EMP Theoretical Notes

Note X
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Prompt Gamma Effects in the Vicinity of A Ground-Air Interface

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Abstract

The accurate calculation of the nuclear electromagnetic pulse (EMP) requires a corresponding accuracy in specifying the Compton current and ionization which produce and shape the EMP. Until recently the EMP inputs have ignored the effects of the ground half-space. Hence, expressions for the Compton current have yielded only a radial component. Moreover, because of symmetry, both the Compton current and ionization expressions have been independent of the polar angle theta above the ground. This paper presents results of Monte-Carlo calculations of the gamma ray effects in the presence of a ground-air interface. These calculations indicate an appreciable theta component of Compton current and significant variation of ionization and both current components with respect to polar angle (theta).

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I. Introduction

Prompt gamma rays, emitted during nuclear reactions, contribute a major share of the current and ionization which generate and shape the electromagnetic pulse associated with nuclear detonations. Compton collisions of these gamma rays yield high energy recoil electrons which can form an appreciable electron current, often called the Compton current. These recoil electrons lose their energy by ionizing some of the atoms in their paths.

Maxwells equations, shown here, relate electromagnetic fields to the Compton current.

$$\nabla \times \vec{B} = \mu \left(\vec{J_c} + \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right) \tag{1-a}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1-b}$$

where Jc can be thought of as the Compton current.

The ionization by the recoil electrons enhances the local conductivity and, hence, contributes to the conductivity, σ.

Descriptions of the Compton current and ionization rates for isotropic sources of gamma rays in a homogenous scattering medium are available*.

In this case, the Compton current is purely radial and exhibits only radial position dependence due to symmetry. In other words, $\vec{J_c} = J\hat{r}\,(R,t)$ where R is the distance from the source. Likewise, the ionization rate, Q ions/meter^3-second, exhibits only radial dependence $Q = Q(R t)$.

In most cases of interest, however, the prompt gamma source is in a region of varying electron density, e.g., near a ground-air interface.

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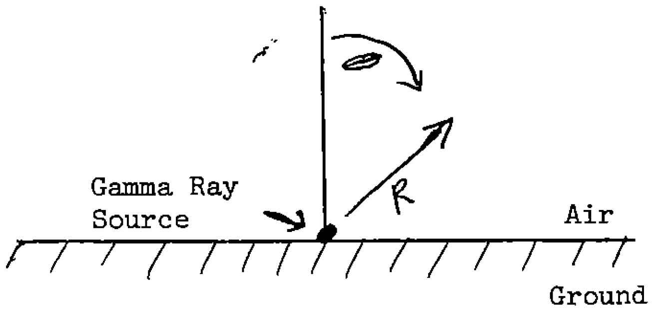
*RM-4151 - PR, June 1964, by R. Lelevier



This inhomogeneity in scattering media results in several complexities. First, all effects are functions of polar angle, θ, as well as radial distance, R. Secondly, a theta component of Compton current, $J_{\hat{\theta}}$ is created. These complexities arise because of the different gamma ray mean free paths in the ground and in the air. At some radius near the ground, then, one might expect more gamma rays to come down from the air than up from the ground. This lack of symmetry results in a theta component of electron current directed toward the ground. The influence of this asymmetry will vary with proximity to the ground-air interface. Consequently,

$$\vec{J}_c = J_{\hat{r}}(R,\theta,t) + J_{\hat{\theta}}(R,\theta,t)$$

$$Q = Q(R,\theta,t)$$

The time dependence of these quantities should be affected by the inhomogeneity, also, because the relative contributions from multiple scattering at any R is a function of theta.

This report presents results of a Monte Carlo calculation of the $J_{\hat{r}}$, $J_{\hat{\theta}}$, and Q, due to a delta function (in time) source of monoenergetic gamma rays. These delta function responses can then be convoluted with any source function to yield the actual Compton currents and ionization. For example,

$$J_{\hat{r}}(R,\theta,t) = \int_0^t S(t-\tau)\left[J^I_{\hat{r}}(R,\theta,\tau) + J^D_{\hat{r}}(R,\theta)\,\delta(t)\right] d\tau \qquad (2)$$

where $J_{\hat{r}}(R,\Theta,t)$ is the Compton current

$S(t-\tau)$ is the source of gamma rays per second from the origin $(S(x)=0$ for $x<0)$

$J^{I}_{\hat{r}}(R,\Theta,t)$ is the indirect delta function radial current response (amps/m^2-gamma)

$J^{D}_{\hat{r}}(R,\Theta,t)$ is the direct delta function radial charge transport response (charge/m^2- gamma)

τ is a dummy time variable. The adjective "direct" refers to a response due to collisions of the direct beam, while "indirect" refers to the response due to all subsequent collisions.

