wave. Figs. 18-23 are plots of dose rate and radial current divided by average neutron energy for three ranges. The purpose of these is to show the effectiveness of various portions of the neutron spectrum normalized to one MEV of source energy.

The scales are labeled in such a way that the results may be used for air of any density. A full discussion of density scaling is given in reference 6, and an example is given below to illustrate the use of density scaling.

PROBLEM: Obtain the gamma dose rate from one 14 MEV neutron at a range of 500 m in air of density .9 mg/cc.

Figs. 1 and 4 apply in this case, but for illustration purposes we will use only Fig. 1 and compute the dose rate at retarded times 0, .5, and 1 microseconds. Let ρ = air density relative to STP.

Compute air density relative to STP: $\rho = \frac{.9}{1.2929} = .696$

Compute scaled distance: $\rho * r = 348\text{m}$

Compute scaled times: $\rho * 0. = 0\ \mu s$

$\rho * .5 = .348\ \mu s$

$\rho * 1. = .696\ \mu s$

The scaled distance is between the 200m and 400m curves, so values must be extrapolated at each of the scaled times. These are listed in the following table:

Time (μs)	Scale Time (μs)	$\frac{r^2 * \text{Dose Rate}}{\rho^2}$	Dose rate (MEV/M^2·sec)
0.0	0.	50.	$9.7 * 10^{-5}$
0.5	.348	75.	$1.45 * 10^{-4}$
1.0	.696	80.	$1.55 * 10^{-4}$

To obtain the final or desired value of dose rate, the extrapolated values must be multiplied by $(\rho/r)^2$, and this value is also listed in the above table. The radial current is similarly computed.

A preliminary investigation indicates that the dose rate and radial current may be accurately given by a function of the form:

$$Q(E,r,\tau) = \frac{F_1\ (E,\tau)\ \exp\ (-rF_2(E,\tau))}{r^2}, \qquad (16)$$

where $\tau = t - r/c$

$$F_1 = c_1 e^{-c_2 \tau} + c_3 e^{-c_4 \tau} \quad (17)$$

$$F_2 = \frac{c_5 \tau^2 + c_6 \tau + c_7}{\tau + c_8} \quad (18)$$

The coefficients c_i are functions of neutron energy only. If early time behavior ($\tau < 2\mu s$) is unimportant, a very good fit is obtained with the F_1 equal to a single exponential in τ, and F_2 a linear function of τ. The equation for radial current has the same form with a different set of c's. This representation of the "rattle model" prediction will be investigated further in a later report.

IV. Conclusions

The "Rattle" model is probably the only practical method for obtaining an accurate time history for the neutron-induced dose rate and radial current within the first microsecond of observer time. In addition, the relative simplicity and speed of the method make it very useful for quickly determining the effect on dose rate and current as neutron and gamma-production cross-sections are updated, without doing a complete transport calculation.

I would like to thank Rodney Lowen and Richard Summey for plotting the graphs contained in this report.

FIG. 1 Early Time behavior of dose rate
and radial current at neutron source energy 12.2-15 Mev.

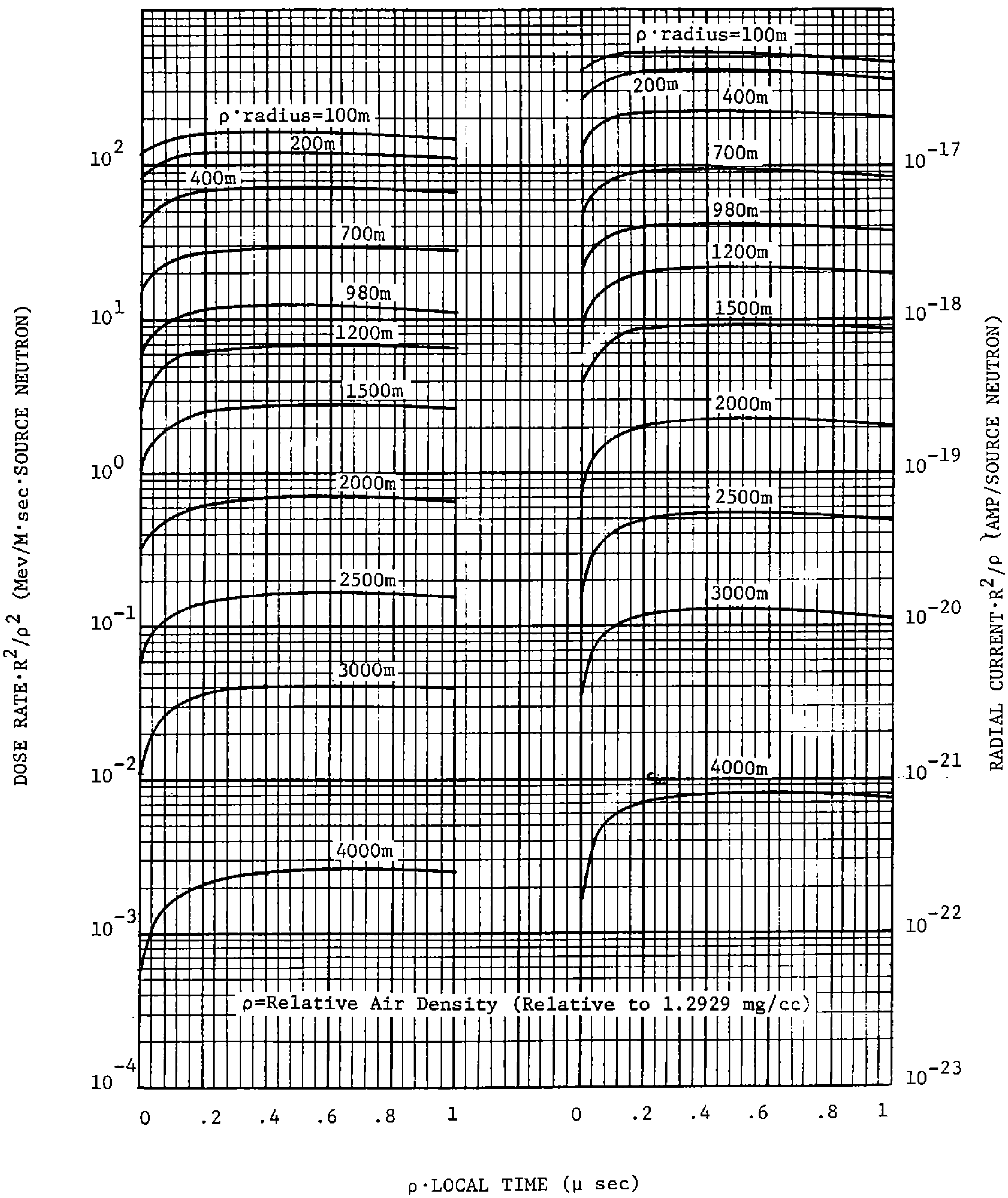


FIG. 2 Early time behavior of dose rate
and radial current at neutron source energy 8.18-10.0 Mev.

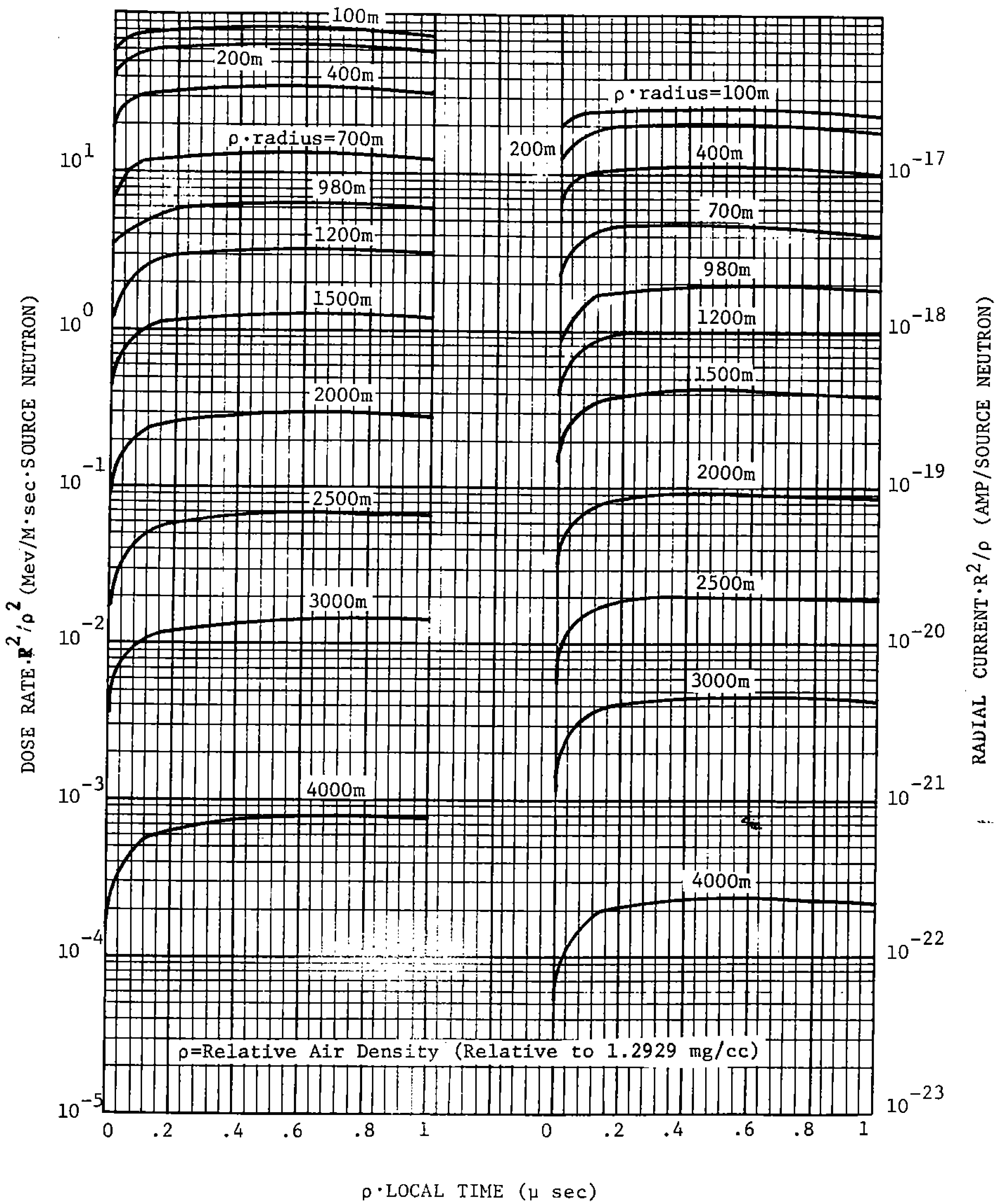


FIG. 3 Early Time behavior of dose rate and radial current at neutron source energy 4.96-6.36 Mev.

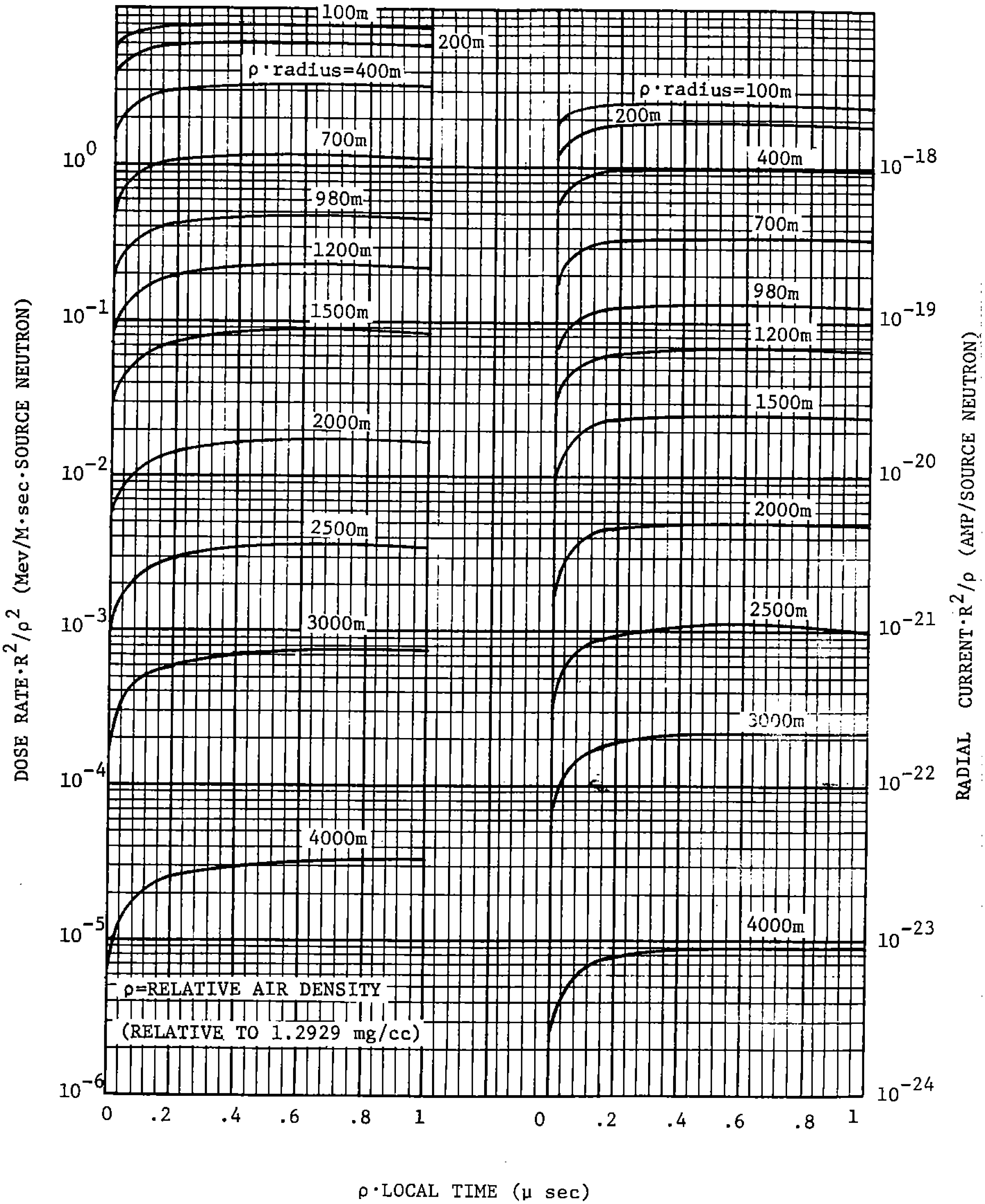


FIG. 4 R^2 times dose rate.
Neutron source energy 12.2-15 Mev.

