EMP THEORETICAL NOTES

Note XX

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Curve Fits to the Electric Current and Ionization Rate Delta Function Responses

by

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ABSTRACT

This note contains curve fits to the radial electric current and ionization rate responses to a temporal delta function, point, isotropic source of gamma rays. The curve fits are based on Monte Carlo calculations performed by R. E. LeLevier for monoenergetic gamma sources at 1, 1.5, 2, 3 and 4 Mev in air.* The fits are continuous functions of gamma energy, distance from the source, and time since direct gamma arrival, and have been adjusted such that the time integrated dose buildup factors match those obtained by the moments method for gamma energies between 1 and 10 Mev and distances between .6 and 10 mean free paths to within approximately 15%.

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* These responses were generously supplied by R. E. Lelevier of the RAND Corporation, and are described in RM-4151-PR, "The Compton Current and Energy Deposition Rate From Gamma Quanta - A Monte Carlo Calculation," June, 1964.

I. Introduction

This note claims to present reasonable fits to the radial electric current and ionization (or energy deposition) rate density responses due to indirect gammas from a delta function (in time), point, isotropic source of gamma radiation.* These delta function responses were estimated by a time-dependent Monte Carlo code as described in RM-4151-PR.

The scattering medium was assumed to be air with the electron density of STP air. The Monte Carlo program was run with source gamma rays at 1.0, 1.5, 2.0, 3.0, and 4.0 Mev. Hence any curve fits in this report are based on the 1 - 4 Mev interval and their accuracy outside of this range other than on a time integrated basis is uncertain.

The units used in this report are chosen so that the results will be readily usable in any unit system including seconds as its unit of time (e.g., cgs, mks). This is accomplished by expressing the delta function responses as fractions of the direct beam contributions. For example, the radial electric current response would normally be expressed in charge-per-unit, area-per-unit time, while the direct beam response would be expressed in charge-per-unit area.** Thus, the indirect contributions divided by the direct would be expressed as seconds^{-1} regardless of the charge or area units.

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* Henceforth, referred to as the delta function responses.

** Under the usual near surface assumption that the recoil electrons travel radially at the speed of light. Strictly speaking, the direct beam effects are also spread over time.

The radial current indirect delta function response, $f(E, R, t)$ is here defined by

$$f(E, R, t) = \frac{J_{indirect}}{J_{direct}} \frac{1}{sec} , \tag{1}$$

where

$$J_{direct} = q \frac{Re}{\lambda} \frac{e^{-R/\lambda}}{4\pi R^2} ,$$

where

q	is the electronic charge,
Re	is the effective forward range of the recoil electrons from collisions of the source gammas,
λ	is the gamma mean free path for collision,
R	is the radial distance,
E	is the source gamma energy,
t	is the time since direct beam gamma arrival at R,

and

$J_{indirect}$	is the radial charge current due to indirect collisions of gammas.

Similarly, the energy deposition delta function response, $g(E, R, t)$ is defined by

$$g(E, R, t) = \frac{Q_{indirect}}{Q_{direct}} \frac{1}{sec} , \tag{2}$$

where

$$Q_{direct} = \frac{Ee}{\lambda} \frac{e^{-R/\lambda}}{4\pi R^2} \quad ,$$

where

Ee — is the average energy lost to recoil electrons on initial collisions of the source gammas, and

$Q_{indirect}$ — is the energy deposition rate due to collisions of other than the direct beam.

Obviously then, the total radial charge current, J_r, due to both direct and indirect gamma collisions from a rest source can be expressed, using (1) as

$$J(R,t) = J_{direct} * \int_{-\infty}^{\infty} [\delta(\tau) + f(E,R,\tau)] \; S_o(t-\tau) \; d\tau \quad , \tag{3}$$

where

$S_o(t)$ — is the source strength in gammas per second, and

$\delta(\tau)$ — is the Dirac delta function.

Note that J_{direct} can be chosen in any system of units. Similarly, the total ionization rate density due to a real source, $Q(R,t)$ can be expressed as

$$Q(R,t) = Q_{direct} * \int_{-\infty}^{\infty} [\delta(\tau) + g(E,R,\tau)] \; S_o(t-\tau) \; d\tau \quad . \tag{4}$$

The resultant expressions for ionization rate and radial charge current are given in their entirety in Section VI.

II. Curve Fit Method

The curve fitting method used here can best be described as "pragmatic". In other words, no physical model was created with unknown parameters, which were then to be optimized according to a least squares criteria. Instead, a functional form was chosen arbitrarily, but which seemed to reasonably approximate the time dependence as evidenced by some histograms. Then, values of some parameters were chosen by an "eyeball" method, while others were objectively evaluated by the familiar least squares criteria. The object was to obtain useful curve fits with a minimum of effort rather than contribute to analytic transport theory.

The analytic forms chosen for both the $f(E, R, t)$ and $g(E, R, t)$ functions were $A(R, E)\, e^{-\sqrt{t}/K(R, E)}$ where A is a fictitious initial value, t is the time since direct beam gamma arrival, and K is a decay parameter. The "initial value"* functions, $A(R, E)$, were fit by an "iterative eyeball" method--which was relatively easy. Values for the initial values obtained by the Monte Carlo calculations are shown in Figs. 1 and 2 for the charge current and ionization rates respectively. Then, assuming that the initial value curve fit functions were accurate, the exponential decay parameters,

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* Actually, the theoretical initial value approaches infinity as $t \to 0$ as $\log(R/t)$; but the area in any time interval, Δt, is finite.

