Theoretical Notes
Note 47

MEMORANDUM
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THE DEVELOPMENT OF A RADIO SIGNAL FROM A NUCLEAR EXPLOSION IN THE ATMOSPHERE

Victor Gilinsky

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The RAND Corporation
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PREFACE

The distant electromagnetic signal from a small atmospheric nuclear explosion is computed numerically on the basis of a simple model. A series of curves are presented which are, in effect, a motion picture of the development of the signal.

This computation is a continuation of work reported in RM-4134, RM-4152, RM-4370, and RM-4934. The details of the numerical program are described in RM-4942 by Glenn Peebles.

SUMMARY

The dipole radio signal from a low altitude atmospheric nuclear explosion of low yield is computed by numerically integrating Maxwell's equations. The source is described by a simple model. The function describing the time history of the rate of gamma output is taken to be an isosceles triangle with a base equal to a fraction of a microsecond. The asymmetry necessary for radiation comes entirely from the variation of air density with altitude.

The dipole fields are shown as functions of position for various times after the explosion. These curves form, in effect, a motion picture of the development of the signal.

ACKNOWLEDGEMENTS

Glenn Peebles designed the numerical program for solving Maxwell's equations that is essential for this report. It was coded by Don MacNeilage.

I. INTRODUCTION

A nuclear explosion in which part of the gamma flux escapes into the atmosphere produces a transient electromagnetic disturbance which radiates electromagnetic waves. These waves, variously called electromagnetic signal, electromagnetic pulse, or radioflash, are of such a magnitude that they can be detected at large distances. Since there is a background of other signals, in particular signals from lightning strokes, it is important for purposes of discrimination to know what kinds of signals can be produced by various types of nuclear explosions. That is to say, it is necessary to consider a number of cases with gamma fluxes of various magnitudes and time histories. In this report we shall take one such simple case and we shall examine the relationship between the gamma pulse emerging from the nuclear device and the subsequent radiated signal by numerically solving Maxwell's equations (in a suitable approximation). We shall present a series of curves which are, in effect, a motion picture of the development of the signal.

We shall take the gamma pulse to contain an energy equivalent to about 1 ton HE ($\approx 2.62 \times 10^{22}$ Mev), which corresponds to a total yield of about 1 KT. The function describing the time history of the gamma rate is taken to be an isosceles triangle with a base equal to a fraction of a microsecond. It was found that the computing time became excessive for gamma yields greater than 1 ton and a pulse duration less than 25×10^{-8} sec.

The mechanism of the signal has been described in many places.* It is essentially the following: The gammas enter the atmosphere with about 1 Mev of energy and scatter electrons in air molecules driving them outward at high speeds. The gamma mean free path for this process is about 300 meters at sea level. The electrons slow down in about a meter by ionizing the air which then becomes conducting. The charge separation produces a large radial electric field which causes a return current to flow through the conducting gas.

*For example, see Refs. 1, 2, and 4.

If the current system is not spherically symmetric it radiates electromagnetic waves. The main asymmetry may arise from an initially asymmetrical gamma flux or an asymmetry in the surrounding region because of the presence of the ground or just the variation of air density with altitude.

We shall consider here the case of a symmetrical explosion in an asymmetrical atmosphere. The calculation is based on a dipole approximation to Maxwell's equations. The solutions are obtained by integrating numerically along characteristics.*

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*The machine program used to integrate the equations was designed by Glenn Peebles and will be described in RM-4942.

II. ELECTROMAGNETIC EQUATIONS

We shall solve Maxwell's equations in dipole approximation.* That is, we shall assume the radial compton current j and the electronic conductivity σ have the form

$$J(r,\theta,t) = j_o(r,t) + j_1(r,t)\cos\theta \tag{2.1}$$

$$\sigma(r,\theta,t) = \sigma_o(r,t) + \sigma_1(r,t)\cos\theta \tag{2.2}$$

with $j_1 << j_o$ and $\sigma_1 << \sigma_o$, and the fields have the form

$$E_r(r,\theta,t) = E_o(r,t) + E_1(r,t)\cos\theta \tag{2.3}$$

$$E_\theta(r,\theta,t) = E_2(r,t)\sin\theta \tag{2.4}$$

$$B_\varphi(r,\theta,t) = B(r,t)\sin\theta . \tag{2.5}$$

If we neglect products of two small quantities then E_o, E_1, E_2, and B satisfy the following differential equations (with $x = r/\lambda$ where λ is the gamma mean free path, and t measured in units of λ/c)

$$\frac{\partial E_o}{\partial t} = -4\pi\,\sigma_o E_o - 4\pi\, j_o \tag{2.6}$$

$$\frac{\partial E_1}{\partial t} = \frac{2}{x} B - 4\pi\,\sigma_o E_1 - 4\pi\,\sigma_1 E_o - 4\pi\, j_1 \tag{2.7}$$

$$\frac{\partial E_2}{\partial t} = \frac{1}{x}\frac{\partial}{\partial x}(xB) - 4\pi\,\sigma_o E_2 \tag{2.8}$$

$$\frac{\partial B}{\partial t} = -\frac{1}{x} E_1 - \frac{1}{x}\frac{\partial}{\partial x}(x E_2) . \tag{2.9}$$

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*See Ref. 3 for a more detailed discussion.

Strictly speaking, the dipole approximation is valid only for very low yield explosions at altitudes low enough so that λ, the gamma mean free path, is much less than H, the scale height of the atmosphere, but yet high enough so that the ionized region surrounding the explosion does not reach the ground. Roughly speaking, this includes explosions of less than a few KT detonated at between, say, 1 and 5 km. In practice, therefore, this approximation is valid for only a small number of explosions. Nevertheless the dipole approximation is very useful because it does permit detailed analysis of some cases without an overly elaborate numerical program. And furthermore, it is hoped that certain conclusions about the nature of the signal will carry over to those cases where the approximation is not strictly valid.

We want particularly to examine a case where the gamma pulse is of short duration but also has a finite rise time. For simplicity we have chosen the following simple time dependence for the gamma rate

$$f(t-x) = \begin{cases} \dfrac{1}{\Delta^2}\,(t-x) & 0 < (t-x) < \Delta \\[2ex] \dfrac{1}{\Delta}\left[2 - \dfrac{1}{\Delta}\,(t-x)\right] & \Delta < (t-x) < 2\Delta \\[2ex] 0 & 2\Delta < (t-x)\ . \end{cases} \tag{2.10}$$

The constants have been chosen so that $\int dt\, f(t-x) = 1$.