II. Analysis of the Problem

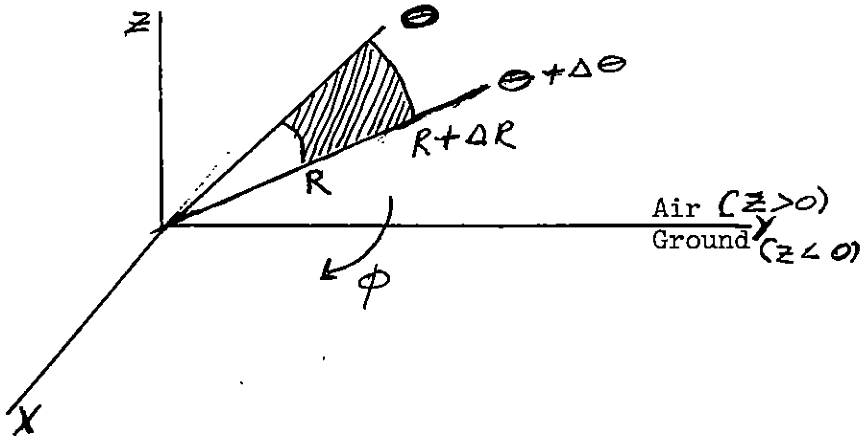
A Monte-Carlo calculation of the time and space-resolved Compton currents and energy deposition rates requires more computer running time than is practical on presently available computers. By reducing the Monte-Carlo calculator to separate time-resolution and space-resolution studies, however, all of the required information can be obtained. Therefore, two distinct codes have been created. The first, the TIGRE code, calculates the time-integrated gamma ray effects in spatial bins. This code gives such information as the relative amount of theta current compared to radial current, the extent of the region where theta dependence is manifest, and the spatial smoothness of the current and ionization functions.

The second code, the PROJ code, is more important in relation to electromagnetic field generation. It is designed to yield the time resolved response to a delta function (in time), point source of monoenergetic gamma rays. This is done by accumulating in time bins (of 10^{-8} seconds) the projected gamma ray effects at several (usually nine) "target" points. These target points are arranged at various angles at each of several radii. It is expected that, on the basis of results from the TIGRE code, the delta function response at several judiciously selected target points will be sufficient basis for a curve fit to the delta function response in all of space. In any event, if doubt arises it can be allayed by rerunning PROJ with targets placed at the suspected points. The following two sections contain descriptions of features of TIGRE and PROJ respectively.

III TIGRE Code

A. Discussion

The TIGRE code yields time integrated gamma ray effects in spatial bins around a monoenergetic, point source of gamma rays. For ease of inserting these results into existing computer codes, which solve Maxwell's equations[1], the spatial bins have been selected in polar coordinates. Since azimuthal, ϕ, symmetry exists bins are independent of ϕ. The following diagram shows the cross section of a representative bin in the y-z plane. The entire bin is the volume defined by the revolution of this cross section about the z axis



The bin increments used are $\Delta R = 25$ meters, $\Delta \cos\theta = .1$. Note that the $\Delta\theta$ is function of θ. The use of a variable theta bin size allows a finer bin size and hence greater θ resolution near the ground ($\theta = 90^\circ$). This is advantageous because the ground-air interface effects should experience their greatest variations near the ground. The origin of the polar coordinates is located at the projection in the x-y plane of the gamma ray source point.

The TIGRE code emits gamma rays from the source isotropically and allows them to pass from one medium to another as the random numbers and probability distributions determine. Upon each collision, the average electron to be expected from the incident gamma ray is used to determine the current contribution. The energy deposition, Q, is determined by the actual collision event. Contributions to both current and energy deposition (ionization) are accumulated in the bin corresponding to the point of collision. No allowance is made for transition of the recoil electron out of their "mother" bin. This procedure may draw criticism because it is not obvious that the effect of the recoil electrons in the ground entering the adjacent air bins is negligible. Identical calculations which include this effect, however, have confirmed its negligibility.

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[1] For example, see AFWL TR 64-153, by W. R. Graham

The TIGRE code is applicable for initial gamma ray energies up to 5 Mev. The cutoff energy is easily altered. For a 2 Mev initial gamma energy decaying to .1 Mev the average number of collisions per history is about 8.

B. Results

The TIGRE code was especially designed to compute gamma ray effects in the vicinity of a ground-air interface. By changing the ratio of ground density to air density, however, an air-air interface or homogeneous scattering medium can be simulated.

Results of this air-air problem agree with results from a simpler Monte-Carlo program[2]. Figure 1 shows the energy deposition build up factors versus radial distances from the source for TIGRE and R. Lelevier's code for a point source of 2 Mev gamma rays at an origin in an air-air interface. Figure 2 shows the corresponding radial current build up factor.[3] Of course these build up factors are not functions of polar angle, theta, for the homogeneous air-air interface situation.