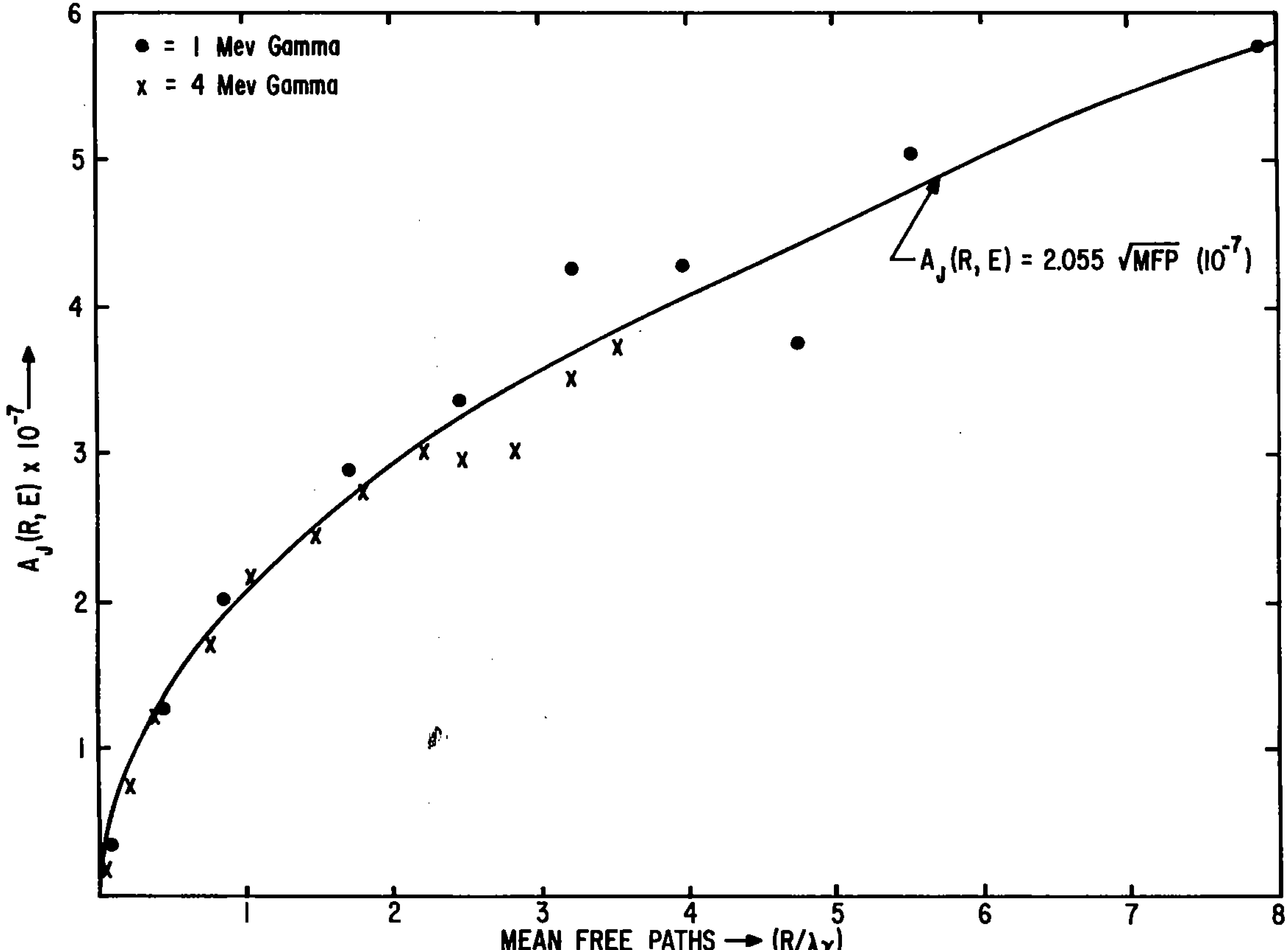

FIGURE I. THE ELECTRIC CURRENT DELTA FUNCTION RESPONSE INITIAL VALUES $A_J(R, E)$ vs MEAN FREE PATHS

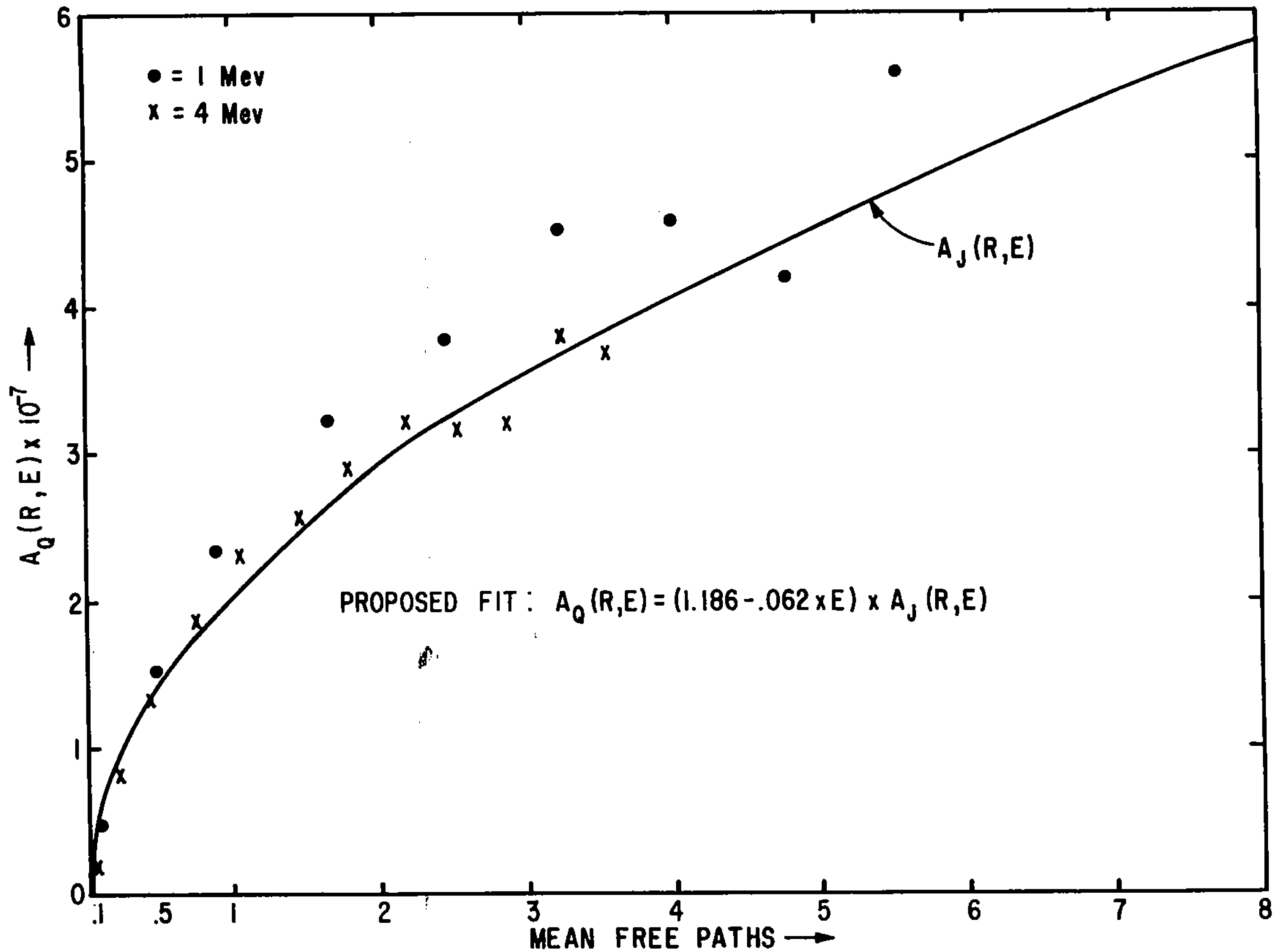

FIGURE 2. THE IONIZATION RATE DELTA FUNCTION RESPONSE INITIAL VALUES VS MEAN FREE PATHS.

K(R, E) were chosen by a least squares evaluation on a computer. After these parameters were chosen, the areas under the fit curves were compared with the areas under the Monte Carlo results, and the curve fit build-up factors were evaluated. The curve fit build-up factors (for energy deposition and radial electric current) are simply

$$1 + \int_0^\infty A e^{-\sqrt{t}/K} \, dt \quad ,$$

which equals $1 + 2K^2 \cdot A$. The energy deposition build-up factor was used, after the A(R, E) and K(R, E) were fit as functions of radius and gamma energy, to judge the validity of the fits for energies outside the 1 - 4 Mev interval and for radii greater than 1 km. Some of the decay parameters for the electric current and ionization rate delta function responses, as determined by the least squares method, are plotted in Figs. 3 and 4 respectively.

III. The Curve Fit Results at Standard Temperature and Pressure for Source Energies Between 1 - 4 Mev

The fits presented in this section were based on calculations made assuming standard temperature and pressure air as the scattering medium (760 mm Hg. pressure, and 273° K temperature). This corresponds to an electron density of approximately 38.75 (10^{25}) electrons/meter^3. The fits are extended to arbitrary air densities in the next section.

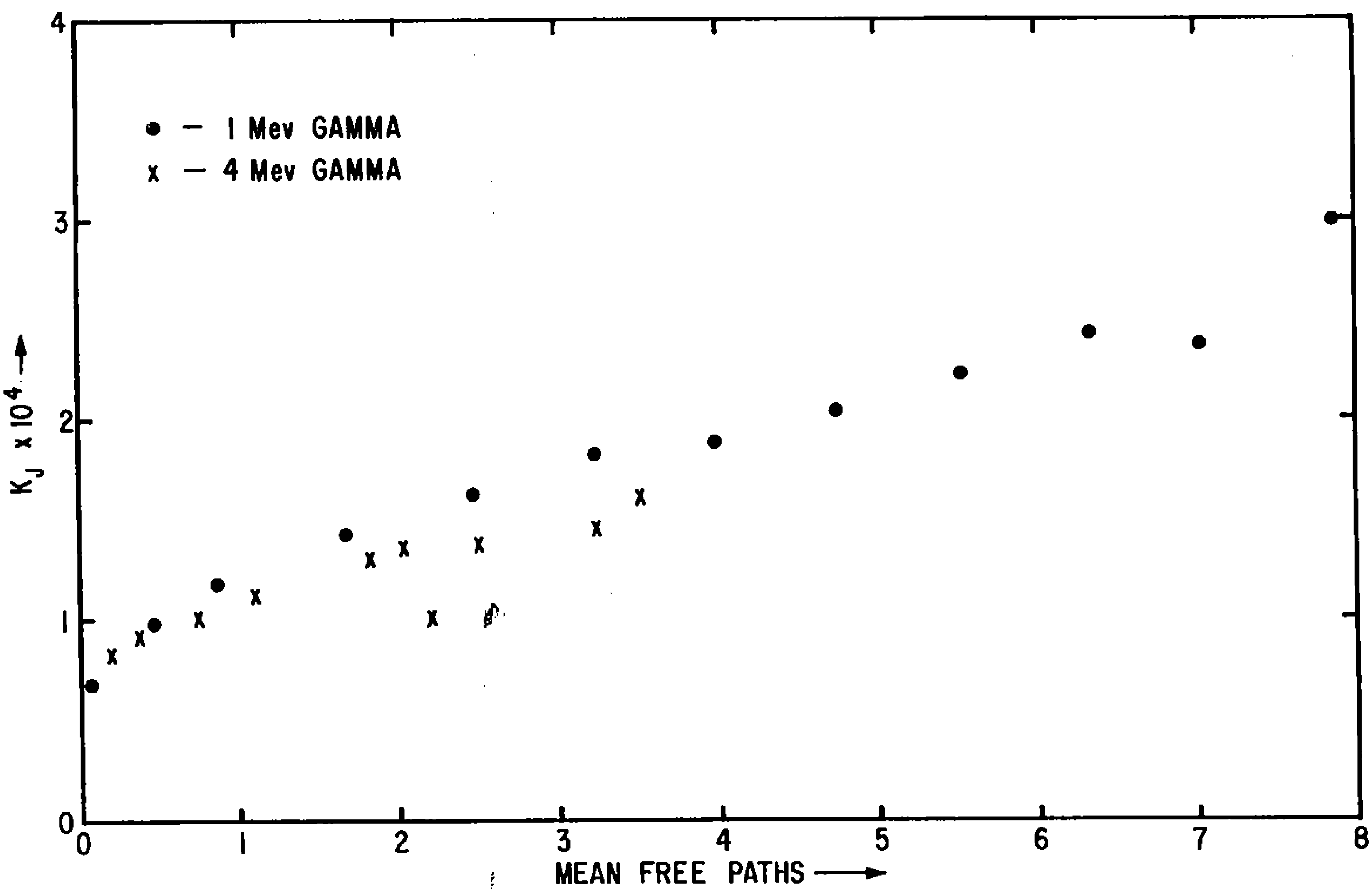

FIGURE 3. ELECTRIC CURRENT DECAY PARAMETER, K_J, VS MEAN FREE PATHS AS DETERMINED BY A LEAST SQUARE FIT.

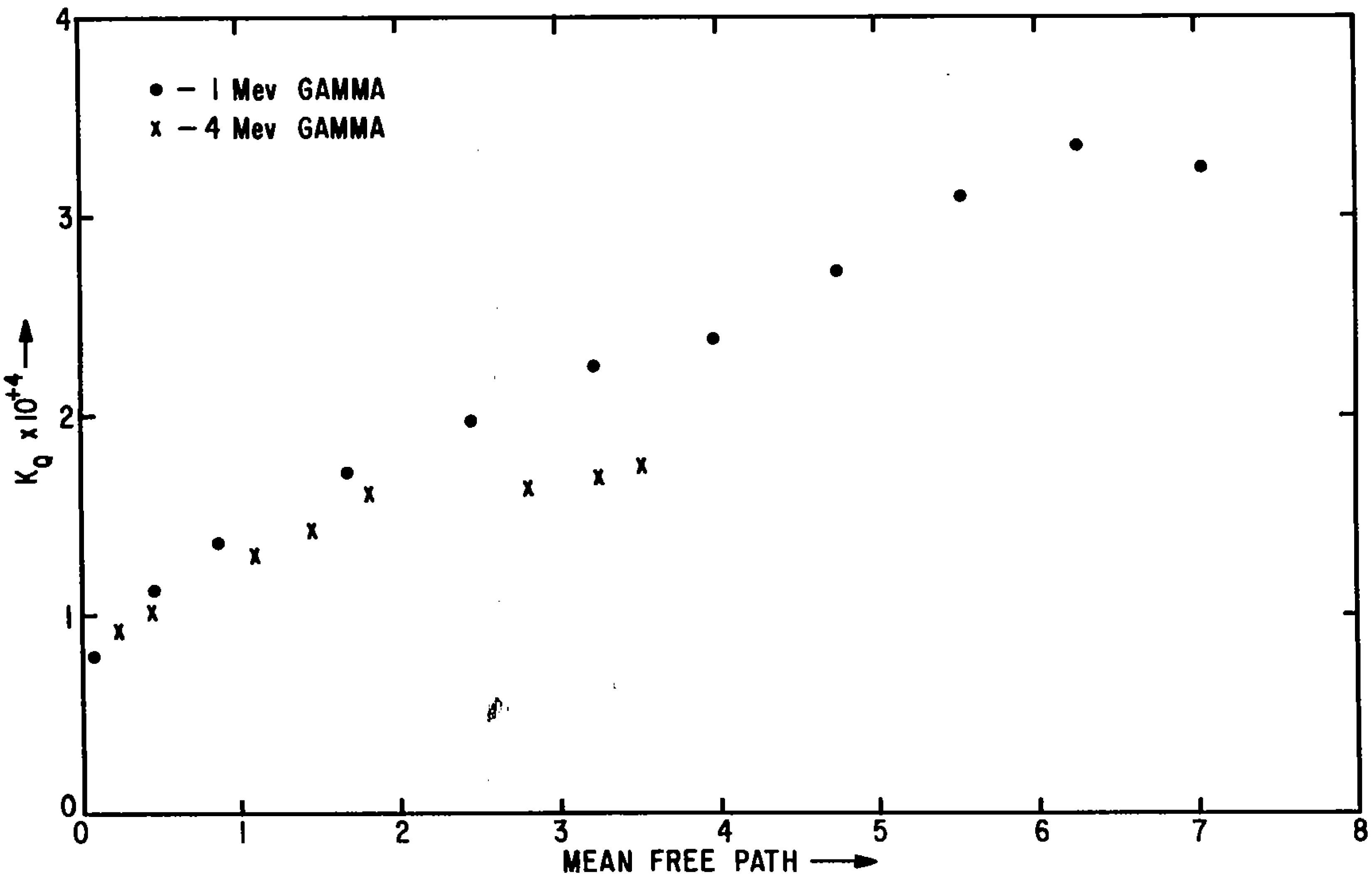

FIGURE 4. IONIZATION RATE DECAY PARAMETERS, K_Q, VS MEAN FREE PATHS AS DETERMINED BY A LEAST SQUARES FIT.

