EMP Theoretical Note XV

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Charge Currents and Conductivity Arising from Inelastic and Fast Capture Collisions of Neutrons in the Air Surrounding a Nuclear Detonation

by

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ABSTRACT

A method of calculating the charge current and conductivity due to inelastic and fast capture collisions of an expanding shell of mono-energetic neutrons is described in EMP Theoretical Note XI*. In this note I intend to describe analytic approximations to evaluations of the charge current and ionization rate expressions contained therein. If the neutrons are emitted from a point source as a square wave or delta function in time, the expressions can be solved analytically at early times, at the neutron arrival time, and at later times. Finally, I present curve fits to the current and ionization based on one neutron emitted in square waves of $2(10^{-8})$ and $1(10^{-9})$ second widths for any air density. These calculations and curve fits all assume no ground; that is, the neutrons expand in a spherical shell rather than a hemispherical shell.

*See Appendix

A. Early Time Approximation

At early times the neutrons will be localized around the source (G.Z.) and will not be attenuated appreciably. Hence, the inelastic collisions will form essentially a point source of gamma rays of magnitude $\sigma_g \cdot Dt \cdot V_N$ gammas/second-neutron, where σ_g is the inelastic gamma producing cross section in air, Dt is the air nuclei density, and V_N is the neutron velocity (5.2 (10^7) meters/second). The current and ionization rates will decay as the number of neutrons in the shell does at very early times, i.e., as exp $(-V_N \cdot t/\lambda_R)$. Thus we can easily calculate the magnitudes and the slope of the ionization rate, QN, and charge current, JN, at early times $(t \simeq R/C)$ using standard point source expressions.

$$(1) \quad QN(R, t \simeq R/C) = \sigma_g \cdot Dt \cdot V_N \frac{E_e(10^6)}{34\lambda_\gamma} \cdot \frac{e^{\frac{-R}{\lambda_\gamma}}}{4\pi R^2} \quad \frac{\text{ion-pairs}}{m^3\text{-sec-neutron}}$$

$$= 3.74\ (10^6) \frac{e^{\frac{-R}{\lambda_\gamma}}}{R^2} \qquad \text{(at STP)*}$$

$$(2) \quad JN\ (R, t \simeq R/C) = \sigma_g \cdot Dt \cdot V_N \frac{q \cdot R_e}{\lambda_\gamma} \cdot \frac{e^{\frac{-R}{\lambda_\gamma}}}{4\pi R^2} \quad \frac{\text{amps}}{m^2\text{-sec-neutron}}$$

$$= 4.52 (10^{-17}) \frac{e^{\frac{-R}{\lambda_\gamma}}}{R^2} \qquad \text{(at STP)}$$

where E_e is the average recoil electron energy

λ_γ is the gamma ray mean free path

q is the electronic charge

R_e is the electron range in meters

R is the observation point radius

σ_g is the neutron cross-section for gamma production

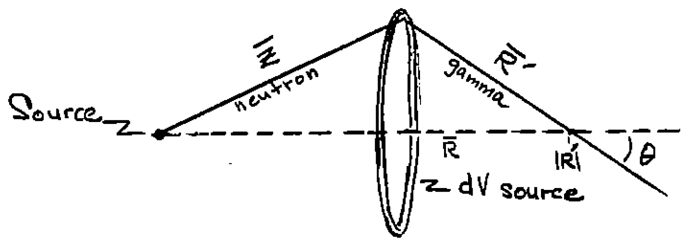
Dt is the air nuclei density

B. Neutron Arrival Time Evaluation

An interesting time to look at current and ionization rates is the neutron arrival time. As was shown in EMP Theoretical Note XI, the current

*STP indicates 273°K temperature and 760 mm of Hg pressure.

and ionization rates peak when the neutrons surround the observation point. This is because the neutron shell creates a gamma ray source very near the observation point, and hence, the $1/R^2$ fall off is suddenly very small. Consider the geometry and notation of Note XI.



All points in dV emit gammas which reach R at the same time. dV is a ring about the dashed axis. The ionization rate at R and any time, t, is, from Note XI:

$$\text{(3)} \quad QN(R,t) = \frac{E_e(10^6)}{34\ \lambda_g} \cdot \frac{\sigma_g \cdot Dt}{4\pi \cdot 2R} \int_0^{R+V_N t} \left[\int_{|R-R'|}^{R+R'} N_o(t-|Z|/V_N - R'/C)\, \frac{e^{-Z/\lambda_R}}{Z}\, dZ \right] \frac{e^{-R'/\lambda_\gamma}}{R'^2}\, dR'$$

where λ_g is the removal cross section

$N_o(t)$ is the number of neutrons emitted per second (changed from Note XI)

The charge current is

$$\text{(4)} \quad JN(R,t) = \frac{1.6(10^{-19})R_e}{\lambda_\gamma \cdot 4\pi} \cdot \frac{\sigma_g Dt}{4R^2} \int_0^{R+V_N t} \left[\int_{|R-R'|}^{R+R'} N_o(t-|Z|/V_N - R'/C)\, \frac{e^{-Z/\lambda_R}}{Z} (R'^2 + R^2 - Z^2)\, dZ \right] \cdot \frac{e^{-\frac{R'}{\lambda_g}}}{R'^2}\, dR'$$

When the neutrons arrive at R, the integral over R' contains contributions for vanishing R' which can cause the integrals to diverge depending on the form of $N_o(t)$. It will be seen that the integrals blow up for a delta function (in time) source, but not for a square wave.