When earth[4] occupies the lower half space, we expect theta variation of variables and a theta components of current. Figure 3 contains a polar plot of the indirect energy deposition versus the polar angle, θ, from the z axis (at 312.5 meters). Also contained in this plot is a polar plot of the indirect energy deposition to be expected from the air-air case. Figure 4 contains a similar treatment of the indirect radial current. Both of these quantities show a decrease near the ground ($\theta = 90^\circ$). This is because there are few contributions from the ground. The next plot, Figure 5, shows the theta component of current (which can only come from indirect collisions). The magnitude of this quantity should approach zero as $\theta \to 0$ because symmetry with respect to theta exists at $\theta = 0$ and no theta direction is preferred. Near the ground ($\theta \to 90^\circ$), however, a maximum asymmetry exists. More gammas will be

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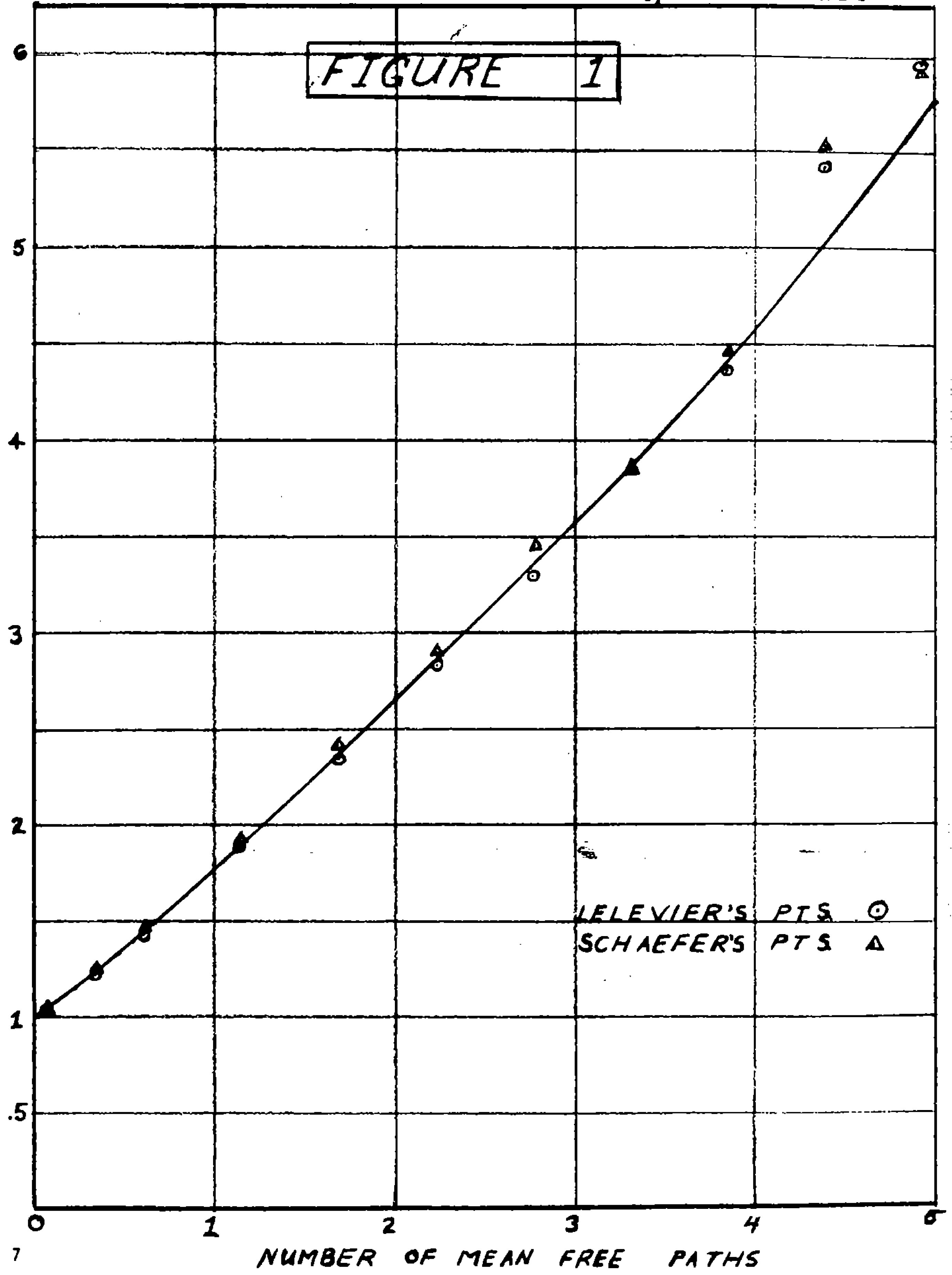
[2] Memorandum RM-4151-PR, "The Compton Current and the Energy Deposition Rate from Gamma-Quanta a Monte-Carlo Calculation", by R. Lelevier

[3] The build up factor = $\frac{X \text{ direct} + X \text{ indirect}}{X \text{ direct}}$, where X is the quantity of interest.

[4] earth (or the ground) is represented by a medium 2000 times as dense (with electrons) as standard air.