The energy deposition or ionization rate delta function response, $g(E, R, t)$ can be expressed as

$$g(E, R, t) = A_Q(E, R) \exp(-\sqrt{t}/K_Q(E, R)) \frac{1}{sec} , \qquad (5)$$

where

E	is the source gamma energy (in Mev),
R	is the radial distance from the gamma source (in meters),
t	is the time since direct gamma arrival (in seconds),
$A_Q(E, R)$	is the ionization rate delta function response "initial value" (see Fig. 2), and
$K_Q(E, R)$	is the ionization rate delta function response decay parameter (see Fig. 4).

The initial value function, $A_Q(E, R)$ was fit by the following function by trial and error using no objective criteria. The fit is evaluated at 3.0 Mev and is plotted in Fig. 2.

$$A_Q(E, R) = (1.186 - .062 \times E)\, 2.055 (10^7) \sqrt{R/\lambda_o(E)} \frac{1}{sec} , \qquad (6)$$

where

λ_o	is the gamma mean free path for collision* in STP air.

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* In this energy range, 1 - 4 Mev, the mean free path for all interactions is very nearly equal to the mean free path for Compton collisions.

The exponential decay factor $K_Q(E, R)$ was fit by straight lines which are obviously not very accurate inside approximately .6 mean free paths.

$$K_Q = (1.1 + a(E) \times R/\lambda_o) \times 10^{-4} \text{ (seconds}^{\frac{1}{2}}\text{)} \quad , \tag{7}$$

where

$a(E)$ is a function of energy only, which is plotted in Fig. 5.

The coefficient $a(E)$ was fit by a polynomial at 1, 1.5, 2, 3, 4, 5, 6, 7, and 8 Mev -- the points at 5, 6, 7, and 8 Mev being extrapolated points (see Fig. 5). Thus, $a(E)$ can be expressed as

$$a(E) = \sum_{i=1}^{9} a_i E^{i-1} \quad . \tag{8}$$

The a_i were determined by numerical solution of the resulting set of nine simultaneous linear equations obtained by enforcing Eq. (8) at the above energies.

TABLE 1

i	a_i	i	a_i
1	1.565706	5	3.875632 (10^{-1})
2	-2.774190	6	-7.185585 (10^{-2})
3	2.529891	7	8.005050 (10^{-3})
4	-1.280640	8	-4.921981 (10^{-4})
		9	1.283994 (10^{-5})

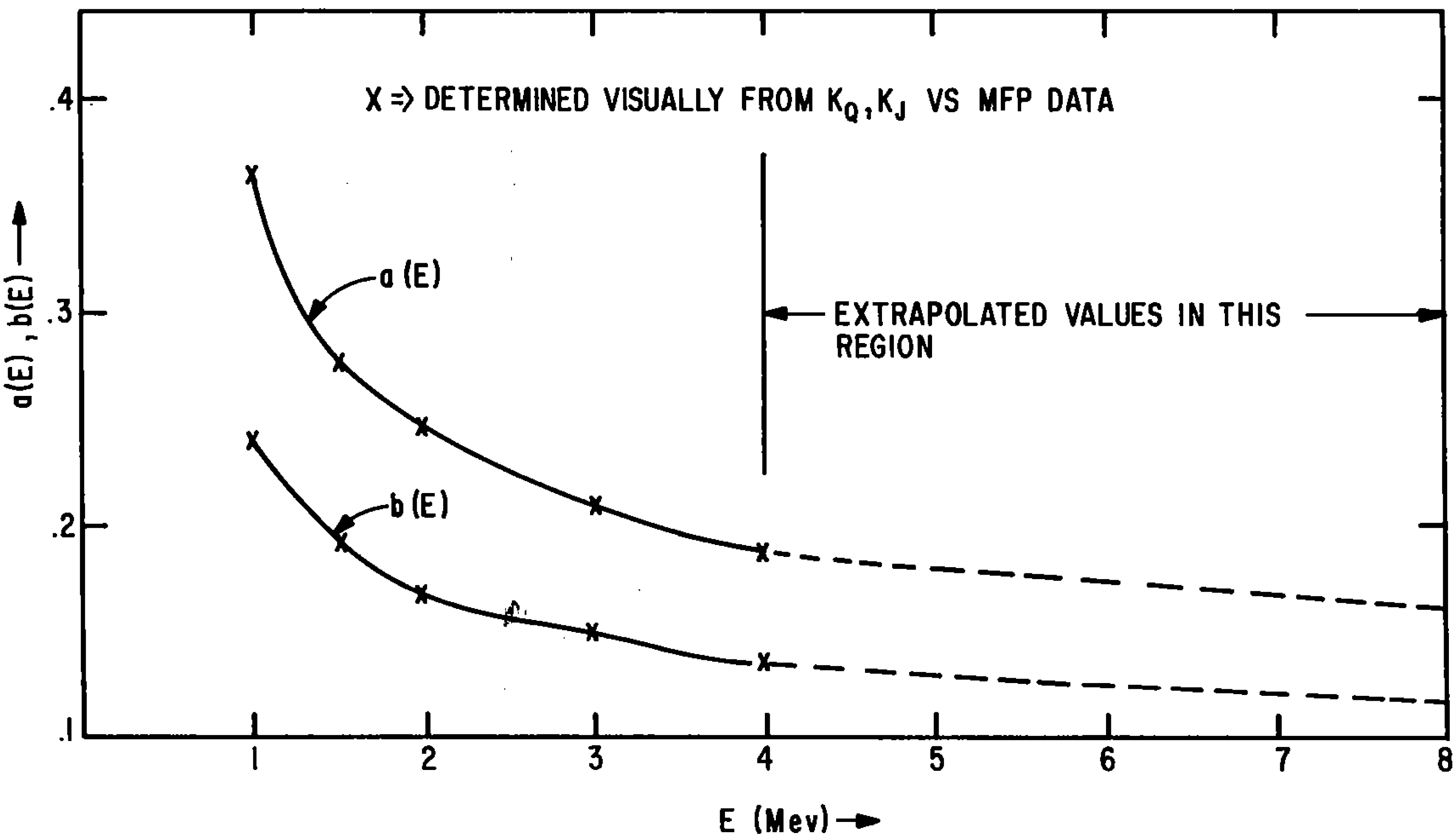

FIGURE 5. EXPONENTIAL DECAY PARAMETER, LINEAR FIT CONSTANTS, a(E), b(E), VS GAMMA ENERGY (E)

Thus, Eqs. (6), (7), and (8) provide everything needed to evaluate Eq. (5) except $\lambda_o(E)$, the source gamma mean free path at STP. This gamma mean free path can be fairly accurately approximated by

$$\lambda_o(E) \begin{cases} = 52.2 + (E - .1) \times 93.5 \text{ meters for } .1 < E < .5 \text{ Mev} \\ = 89.6 + (E - .5) \times 61.1 \text{ meters for } .5 < E < 1.5 \text{ Mev} \\ = 150.7 + (E - 1.5) \times 48.3 \text{ meters for } 1.5 < E < 4 \text{ Mev} \\ = 271.4 + (E - 4) \times 39.4 \text{ meters for } 4 < E < 8 \text{ Mev} \end{cases} \quad (9)$$

Similarly, the radial electric current delta function response, $f(E, R, t)$ can be expressed as

$$f(E, R, t) = A_J(E, R) \exp(-\sqrt{t}/K_J(E, R)) \frac{1}{\text{sec}} \quad , \quad (10)$$

where

$A_J(E, R)$ is the radial charge current delta function response "initial value" (see Fig. 1),

and

$K_J(E, R)$ is the radial charge current delta function response decay parameter (see Fig. 3),

and all other variables are defined above.

The initial value function, $A_J(E, R)$ can be approximated by

$$A_J(E, R) = 2.055(10^7) \sqrt{R/\lambda_o(E)} \frac{1}{\text{sec}} \quad , \quad (11)$$

and the exponential decay parameter, $K_J(E, R)$ is given by

$$K_J = (1.0 + b(E) \times R/\lambda_o) \times 10^{-4} \text{ (seconds}^{\frac{1}{2}}) \quad . \quad (12)$$

Here, also, the coefficient b(E)* was fit by a polynomial.

$$b(E) = \sum_{i=1}^{9} b_i E^{i-1} \quad . \tag{13}$$

The b_i are given in the following table.