B.1 A Delta Function Source at $t = R/V_N$

If $N_o(t-|Z|/V_N - R'/C) = \delta(t-|Z|/V_N - R'/C)$,

then from (3), $QN = K \int_0^{2R} \left[\frac{e^{-(R-R'V_N/C)/\lambda_R}}{(R-R'V_N/C)} \right] \frac{e^{-R'/\lambda_g}}{R'}\, dR'$

$$\text{where } K = \frac{E_e(10^6)}{34\lambda_g} \cdot \frac{\sigma_g \cdot Dt}{4\pi \cdot 2R} \cdot V_N{*}$$

$$(5) \quad QN = K e^{-R/\lambda_R} \int_0^{R\cdot\frac{2C}{C+V_N}} \frac{e^{-R'\left(\frac{1}{\lambda_\gamma} - \frac{V_N}{C\cdot\lambda_R}\right)}}{R'(R-R'V_N/C)}\, dR'$$

The upper bound on R' in (3) was determined geometrically. In reality more stringent limits are imposed by the form of the source. We cannot integrate where the delta function cannot exist for any Z at R'. This condition is that R' be smaller than or equal to $R \cdot C/V_N$. The integral has the form

$$e^{-aR'}/(R'+bR'^2)$$

which can be expanded to

$$e^{-aR'}\left(\frac{1}{R'} - \frac{1}{\frac{1}{b}+R'}\right),$$

the second term of which is negligible at small R'. The integral of $e^{-aR'}/R'$ includes a term log R' which is infinite when evaluated at R' = 0. Thus, $QN(R,t = R/V_N) = \infty$ for a delta function (in time) source.

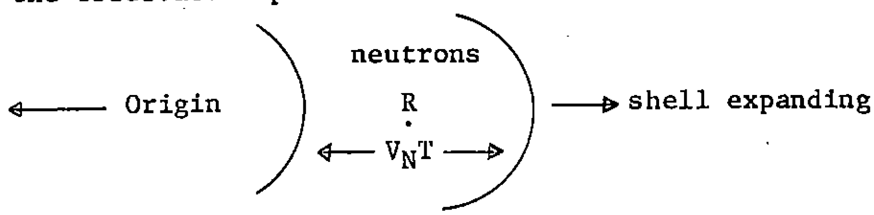
$JN(R,t = R/V_N)$ will not blow up because at $t = R/V_N$ and $R' = 0$, R = Z, and hence, the cosine factor goes to zero.

B.2 A Square Wave Source at $t = R/V_N + T/2$.

A square wave source is represented by

$$N_o(t) = \begin{cases} 0 \text{ for } t < 0 \\ 1/T \text{ for } 0 < t < T \\ 0 \text{ for } t > T \end{cases}$$

where T is the duration of the square wave. We want to evaluate QN and JN of equations (3) and (4) at $t = R/V_N + T/2$, when the neutron shell surrounds the observation point at R.



The upper and lower bounds of integration of equations (3) and (4) were determined by the geometry, i.e., no Z can exist that is greater than R + R' or less than |R - R'| for any R - otherwise R, Z, and R' will

* $N_o(t)$ is a delta function in time, whereas we are integrating over space in which the source looks like a delta function in space times V_N. Consequently, I have placed V_N in the constant coefficient.

not form a triangle as required (see figure above). If we wish $N_o(t)$ to be a constant rather than a discontinuous function, further requirements must be placed on Z and R'. The upper bound on Z, Z_u, must be such that the argument of N_o is positive.

$$R/V_N + T/2 - Z/V_N - R'/C \geq 0$$

$$(6) \quad Z_u = R + \frac{T \cdot V_N}{2} - \frac{R' \cdot V_N}{C} .$$

The lower bound Z_L, must satisfy $R/V_N + T/2 - Z/V_N - R'/C \leq T$

$$(7) \quad Z_L = R - \frac{T \cdot V_N}{2} - \frac{R' \cdot V_N}{C}$$

Depending on R', either these bounds or the geometric bounds are more restrictive. When

$$\frac{R' \cdot V_N}{C} + \frac{T \cdot V_N}{2} \geq R', \text{ then } Z_L \leq |R-R'|$$

and the lower bound is $|R-R'|$. Let us define the largest R' satisfying this condition as

$$R'_a = \frac{T \cdot V_N}{2} / (1 - \frac{V_N}{C}).$$

When $-\frac{R' \cdot V_N}{C} + \frac{T \cdot V_N}{2} \geq R'$, then $Z_u \geq R + R'$

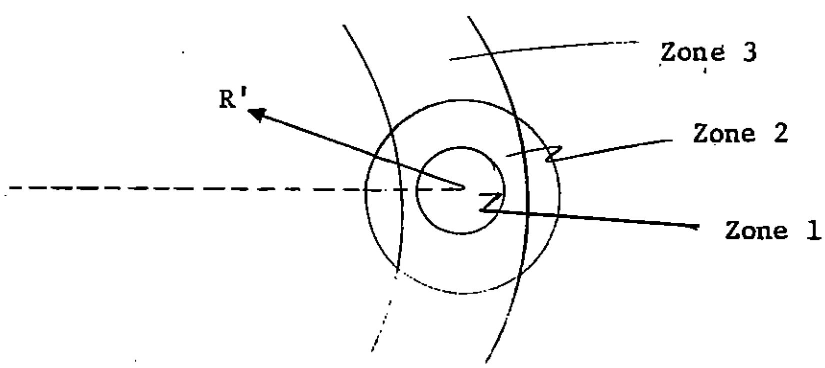
and the upper bound is R + R'. Let us define R'_b as the largest R' satisfying this condition:

$$R'_b = \frac{T \cdot V_N}{2} (1 + \frac{V_N}{C})$$

Note that $R'_b < R'_a$. The regions of applicability of the various bounds of integration are:

R-R', R+R'	for	$R' < R'_b$	Zone 1
R-R', Z_u	for	$R'_b < R' < R'_a$	Zone 2
Z_L, Z_u	for	$R > R'_a$	Zone 3

Graphically, these zones are seen below



Zone 1 is an interesting zone because it includes the smallest R' values and possibly the major contributions to QN and JN at $t = R/V_N + T/2$.

B.3 Evaluation of the contributions due to a square wave $2(10^{-8})$ seconds wide.