ENERGY BUILD UP FACTOR Vs MEAN FREE PATH
2 Mev AIR-AIR INTERFACE 400,000 HISTORIES

FIGURE 1

RADIAL CHARGE TRANSPORT BUILD UP FACTOR Vs. M.F.P.
2 Mev AIR-AIR INTERFACE 400,000 HISTORIES

FIGURE 2

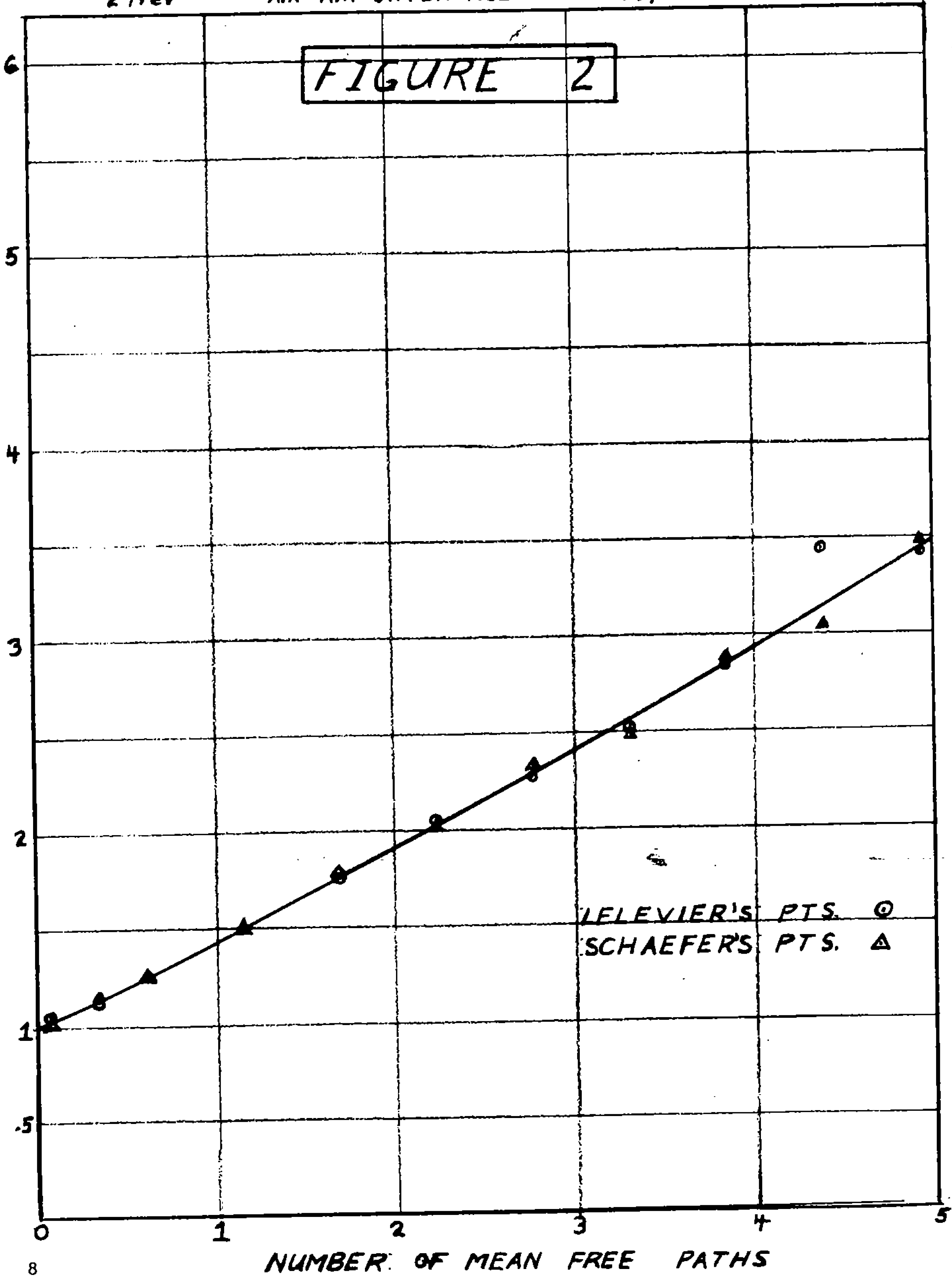


FIGURE 3

$\int Q_{indirect} \cdot dt$ Vs THETA AT 312.5 METERS
FOR AN INITIAL GAMMA RAY AT 2 Mevs
(400,000 HISTORIES)