In terms of $f(t-x)$ we have

$$4\pi\, j_o = -\, y(x)\, f(t-x) \tag{2.11}$$

where in our units (see Ref. 3)

$$y(x) \approx 100\, Y\, \frac{e^{-x}}{x^2} \tag{2.12}$$

and Y in the gamma yield in tons. More precisely

$$y(x) = \frac{E_b}{E_a} \frac{e^{-x}}{x^2} \tag{2.13}$$

where the fields E_b and E_a are given by

$$E_b = \frac{Ne\ell}{\lambda^3} \quad \text{and} \quad E_a = \frac{\alpha}{\mu} \frac{\ell}{\nu}, \tag{2.14}$$

and the symbols have the following meaning: N is the gamma yield in Mev (2.62×10^{22}Y), $e = 4.8 \times 10^{-10}$ cgs, ℓ is the compton range ($\approx 10^2 \; e^{z/H}$ cm at z km), μ is the electron mobility ($\approx 10^6 \; e^{z/H}$ cgs at z km), $\nu \approx 3 \times 10^4$ is the number of secondary electrons per Mev primary energy, and α is the rate for electron attachment to O_2 ($\approx 10^8 \; e^{-2z/H} \; sec^{-1}$ at z km). We measure the fields in units of E_a, the "saturation field."

The dipole part of the radial current is given by

$$4\pi \, j_1 = -\tfrac{1}{2} x^2 \frac{\lambda}{H} y(x) \; f(t-x) \tag{2.15}$$

so that

$$4\pi \, j = 4\pi \, j_o \, (1 - \tfrac{1}{2} x^2 \frac{\lambda}{H} \cos \theta).$$

The electronic conductivity* is obtained from the differential equation for the electron density n,

$$\frac{dn}{dt} + \alpha n = \frac{1}{4\pi} \frac{N\nu}{\lambda^3} \frac{e^{-x}}{x^2} \; f(t-x) \; . \tag{2.16}$$

Since $\sigma = \mu en$ we get

$$4\pi \, \sigma = \alpha y(x) \int_0^t dt' \, \exp\left[-\alpha(1-2x \frac{\lambda}{H} \cos \theta)(t-t')\right] f(t') \; , \tag{2.17}$$

*We shall neglect the ionic conductivity.

and the spherically symmetric part of the conductivity is given by

$$4\pi\,\sigma_o = \frac{\alpha y(x)}{(\alpha\Delta)^2}\begin{cases} e^{-\alpha(t-x)} - 1 + \alpha(t-1) & \text{for } 0 < (t-x) < \Delta \\ -2e^{-\alpha(t-x-\Delta)} + e^{-\alpha(t-x)} + 1 + 2\alpha\Delta - \alpha(t-x) & \\ & \text{for } \Delta < (t-x) < 2\Delta \\ e^{-\alpha(t-x-2\Delta)} - 2e^{-\alpha(t-x-\Delta)} + e^{-\alpha(t-x)} & \\ & \text{for } 2\Delta < (t-x)\ . \end{cases} \tag{2.18}$$

If we write $4\pi\,\sigma_o = \alpha y(x)\ I(\alpha)$ then we can write the dipole part of the conductivity in the following form

$$4\pi\,\sigma_1 = \tfrac{1}{2}\,x^2\,\frac{\lambda}{H}\,4\pi\,\sigma_o - 2x\,\frac{\lambda}{H}\,y(x)\ \alpha^2\,\frac{\partial}{\partial\alpha}\,I(\alpha)\,. \tag{2.19}$$

With these currents and conductivities we are now in a position to integrate Maxwell's equations in dipole approximation. For completeness we shall write out Eq. (2.19) for $4\pi\,\sigma_1$,

$$4\pi\,\sigma_1 = \tfrac{1}{2}\,x^2\,\frac{\lambda}{H}\,4\pi\,\sigma_o + 2x\,\frac{\lambda}{H}\,\frac{\alpha y(x)}{(\alpha\Delta)^2}\,\Sigma$$

where

$$\Sigma = \begin{cases} [2+\alpha(t-x)]\,e^{-\alpha(t-x)} - [2-\alpha(t-x)] & \\ \qquad \text{for } 0 < (t-x) < \Delta & \\ [2+\alpha(t-x)]\,e^{-\alpha(t-x)} - [4+2\alpha(t-x-\Delta]\,e^{-\alpha(t-x-\Delta)} & \\ \qquad + [2-\alpha(t-x-2\Delta] & \\ \qquad \text{for } \Delta\ (t-x) < 2\Delta & \\ [2+\alpha(t-x)]\,e^{-\alpha(t-x)} - [4+2\Delta(t-x-\Delta)]\,e^{-\alpha(t-x-\Delta)} & \\ \qquad + [2+\alpha(t-x-2\Delta)]\,e^{-\alpha(t-x-2\Delta)} & \\ \qquad \text{for } 2\Delta < (t-x)\ . & \end{cases} \tag{2.20}$$

III. PICTURES OF THE FIELDS

Let us first consider the radiated fields. In the radiation zone we have only the fields E_θ and B_φ which can then be expressed in terms of a single function $\ddot{Z}$ (t-x)

$$E_\theta = B_\varphi = \frac{\ddot{Z}(t-x)}{x} \sin \theta \tag{3.1}$$

where Z is the effective dipole moment.* We shall present curves for the quantity $\ddot{Z}$ for the two cases: $\Delta = 1/2$ and $\Delta = 1/8$. In both cases $Y = 1$ so we have only varied the duration of the gamma pulse. The results are shown in Fig. 1a where we can see the effect of the gamma pulse shape on the early part of the signal. The peak of the signal comes roughly at the point where the gamma pulse ends.

In Fig. 1b, we compare these calculations with the results of an analytic approximation developed by Karzas and Suydam (Ref. 4), which gives the radiated field at early times. This approximation says that, for the dipole field,

$$\ddot{Z}(t-x) \approx -\tfrac{1}{2} E_1(t-x,x_s) ,$$

where x_s is the distance for which

$$4\pi\sigma_o(t-x,x_s) = 2c/\lambda ,$$

and E_1 is the field defined earlier.

The development of E_o, the spherically symmetric part of the radial field, is shown in Fig. 2. The field reaches an asymptotic form because we have neglected the ionic conductivity.