TABLE 2

i	b_i	i	b_i
1	2.072634 (10^{-1})	6	5.845509 (10^{-2})
2	4.621258 (10^{-1})	7	-7.705808 (10^{-3})
3	-8.721357 (10^{-1})	8	5.449037 (10^{-4})
4	6.476889 (10^{-1})	9	-1.599049 (10^{-5})
5	-2.562205 (10^{-1})		

IV. Extension of the Curve Fits to Other Than STP Air Electron Densities

Let us define ρ_r as the electron density relative to that in STP air ($\sim 38.75\,(10^{25})$ electrons/meter^3), and, also define a distance R' which equals R/ρ_r meters from the gamma source. Then we would expect the same number of gammas and electrons to reach R' in a differential solid angle as reached R at STP in the same solid angle. This is because the same numbers of mean free paths are traversed with scattering at the same angles in both cases. Hence, the time integrated fluxes at R' will be

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* See Fig. 5.

multiplied by ρ_r^2 over those at R and STP because of the area scaling. The time dependence is also affected because gammas take different times traversing the "equivalent" distances. The rates are altered by an additional ρ_r factor because groups of gammas or electrons arriving in Δt at STP arrive in intervals equal to $\Delta t/\rho_r$ at R'. Also, the time of arrival of the gammas is changed to t/ρ_r rather than t. Consequently, the non-STP, indirect contributions to electric current density, $J'_{indirect}$, are related to those at STP by

$$J'_{indirect}(R', t') = \rho_r^3 J_{indirect}(R, t) \quad , \tag{14}$$

and the indirect contributions to ionization rate density $Q'_{indirect}$ are related to those at STP by

$$Q'_{indirect}(R', t') = \rho_r^4 Q_{indirect}(R, t) \quad , \tag{15}$$

where

$$R' = R/\rho_r$$

$$t' = t/\rho_r \quad .$$

But, these relationships do not describe the dependence of our delta function responses f(E, R, t) and g(E, R, t) because we have not considered the effects on the direct beam contributions. The direct beam contributions to current and ionization rate at R' will equal those at R at STP multiplied by ρ_r^2 and ρ_r^3 respectively because of the area and volume increments in the scaled geometry.

$$J'_{direct}(R') = \rho_r^2 \, J_{direct}(R) \tag{16}$$

and

$$Q'_{direct}(R') = \rho_r^3 \, Q_{direct}(R) \quad . \tag{17}$$

Therefore the $f(E, R', t')$, $g(E, R', t')$ responses are related to $f(E, R, t)$, $g(E, R, t)$ at STP by the following equations:

$$f(E, R', t') = \frac{\rho_r^3 \, J_{indirect}(R, t)}{\rho_r^2 \, J_{direct}(R)} \quad , \tag{18}$$

$$f(E, R', t') = \rho_r \times f(E, R, t) \quad ,$$

and

$$g(E, R', t') = \rho_r \times g(E, R, t) \quad , \tag{19}$$

where

$$R' = R/\rho_r$$

$$t' = t/\rho_r \quad .$$

More explicitly, the indirect beam responses to a delta function source at any electron density in a monogeneous Compton scatterer, are*

$$g(E, R, t) = \rho_r (1.186 - .062E)(2.055(10^7)) \sqrt{R \cdot \rho_r / \lambda_o(E)}$$

$$\times \;\; \exp(-\sqrt{t \cdot \rho_r}) \Big/ K_Q(E, R) \; \frac{1}{sec} \quad , \tag{20}$$

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* The primes have been dropped for clarity.

where

$$K_Q(E, R) = \left(1.1 + a(E) \times \left(\frac{R \cdot \rho_r}{\lambda_o}\right)\right) 10^{-4} \quad ,$$

and a(E) is as given in Eq. (8), and

$$f(E, R, t) = \rho_r \cdot 2.055 (10^7) \sqrt{R \cdot \rho_r / \lambda_o(E)} \; \exp(-\sqrt{t \cdot \rho_r} / K_J) \; \frac{1}{\text{sec}} \quad , \qquad (21)$$

where

$$K_J(E, R) = \left(1.0 + b(E) \times \left(\frac{R \cdot \rho_r}{\lambda_o}\right)\right) 10^{-4} \quad ,$$

and b(E) is as given in Eq. (13). Note that

$$\sqrt{R \cdot \rho_r / \lambda_o(E)} = \sqrt{R / \lambda(E)}$$

where

$\lambda(E)$ is the mean free path length at relative density ρ_r.

V. Comparison of Curve Fits with the Monte Carlo Data and Analytic Buildup Factors and Extension of Applicability to 8 Mev and 10 Mean Free Paths

Figures 6 - 9 contain histograms representing the thrice-smoothed* Monte Carlo results compared with the curve fits as expressed in Eqs. (20) and (21). The histograms were made compatible with the units of the curve fits by multiplying the curve fits by the appropriate direct beam effects.

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* For an explanation of smoothing, see page 9 of RM-4151-PR cited earlier.