For a sample calculation, let R = 100 meters, $T = 2(10^{-8})$ seconds, and consider QN for Zone 1. Equation (3) becomes

$$QN = K(R)\int_0^{R'_b}\left[\int_{R-R'}^{R+R'} \frac{1}{T}\cdot\frac{e^{-Z/\lambda_R}}{Z}\,dZ\right]\frac{e^{-R'/\lambda_\gamma}}{R'}\,dR'$$

$$\text{where } K(R) = \frac{E_e(10^6)\sigma_g\, Dt}{34\lambda_\gamma\cdot 4\pi\cdot 2R}$$

$$\lambda_R = 202 \text{ m.}$$

The integral within the brackets, I, can be evaluated as

$$I = \int_{R-R'}^{R+R'} \frac{1}{T}\,\frac{e^{-Z/\lambda_R}}{Z}\,dZ = \frac{1}{T}\left[\log Z - \frac{Z}{\lambda_R} + \frac{Z^2}{2\cdot 2!(\lambda_R)^2} - \frac{Z^3}{3.3!(\lambda_R)^3} + \text{---}\right]_{R-R'}^{R+R'}$$

$$(8)\quad I = \frac{1}{T}\left[\log\left(\frac{R+R'}{R-R'}\right) - \frac{2R'}{\lambda_R} + \frac{4RR'}{2\cdot 2!\lambda_R^2} - \frac{(6R^2R'+2R'^3)}{3.3!\lambda_R^3} + \text{----}\right]$$

For $R' << R$, which is the case in Zone 1:

$$I \cong \frac{1}{T}\left[\frac{2R'}{R} - \frac{2R'}{\lambda_R} + \frac{2RR'}{2!\lambda_R^2} - \frac{2R^2R'}{3!\lambda_R^3} + \frac{2R^3R'}{4!\lambda_R^4} - \frac{2R^4R'}{5!\lambda_R^5} + \text{----}\right]$$

$$= \frac{2R'}{T}\left[\frac{1}{R} - \frac{1}{\lambda_R} + \frac{R}{2!\lambda_R^2} - \frac{R^2}{3!\lambda_R^3} + \frac{R^3}{4!\lambda_R^4} - \frac{R^4}{5!\lambda_R^5} + \text{---}\right]$$

$$= \frac{2R'}{RT}\left[1 - \left(\frac{R}{\lambda_R}\right) + \frac{(R/\lambda_R)^2}{2!} - \frac{(R/\lambda_R)^3}{3!} + \text{----}\right]$$

So $I \cong \frac{2R'}{RT}\, e^{-R/\lambda_R}$. Evaluating $K(R) = .0359/R$ we have

$$QN \cong \frac{.0359}{R}\int_0^{R'_b} \frac{2R'}{RT}\, e^{-R/\lambda_R}\cdot\frac{e^{-R'/\lambda_\gamma}}{R'}\,dR' = \frac{.0718}{T}\,\frac{e^{-R/\lambda_R}}{R^2}\left[-e^{-R'/\lambda_\gamma}\right]_0^{R'_b}.$$

$$(9)\quad QN = \frac{.0718}{T}\ \frac{e^{-R/\lambda_R}}{R^2}\ \lambda_\gamma \left[1 - e^{-R'_b/\lambda_\gamma}\right]$$

Note that $R'_b \ll \lambda_\gamma$; therefore, $e^{-R'_b/\lambda_\gamma} \cong 1 - R'_b/\lambda_\gamma$

Hence, for Zone 1,

$$(10)\quad QN(R, t = R + V_N + .5T) \cong \frac{.0718}{T}\ \frac{e^{-R/\lambda_R}}{R^2} \cdot R'_b$$

Since $T = 2(10^{-8})$ sec and $R = 100$ meters, we have $R'_b = \frac{T \cdot V_N}{2} \Big/ (1 + \frac{V_N}{C}) = .442$ m.

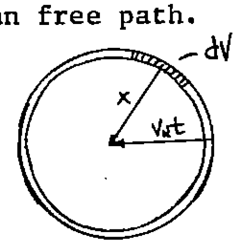
This, together with $\lambda_R = 202$ meters allows one to evaluate

(11) $QN\ (100,\ t = 100/5.2(10^7) + 1(10^{-8})) = .967\ (10^2)$ ion-pairs/m^3-sec-neutron for Zone 1.

Evaluation of the contributions from Zone 2 is complicated because of the form of Z_u. The I function is not a constant times R', hence the R' integration is complicated. This exposition should indicate that evaluation of (3) and (4) is messy by analytic methods. However, the Zone 1 analysis indicates that QN and JN have finite values at the troublesome neutron arrival time* for a finite square wave source, and hence for any real source that can be represented by a sum of finite square waves. Numerical evaluation of (3) and (4) is practical on a digital computer and results will be given for various sources in section D. The values of QN and JN at neutron arrival are linearly dependent on the neutron density in the shell; therefore, the peak values fall as $\exp\ (-R/\lambda_R)/4\pi R^2$.**

C. The Later Time Approximations by Analogy.

At later times ($t > R/V_N + 2(10^{-6})$) the values of ionization rates at all points inside of R are approximately the same. This is because the contributions from the expanding shell fall off as $\exp\ (-X)/X^2$ where X is the distance from the shell element to the point well inside of the shell divided by the gamma ray mean free path.



* We have shown that the contributions to QN from source elements including R' = 0 are finite, and are given by (10). These contributions caused the blow up in Section B.1.

** In any fusion neutron source we can expect a gaussian distribution of initial neutron velocities with distribution about 14.1 MeV depending upon the burn temperature, and thus, the shell will expand in thickness linearly with distance traveled. Therefore, the neutron arrival time ionization rate and charge current densities will fall somewhat faster than $\exp\ (-R/\lambda_R)/R^2$.

If we expand e^{-x}/x^2 we obtain $(1-x+x^2/2!-x^3/3!+ ---)/x^2$

or $\frac{1}{x^2} - \frac{1}{x} + \frac{1}{2!} - \frac{x}{3!} + ---$

Note that the first term varies as the electric field, except that it is a scalar, the second as the electrical potential and the third is constant. Inside a spherical shell of charge we would expect a zero field or a constant scalar quantity, and a constant potential. Analogously, then, we can expect a constant ionization rate if the terms after the third are negligible. This condition will certainly hold for an appreciable region of space. Oh, what will be the magnitude of the constant ionization rate? Since the ionization rate is independent of position, let us calculate it for R = 0:

$$QN(R=0,t\ R/V_N+2(10^{-6})) \triangleq QN^*$$
$$QN^* = \sigma_g \cdot Dt \cdot V_N \cdot e^{-R/\lambda_R} \cdot \left(\frac{e^{-R/\lambda_\gamma}}{\lambda_\gamma 4\pi R^2}\right) \cdot \frac{4.55(10^6)}{34}$$