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*The radiated electric field at a distance r meters from an explosion at z km is roughly given by

$$E_2 \approx 45 \times 10^5 e^{-z/H} \frac{\ddot{Z}}{r} \quad v/m$$

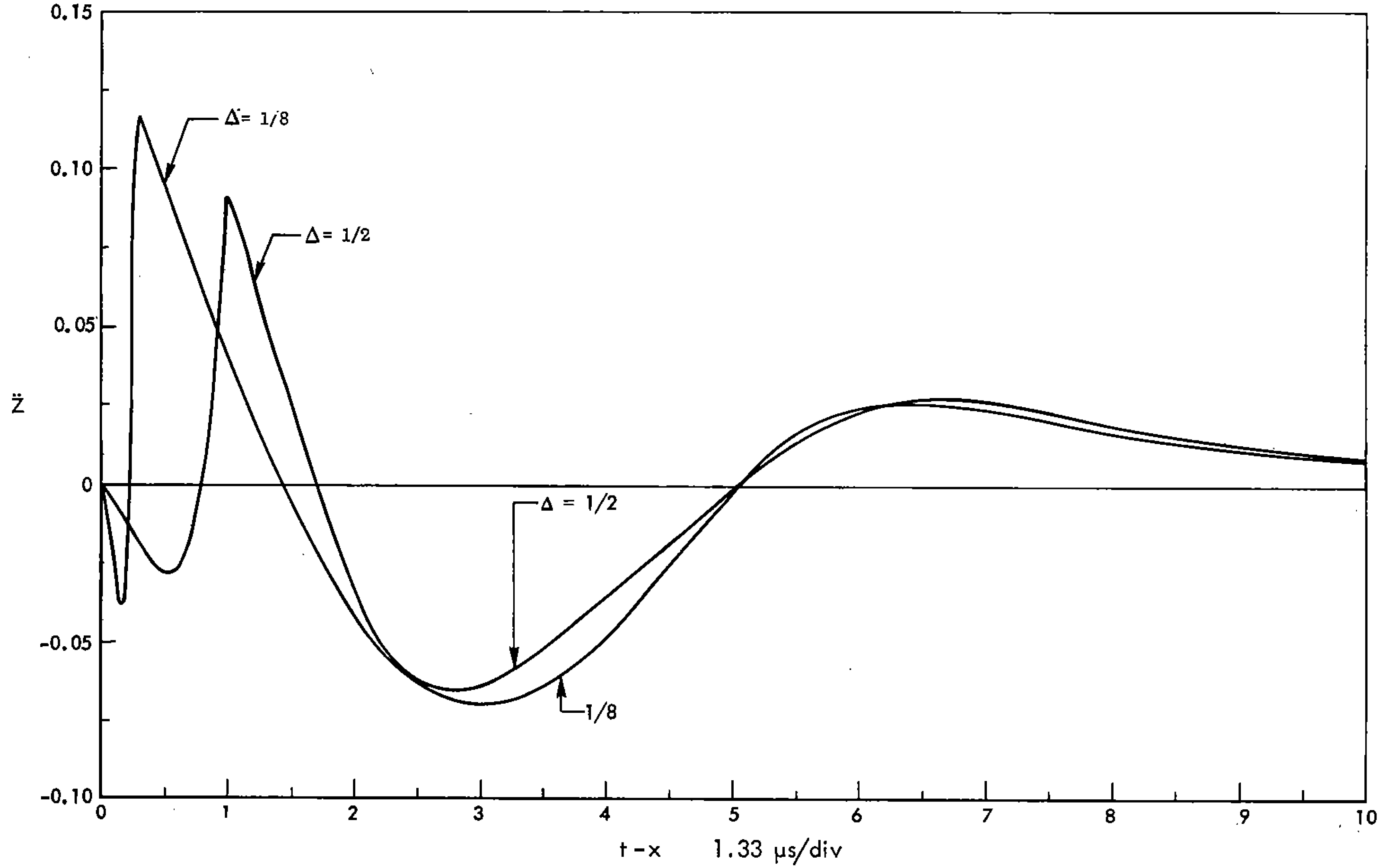


Fig. 1a--The radiated electric field, in terms of the function $\ddot{Z}$, for two values of the parameter Δ which corresponds roughly to the duration of the gamma pulse. The gamma yield in both cases is Y = 1 ton and the altitude is z = 2 km.

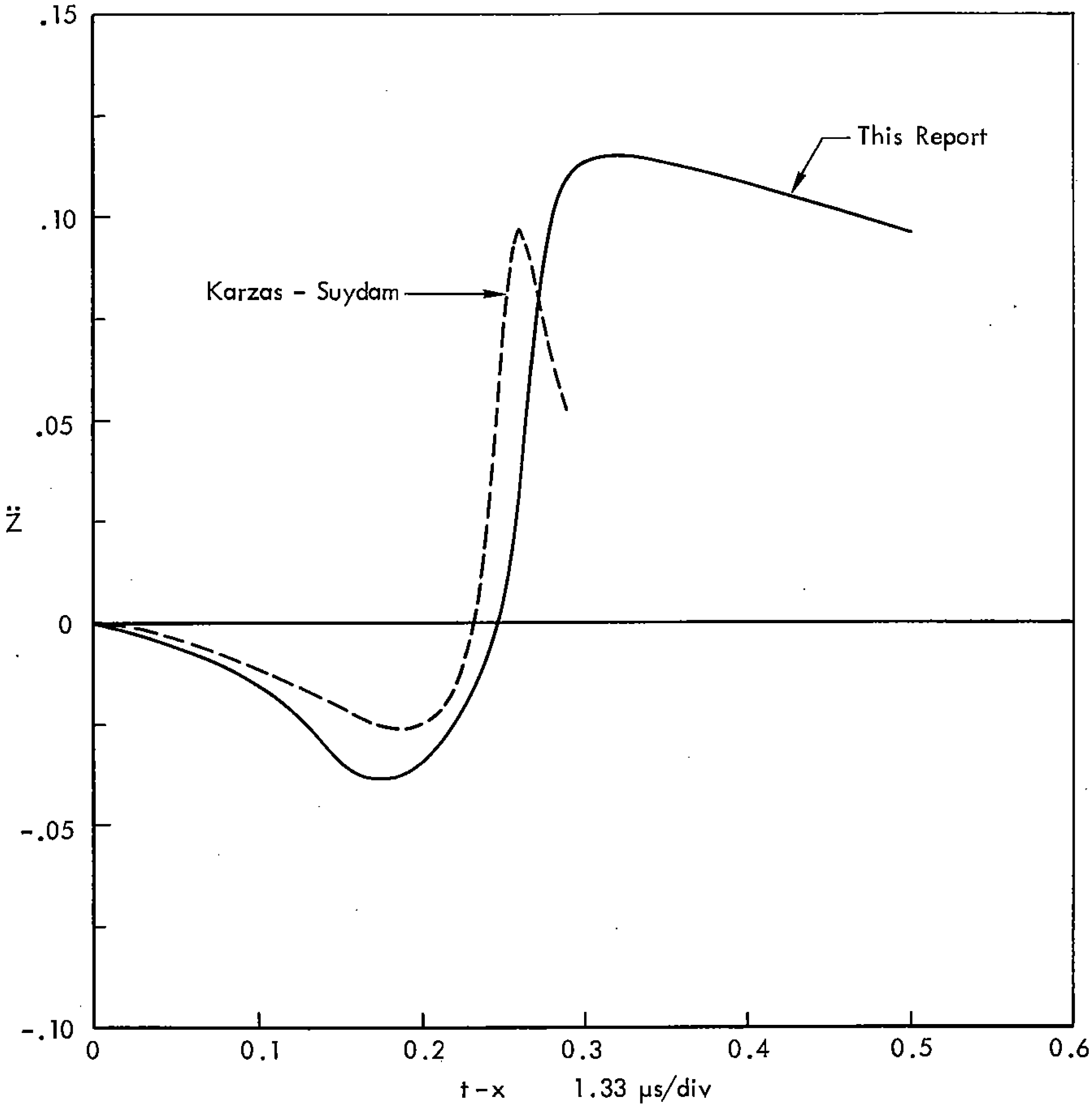


Fig. 1b--Comparison of our result for the early part of the signal for the $\Delta = 1/8$ case with the result of an analytic approximation developed by Karzas and Suydam (Ref. 4).