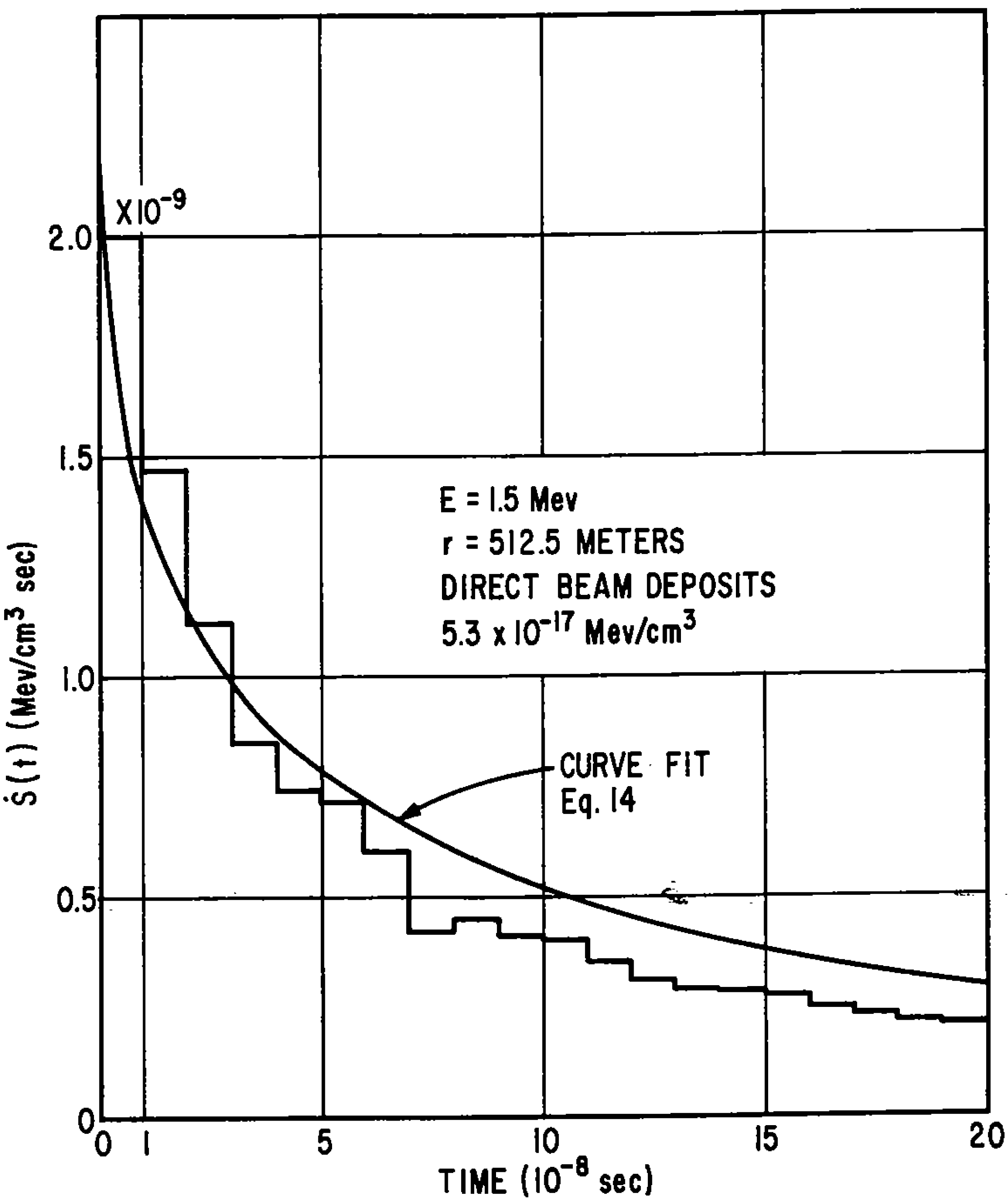

FIGURE 6. ENERGY DEPOSITION RATE AS A FUNCTION OF TIME

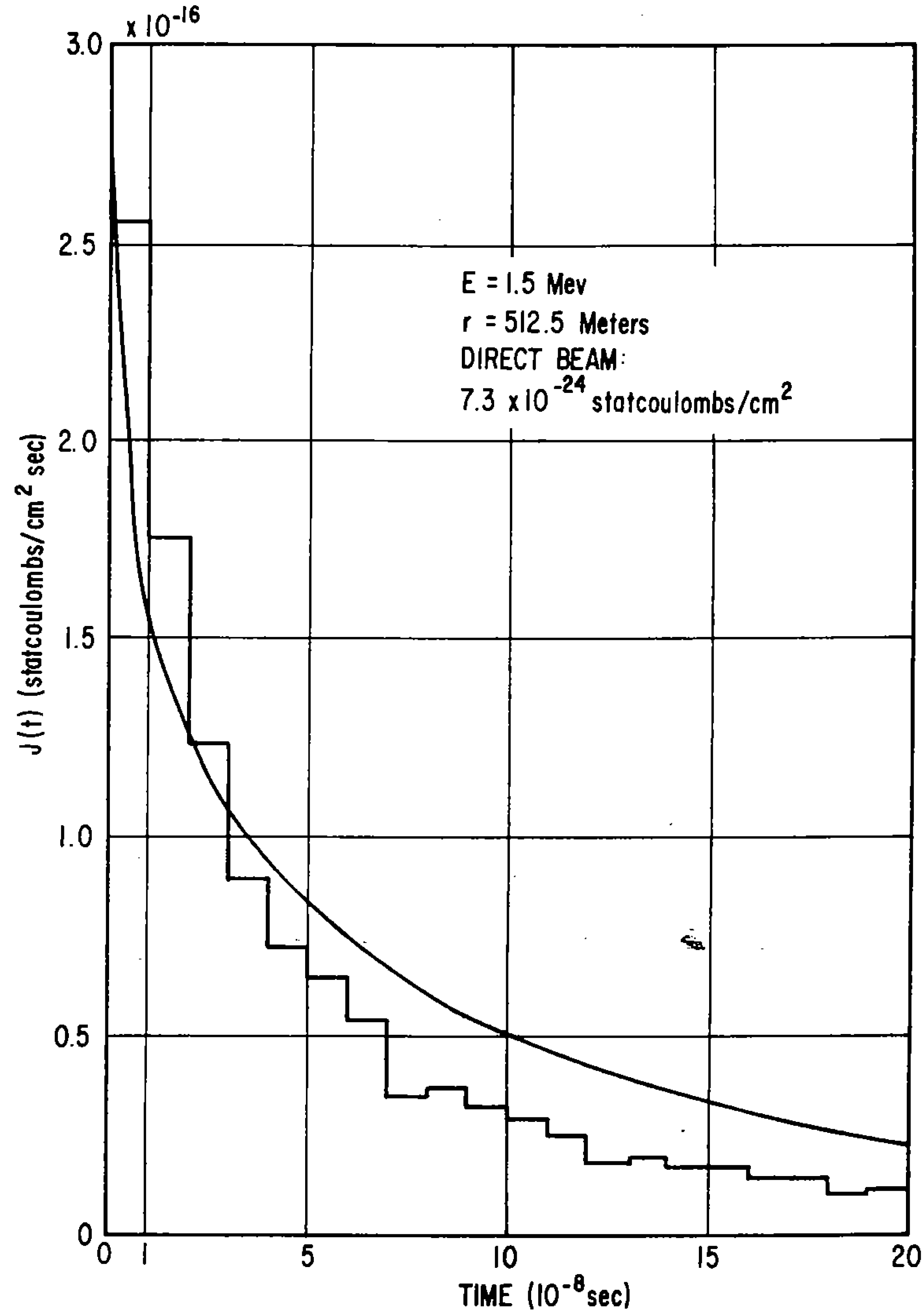

FIGURE 7. RADIAL COMPTON CURRENT AS A FUNCTION OF TIME

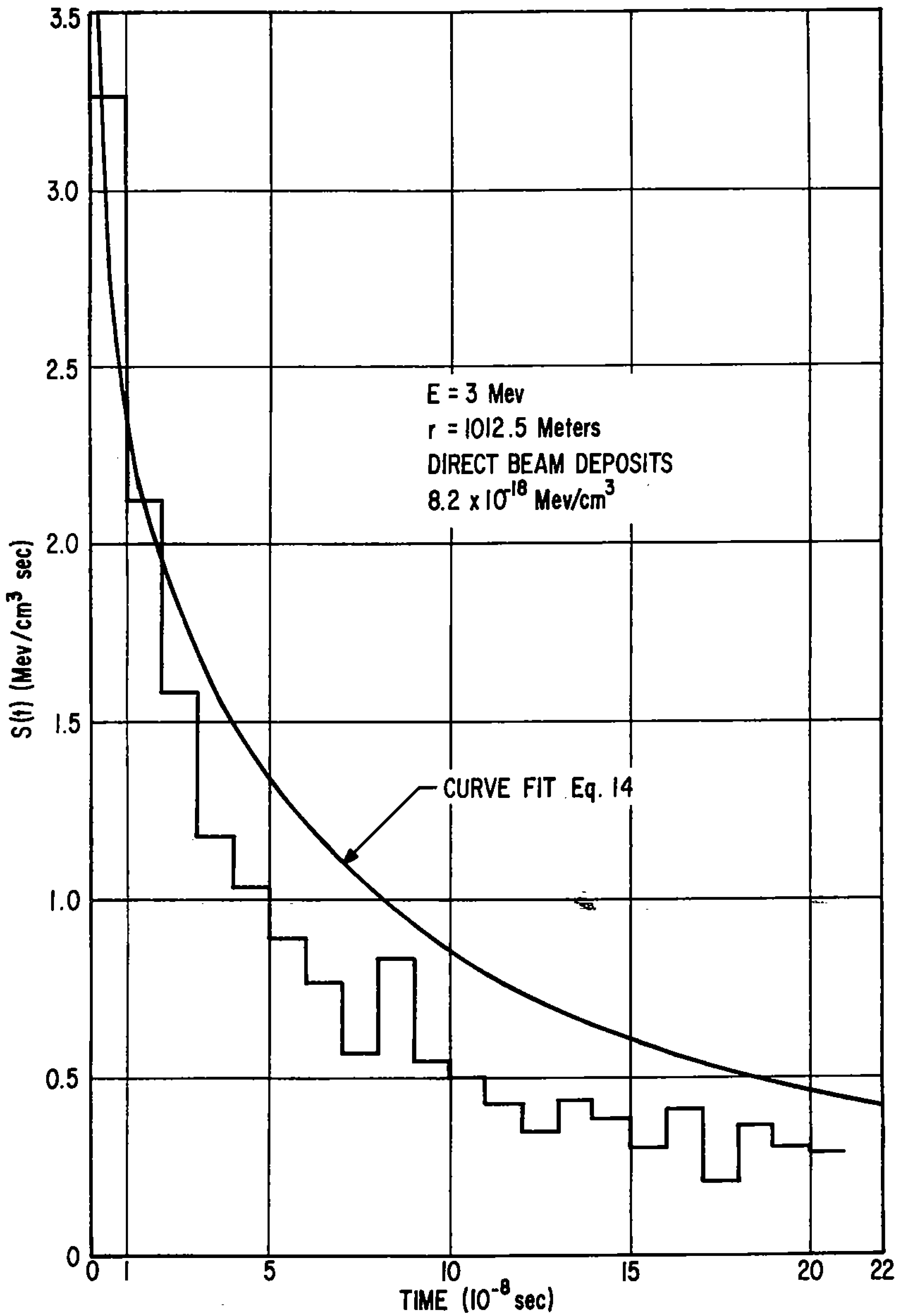

FIGURE 8. ENERGY DEPOSITION RATE AS A FUNCTION OF TIME

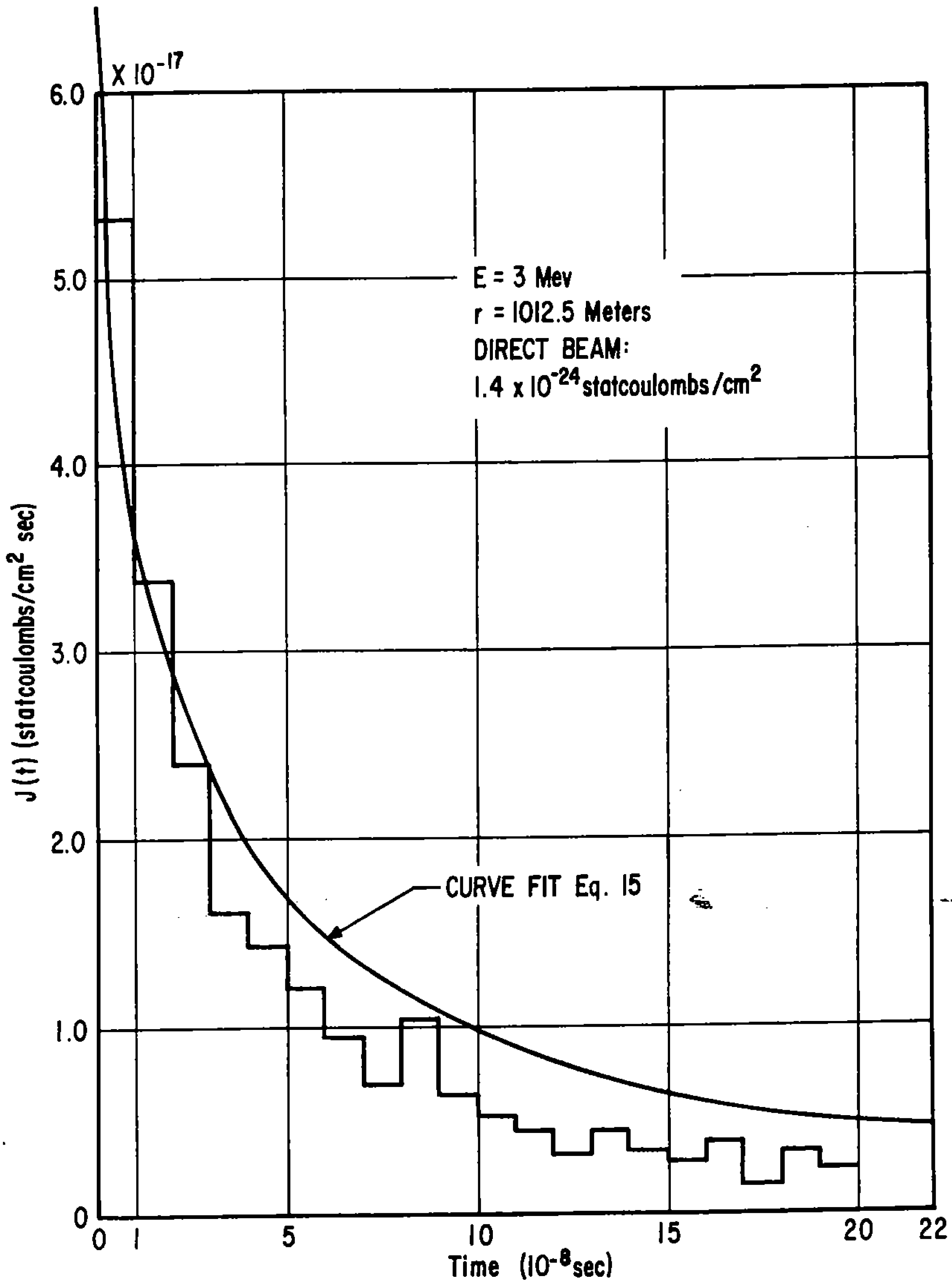

FIGURE 9. RADIAL COMPTON CURRENT AS A FUNCTION OF TIME

The buildup factors implied by the curve fits expressed in Eqs. (20) and (21) can be easily calculated. The buildup factor for energy deposition or radial charge transport is defined as the sum of contributions due to the direct beam only. Examining Eq. (3) for the case that $S_o(t) = \delta(t)$, we see that the buildup factor for radial charge transport, B_J, is

$$B_J = \int_{-\infty}^{\infty} \frac{J(R, t)\, dt}{J_{direct}}$$

$$= J_{direct} \int_{-\infty}^{\infty} \frac{|\delta(t) + f(E, R, t)\, dt|}{J_{direct}}$$

$$= 1 + \int_{-\infty}^{\infty} f(E, R, t)\, dt$$

$$B_J = 1 + \int_{-\infty}^{\infty} f(E, R, t)\, dt \quad . \tag{22}$$

Similarly, the buildup factor for energy deposition, B_Q, can be expressed as

$$B_Q = 1 + \int_{0}^{\infty} g(E, R, t)\, dt \quad . \tag{23}$$

Note that the form of $f(E, R, t)$ and $g(E, R, t)$ is $A \cdot \exp(-\sqrt{t}/K)$. The value of this integral can be expressed in closed form as $2 \cdot K^2 \cdot A$. Thus,

$$B_J = 1 + 2 \cdot \rho_r \cdot 2.055(10^7) \; \overline{\sqrt{R/\lambda(E)} \cdot K_J(E,R)}^{\,2} \tag{24}$$

and

$$B_Q = 1 + 2 \cdot \rho_r (1.186 - .062E)\; 2.055(10^7) \; \overline{\sqrt{R/\lambda(E)} \cdot K_Q(E,R)}^{\,2} \quad . \tag{25}$$

These buildup factors are plotted as function of mean free path $\left(\sqrt{R}/\lambda(E)\right)$ with moments method results of Goldstein and Wilkins* for the dose buildup factor from a point source in water) in Figs. 10 - 14.