But $t = R/V_N + R/C$, $R = t \cdot V_N C/(V_N+C) = 4.43(10^7)t$**

$$\text{So}\quad QN^* = \frac{\sigma_g \cdot Dt \cdot V_N \cdot 4.55(10^6)}{\lambda_\gamma \cdot 4\pi R^2 \cdot 34}\, e^{-\frac{t(V_N \cdot C)}{V_N+C}\left(\frac{1}{\lambda_R}+\frac{1}{\lambda_\gamma}\right)}$$

$$(12)\quad QN^* = \frac{3.74(10^6)e^{-4.43(10^7)t/\lambda_{eff}}}{[t(4.43\times10^7)]^2}, \text{ where } \lambda_{eff} = 134.2 \text{ at STP.}$$

D. Results of 2 Shake Square Wave Source Computer Calculations Compared With Analytic Predictions.

The curves in figures 1 and 2 represent the ionization rate and charge current densities resulting from a square wave shell of 14 MeV neutrons expanding from a point source. The square wave was $2(10^{-8})$ seconds wide and the atmosphere was assumed to be at STP†. The predicted initial values (see equations (1) and (2)) are indicated by the circles at $t_R = 0$, t_R is t - R/C where t = 0 is the neutron emission time. The later time values computed from (12) are indicated by the dashed line. The solid lines represent the computer numerical integrations of (3) and (4) in section B. The X's represent the peak values during the neutron arrival time based on an exp $(-R/\lambda_R)/4\pi R^2$ dependence through the 300 meter value.

† The relative air density, ρ_r, dependence is presented in section F.
** As will be seen later, a more accurate version should replace by $4.9(10^7)$ meters/sec.

The only discrepancy with the analyses of sections A, B, and C is in the magnitude and time dependence of the later time approximation (section C). The discrepancy noted in figure 1 seems to indicate that the ionization rate well within the shell does not exactly approach a value independent of position. As a close approximation, however, we can alter expression (12) to yield an ionization rate which adequately represents the later time value within most of the significant EMP generation region. I proposed to change the $V_N \cdot C/(V_N + C)$ term of expression (12) from $4.43(10^7)$ meters/sec to $4.9(10^7)$ meters/sec.

E. Curve Fits to the 2 Shake Square Wave Source Results at STP

Curve fits (in several regions) to the curves in figures 1 and 2 are given here. These curve fits are adequate descriptions of the ionization rate and charge current densities from 14 MeV neutrons interacting with air nuclei for weapons undergoing fusion for about $2(10^{-8})$ seconds. This rather loose requirement is necessary in order that the peak values at the neutron arrival time are accurate. These peak values are not very strongly dependent on source width.

Consider the ionization rate densities in figure 1. We already have the initial values given by equation (1). Let this value decay as the exponential of t_R to a minimum. What values should we choose for the minimums? The minimums seem to be a factor of three below the neutron arrival peaks and they seem to occur 1.4 μseconds before the neutron arrival time.

(13) Then $QN(R, t_R < R/V_N - R/C - 1.4\mu sec) = QN_o \cdot e^{-Kt_R}$
in the first region, where QN_o is the QN given by equation (1). $QN_o = 3.74(10^6)$

What is the attenuation constant? The neutron arrival ionization rate peak can be expressed as $1.42(10^7)\ \exp\ (-R/\lambda_R)/R^2$; therefore

$$e^{-KR/V_N - R/C - 1.4\mu sec} = \frac{3.74(10^6)\ e^{-R/\lambda_Y}}{1.42(10^7)\, e^{-R/\lambda_R}} = e^{-1.334} \cdot e^{-R/\lambda_Y + R/\lambda_R}$$

So,

(14) $K = \left(\frac{R}{\lambda_Y} - \frac{R}{\lambda_R} + 1.334\right) \Big/ \left(R/V_N - R/C - 1.4(10^{-6})\right)$

The region of applicability of expression (13) and (14) is for $t_R > 0$ and $t_R < R/V_N - R/C - 1.4\mu$seconds.

The next region to consider is between $R/V_N - R/C - 1.4(10^{-6})$ seconds and $R/V_N - R/C$, the neutron arrival time. The value of QN must rise from its minimum, $.473(10^7)\exp(-R/\lambda_R)/R^2$. Let QN min rise as the $\exp(K'(\Delta t)^6)$ where $\Delta t = t_R - (R/V_N - R/C - 1.4\ \mu\text{sec})$, that is, the time since the minimum.

$$e^{K'(1.14\times10^{-6})^6} = 3 \doteq e^{1.1}$$

$$\text{or } K' = \frac{1.1\times10^{36}}{(1.4)^6} = .1477(10^{36})$$

Thus, between $R/V_N - R/C - 1.4$ μsec and $R/V_N - R/C$,

$$(15)\quad QN(R,t) = 4.73(10^6)\ \frac{e^{-R/\lambda_R}}{R^2}\ \cdot\ e^{-.1477(10^{36})(t_R - (R/V_N - R/C - 1.4(10^{-6})))^6}$$

After the neutron arrival peak ($t_R = R/V_N - R/C$), the value of QN should decay from $1.42(10^7)\exp(-R/\lambda_R)/R^2$ to the later time value given by (12) (with $4.9(10^7)$ substituted for $4.43(10^7)$). Let the modified later time value be designated by QN*

$$QN^* = \frac{3.74(10^6)\ e^{-\frac{4.9(10^7)t_R}{134.2}}}{[4.9(10^7)t_R]^2}$$

Then for $t_R \geq R/V_N - R/C$, let

$$(16)\quad QN(R,t_R) = \left[1.42(10^7)\ \frac{e^{-R/\lambda_R}}{R^2} - QN^*\right]\ e^{-3.12(10^3)(t_R - (R/V_N - R/C))^{1/2}} + QN^*$$

Note that when $t_R = R/V_N - R/C$, $QN = 1.42(10^7)e^{-R/\lambda_R}/R^2$ as is to be expected, and that as $t_R - (R/V_N - R/C)$ increases, QN approaches QN*.