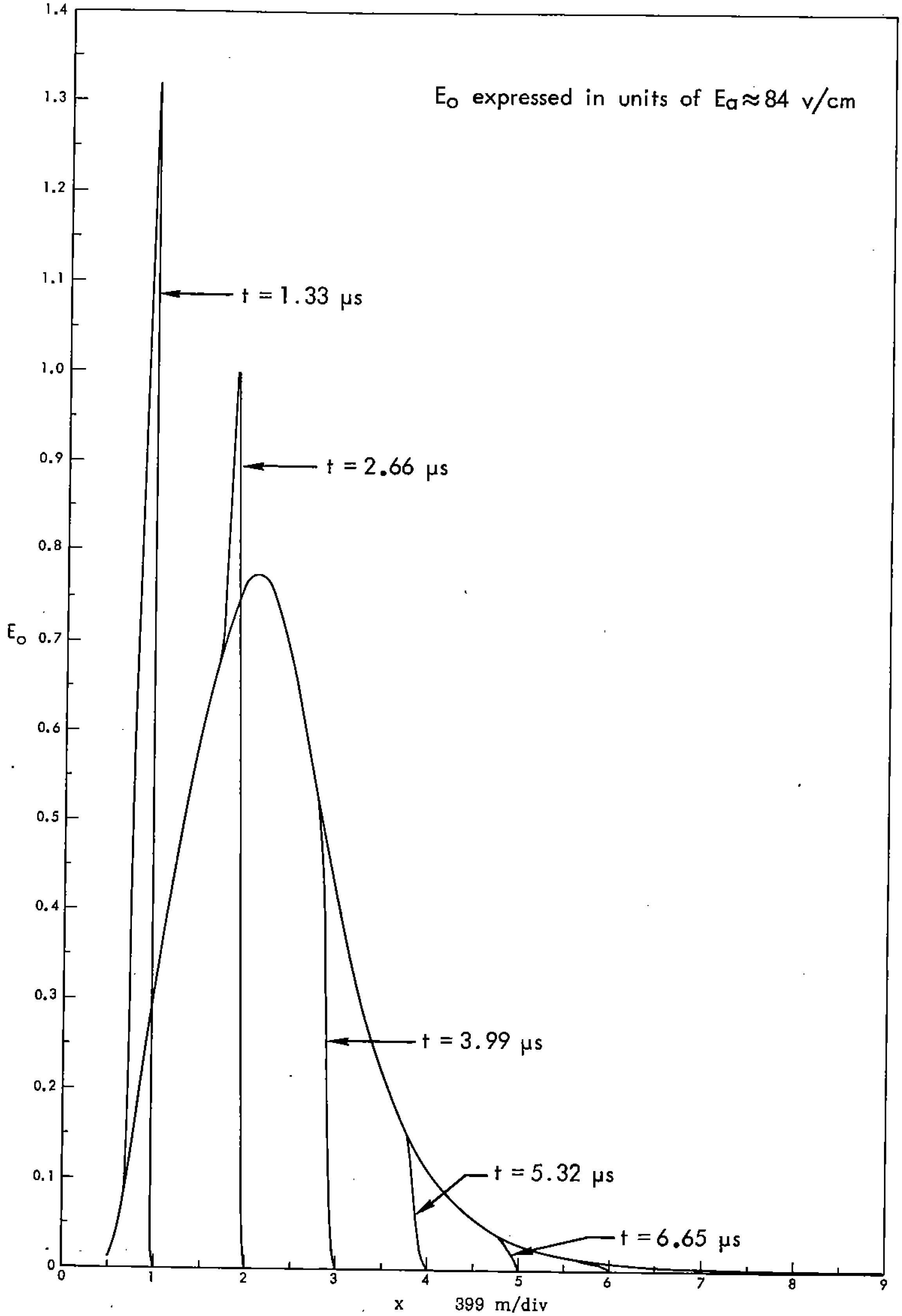


Fig. 2--E_o, the spherically symmetric part of the radial field as a function of position for various times for the case $\Delta = 1/8$. The field approaches an asymptotic form because ionic conductivity is neglected.

The curves shown in Figs. 3a-3g follow the development of the signal. These curves are for the case $\Delta = 1/8$. That is, the function describing the time behavior of the gamma pulse is an isosceles triangle with a base of 1/4 (~ 0.33 μs). The gamma pulse therefore extends 1/4 space unit ($\sim$ 100 meters) behind the front.

Since we have included only the electronic part of the conductivity it is closely tied to the gamma pulse, and it dies away rapidly after the gamma pulse ends. The electrons attach to oxygen molecules in about a shake so the conducting region ends only a few meters behind the end of the gamma pulse. In a more realistic case the conductivity would persist longer because the gamma pulse has a long tail and because there would later remain some ionic conductivity.

We have applied the boundary condition $E_2 = 0$ at the surface of a (perfectly conducting) sphere of radius $a = \frac{1}{2}$. Strictly speaking we should choose the radius a to be much smaller. However, our approximation should be satisfactory because we can see from Fig. 2 that the outer radius of the plasma surrounding the explosion is about $x = 3$ or 4 so that the sphere of radius $a = \frac{1}{2}$ takes up a small fraction of the radiating volume.

The conducting region is clearly defined in Fig. 3a because the transverse field E_2 is almost zero there. Note that in Fig. 3a the magnetic field B is directed counter clockwise around the z - axis everywhere except at the very front of the gamma pulse. That is, the current reverses so fast behind the front that the magnetic field is mainly the field of the return current.

The following figures show the subsequent development of the signal. The first peak in the signal comes at the end of the gamma pulse, in our case about a quarter of a mean free path behind the front. In the later figures we follow the signal into the radiation zone. The radial part E_1 of the dipole field becomes small and the transverse electric field E_2 and the (transverse) magnetic field become identical in magnitude.