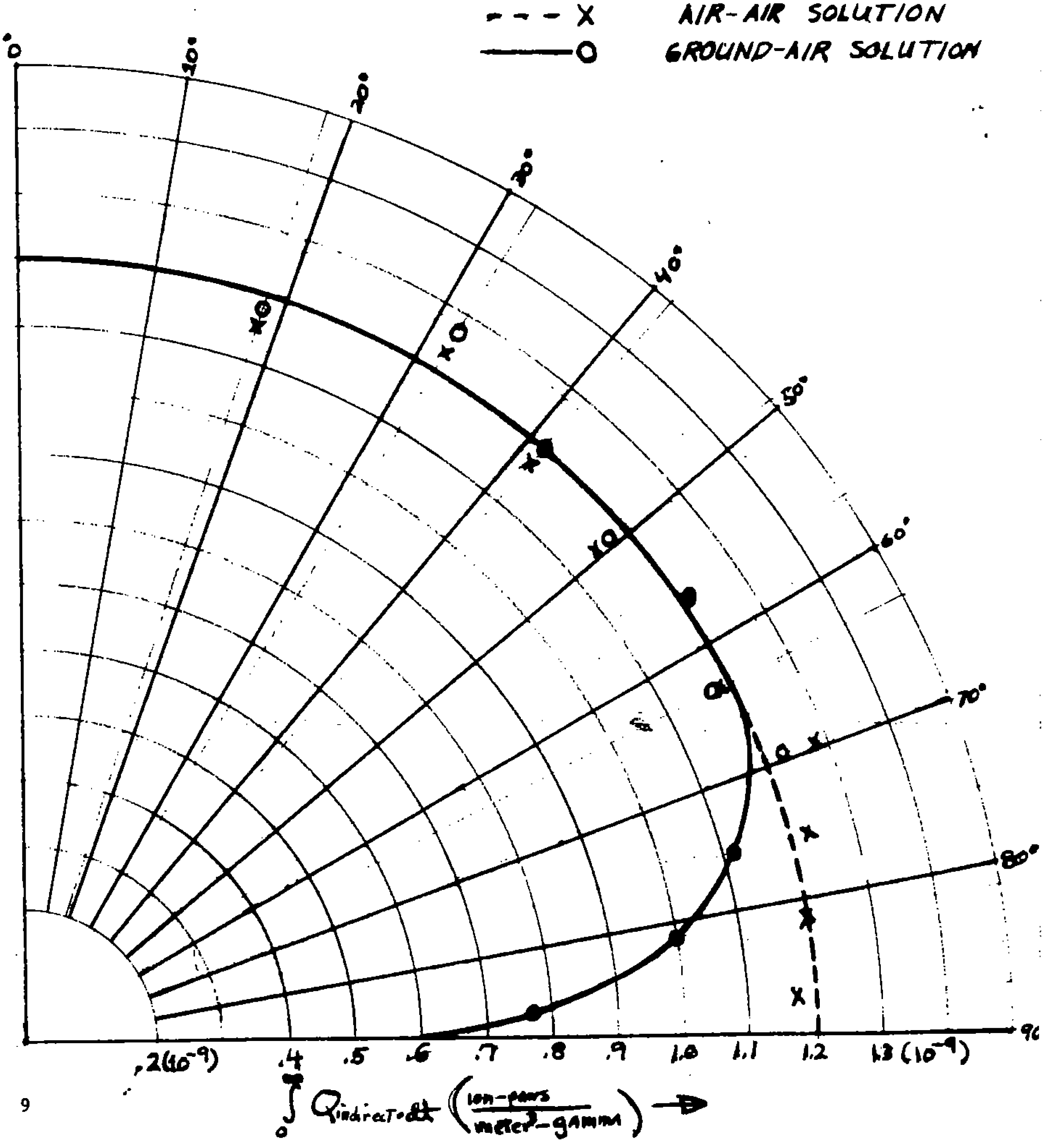


FIGURE 4

$\int J_{\phi}^{\text{indirect}} \cdot dt$ Vs. THETA AT 312.5 METERS
FOR AN INITIAL GAMMA RAY AT 2 Mevs
(400,000 HISTORIES)