The delta function response fits given by Eqs. (20) and (21) were based on Monte Carlo data assuming source gamma energies of 1, 1.5, 2, 3, and 4 Mev. However, the K_J and K_Q decay parameters were extrapolated to 5, 6, 7, and 8 Mev -- see Fig. 5. Figures 10 - 13 indicate that the buildup factors based on Eqs. (24) and (25) are too high for mean free paths greater than about 10^3 meters/$\lambda_o(E)$. This inaccuracy is particularly obvious at higher energies (Figs. 12 - 14). Therefore, a correction factor to K_Q and K_J as given in Eqs. (7) and (12) was devised which insures sufficient agreement between the curve fit buildup factors and those calculated by the moments method. This correction factor, F_c, for both K_Q and K_J, is

$$F_c = 1 \Big/ \left[1 + F(E) \times \left|R/\lambda(E) - 10^3/\lambda_o(E)\right|\right] \quad . \tag{26}$$

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* Goldstein, H. and J. Ernest Wilkins, Jr., Calculations of the Penetration of Gamma Rays, NYO-3075, Nuclear Development Associates, Inc., White Plains, New York, June 30, 1954.

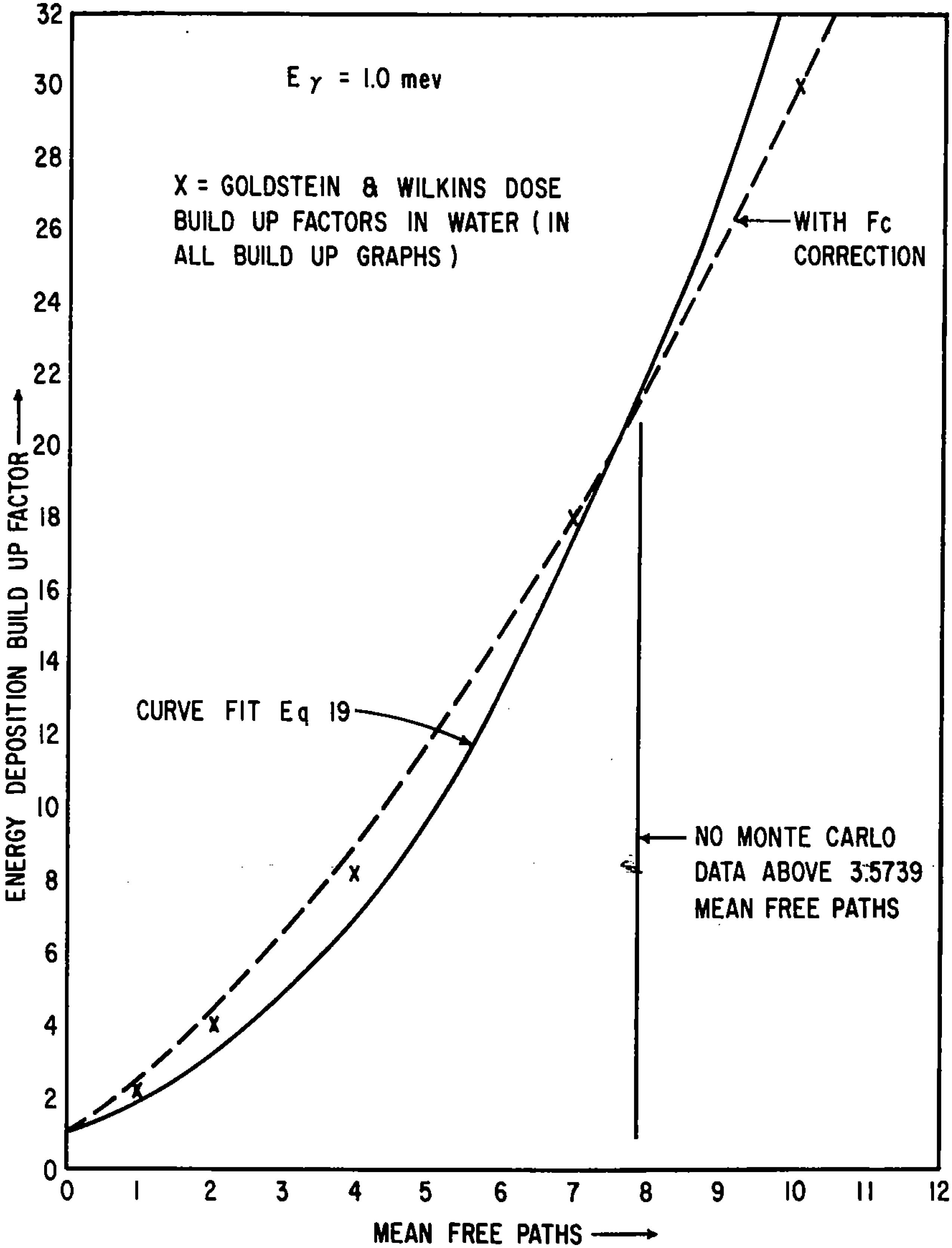