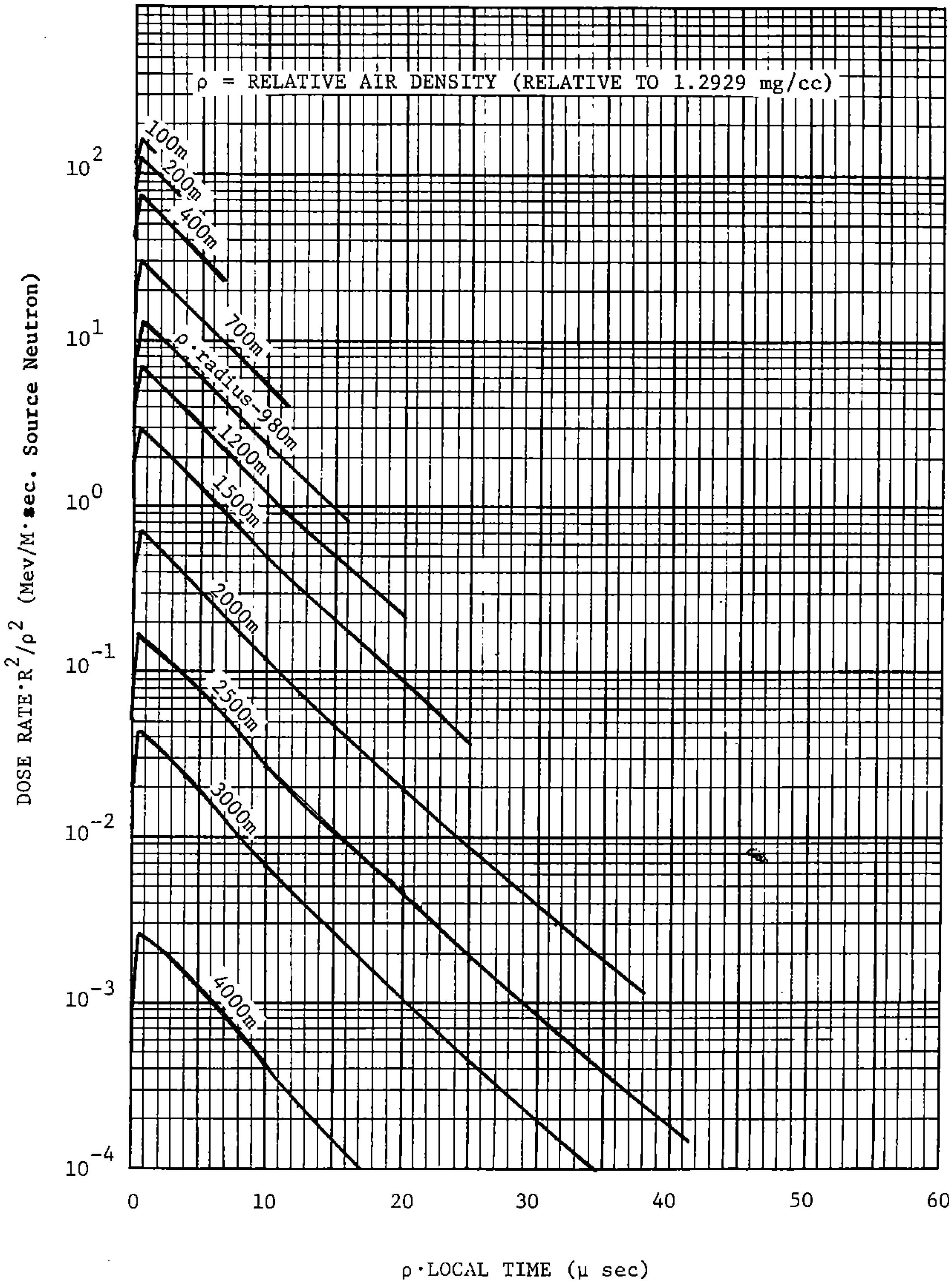


FIG. 5 R^2 times radial current.
Neutron source energy 12.2-15 Mev.

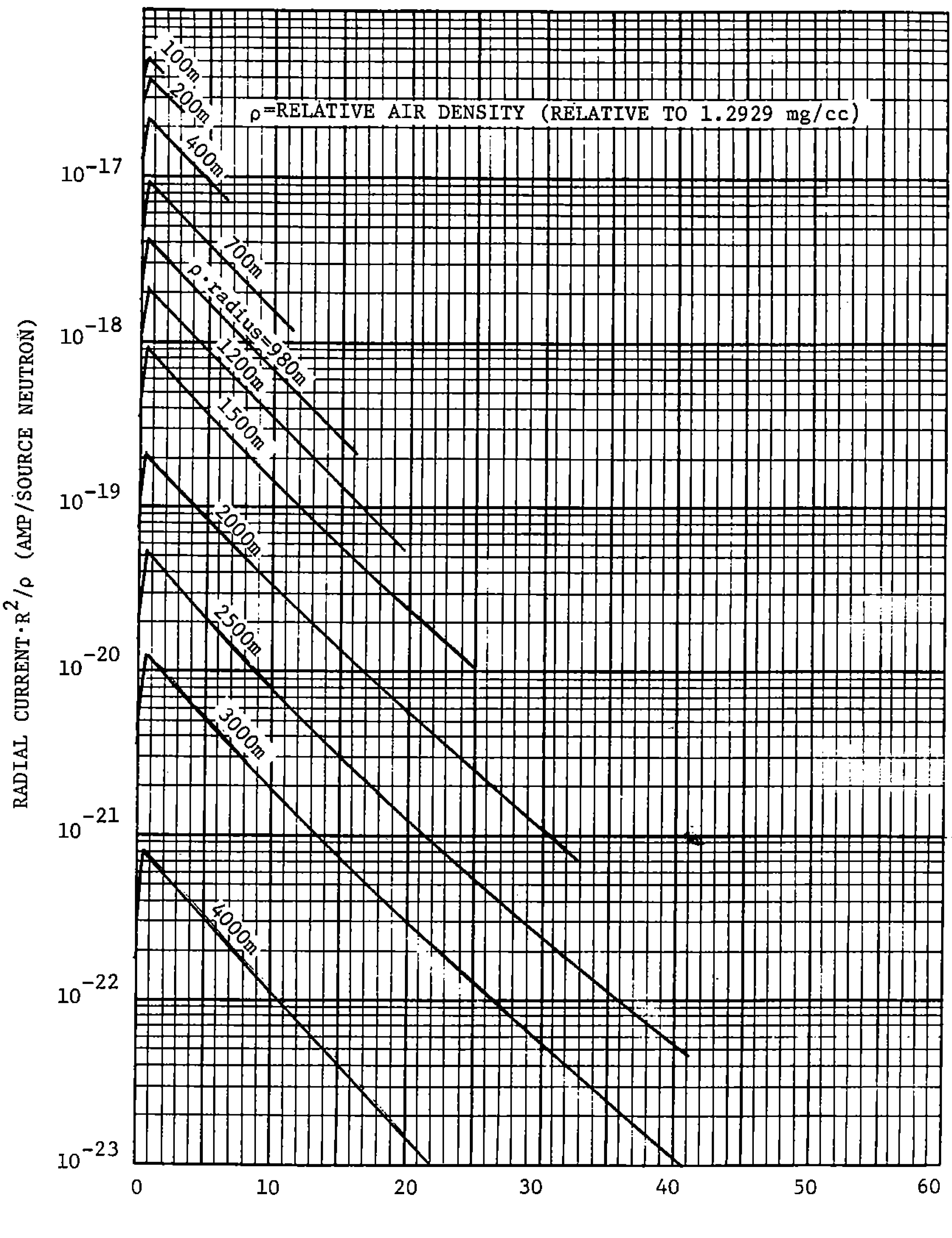


FIG. 6 R^2 times dose rate.
Neutron Source energy 10.0-12.2 Mev.

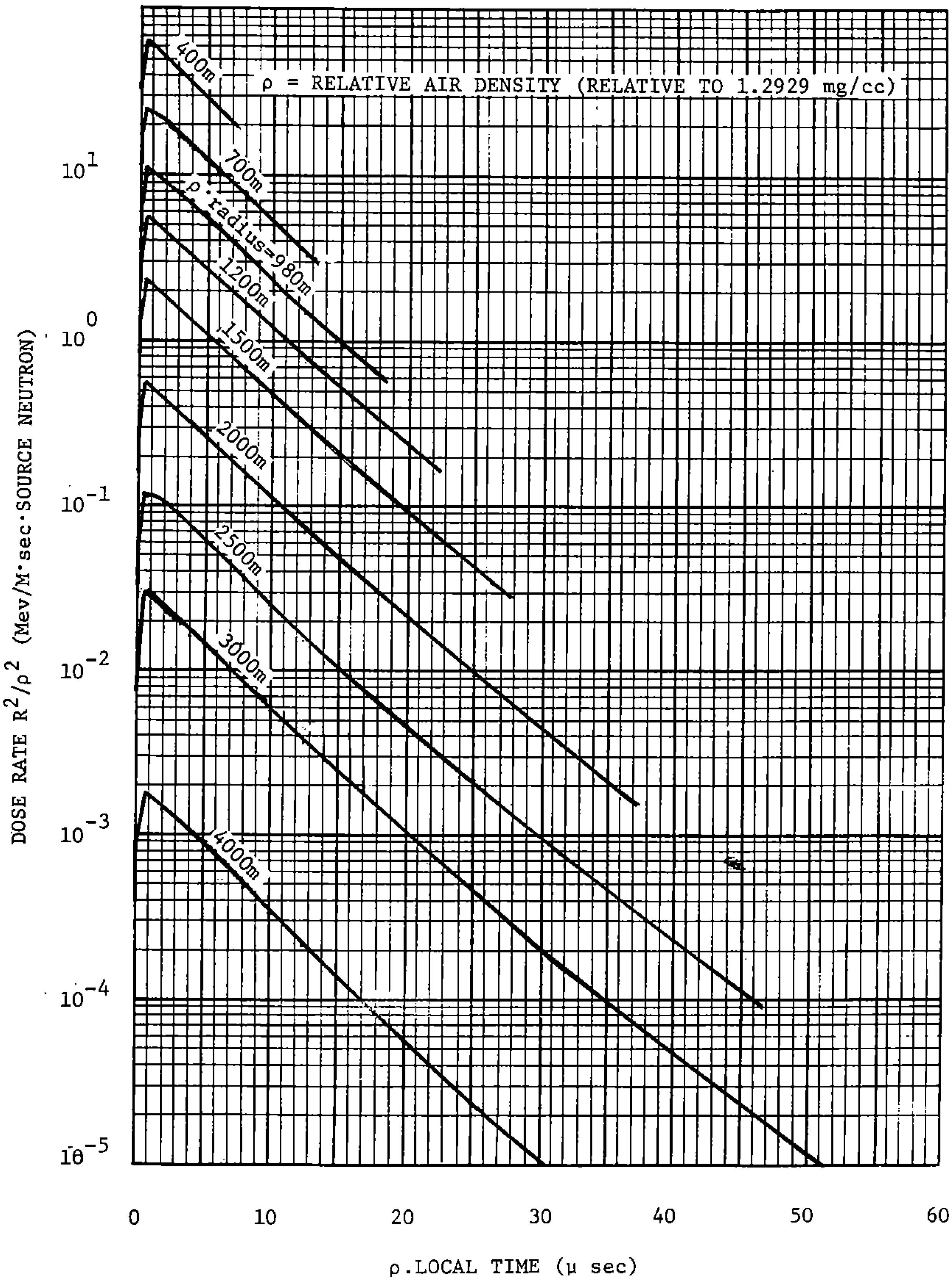


FIG. 7 R^2 times radial current.
Neutron source energy 10.0-12.2 Mev.