Similarly, we have the initial values for JN, given by expression (2). $JN(R, t_R < R/V_N - R/C - 1(10^{-8}))$ is adequately fit by

$$(17)\quad JN(R,t_R) = 4.52(10^{-17})\ \frac{e^{-R/\lambda_\gamma}}{R^2}\ e^{-\alpha t_R}$$

where α is a function of R such that JN approaches 2/3 of the neutron arrival $(t_R=R/V_N-R/C+1(10^{-8})$ peak value. This peak value is determined numerically to be $6.08(10^{-17})\ \exp\ (-R/\lambda_R)/R^2$.

$$e^{-\alpha(R/V_N-R/C)} = \frac{6.08(10^{-17})e^{-R/\lambda_R}}{4.52(10^{-17})e^{-R/\lambda_\gamma}} \cong e^{.297-R/\lambda_R+R/\lambda_\gamma} \quad .$$

So,

(18) $\alpha = (R/\lambda_R-.297-R/\lambda_\gamma)/(R/V_N-R/C-1(10^{-8}))$

For a brief interval of time around $R/V_N-R/C$ we must add a peak to boost this QN (exp 17) by a third of $6.08\ (10^{-17})\ \exp\ (-R/\lambda_R)/R^2$.

If we define JN* as $Ae^{Bt^*}/(1-e^{C(t^*-t_p)})$

and define $t^* = t_R-(R/V_N-R/C-1\mu sec)$

$t_p = 1\ \mu sec$

$B = .6(10^7)$

$C = 10^8$

and $A = 1(10^{-19})\ \frac{e^{-R/\lambda_R}}{R^2}$,

and, if we add this function JN* to (17), we get a peak in JN rising to the appropriate value and decaying quickly.

For $t_R>R/V_N-R/C-1(10^{-8})$, (17) should be dropped and

(19) $JN(R,t_R) = 3.JN^*$

This is as far as we should both fitting JN because in reality the neutrons will undergo multiple scattering and make invalid our JN curves for $t_R>R/V_N-R/C$

F. Relative Air Density Dependence

If the relative air density, ρ_r, is not unity, how are the values of QN and JN altered? The equations presented in this paper, where not evaluated at STP, contain Dt, $1/\lambda_g$, $1/\lambda_R$, and $1/R_e$ which are linearly dependent on relative air density. If these quantities are correctly evaluated, then the relative air density dependence of the QN and JN expressions is obtained. At the end of this section, however, the results expressed in (13),(14),(15), (16), (17), (18), and (19) are rewritten as the STP values with the ρ_r dependence explicitly inserted.

How can the numerical results presented in figures 1 and 2 be interpreted so as to render them valid for any ρ_r? These curves were calculated at STP. Therefore, if we increase all dimensions of any neutron-gamma-electron history by $1/\rho_r$ such that equal numbers of mean free paths are traversed as in the STP case, then we can expect the same number of gammas or electrons reaching the observation point as in the STP case. The expanded radius is R/ρ_r and the area of a sphere at the observation point is R^2/ρ_r. Therefore, the electron current density, JN, at R/ρ_r is altered by ρ_r^2, reflecting the area change with the same number of electrons. The time of arrival (not the rate) of these electrons, however, is increased to t/ρ_r.

Similarly, the QN is altered except that the energy deposition is accomplished in a volume shell which is altered by V/ρ_r^3 and hence QN is altered by ρ_r^3.

The QN, JN, R, and t_R $(=t-\frac{R}{C})$ in figures 1 and 2 have been replaced by QN/ρ_r^3, JN/ρ_r^2, $R\cdot\rho_r$ and $t_R\cdot\rho_r$, so that, when $\rho_r \neq 1$, we remember to interpret the results calculated at X meters as giving us information at $R = X/\rho_r$ meters, and the values at Y μseconds as telling us what to expect at $t_R=Y/\rho_r$ μseconds. Similarly, we are reminded that Z ion-pairs/meter³-second become $QN = Z\cdot\rho_r^3$ and W amps per meter² become $JN = W\cdot\rho_r^2$ amps/meter².

The curve fits of section E are given here with the correct ρ_r dependence.

(20) $$QN(R,t_R \leq R/V_N-R/C-1.4\ \mu sec) = 3.74(10^6)\ \frac{e^{-\frac{R\cdot\rho_r}{400}}}{R^2}\cdot e^{-Kt_R}\cdot\rho_r^2$$

$$\text{where } K = \left(\frac{R\cdot\rho_r}{400} - \frac{R\cdot\rho_r}{202} + 1.334\right)\Big/\left(R/V_N-R/C-1.4\ \mu sec\right)$$

(21) $QN(R, R/V_N-R/C-1.4\ \mu sec \leq t_R \leq R/V_N-R/C)=$

$$4.73(10^6)\rho_r^2\frac{e^{-R/\lambda_R}}{R^2}\quad \exp\left\{-1.477(10^{36})\left[t_R-(R/V_N-R/C-1.4(10^{-6}))\right]^6\right\}$$

(22) $$QN(R,t_R \geq R/V_N-R/C) = \rho_r^2\left[\left(1.42(10^7)\frac{e^{-\frac{R\cdot\rho_r}{202}}}{R^2} - QN^*\right) e^{-5.12(10^3)(t_R-(R/V_N-R/C))^{1/2}} + QN^*\right]$$

$$\text{where } QN^* = \frac{3.74(10^6)e^{-4.9(10^7)t_R\cdot\rho_r/134.2}}{[4.97(10^7)t_R]^2}$$

and

(23) $$JN(R,t_R<R/V_N-R/C-1(10^{-6})) = 4.52(10^{-17})\ \frac{e^{-\frac{R\cdot\rho_r}{400}}}{R^2}\ e^{-\alpha t_R}\cdot\rho_r$$

where $\alpha = \left(\frac{R \cdot \rho_r}{202} + .195 - \frac{R \cdot \rho_r}{400} \right) \Big/ (R/V_N - R/C - 1(10^{-8}))$

(24) $J_N(R, (R/V_N - R/C - 1(10^{-6}) \leq t_R \leq (R/V_N - R/C - 1(10^{-8})))$

$$= \text{the expression (23)} + 1(10^{-19})\ \rho_r \cdot \frac{\exp(-R \cdot \rho_r/202)}{R^2} \left[\frac{e^{6(10^7)t^*}}{1 + e^{10^8(t^* - t_p)}} \right]$$

where $t^* = t_R - (R/V_N - R/C - 1\ \mu sec)$

$t_p = 1\ \mu sec$

and finally

(25) $JN(R, t_R \geq R/V_N - R/C - 1(10^{-8})) = 3$ times the second term of (24).