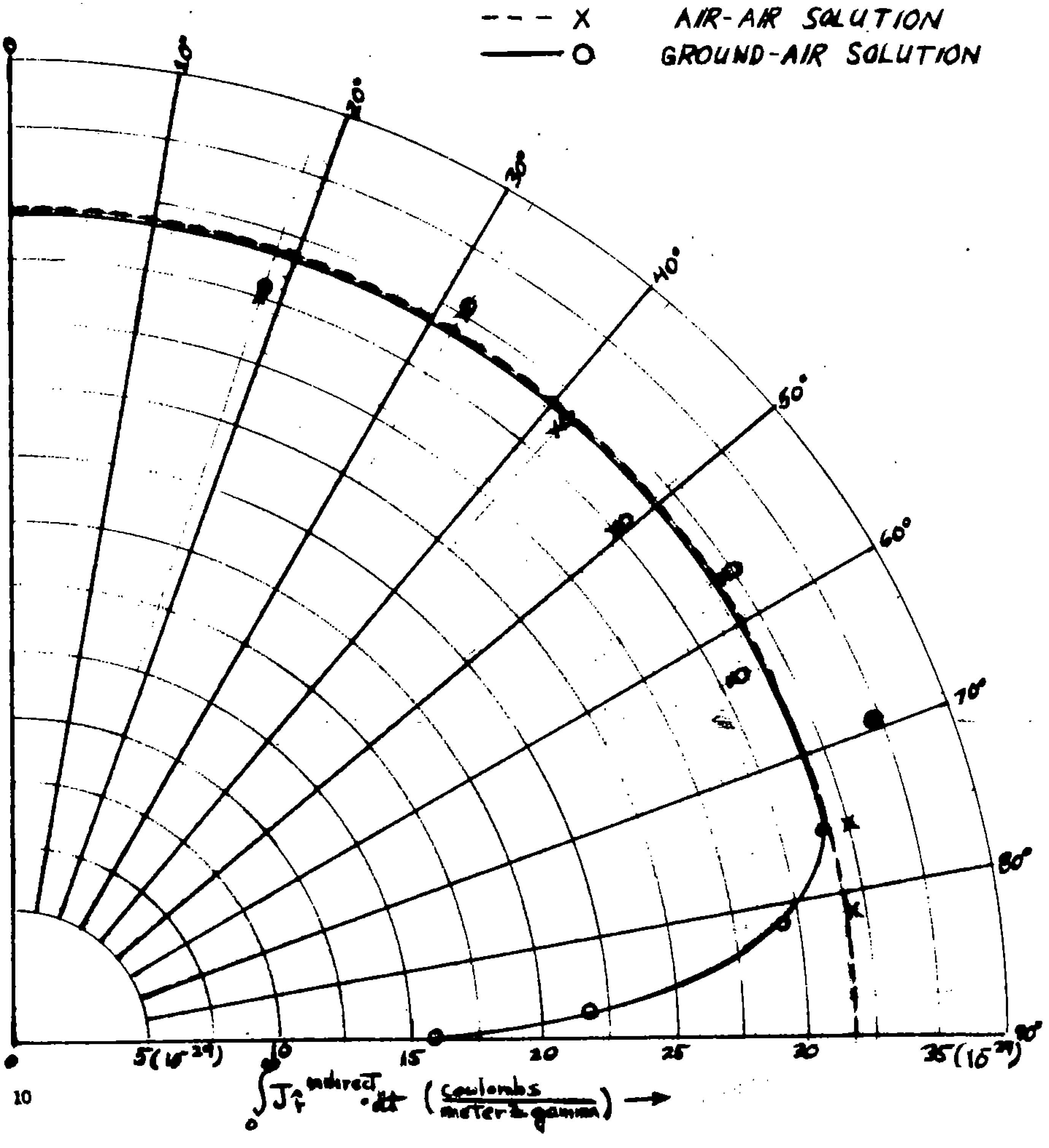
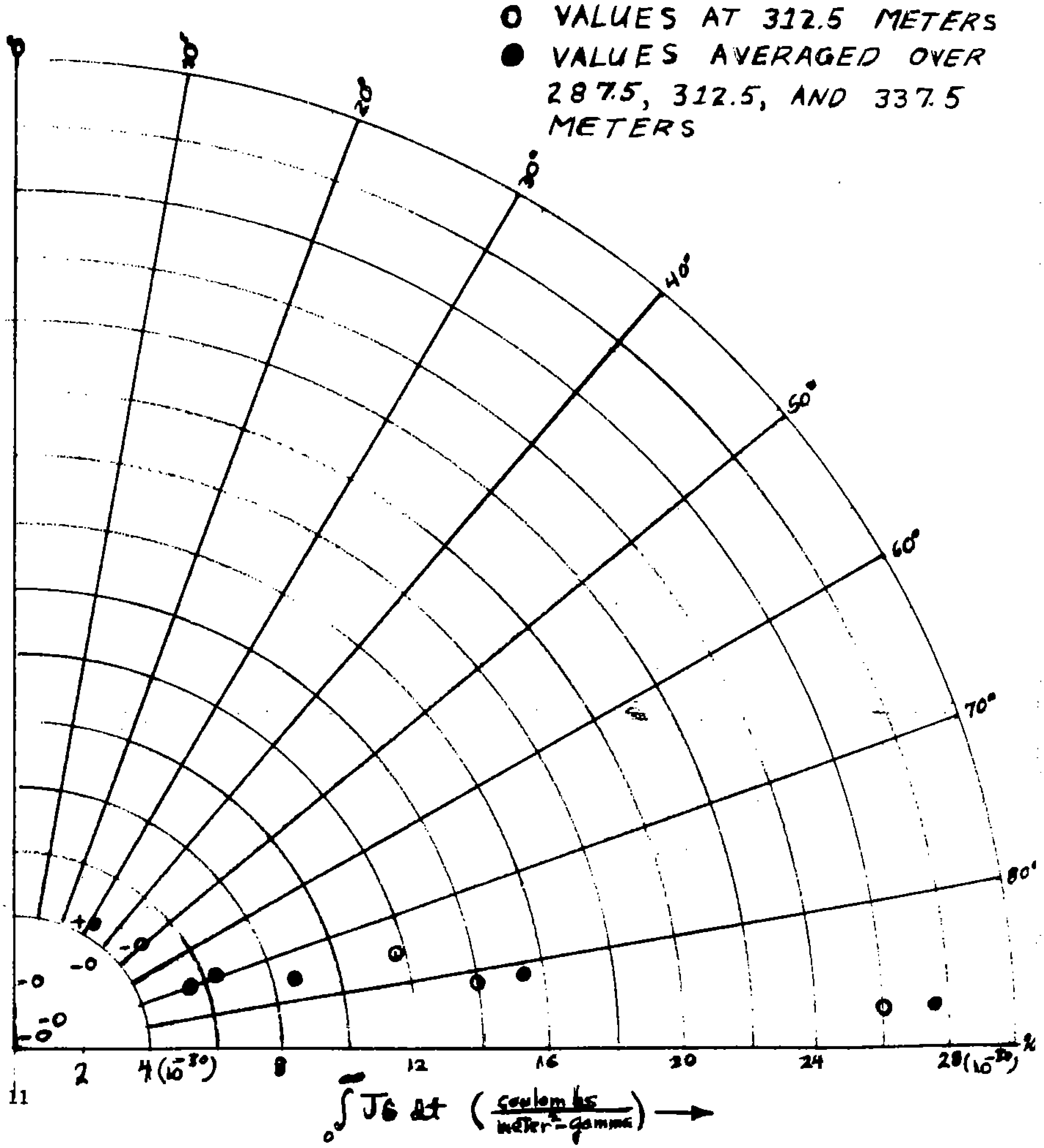