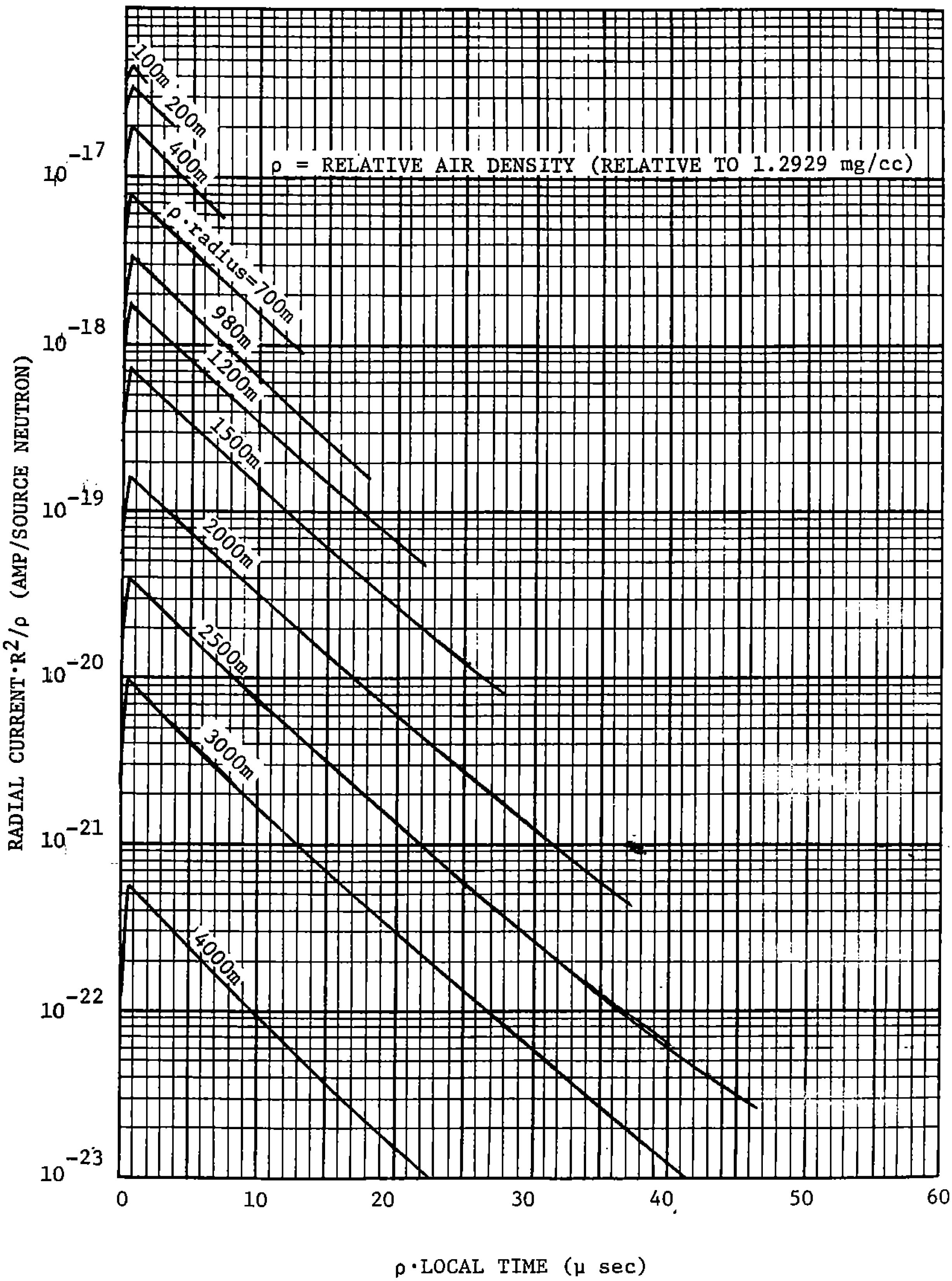


FIG. 8 R^2 times dose rate.
Neutron source energy 8.18-10.0 Mev.

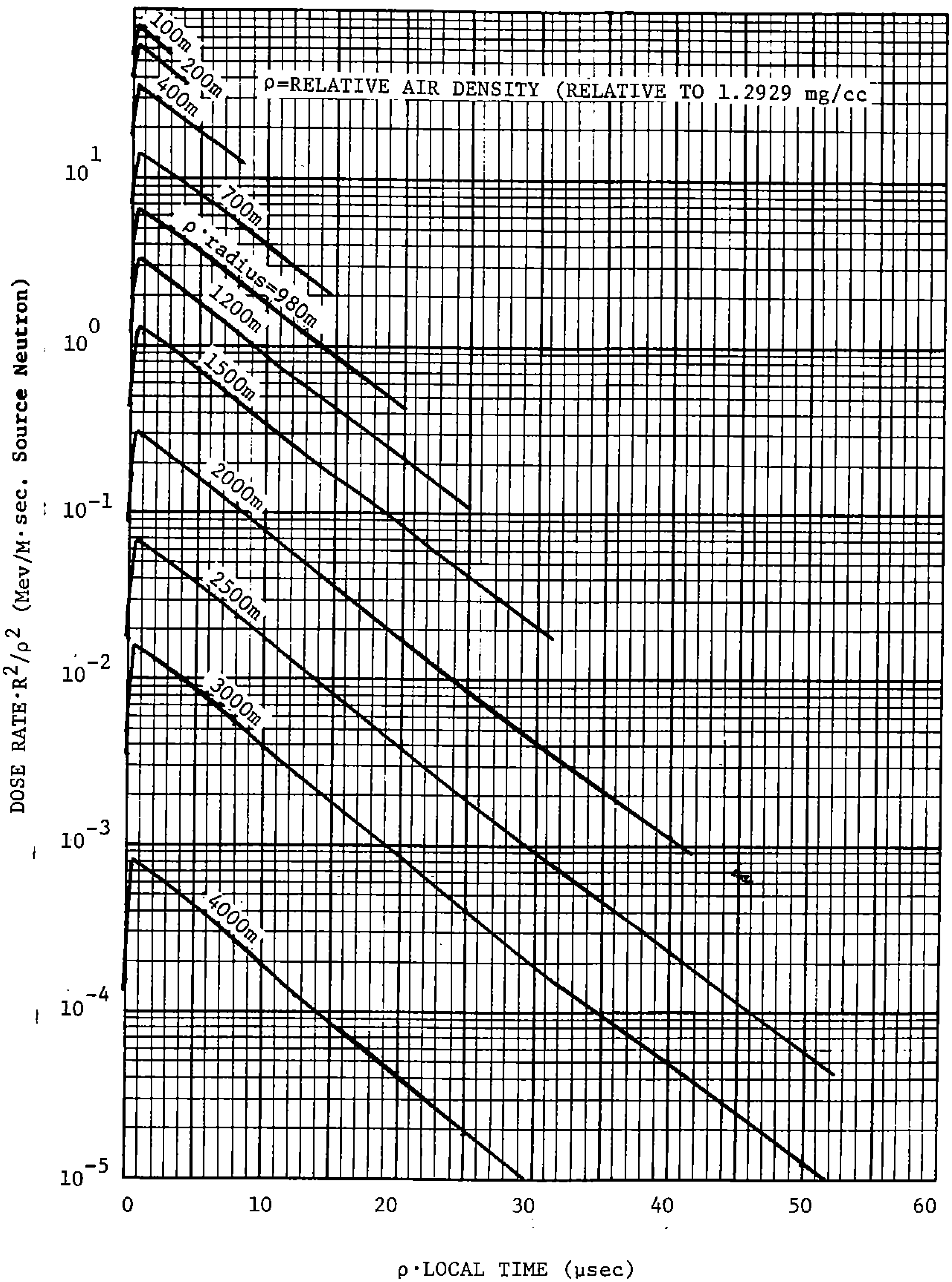


FIG. 9 R^2 times radial current.
Neutron source energy 8.18-10.0 Mev.

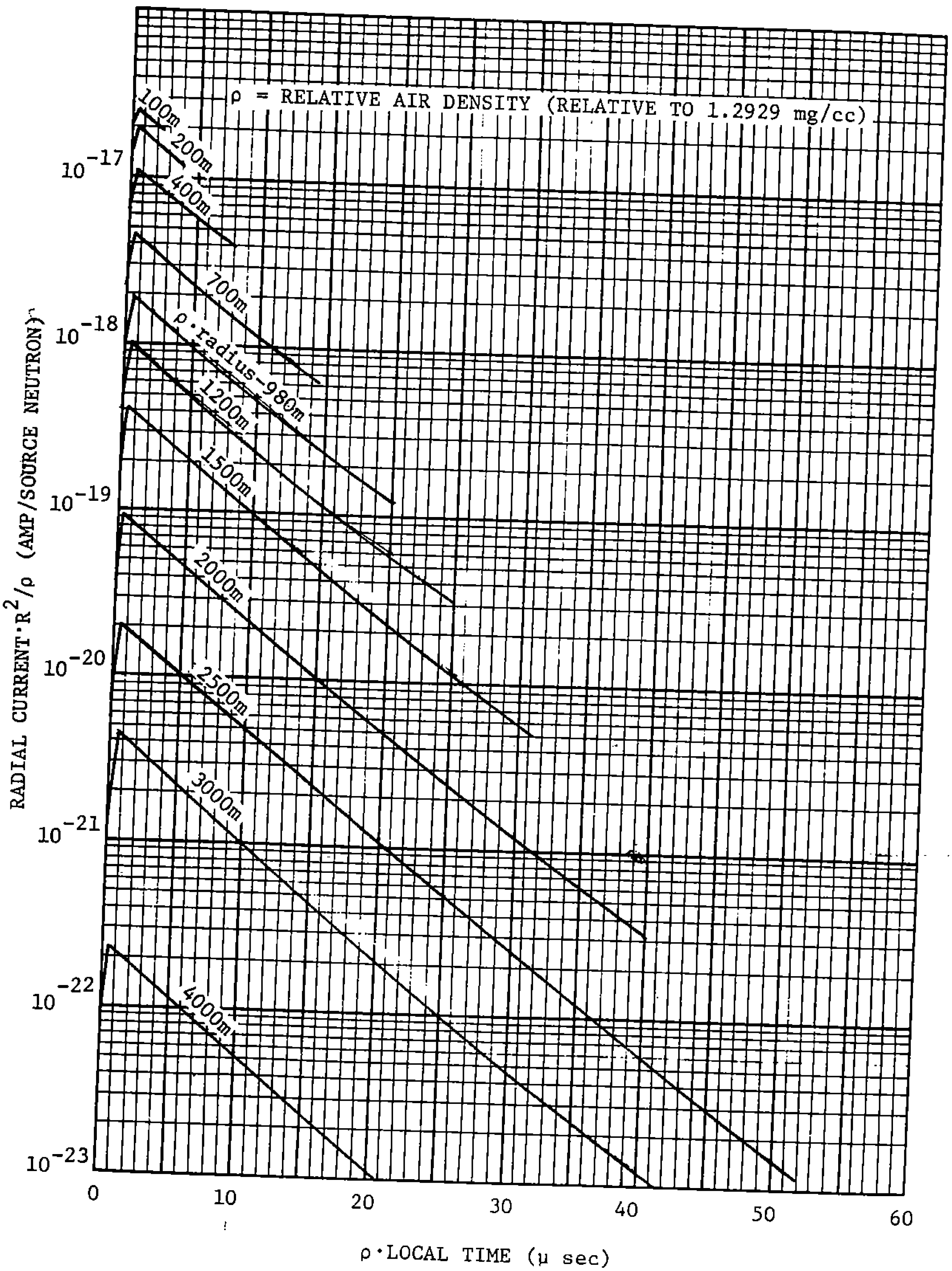


FIG. 10 R^2 times dose rate.
Neutron source energy 6.36-8.18 Mev.