G. A Nanosecond Width Source

For many applications, the curves shown in figures 1 and 2 adequately represent the ionization rate and charge current from a nuclear fusion reaction. In other cases it may be more accurate to convolute a delta function response with a real source, instead of a square wave. In this event one can often, quite accurately, substitute a nanosecond square wave response for a delta function response. The nanosecond square wave responses are practically identical to those in figures 1 and 2. The only significant difference being that the neutron arrival peaks are increased by a factor of approximately 1.3.

H. Extension to Other Energy Groups

The methods used in determining the foregoing ionization rate and charge current densities can be easily extended to neutron energies other than 14 MeV. The neutron to gamma cross sections (including both inelastic and capture collisions), σ_g, and neutron velocity, V_N, and representative gamma energy must be chosen.[8] Once these are determined, the curve fits (20)-(25) can be easily adjusted to any neutron energy group.

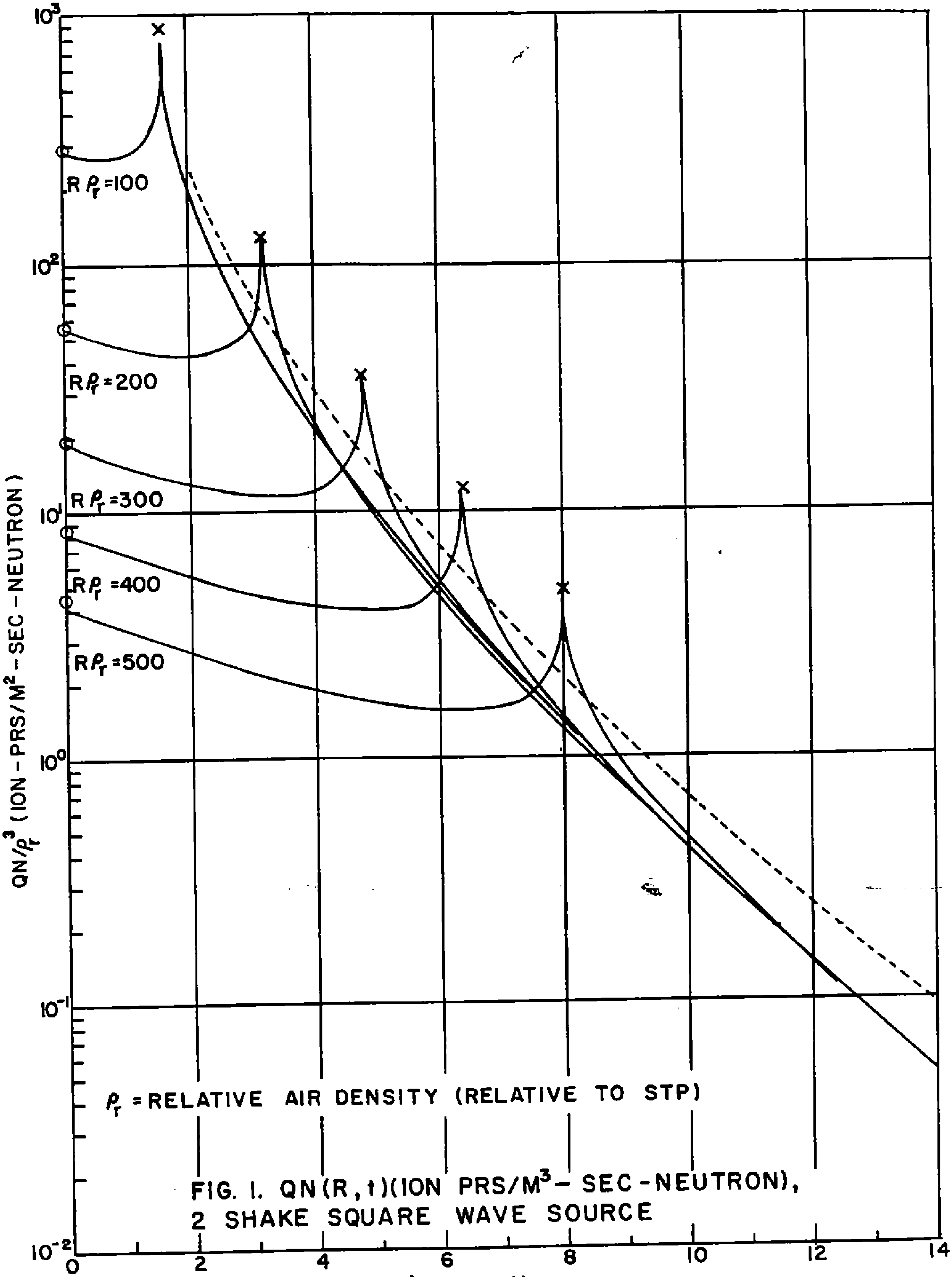


FIG. 1. QN(R, t)(ION PRS/M^3 – SEC - NEUTRON), 2 SHAKE SQUARE WAVE SOURCE

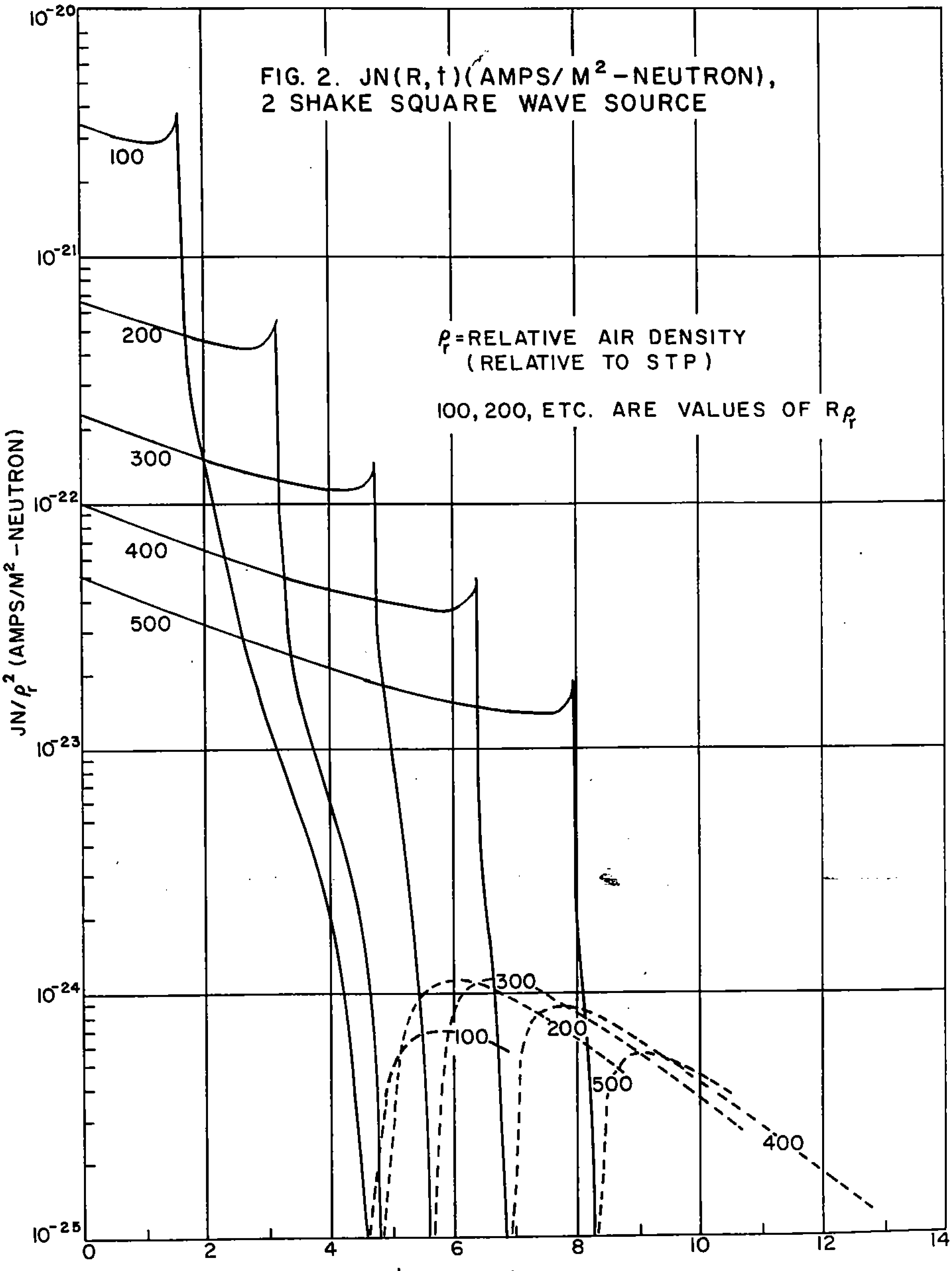


FIG. 2. JN(R, t)(AMPS/ M² -NEUTRON),
2 SHAKE SQUARE WAVE SOURCE
ρr = RELATIVE AIR DENSITY
(RELATIVE TO STP)
100, 200, ETC. ARE VALUES OF Rρr
JN/ρr² (AMPS/M² -NEUTRON)
tR ρr (μSEC)
10⁻²⁰
10⁻²¹
10⁻²²
10⁻²³
10⁻²⁴
10⁻²⁵
0
2
4
6
8
10
12
14
100
200
300
400
500

APPENDIX:

The Neutron-Air Interaction Model as presented in EMP Theoretical Note XI.

For convenience, the neutron and resultant gamma collision model presented in EMP Theoretical Note XI, is reproduced here.

The Air-Inelastic Neutron Collision Model