FIGURE 10. DOSE BUILD UP FACTOR VS. MEAN FREE PATHS FROM A 1. MEV GAMMA SOURCE

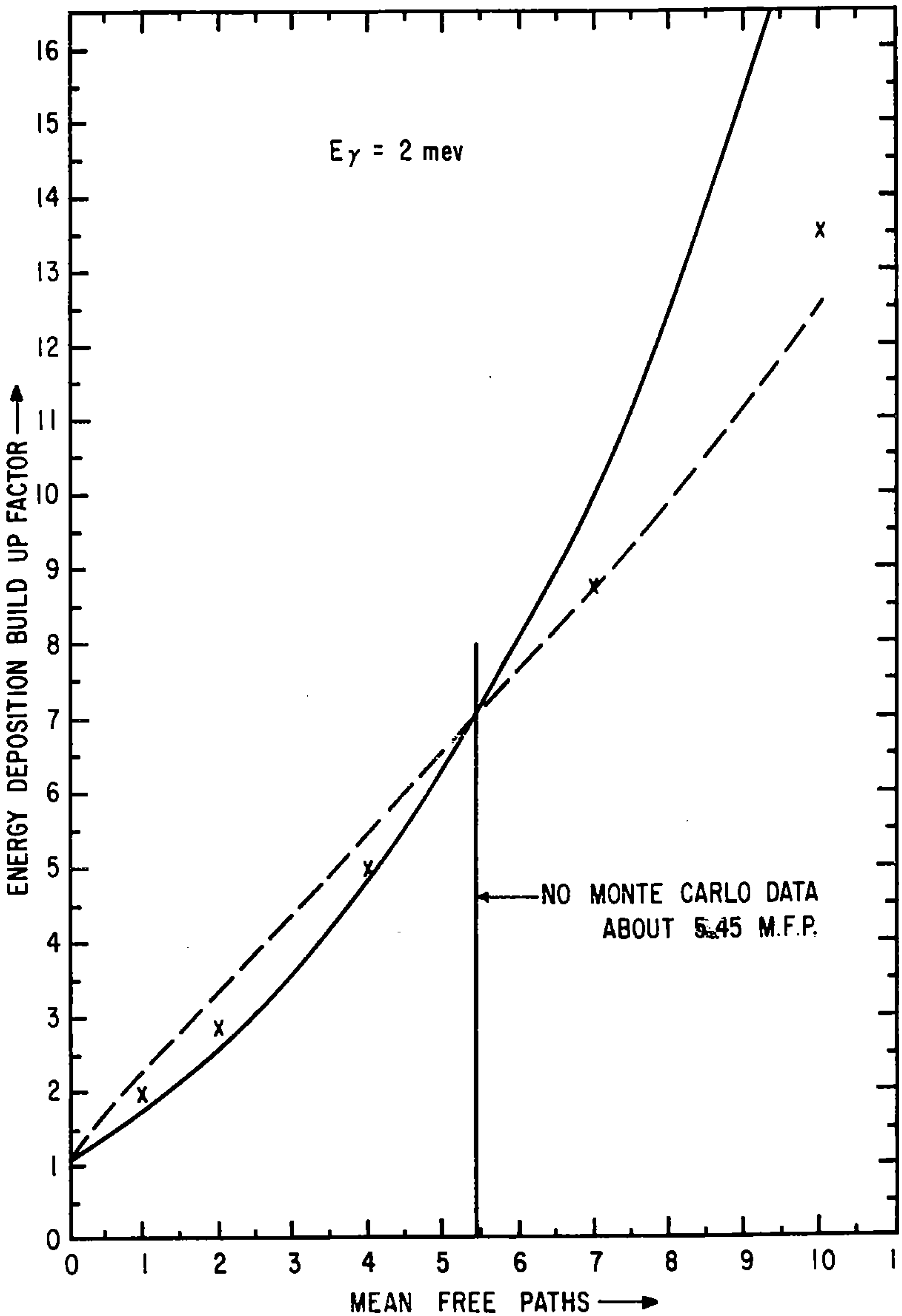

FIGURE 11. DOSE BUILD UP FACTORS VS. MEAN FREE PATHS FROM A 2 MEV GAMMA SOURCE

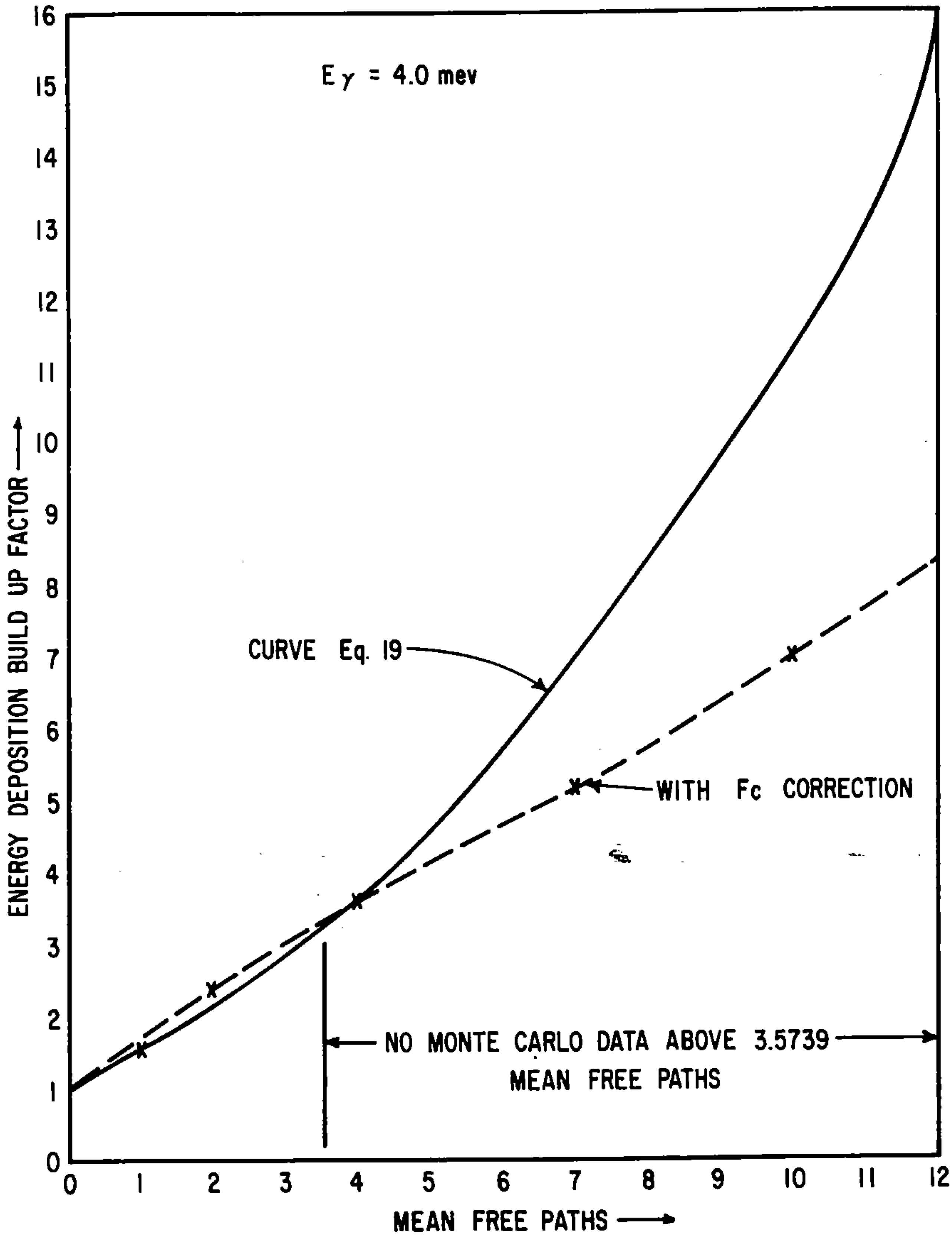

FIGURE 12. DOSE BUILD UP FACTORS VS. MEAN FREE PATHS FROM A 4. MEV GAMMA SOURCE

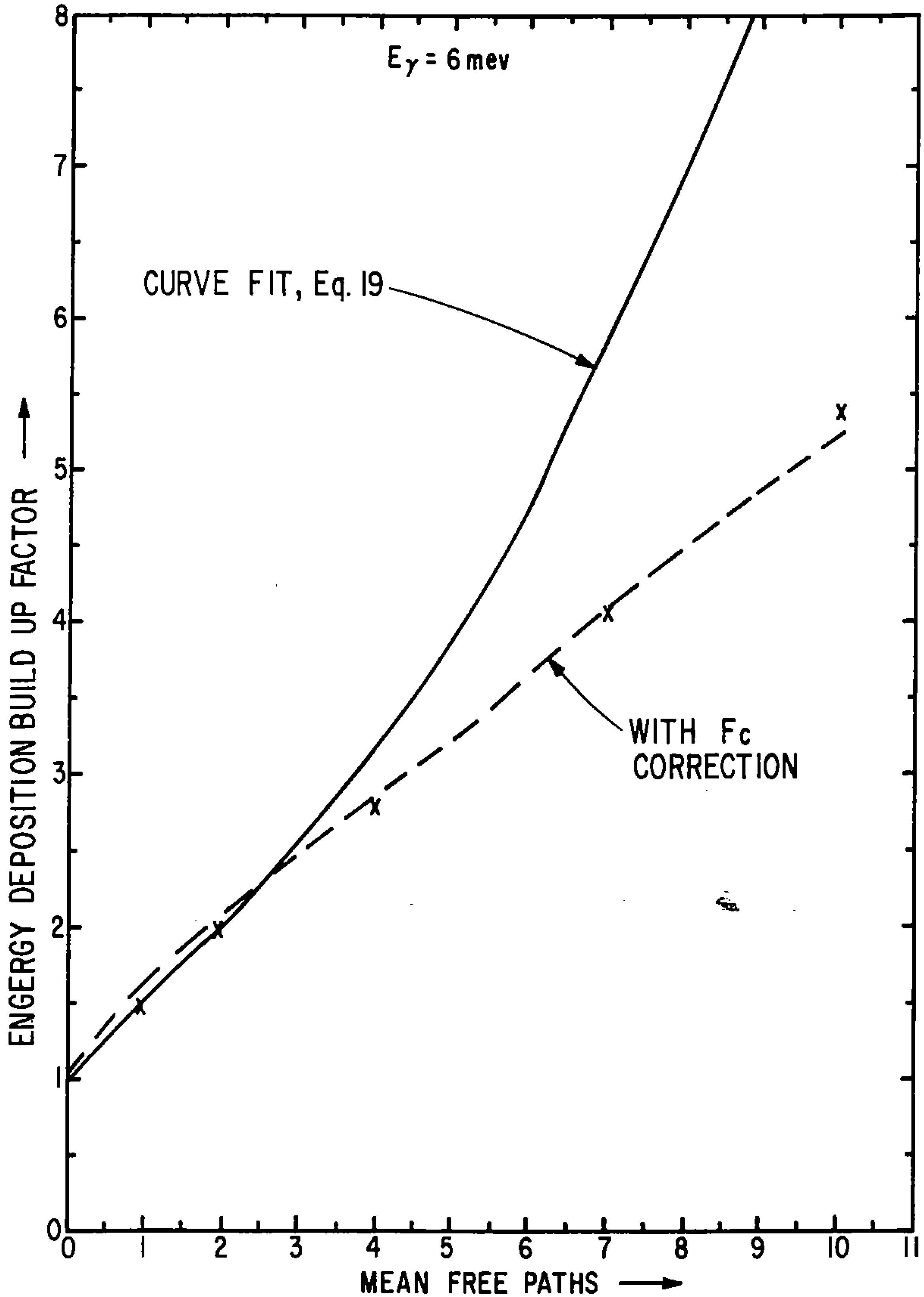

FIGURE 13. DOSE BUILD UP FACTOR VS MEAN FREE PATHS FROM 6 Mev. GAMMA SOURCE.

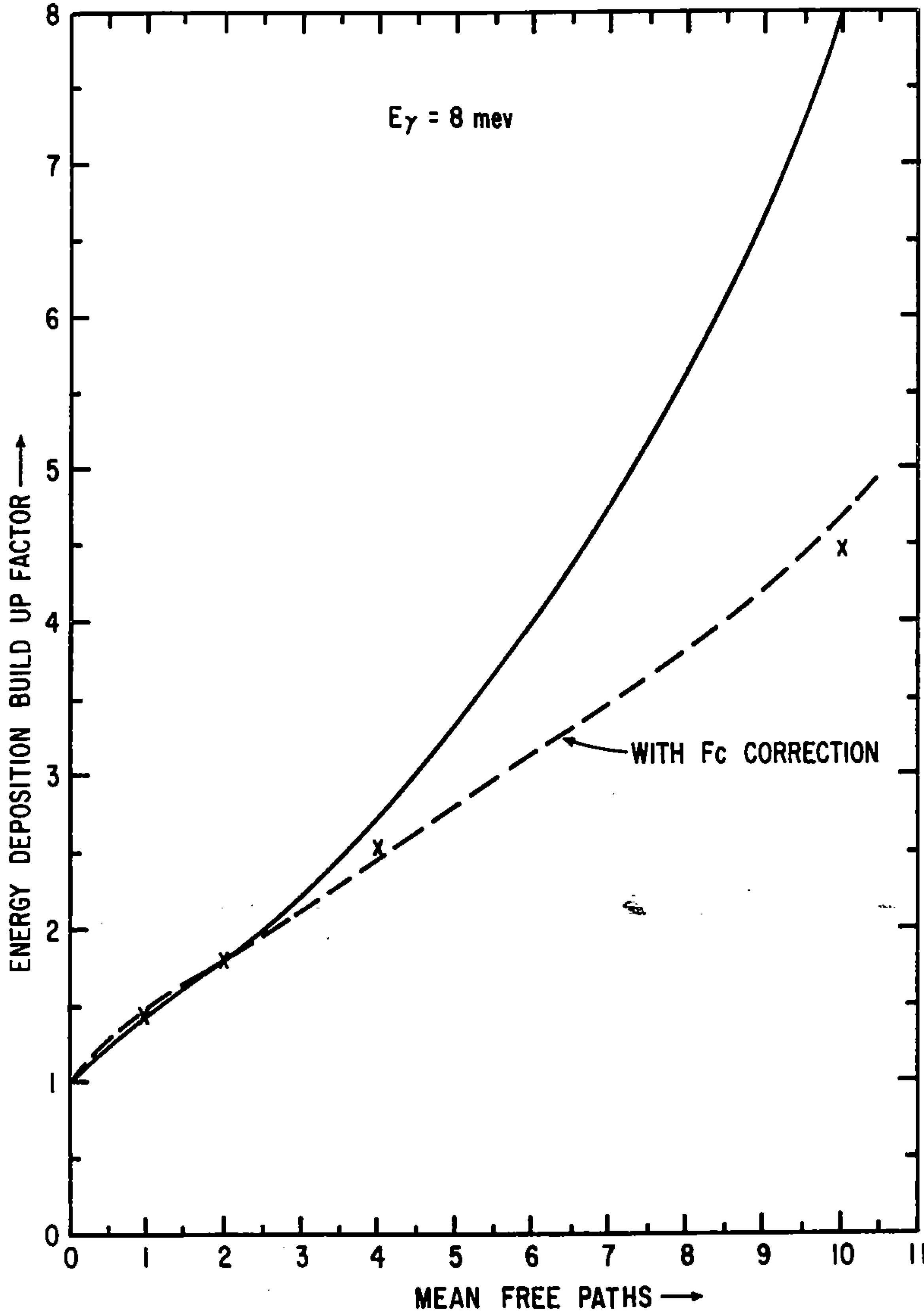

FIGURE 14. DOSE BUILD UP FACTOR VS. MEAN FREE PATHS FROM AN 8 MEV GAMMA SOURCE

The coefficient in the denominator, F(E), was fit by a polynomial as were a(E) and b(E) -- (see Eqs. (8) and (13)).

$$F(E) = \sum_{i=1}^{9} f_i E^{i-1} \quad . \tag{27}$$

The f_i are given in the following table.

TABLE 3

i	f_i	i	f_i
1	$-1.955758\ (10^{-2})$	6	$5.632172\ (10^{-3})$
2	$1.237809\ (10^{-1})$	7	$-6.678283\ (10^{-4})$
3	$-1.323847\ (10^{-1})$	8	$4.279942\ (10^{-5})$
4	$8.040323\ (10^{-2})$	9	$-1.144781\ (10^{-6})$
5	$-2.774780\ (10^{-2})$		

This F(E) function is plotted in Fig. 15 along with the fixed values upon which the polynomial fit was based.