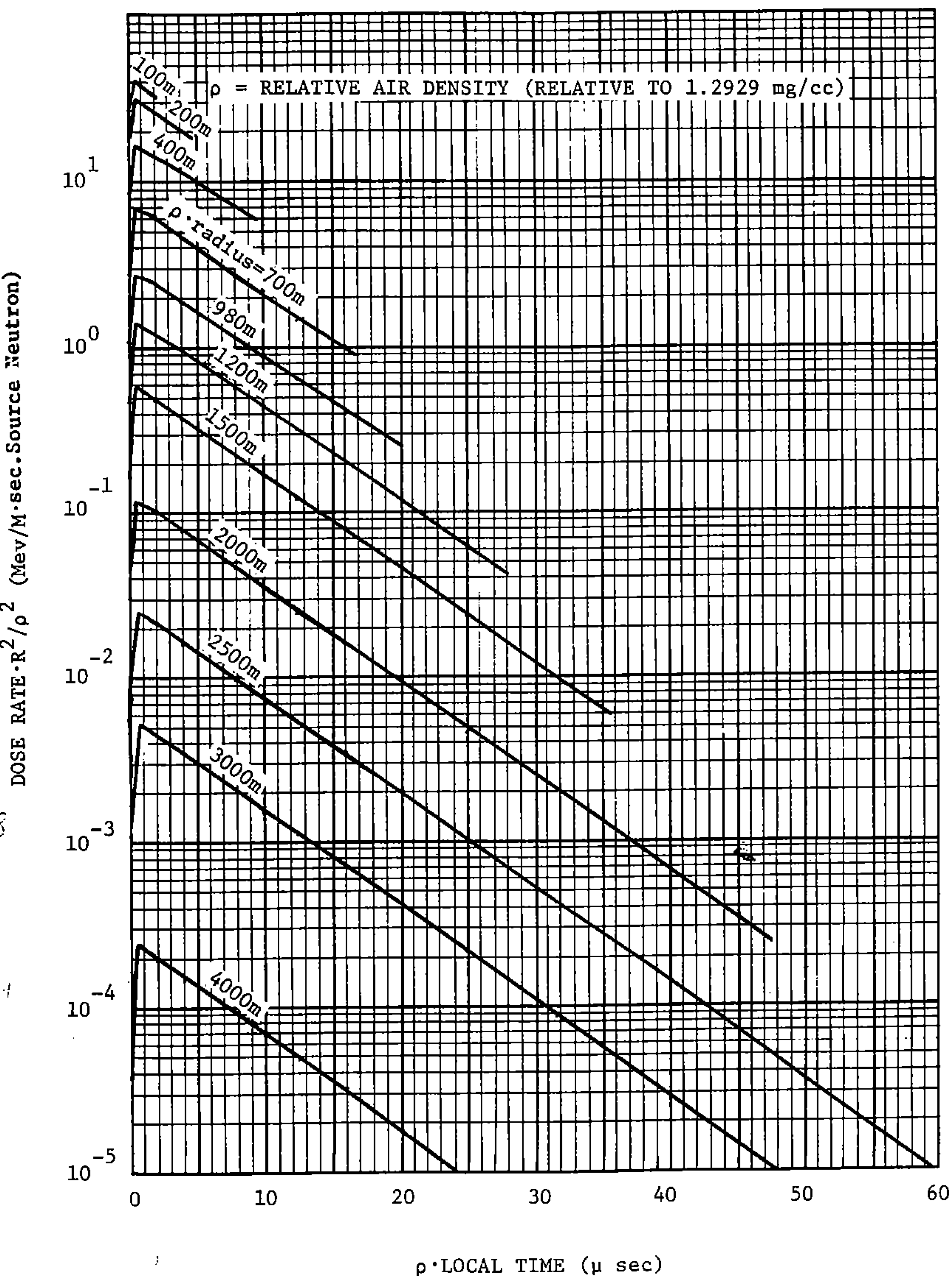


FIG. 11 R^2 times radial current.
Neutron source energy 6.36-8.18 Mev.

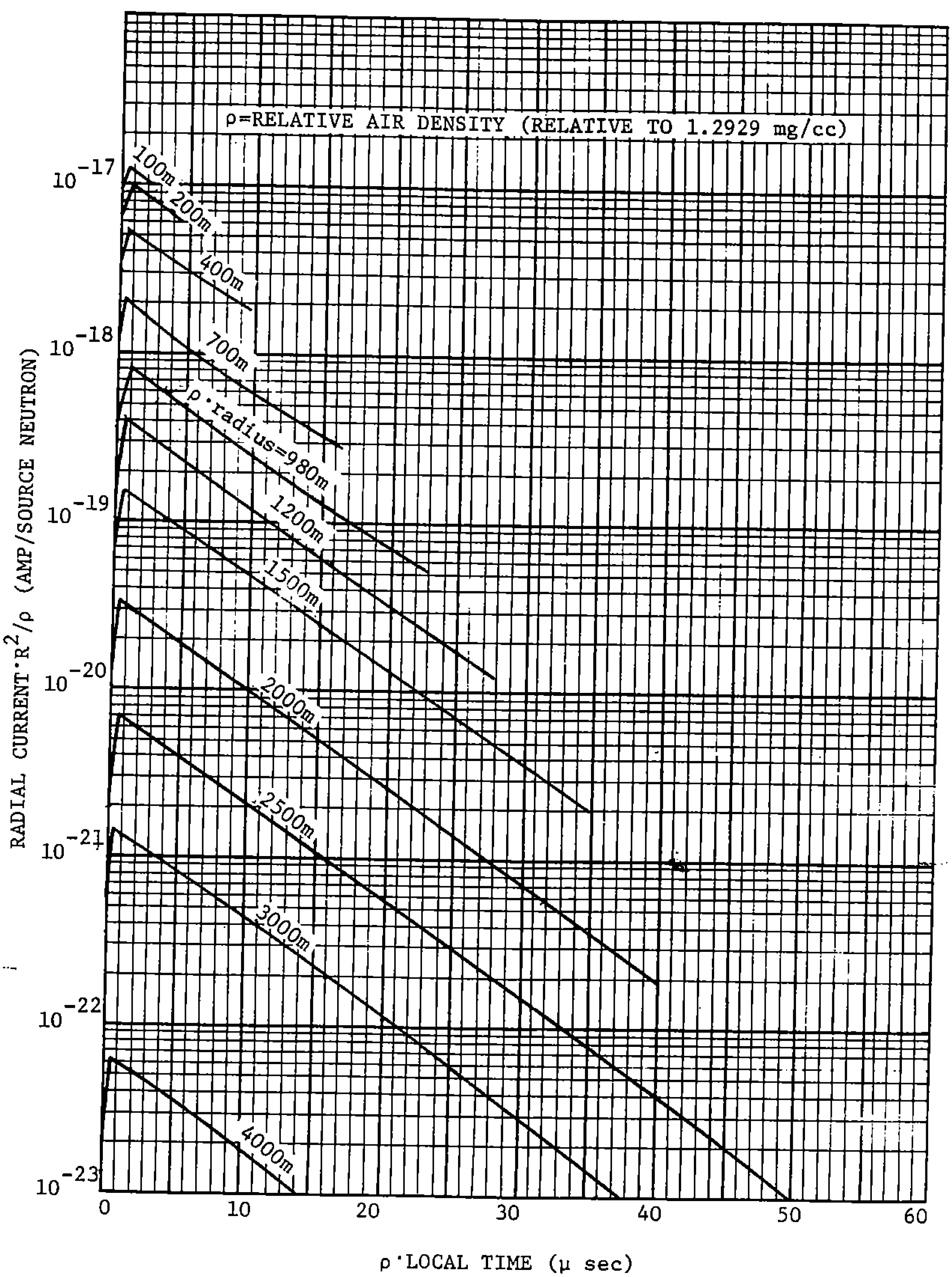


FIG. 12 R^2 times dose rate.
Neutron source energy 4.96-6.36 Mev.

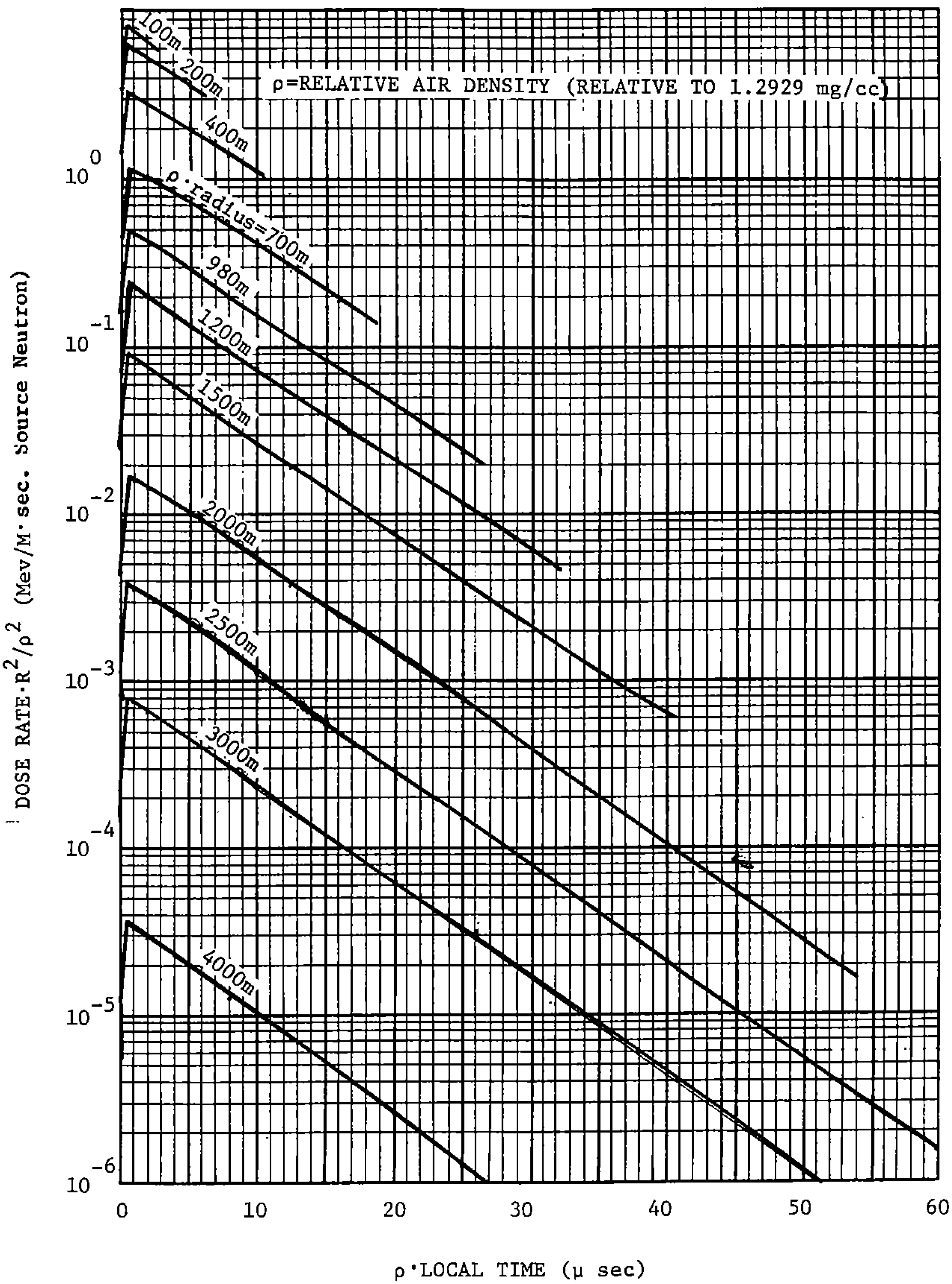


FIG. 13 R^2 times radial current.
Neutron source energy 4.96-6.36 Mev.

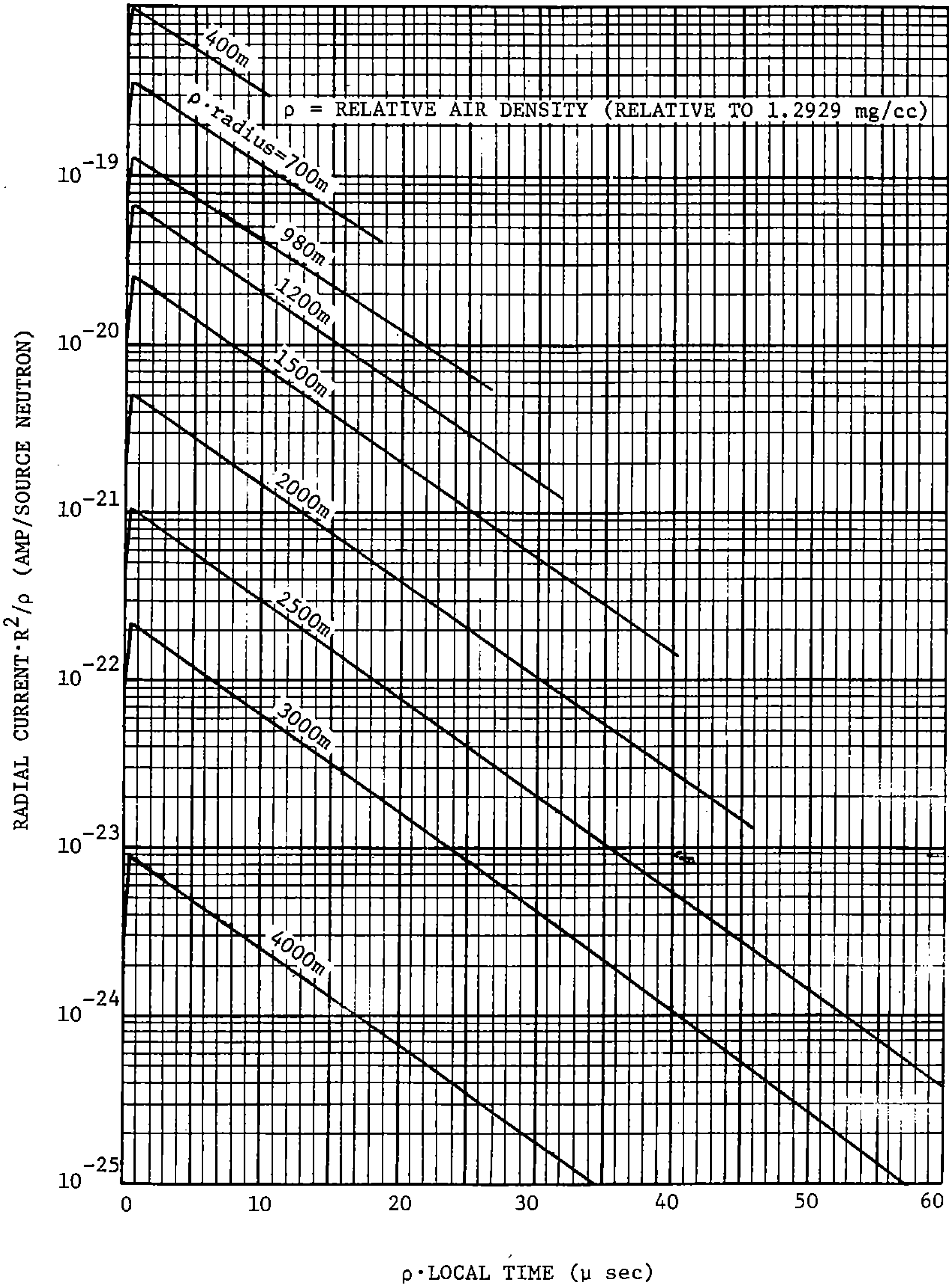


FIG. 14 R^2 times dose rate.
Neutron source energy 4.06-4.96 Mev.