Fast Neutrons (those with energy above a few MeV) are produced during both fission and fusion reactions. However, since the fusion reactions produce many more fast neutrons per kiloton of energy, this analysis will be limited to fusion neutrons. These neutrons are further assumed to be mono-energetic at 14.1 MeVs.

Consider an isotropic source of neutrons represented by $N_o(t)$ neutrons/steradian-second. These neutrons spend their energy in capture, elastic, and wide-angle inelastic collisions and are lost from the "fast" category. The "mean free path", λ_R, for removal from the fast category is about 202 meters at STP. Thus, at any distance, R meters, from the origin, the number of fast neutrons is

$$N(R,t) = N_o[t-R/V_N]e^{-R/\lambda_R} \frac{\text{neutrons}}{\text{steradian-second}}$$

where V_N is the neutron velocity ($\approx 5.2(10^7)$ meters/sec)

Inelastic collisions of these fast neutrons produce gamma rays in several energy ranges. To approximate the total effect, however, a single gamma energy, E_g, with a single mean free path, λ_g, is assumed. A total cross section for gamma ray production, σ_g, is also used. The values to be used in subsequent calculations are as follows:

E_g = 7 MeV

λ_g = 400 meters/REL

σ_g = 500 (10^{-31}) meter^2/target nuclei (500 millibarns)

Dt = 5.39 10^{25} REL nuclei/meter^3

= the target nuclei density

REL = the relative air density

The number of gamma rays produced per steradian-meter-second, NG, is

(2) $$NG(R,t) = \sigma_g \cdot Dt \cdot N(R,t) = \sigma_g \cdot Dt \cdot N_o(t-R/V_N)e^{-R/\lambda_R}$$

These gamma rays are assumed to be scattered isotropically from their point of creation[1].

The problem remaining consists of calculating the amount of current and ion-pair production due to the distributed source of isotropically scattered gammas expressed in equation (2).