E_1, E_2, and B expressed in units of $E_a \approx 0.28$ cgs (84 v/cm or 0.28 gauss)

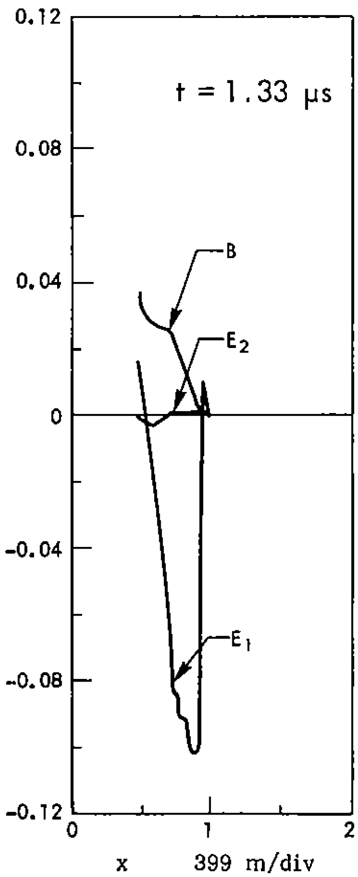
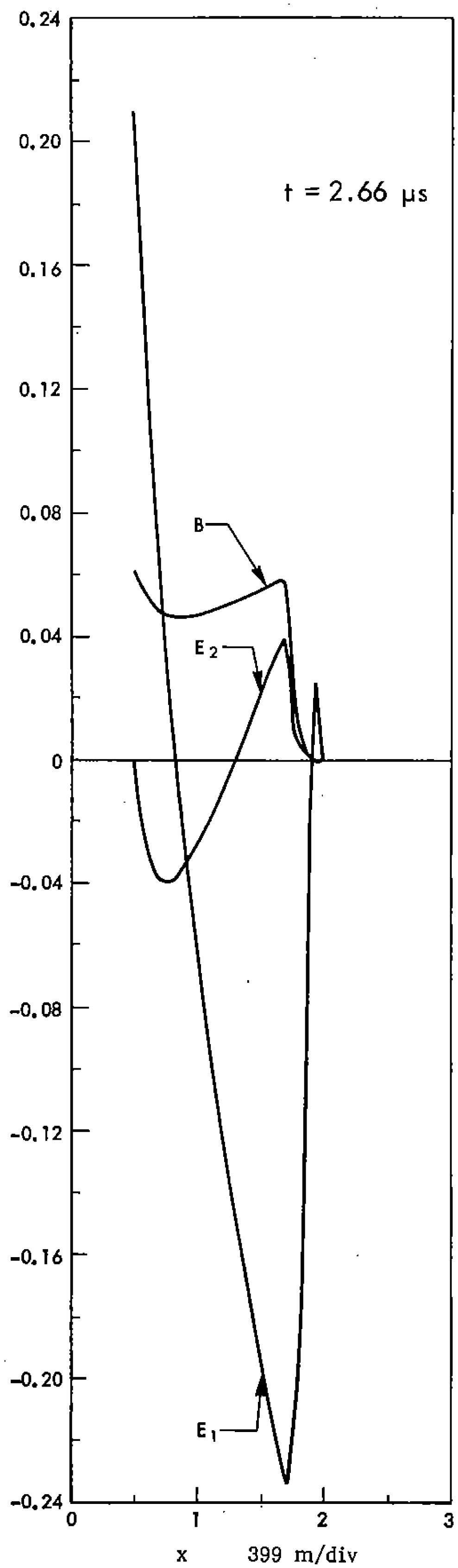


Fig. 3a--The dipole fields as functions of radius for several times for the $\Delta = 1/8$ case. As before Y = 1 ton and z = 2 km.



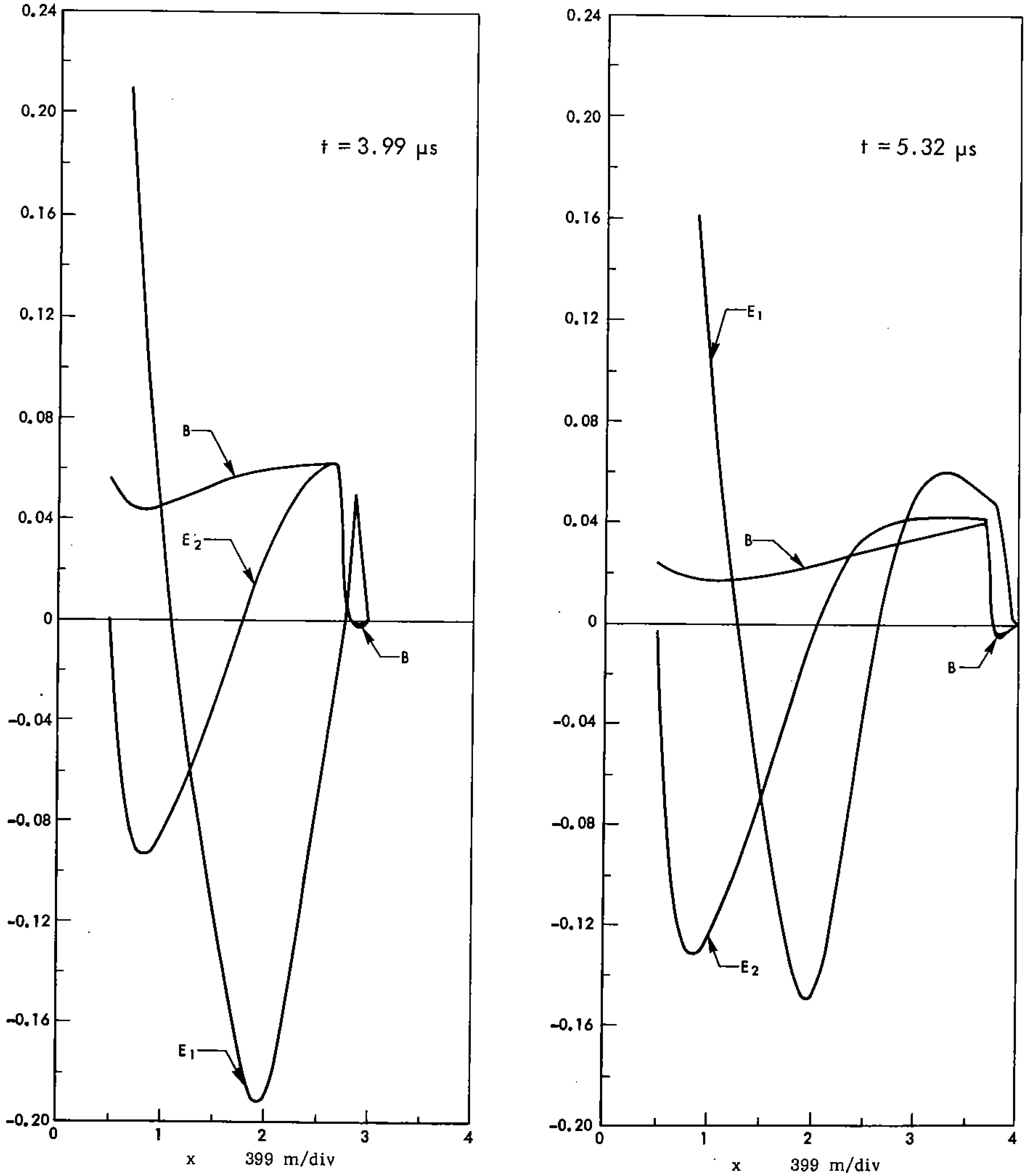


Fig. 3b--The dipole fields at t = 3.99 and 5.32 μs.