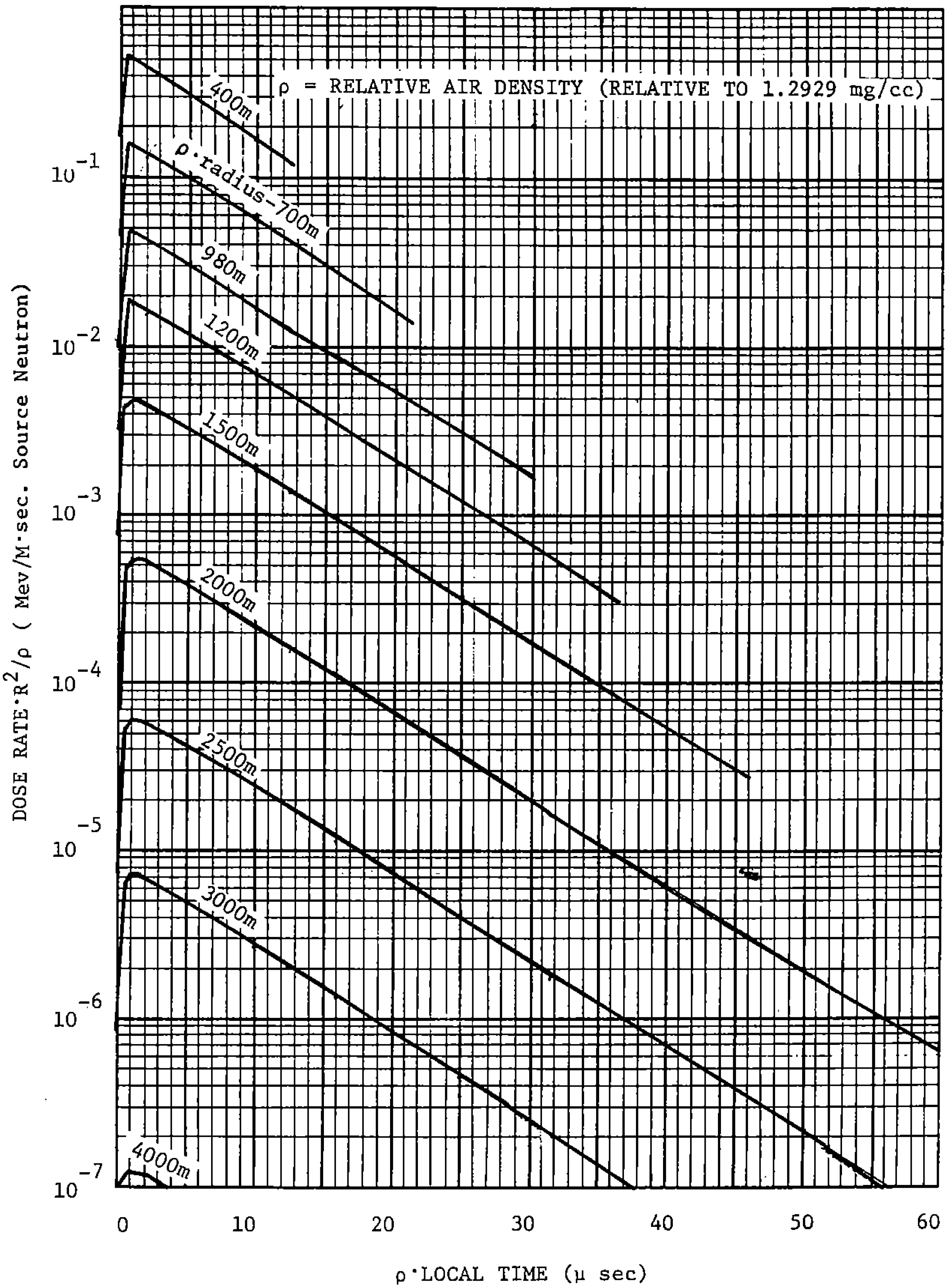


FIG. 15 R^2 times radial current.
Neutron source energy 4.06-4.96 Mev.

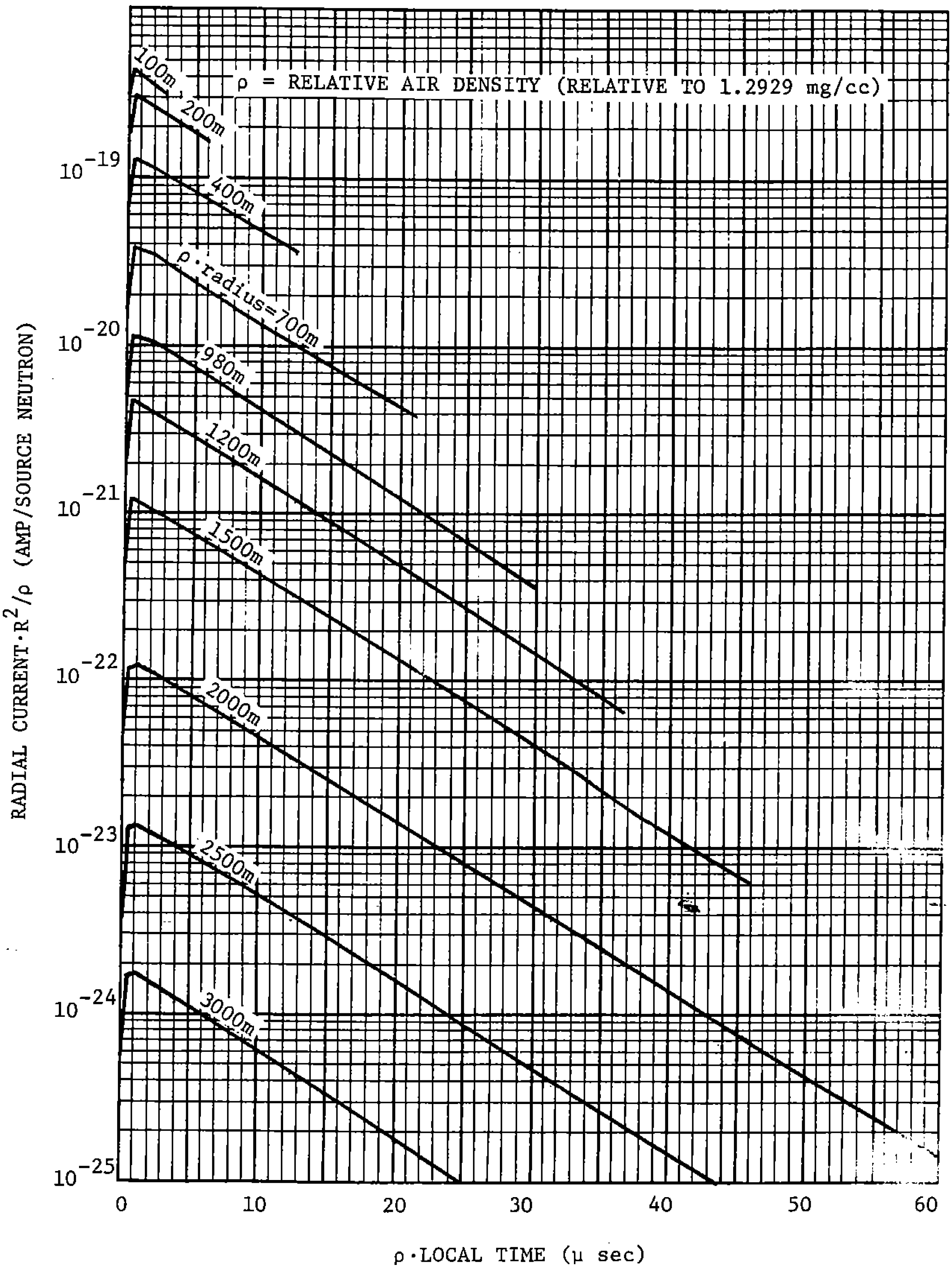


FIG. 16 R^2 times dose rate.
Neutron source energy 3.01-4.06 Mev.

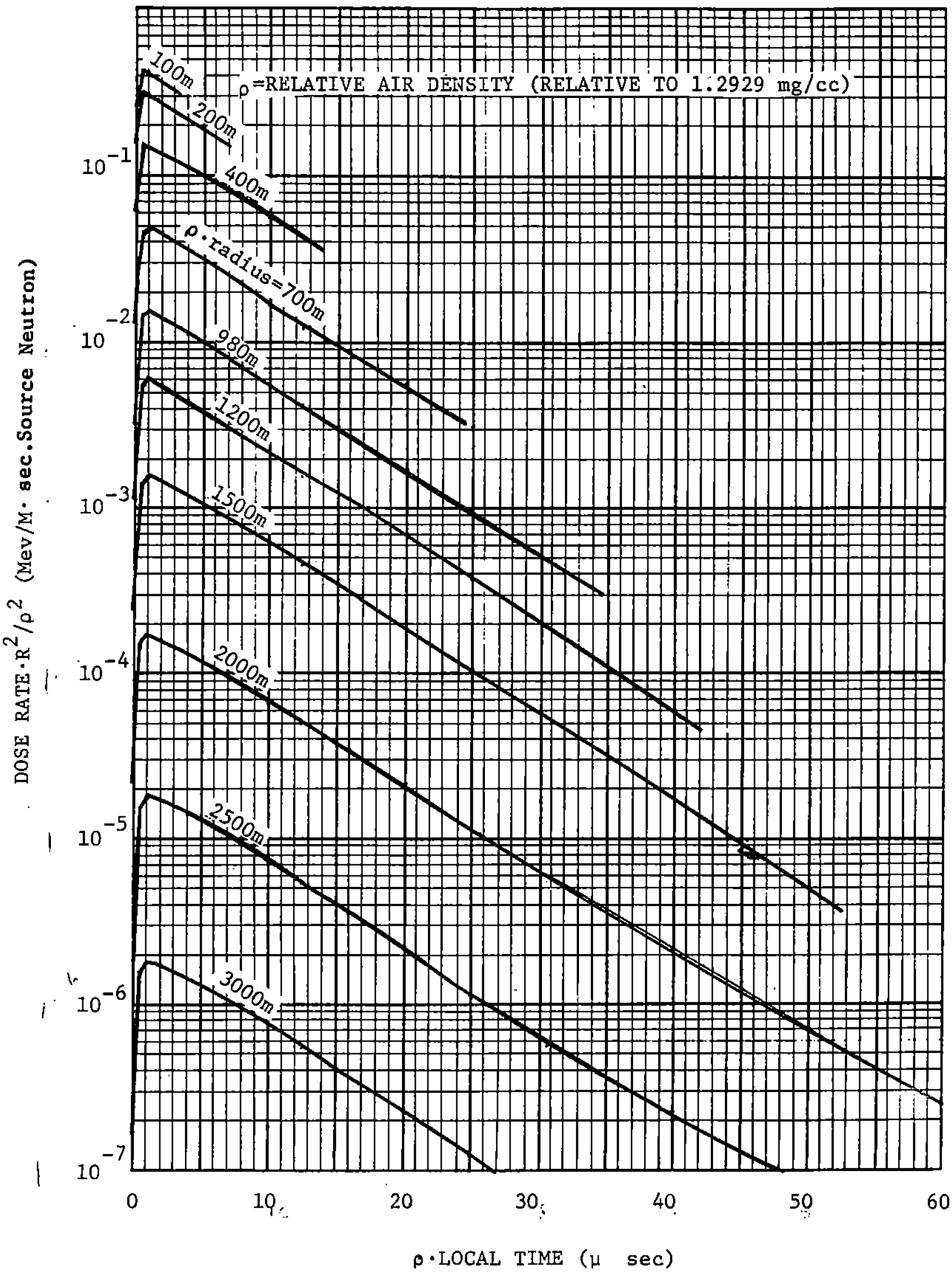


FIG. 17 R^2 times radial current.
Neutron source energy 3.01-4.06 Mev.