A. Ion-Pair[2] Production from 14 MeV Neutrons.

A volume element at point R, at time t (t = o at detonation), receives ion-pairs from all elemental volumes, dV, in the source region (for which $t > Z/V_n + R'/C$). Let QN be the number of ion-pairs produced per unit volume per second due to the prompt neutron induced gamma rays.

$$(3)\quad QN(R,t) = \int_{V\ source} \frac{NG[Z, t-R'/C]}{Z^2} \cdot \frac{QG(R')}{4\pi R'^2}\, dV \quad \frac{\text{ion-pairs}}{m^3\text{-sec}}$$

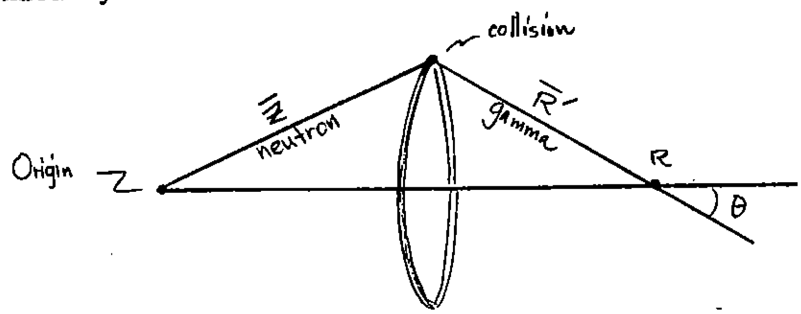
where QG(R') is the ion-pair production per meter for a gamma at R' meters from its origin

Z is the point of creation of the 7-MeV gamma

$\overline{R}' = \overline{R} - \overline{Z}$

V source is the volume occupied by neutrons. It is defined by $|Z| \leq V_N \cdot t$

Graphically the situation is as shown:



The assumptions for the one gamma collision model will be that the average electron energy is deposited in each first collision and that the scattered photon

[1]This is not a necessary condition for the calculation; however, it is convenient.

[2]It is here assumed that an ion-pair, consists of one electron and one positive N_2^+ or O_2^+ ion, for each 34 e.v. deposited.

is ignored. The volume element, dV, can be represented by $(2\pi R'\sin\theta)(R'd\theta)\cdot dR'$, NG as given in equation (2) can also be substituted into equation (3) to yield:

$$(4) \quad QN(R,t) = \int_{o}^{R+V_N t} \int_{o}^{\pi} \sigma_g \cdot DtN_o(t-Z/V_N-R'/C)\frac{e^{-Z/\lambda_R}}{Z^2} \cdot \frac{QG(R')}{4\pi R'^2} \cdot 2\pi R'^2 \sin\theta d\theta dR'$$

θ, Z, R, and R' are not independent. They are related as in the following equation:

$$Z^2 = (R' \sin\theta)^2 + (R-R' \cos \theta)^2$$

To change the theta variable of integration in (4) from θ to Z, $d\theta$ must be expressed as a function of dZ.

$$Z = \sqrt{R'^2 \ (\sin^2\theta + \cos^2 \theta) + R^2-2RR' \cos \theta}$$

$$Z = \sqrt{R'^2 + R^2} = 2RR' \cos \theta$$

$$dZ = \frac{1}{Z} (- RR' \ (-\sin\theta d\theta))$$

$$\frac{ZdZ}{RR'} = \sin \theta d\theta$$

Then new limits of integration are

when $\theta = 0$, $Z = R-R'$

when $\theta = \pi$, $Z = R+R'$

Upon substituting the new limits and changing variable of integration, equation (4) becomes

$$QN(R,t) = \int_{o}^{R+V_N t} \int_{R-R'}^{R+R'} \sigma_g DtN_o(t-Z/V_N-R'/C) \frac{e^{-Z/\lambda_R}}{Z^2} \frac{QG(R')}{4\pi R'^2} 2\pi R'^2 \left(\frac{ZdZ}{RR'}\right) dR'$$

which, upon cancellation of like terms and removal of constants from the integrand, becomes

$$QN(R,t) = \frac{\sigma_g \cdot Dt}{2R} \int_{o}^{R+V_N t} \left[\int_{|R-R'|}^{R+R'} N_o(t-|Z|/V_N-R'/C) \frac{e^{-Z/\lambda_R}}{Z} dZ \right] \frac{QG(R')}{R'} dR'$$

The absolute value of R-R' must be used if the lower limit is to hold for R' > R. It is also necessary for the exponential decay and time retardation QG(R'), which is the ion-pair production per meter-steradian at R' meters from a 7 MeV gamma source, is:

$$QG(R') = \frac{4.55\ (10^6)}{34 \cdot \lambda_g} e^{-R'/\lambda_g} \quad \frac{\text{ion-pairs}}{\text{gamma-meter}}$$

where it has been assumed that:

1. 34 e.v. are absorbed per ion-pair

2. 4.55 MeV are lost to the recoil electron

B. Compton Current Production from 14 MeV Neutron

The distributed source of gamma rays, NG(R,t), as given in equation (2), produces a "lingering" Compton electron current in addition to the ionization mentioned above. This lingering current, together with currents due to neutron scattering in the ground, represents the dominant driving term for EMP generation at later times.

The fast neutron induced current, JN(R,t), can be expressed in the form of equation (6) with JG(R') substituted for QG(R') and a cos(θ) factor inserted in the integrand to account for the non-radial direction of the gamma rays (see picture after equation (3)). JG(R') is defined as the charge displacement (per steradian) at R' meters from a 7 MeV gamma source.

$$JG(R') = \frac{1.6(10^{-19})\lambda_e}{\lambda_g} \; e^{-R'/\lambda_g} \; \frac{\text{coulombs}}{\text{gamma-steradian}}$$

where λ_e is the electron range of a 4.55 MeV electron

$\lambda_e \cong 10.15$

The cos (θ) factor can be expressed in terms of the R, R', and Z variables

$$\cos(\theta) = \frac{R'^2 + R^2 - Z^2}{2RR'}$$

The neutron induced current JN(R,t), is

$$JN(R,t) = \frac{\sigma_g \cdot Dt}{2R} \int_o^{R+V_N t} \left[\int_{|R-R'|}^{R+R'} N_o(t - |Z|/V_N - R'/C) \, \frac{e^{-Z/\lambda_R}}{Z} \left(\frac{R'^2 + R^2 - Z^2}{2RR'} \right) dZ \right] \frac{JG(R')}{R'} \, dR'$$