FIGURE 5

$\int_0^\infty J_\theta \cdot dt$ Vs. THETA AT 312.5 METERS
FOR AN INITIAL GAMMA RAY AT 2 Mevs
(400,000 HISTORIES)

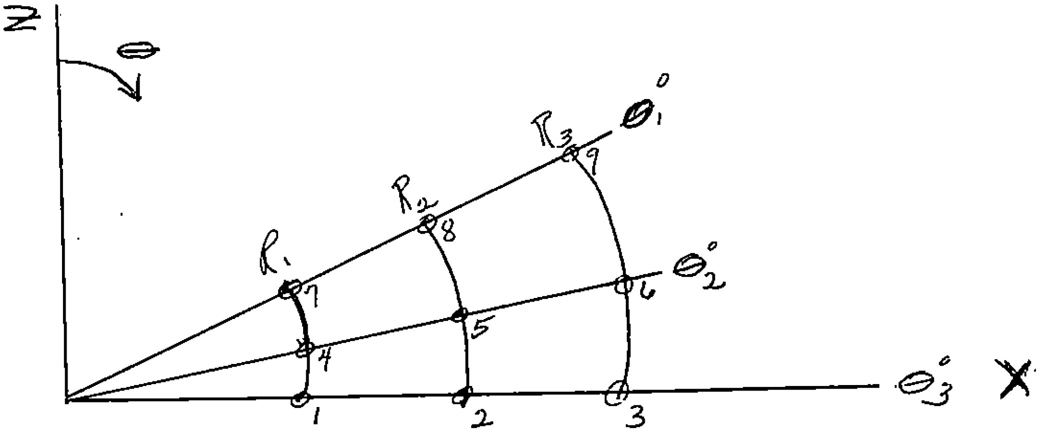


arriving at the observation point from above than from below. Thus a theta current of electron will be created with a maximum value at $\theta = 90^\circ$. The curves presented here represent results at 312.5 meters, however, the results do not seem to be strong functions of radius.

IV The PROJ Code

A Discussion

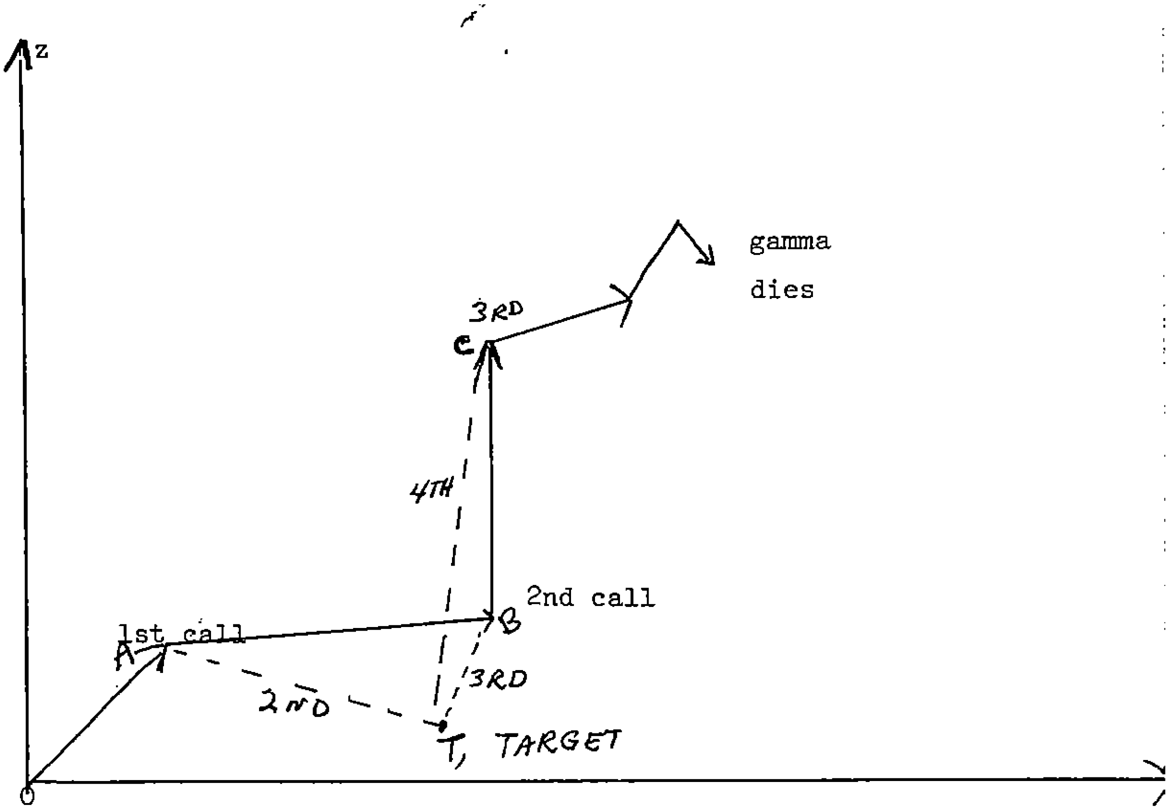
This code has been designed to yield time-resolved responses, at several target points, due to a delta function (in time) point source of monoenergetic gamma rays. The target points are located, for convenience, in the x-z plane as shown below