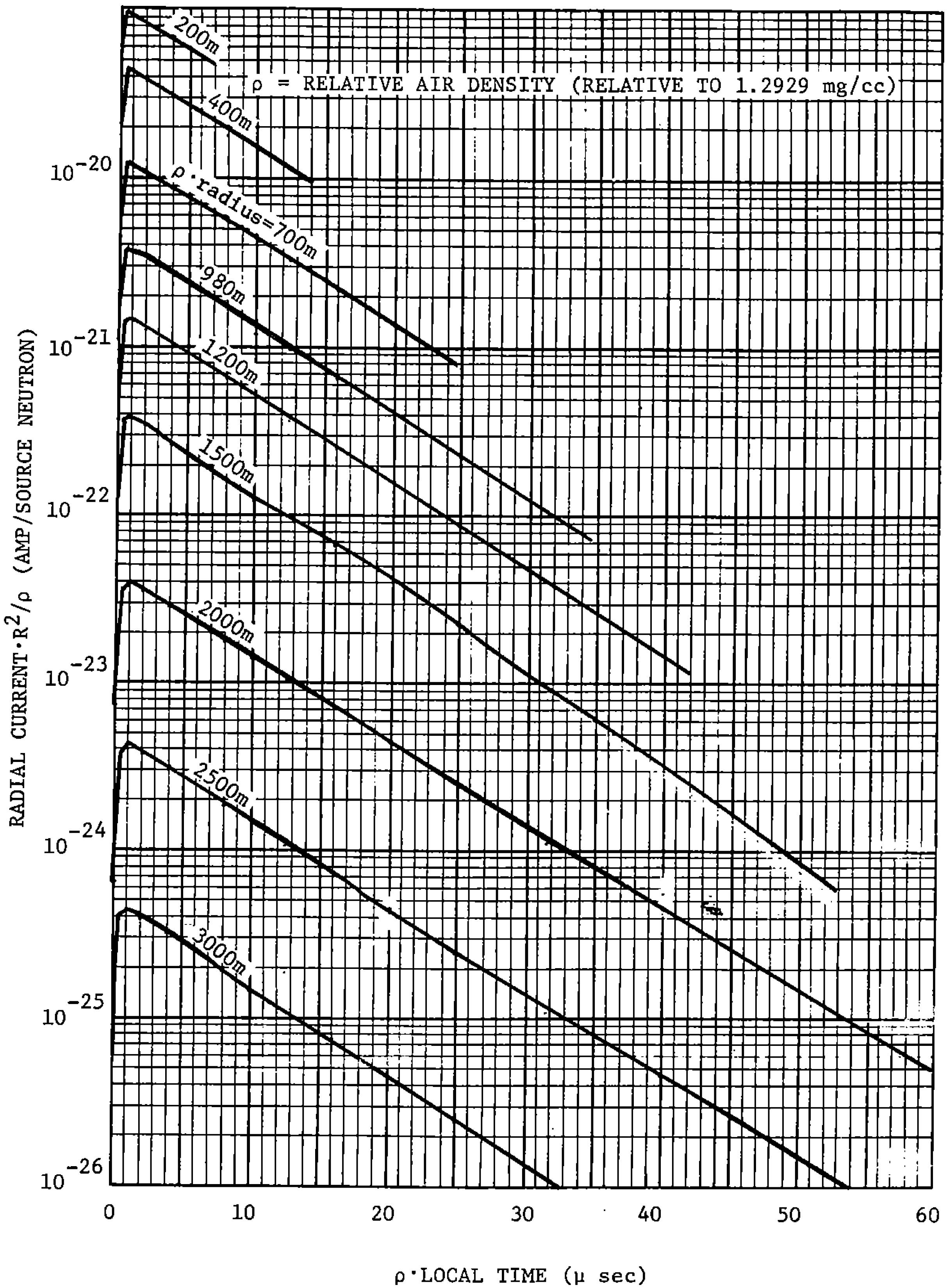


FIG. 18 Dose rate per Mev source energy.
Radial distance 400 meters.

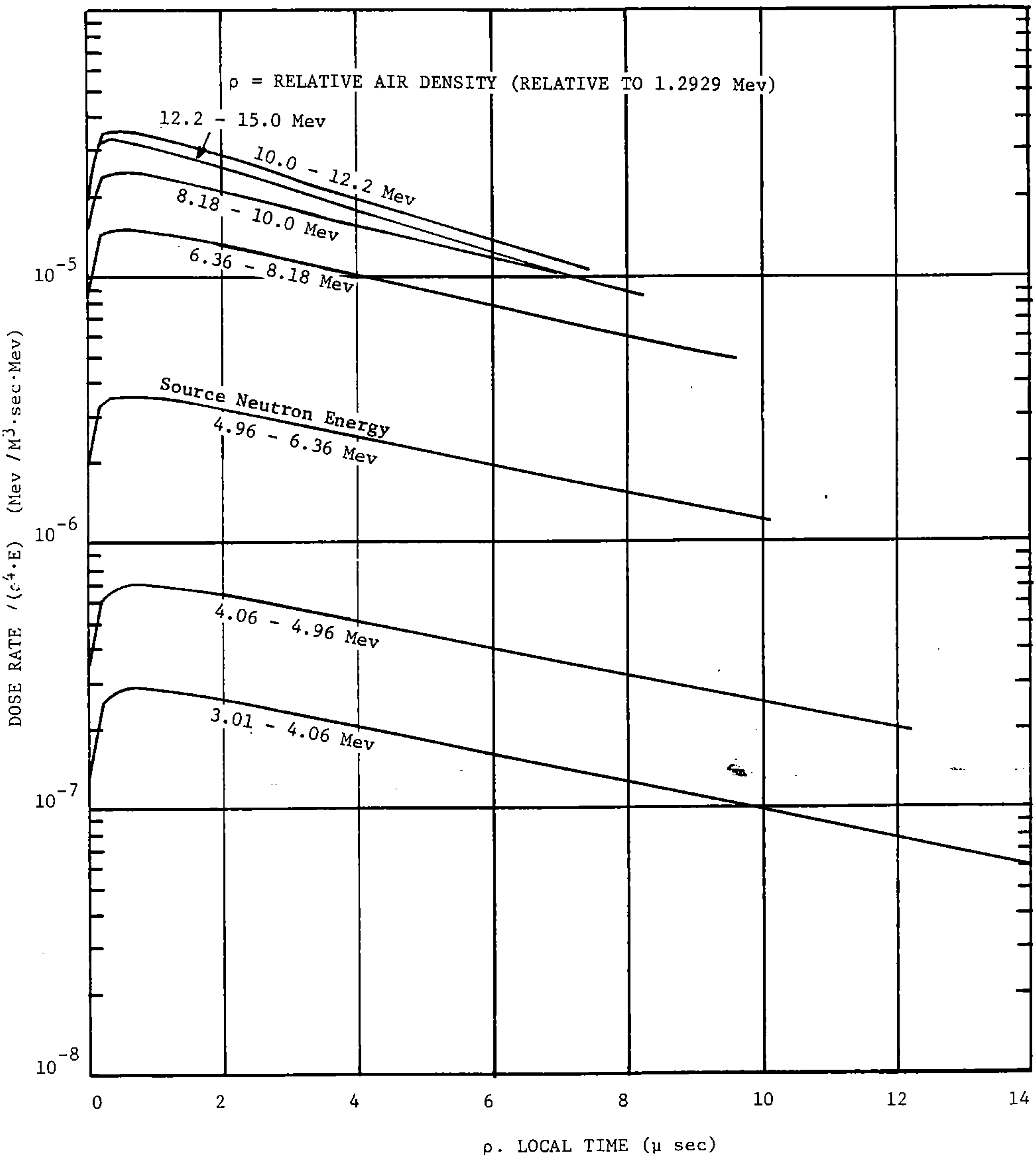


FIG.19 Radial current per Mev source energy. Radial distance 400 meters.

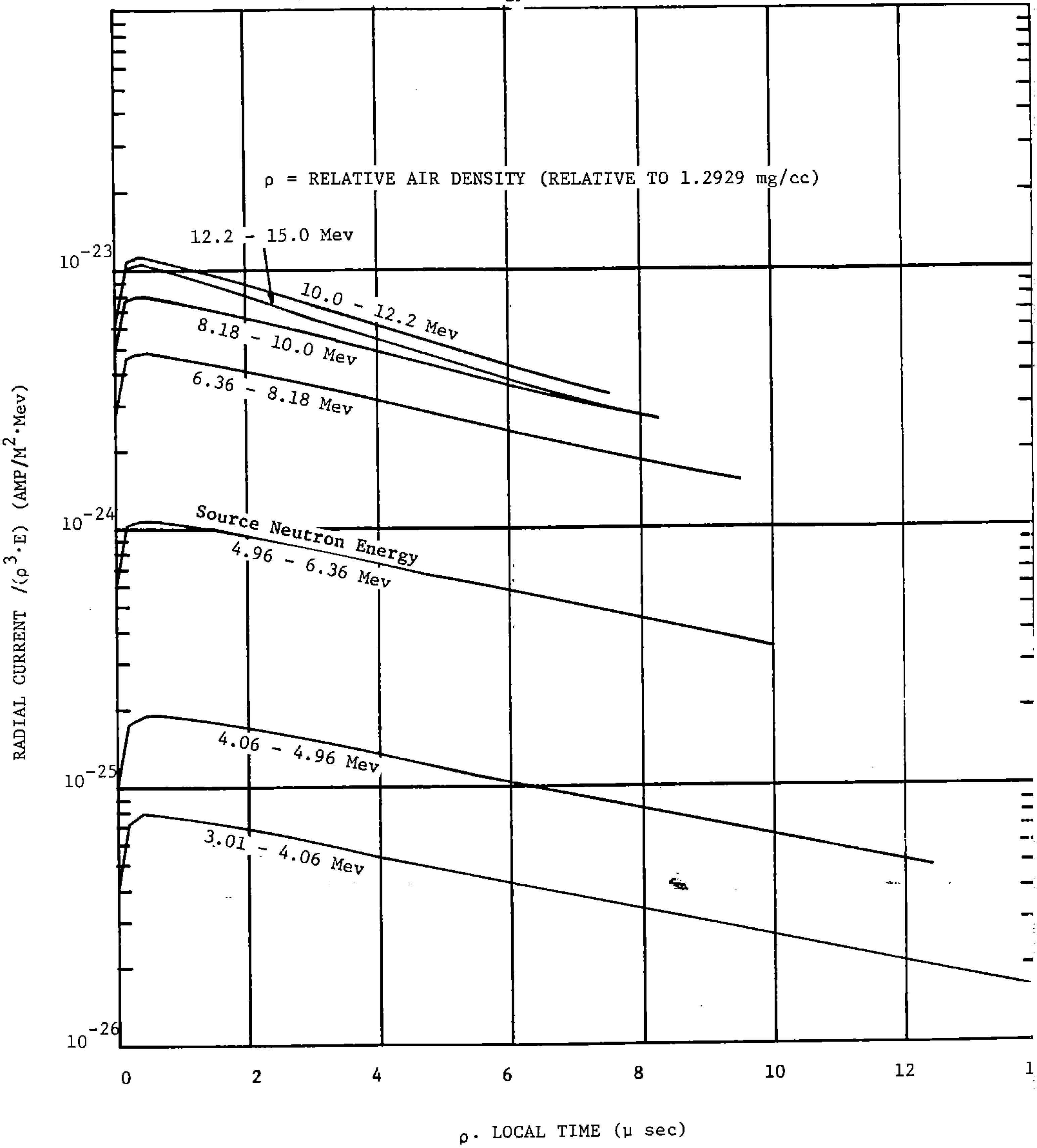


FIG. 20 Dose rate per Mev source energy.
Radial distance 1200 meters.