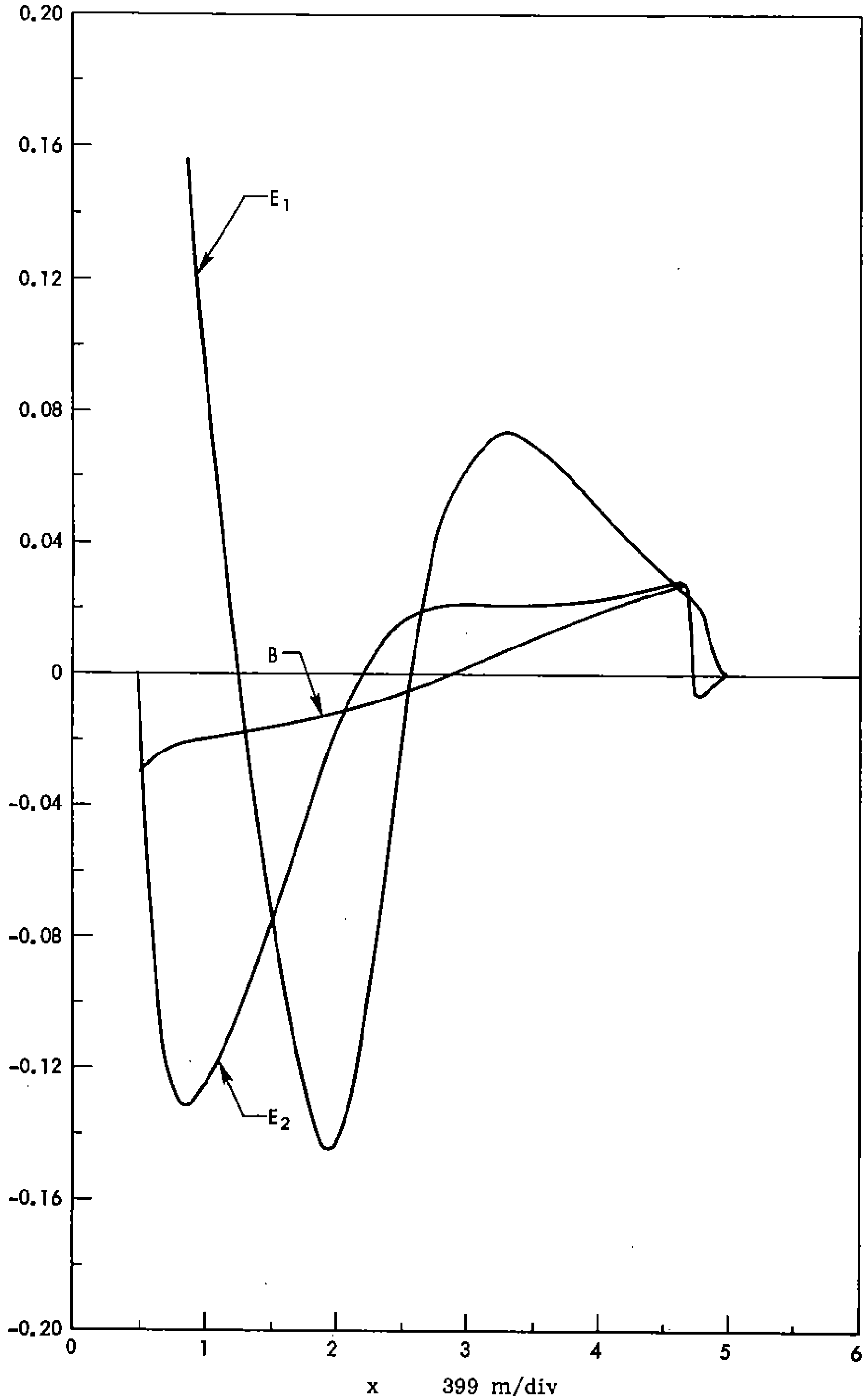


Fig. 3c--The dipole fields at t = 6.65 μs.