VI. Conclusions and Summary

Curve fits to the radial charge (or electron) current and ionization (or energy deposition) rate due to a point, isotropic, monoenergetic, source of gamma rays in homogeneous air are given in Eqs. (21) and (20) respectively, where the K_Q and K_J parameters are to be corrected by the factor, F_c, given in Eq. (26). These curve fits are adequate for gamma energies

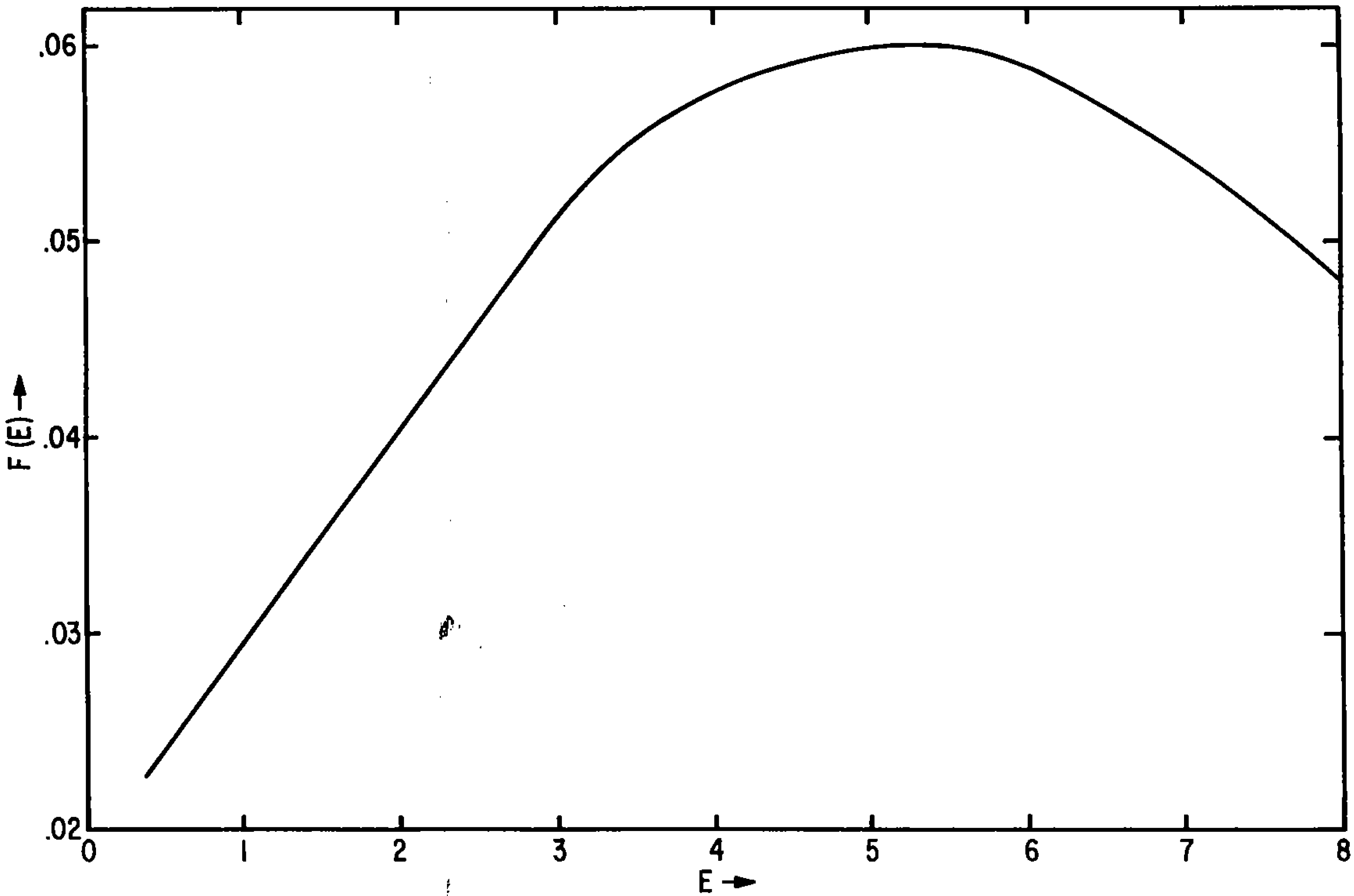

FIGURE 15. LINEAR COEFFICIENT IN CORRECTION FACTOR, F (E), VS GAMMA ENERGY, E.

between 1 and 8 Mev and ranges between .6 and 10 mean free paths. They are based on gamma scattering in air, and hence are not necessarily adequate for scattering in more dense materials where photoelectric and pair production collisions are more probable.

Given an isotropic, monoenergetic, gamma ray emission from a point source into a homogeneous scattering medium of low A (e.g., air), with relative electron density, ρ_r, relative to STP air, one can express the air ionization rate, Q, in pairs/meter^3-sec, as

$$Q(R,t) = \frac{Ee \cdot \rho_r \cdot e^{-R \cdot \rho_r/\lambda_t}}{E_{i\rho} \lambda_t \cdot 4\pi R^2} \int_0^t [\delta(\tau) + g(E, R, \tau)] \cdot S_o(t-\tau)\, d\tau \qquad (28)$$

(ion-pairs/meter^3-second),

where

Ee	is the average energy lost to the recoil electron in first collisions of E Mev gamma rays (in Mev),
$E_{i\rho}$	is the average energy required to form an electron-ion pair in air or other low Z material (in Mev),
ρ_r	is the electron density relative to STP air (unitless),
λ_t	is the gamma mean free path for all types of collisions including pair production ($\lambda_t < \lambda_o$),*

* These total cross sections can be found in National Bureau of Standards Circular 583, April 30, 1957.

R	is the distance from the gamma source (in meters),
t	is the time since initial direct beam gamma ray arrival from the source (in seconds),
$\delta(\tau)$	is the Dirac delta function (in seconds^{-1}),
$S_o(t)$	is the rate of gamma emission from the source (in γ's / second), and
$g(E, R, t)$	is the ionization rate delta function response, which this note purports to have fit (in seconds^{-1}).

For scattering in air the total collision mean free path length can be approximated by

$$\lambda_t(E) \begin{cases} = \lambda_o \text{ for } E < 2 \text{ Mev} \\ = 174.8 + (E-2) \times 40.3 \text{ meters for } 2 < E < 4 \text{ Mev} \\ = 255.4 + (E-4) \times 28.6 \text{ meters for } 4 < E < 6 \text{ Mev} \\ = 312.6 + (E-6) \times 20.7 \text{ meters for } 6 < E < 8 \text{ Mev} \end{cases}$$

The adjusted version of $g(E, R, t)$ to use in Eq. (28) is given here:

$$g(E, R, t) = \rho_r \cdot (1.186 - .062 \times E) \times 2.055 (10^7) \times \sqrt{R \cdot \rho_r / \lambda_t(E)} \qquad (29)$$

$$\times \exp(-\sqrt{t \cdot \rho_r} / K_Q(E, R) \quad ,$$

where

$$K_Q(E, R) = \left| 1.1 + a(E) \times (R \cdot \rho_r / \lambda_t) \right| 10^{-4} \times F_c \qquad (30)$$

and

$$a(E) = \sum_{i=1}^{9} a_i E^{i-1} \tag{31}$$

i	a_i	i	a_i
1	1.565706	5	3.875632 (10^{-1})
2	-2.774190	6	-7.185585 (10^{-2})
3	2.529891	7	8.005050 (10^{-3})
4	-1.280640	8	-4.921981 (10^{-4})
		9	1.283994 (10^{-5})

and

$$F_c = 1 \Big/ \left(1 + F(E) \times (R \cdot \rho_r/\lambda_t - 10^3/\lambda_o)\right) \tag{32}$$

where

$$F(E) = \sum_{i=1}^{9} f_i E^{i-1} \tag{33}$$

i	f_i	i	f_i
1	-1.955758 (10^{-2})	6	5.632172 (10^{-3})
2	1.237809 (10^{-1})	7	-6.578283 (10^{-4})
3	-1.323847 (10^{-1})	8	4.279942 (10^{-5})
4	8.040323 (10^{-2})	9	-1.144781 (10^{-6})
5	-2.774780 (10^{-2})		

Similarly, the radial charge current, $J(R, t)$ amps/meter^2, can be expressed as

$$J(R,t) = q \cdot \frac{Re}{\lambda_t} \frac{e^{-R \cdot \rho_r / \lambda_t}}{4\pi R^2} \int_0^t [\delta(t) + f(E,R,\tau)] \, S_o(t-\tau) \, d\tau \quad , \tag{34}$$

where

q	is the electron charge (in coulombs),
Re	is the average radial distance traveled by recoil electrons from first collisions of E Mev gamma rays at STP (in meters), and
$f(E, R, t)$	is the radial charge current delta function response (in seconds^{-1}),

and all other variables are as defined for Eq. (28).

The adjusted version of $f(E, R, t)$ to be used in Eq. (34) is given here:

$$f(E,R,t) = \rho_r \cdot 2.055\,(10^7) \cdot \sqrt{R \cdot \rho_r / \lambda_t} \cdot \exp(-\sqrt{t \cdot \rho_r}/K_J) \quad , \tag{35}$$

where

$$K_J = \left(1.0 + b(E) \times (R \cdot \rho_r / \lambda_t)\right) 10^{-4} \cdot F_c \tag{36}$$

and

$$b(E) = \sum_{i=1}^{9} b_i E^{i-1} \quad . \tag{37}$$

i	b_i	i	b_i
1	2.072634 (10^{-1})	6	5.845509 (10^{-2})
2	4.621258 (10^{-1})	7	-7.705808 (10^{-3})
3	-8.721357 (10^{-1})	8	5.449037 (10^{-4})
4	6.476889 (10^{-1})	9	-1.599049 (10^{-5})
5	-2.562205 (10^{-1})		

Equations (28) and (34) assume a monoenergetic gamma ray source. For use with non-monoenergetic gamma sources, $S_o(t)$ should be replaced by $S_o(E, t)$ gammas/Mev-second, and the right-hand sides of Eqs. (28) and (34) should be integrated over energy.

The limits of integration in Eqs. (28) and (34) assume that the gamma source function is zero for negative arguments. The ionization rate and charge current expressions assume that the air density is high enough that the recoil electron can be assumed to travel radially at the speed of light without distorting the time dependences significantly. This condition is met if $\left(Re/\rho_r\right)\left(\frac{1}{v} - \frac{1}{c}\right) <$ the time in which $S_o(t)$ varies significantly.

THE END