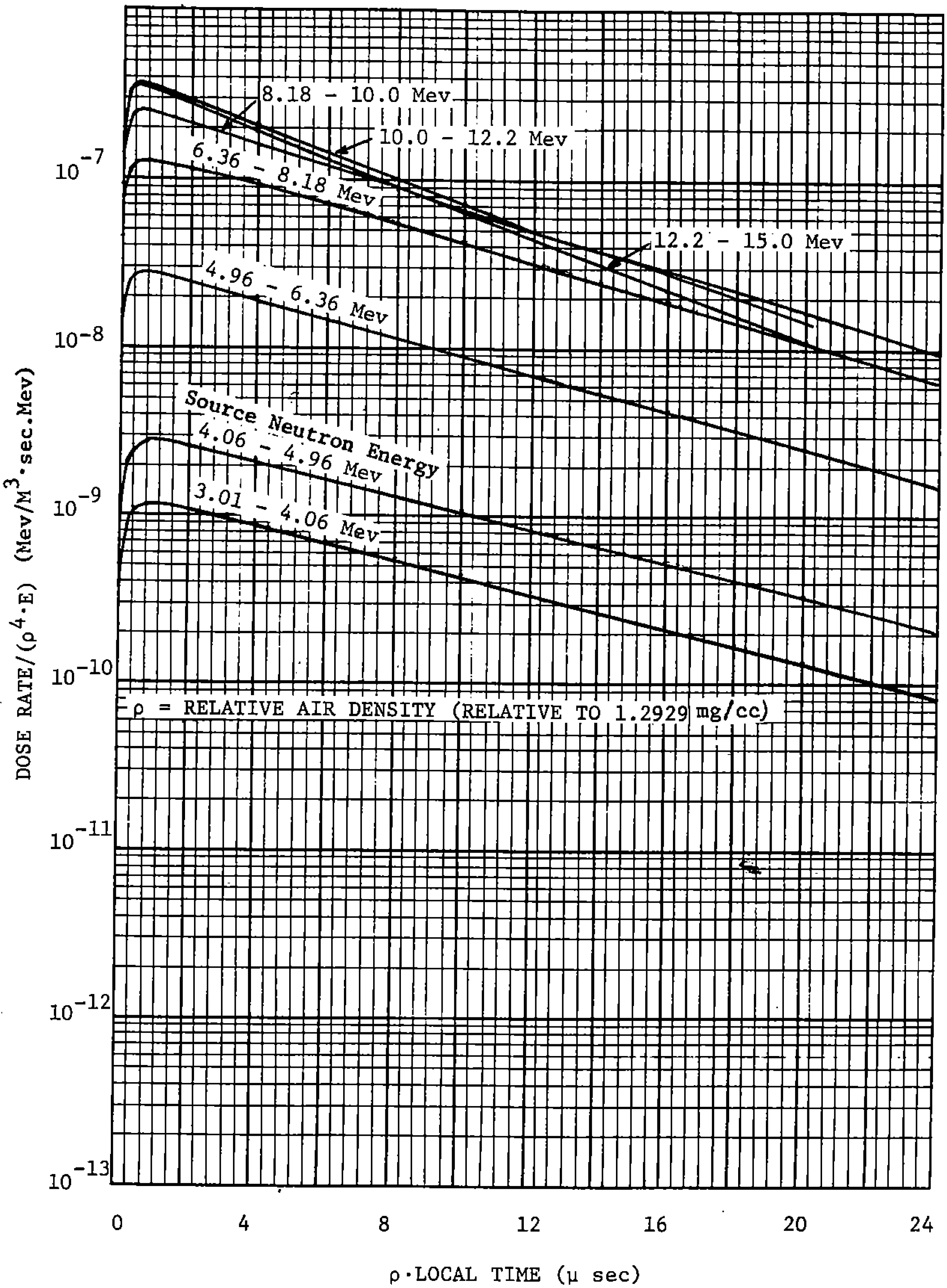


FIG. 21 Radial current per Mev source energy
Radial distance 1200 meters

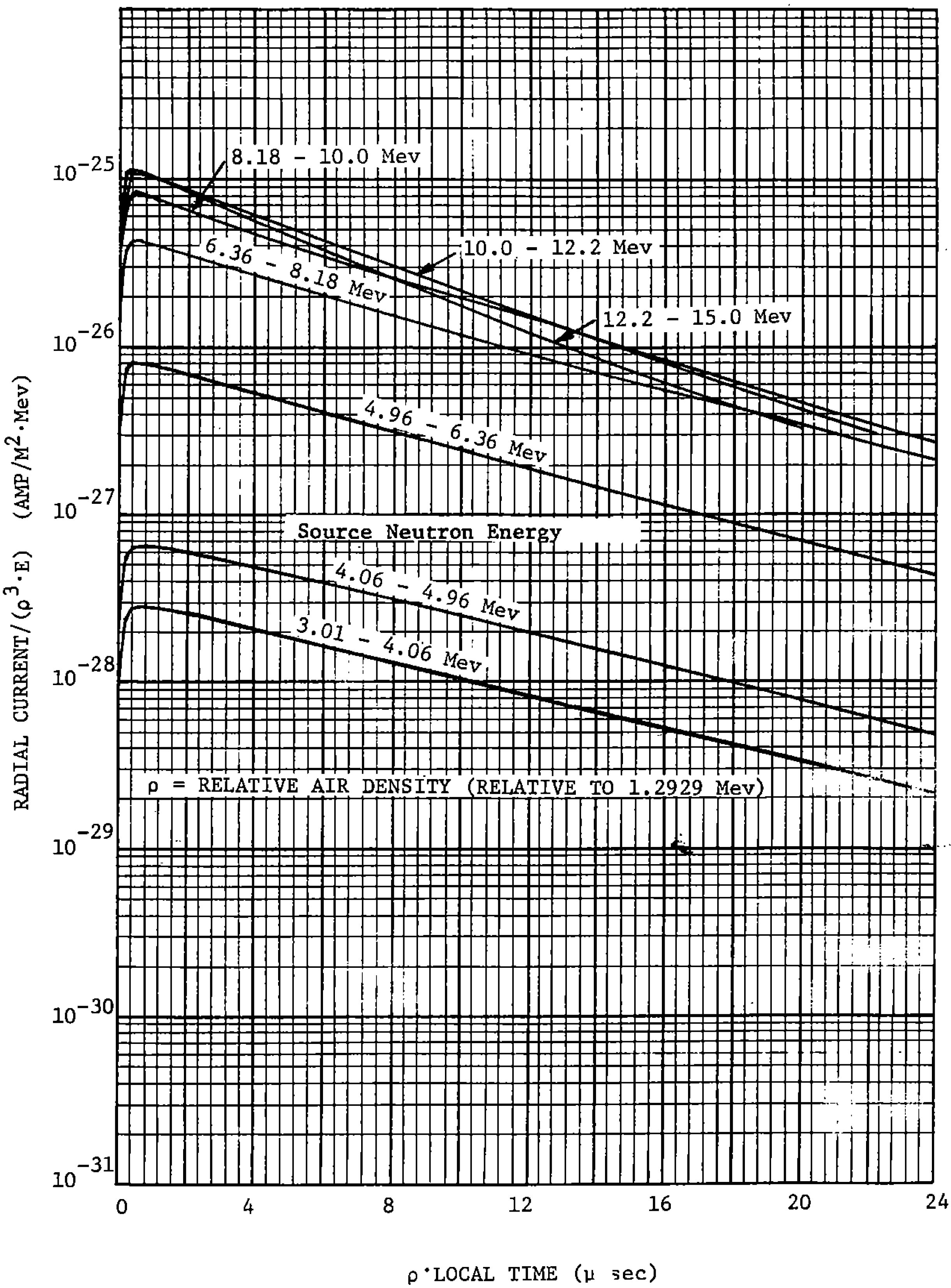


FIG. 22 Dose rate per Mev source energy.
Radial distance 3000 meters.

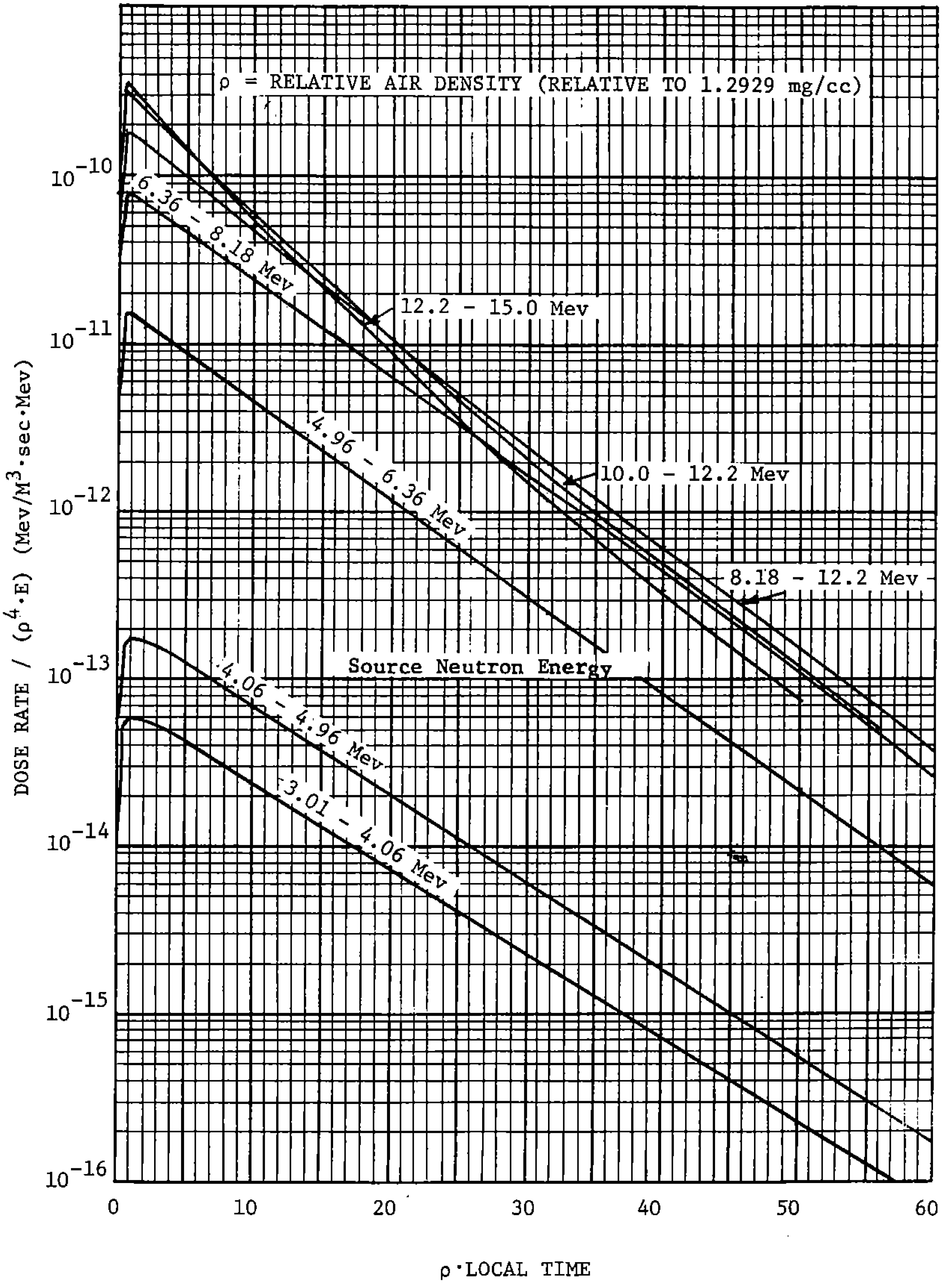
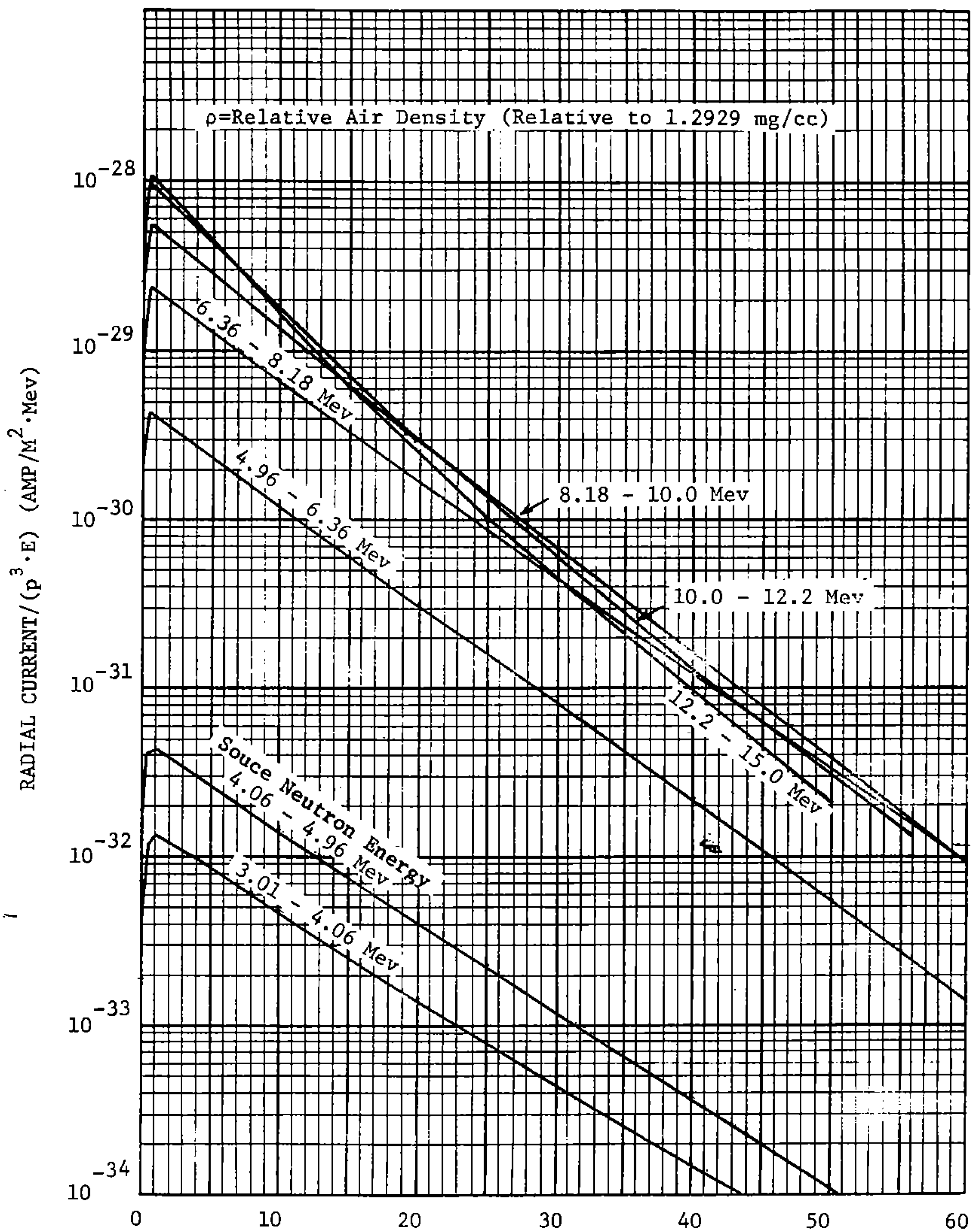


FIG. 23 Radial current per Mev source energy.
Radial distance 3000 meters.