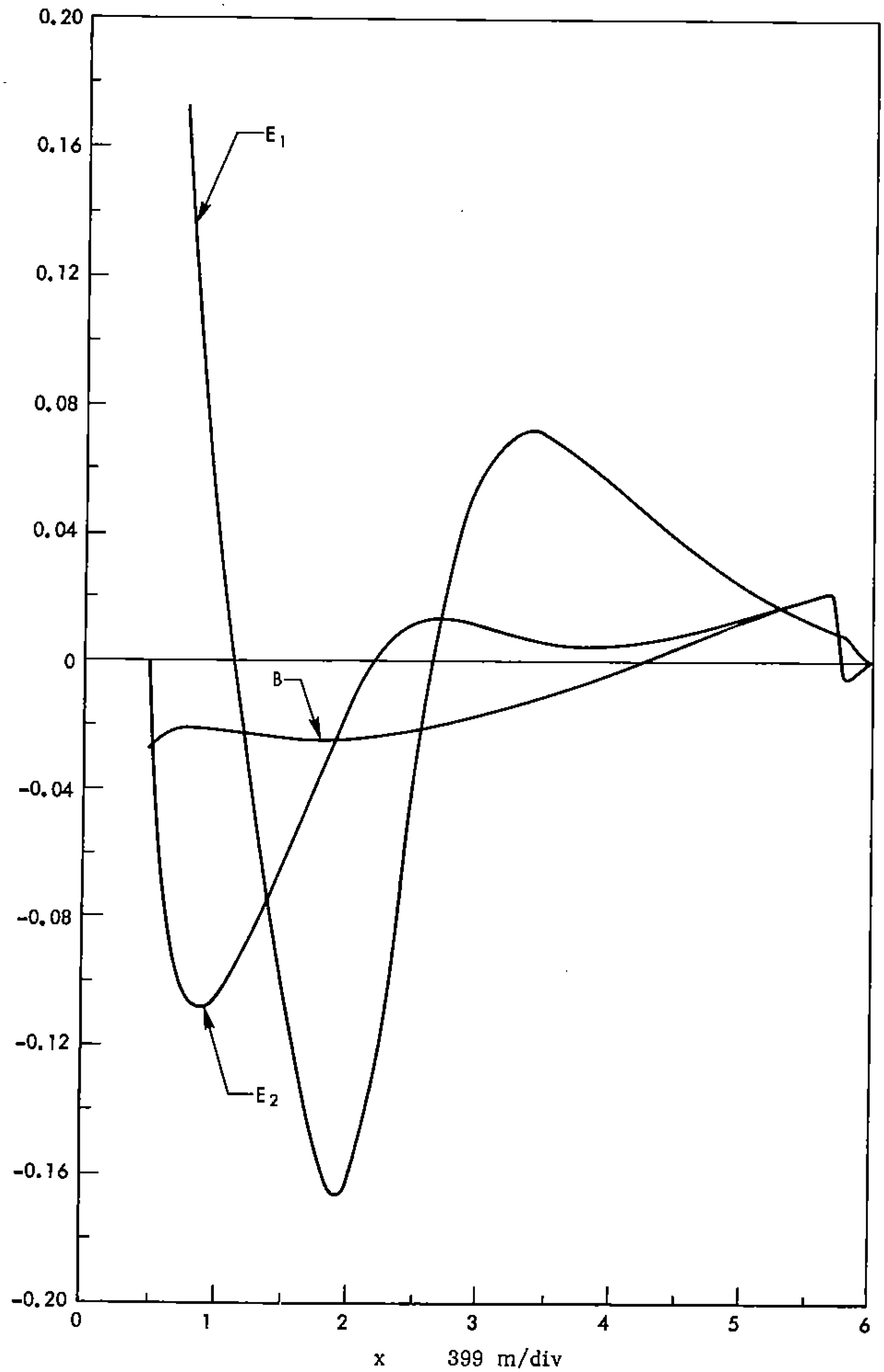


Fig. 3d--The dipole fields at t = 7.98 μs.

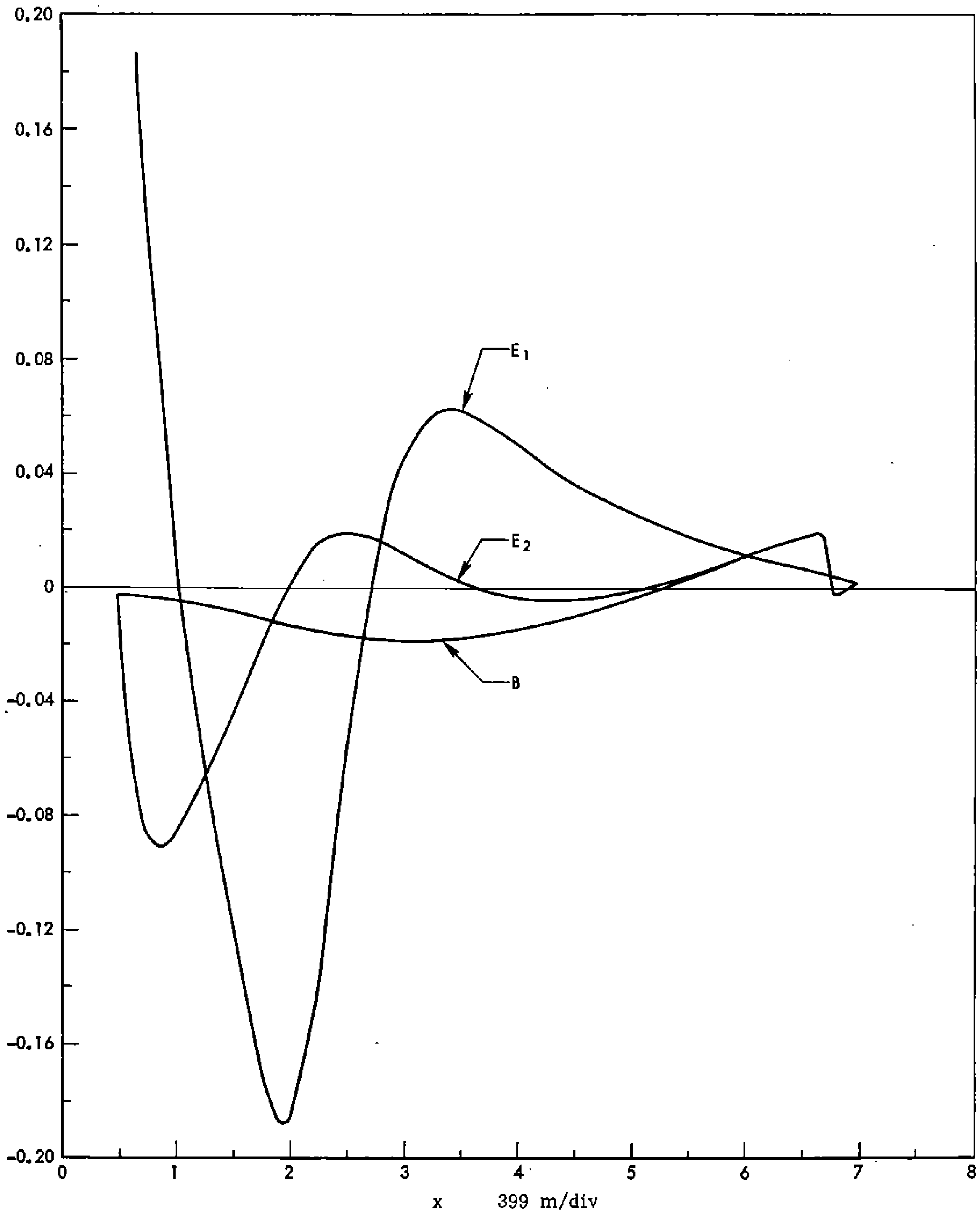


Fig. 3e--The dipole fields at t = 9.31 μs.

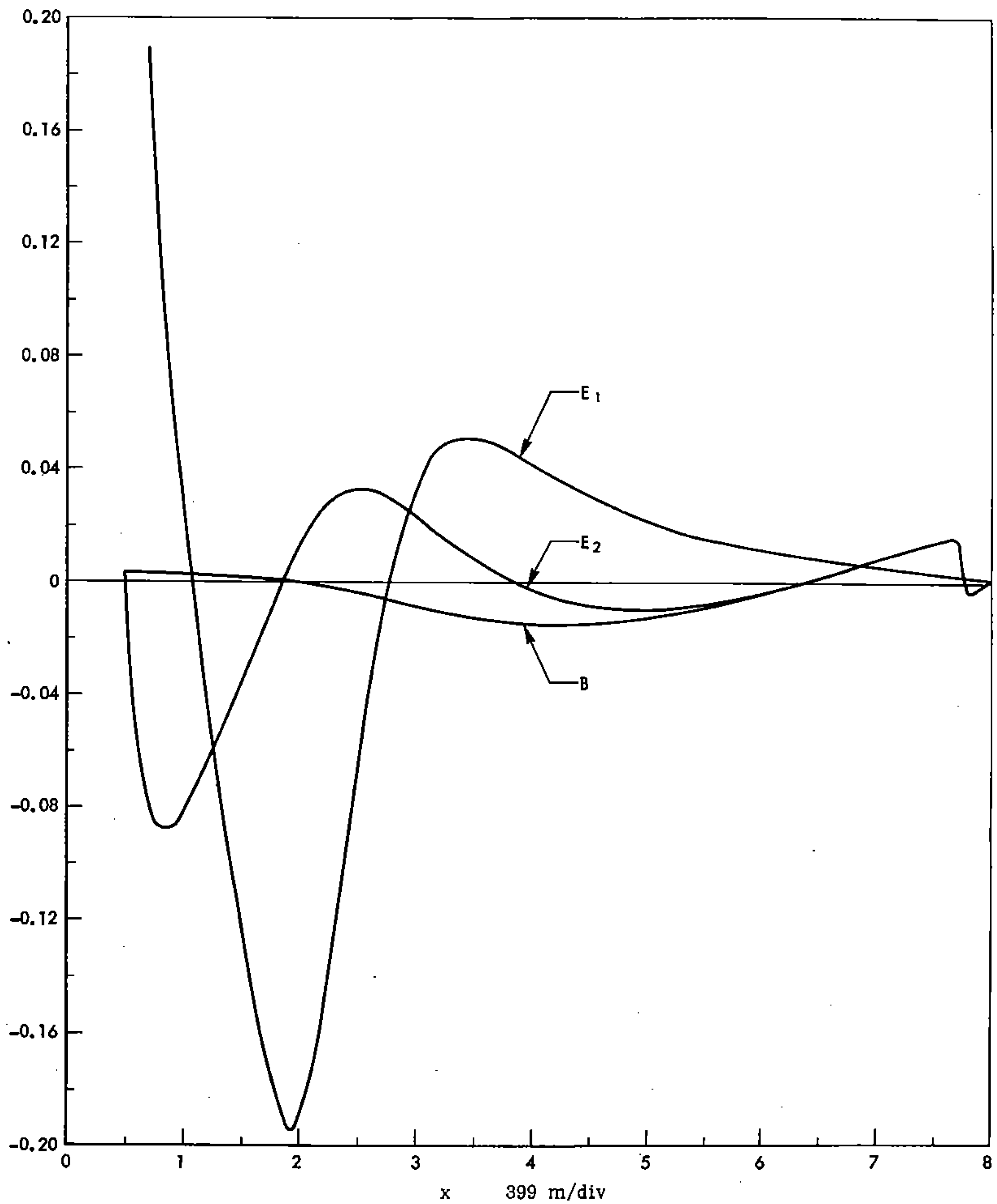


Fig. 3f--The dipole fields at t = 10.64 μs.

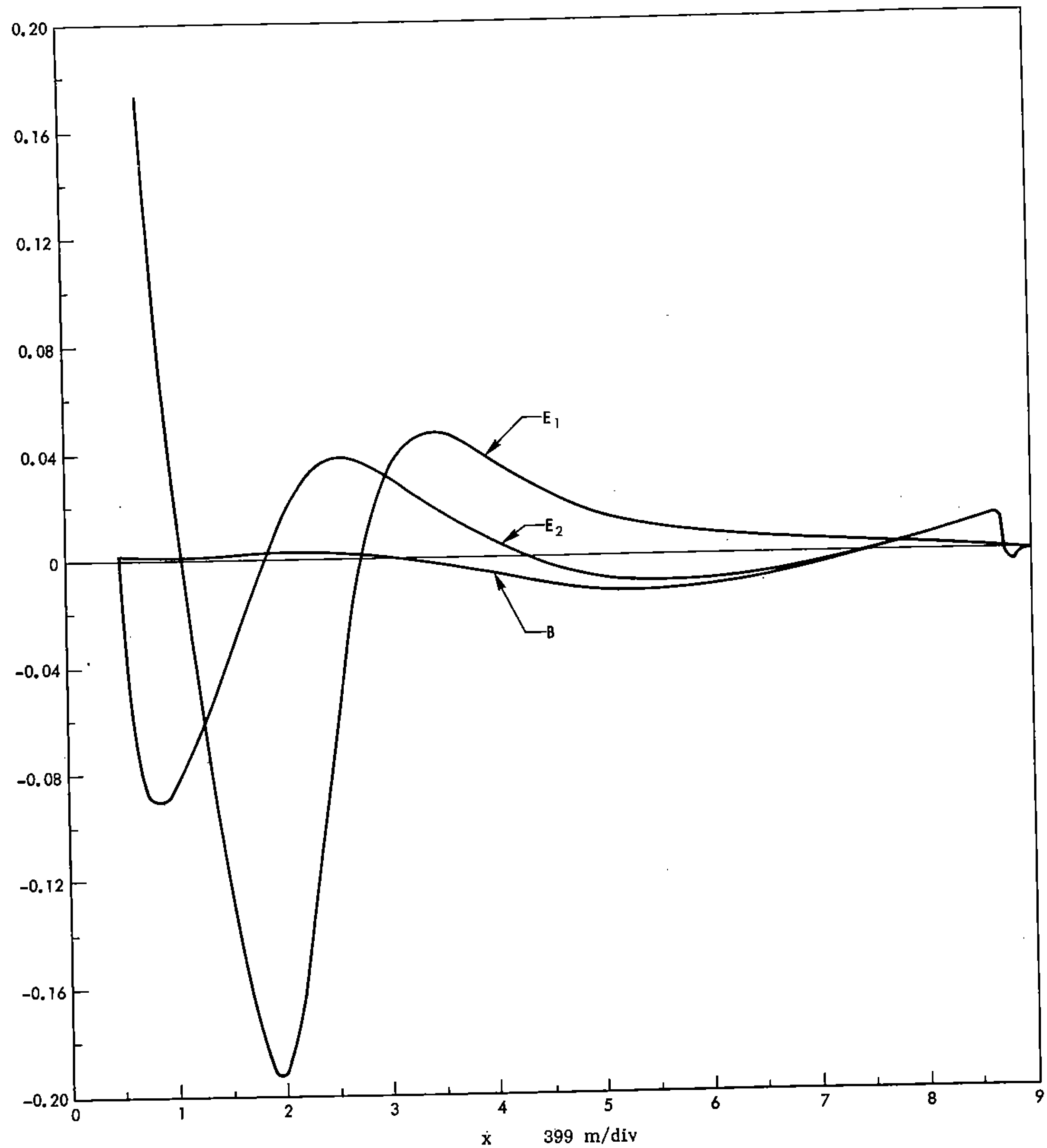


Fig. 3g--The dipole fields at t = 11.97 μs.

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