APPENDIX A

Reduction of the Transport Equation to the Spatially Independent Case

The general form of the Boltzmann equation for neutral particle transport may be written:

$$\frac{\partial N}{\partial t} + \nabla \cdot (\hat{\Omega} v N) + \sigma_t(\bar{r},v)\, vN$$
$$= \int dv' \int d\Omega'\, v'\, N(\bar{r},v'\hat{\Omega}',t)\, \sigma_s(\bar{r},v')\, f(v'\hat{\Omega}' \to v\hat{\Omega})$$
$$+ S(\bar{r},v\hat{\Omega},t) \qquad (1)$$

Where $N(\bar{r},v\hat{\Omega},t)$ = distribution of particles in space & velocity as a function of time,

t = time,

$\hat{\Omega}$ = unit vector in direction of motion,

v = particle velocity,

$\sigma_t(\bar{r},v)$ = total reaction cross section,

$\sigma_s(\bar{r},v)$ = total scatter cross section,

$\bar{r}$ = position vector,

$f(v'\hat{\Omega}' \to v\hat{\Omega})d\ 'dv'$ = probability that a particle originated in the interval $d\Omega' dv'$ at $v'\hat{\Omega}'$ when scattered to $v\hat{\Omega}$, and

$S(\bar{r},v\hat{\Omega},t)$ = independent source.

Now we assume that N and $v\hat{\Omega}$ are continuous and have continuous partial derivations over the domain of interest and integrate (1) term by term over all solid angle. Appeal is made to Leibnitz's rule to reverse the order of integration and differentiation.

$$\int \frac{\partial N}{\partial t} d\Omega = \frac{\partial}{\partial t} \int N d\Omega = \frac{\partial P(\bar{r},v,t)}{\partial t} \qquad (2)$$

$$\int \nabla \cdot v\hat{\Omega} N d\Omega = \nabla \cdot \int v\hat{\Omega} N d\Omega = \nabla \cdot \bar{J}(\bar{r},v,t) \qquad (3)$$

$$\int \sigma_t(\bar{r},v)\, vN d\Omega = \sigma_t(\bar{r},v)\, vP(\bar{r},v,t) \qquad (4)$$

$$\int d\Omega \left[\int dv' \int d\Omega' v' N(\bar{r},v'\hat{\Omega}',t)\, \sigma_s(\bar{r},v') f(v'\hat{\Omega}' \to v\hat{\Omega}) \right]$$

$$= \int dv' \int d\Omega' N(\bar{r}, v'\hat{\Omega}', t)\ \sigma_s(\bar{r}, v') \int d\Omega f(v'\hat{\Omega}' \rightarrow v\hat{\Omega}), \tag{5}$$

Where by definition,

$$P(\bar{r}, v, t) \equiv \int N(\bar{r}, v\hat{\Omega}, t) d\Omega \tag{6}$$

$$\bar{J}(\bar{r}, v, t) \equiv \int v\hat{\Omega} N(\bar{r}, v\hat{\Omega}, t) d\Omega \tag{7}$$

$$Q(\bar{r}, v, t) \equiv \int S(\bar{r}, v\hat{\Omega}, t) d\Omega \tag{8}$$

In general the scatter kernel may be written as an infinite Legendre expansion in the cosine of the scatter angle.

$$f(v'\hat{\Omega}' \rightarrow v\hat{\Omega}) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2}\, g_\ell(v', v)\ P_\ell\ (\cos\theta_o) \tag{9}$$

Now let the coordinates of $\hat{\Omega}$ and $\hat{\Omega}'$ be respectively (θ, ϕ) and (θ', ϕ'), and apply the addition theorem[3]:

$$P_\ell(\cos\ \theta_o) = P_\ell(\cos\theta)\ P_\ell\ (\cos\theta') + \sum_{m=1}^{\ell} \frac{(\ell-m)!}{(\ell+m)!} P_\ell^m(\cos\theta)\ P_\ell^m\ (\cos\theta')\ \cos\ m\ (\phi-\phi') \tag{10}$$

Now if (10) is substituted in (9) and the integration in(5) performed, we have:

$$\int d\Omega f\ \ (v'\hat{\Omega}' \rightarrow v\hat{\Omega}) = 2\pi g_o(v', v) \tag{11}$$

The second integration in (5) over solid angle may now be performed.

$$\int dv' \int d\Omega' N(\bar{r}, v'\hat{\Omega}', t)\ \sigma_s(\bar{r}, v')\ 2\pi g_o(v', v) = 2\pi \int dv' v' g_o(v', v)\ P(\bar{r}, v', t)\ \sigma_s(\bar{r}, v') \tag{12}$$

The transport equation, integrated over solid angle may now be written:

$$\frac{\partial P}{\partial t} + \nabla \cdot \overline{J} + \sigma_t vP$$
$$= 2\pi \int dv'v'g_o(v',v)\ P(\overline{r},v',t)\sigma_s(\overline{r},v') + Q \tag{13}$$

Now if spatially independent cross-sections are assumed we may integrate (13) over all volume to obtain:

$$\frac{\partial n}{\partial t} + \sigma_t(v)vn = 2\pi \int dv'v'g_o(v',v)n(v',t)\sigma_s(v') + s(v,t) \tag{14}$$

Where by definition:

$$n(v,t) \equiv \int d^3r\ P(\overline{r},v,t) \tag{15}$$

$$s(v,t) \equiv \int d^3r\ Q(\overline{r},v,t) \tag{16}$$

The vector current term vanishes by application of the divergence theorem to convert the volume integral to a surface integral, and the assumption of vanishing particle density at infinity.

Lastly, we make the multi-group approximation to equation (14) to obtain a discrete velocity (or energy) dependence. To reduce the amount of writing, we define the linear operator:

$$V_i \langle F(v) \rangle = \int_{v_{i-1}}^{v_i} F(v)\ dv = F_i \tag{17}$$

The "group" variables may now be defined in terms of the linear operator $V_i \langle\ \rangle$.

$$n_i(t) = v_i \langle n(v,t) \rangle \tag{18}$$

$$s_i(t) = V_i \langle s(v,t) \rangle \tag{19}$$

$$\sigma_{ti} = \frac{V_i \langle \sigma_t(v)\ vn(v,t) \rangle}{V_i \langle vn(v,t) \rangle} \tag{20}$$

$$v_i = \frac{V_i \langle vn(v,t) \rangle}{n_i} \tag{21}$$

$$g_i(v') = V_i \langle g_o(v',v) \rangle \tag{22}$$

Thus, we may write

$$V_i \langle \text{equation (14)} \rangle \tag{23}$$

$$= \frac{\partial n_i}{\partial t} + \sigma_{ti}\ v_i\ n_i\ (t) = 2\pi \int dv'v'\sigma_s(v')g_i(v')n(v',t) + s_i(t).$$

$$\text{Let } \sigma_{ji} = \frac{2\pi\ V_j \langle v\ \sigma_s(v)\ g_i(v)n(v,t) \rangle}{v_j\ n_j(t)} \tag{24}$$

Then the final result may be written:

$$\frac{\partial n_i(t)}{\partial t} + \sigma_{ti}v_i n_i(t) = \sum_j \sigma_{ji}v_j n_j(t) + s_i(t) \tag{25}$$

In most practical cases the integrations in equations (17) to (25) are performed on energy rather than velocity, and if that were done the form of (25) would be the same. Also, due to the assumed continuity of the distribution function N $(\bar{r},v\hat{\Omega},t)$, a different order of the three integrations would not alter the final result.

APPENDIX B

Solution of Coupled Linear Differential Equations

In Appendix A the transport equation was reduced to a system of coupled linear differential equations (LDE) with time as the only independent variable. In the absence of up-scatter (particle gaining energy in a collision), the system may be arranged in order of decreasing energy and solved successively starting with the LDE representing neutrons of highest energy. In what follows we will begin with the system of LDE's found in Appendix A and obtain an iterative solution by means of Laplace transforms.

From Appendix A we have:

$$\frac{dn_i(t)}{dt} + \sigma_{ti} \, v_i n_i(t) = \sum_{j=1}^{NN} \sigma_{ji} v_j n_j(t) + s_i(t) \qquad i=1, NN \tag{25-A}$$

To simplify the mathematics we define the following:

$$b_{ji} = \sigma_{ji} v_j \qquad i,j = 1,NN \tag{1}$$

$$a_j = \sigma_{tj} v_j - \sigma_{jj} v_j \qquad j=1,NN \tag{2}$$

$\sim$ indicates Laplace transformed variable

Further, we make the following assumptions:

$$s_i(t) = 0 \qquad i = 1,NN \tag{3}$$

$$n_i(0) = N_i \qquad i = 1,NN \tag{4}$$

$$\sigma_{ji} = 0 \ , \quad j > i \quad \text{(no up-scatter)} \tag{5}$$

Equation (25-A) may now be written:

$$\frac{dn_i}{dt} + a_i n_i = \sum_{j=1}^{i-1} b_{ji} n_j \qquad i=1,NN \tag{6}$$

Solutions to (6) will be of the form:*

$$n_i(t) = \sum_{j=1}^{i} A_{ji}\, e^{-a_j t} \qquad i=1,NN \tag{7}$$

The object then is to obtain the coefficients Aji as explicit functions of the initial values Ni, the decay constants ai, and the transfer constants bji defined in equations (1) and (2).

Now perform a Laplace transfrom on (6):

$$s\tilde{n}_i - N_i + a_i\tilde{n}_i = \sum_{j=1}^{i-1} b_{ji}\tilde{n}_j \qquad i=I,NN \tag{8}$$

$$\tilde{n}_i = \frac{1}{s+a_i} \sum_{j=1}^{i-1} b_{ji}\tilde{n}_j + \frac{N_i}{s+a_i} \tag{9}$$

Next, transform (7) and substitute the result into (9):

$$\tilde{n}_j = \sum_{i=1}^{j} A_{ij} \frac{1}{s+a_i} \tag{10}$$

$$\tilde{n}_i = \frac{1}{s+a_i} \sum_{j=1}^{i-1} b_{ji} \sum_{k=1}^{j} A_{kj} \frac{1}{s+a_k} + \frac{N_i}{s+a_i} \tag{11}$$

Equation (11) may be rearranged after separation of the product terms in s by partial fractions to obtain:

$$\tilde{n}_i = \frac{1}{s+a_i} \left[N_i + \sum_{j=1}^{i-1} b_{ji} \sum_{k=1}^{j} \frac{A_{kj}}{a_k - a_i} \right]$$

$$+ \sum_{k=1}^{i-1} \frac{1}{s+a_k} \sum_{j=k}^{i-1} \frac{b_{ji}A_{kj}}{a_i - a_k} \qquad i=1,NN \tag{12}$$

* This may be shown by sucessively solving the first few of equations (6) and then applying induction.

Now apply the inverse Laplace transform to (12) to obtain the time domain solution.

$$n_i(t) = \left[N_i + \sum_{j=1}^{i-1} b_{ji} \sum_{k=1}^{j} \frac{A_{kj}}{a_k - a_i} \right] e^{-a_i t}$$

$$+ \sum_{k=1}^{i-1} e^{-a_k t} \sum_{j=k}^{i-1} \frac{b_{ji} A_{kj}}{a_i - a_k} \qquad i=1,NN \qquad (13)$$

The solution must be unique, and the exponentials are linearly independent, so the coefficients on the left hand sides of (7) and (13) must be equal. Therefore,

$$A_{ki} = \begin{cases} N_i + \sum_{j=1}^{i-1} b_{ji} \sum_{\ell=1}^{j} \dfrac{A_{\ell j}}{a_\ell - a_i}, & k=i \\ \dfrac{1}{a_i - a_k} \sum_{j=k}^{i-1} b_{ji} A_{kj}, & k < i \end{cases} \qquad (14)$$

Now, starting with $A_{11} = N_1$, all other A_{ki} may be determined from (14).

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