The target points, labled 1 through 9, are typically located at several radii at each angle and at several angles at each radius. These target points are arranged to yield information on angular dependence of gamma ray effects as a function of range, and time dependence as a function of angle. Each target has associated with it a set of 50 time bins. These time bins are all 10^{-8} seconds wide excepting the last which includes all time after 49 shakes (1 shake = 10^{-8} seconds).

The general procedure for accumulating effects in these bins consists of emitting gamma rays from the point source and allowing them to multiply scatter as in the TIGRE code. After each collision, however, the expected gamma effect contribution, projected to each target point, is calculated and accumulated. The "expected" projected value is weighted according to the probability that the gamma ray will scatter from the present collision toward the target point and collide there. Thus, after the nth collision, the expected contribution due to the (n-1) collisions at the target is accumulated in the appropriate time bin. By "expected contribution" is meant the average contribution that would be expected at the target if an infinite number of nth collisions (from which we are projecting) occurred.

The following picture represents the technique used.



A gamma ray leaving the origin, 0, collides at A (first collision). The projected effect from A to T simulates a second collision. Next, the gamma ray, scattering from A, experiences a second collision at B. The projected effect from B to T simulates a third collision, and so on. The effect of many such histories is to simulate all indirect contributions to the target. The direct contribution is analytically calculable.

Since the theta component of current, $J_\theta(R,\theta)$, is the difference of large contributions in the positive theta and negative theta directions, a single technique is used to encourage the statistics to settle down more rapidly. This technique[5] is to ignore any proposed contributions to the theta current if there exists an equally probable cancelling path.

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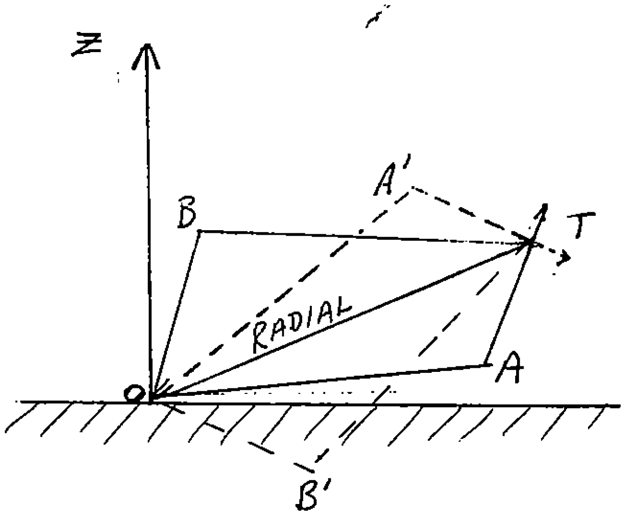
[5] This technique was suggested by W.R. Graham at 5000 feet above the rugged northern New Mexico terrain.

For example, consider the following diagram.



If a gamma travels from O to A, the projection to T would yield a theta component of current. There is, however, an equally probably path which would cancel this theta component, namely O → A' → T. A' is found by rotating A 180 degrees about the radial. For path O→ B→ T, no such equally probable path exists because B' is reached by traveling through a medium of different mean free path (the ground).

IV Results

A comparison of various responses to a delta function (in time) point source of monoenergetic gamma rays is shown in Figure 6. This graph contains the energy deposition rate response for the homogeneous (air) scattering medium and for several angles in the vicinity of an air-ground interface. All responses are 300 meters from a pt source of 2 Mev gamma ray. Figure 7 treats the radial current response similarly. Figure 8 contains a representation of the theta current response at 300 meters and 90° from the normal. Note that the theta current delta function response starts at zero and rises to maximum before decaying. This is because the early arriving, multiply-scattered gammas are traveling nearly radially and hence contribute little theta current.

Figures 9, 10, and 11 contain delta function responses at 100 meters and 90° (at the ground-air interface).

V Summary and Conclusions

From the TIGRE code the magnitudes of the time integrated ionization rate and Compton currents are known in all space. At the same time, the time resolved responses are known at certain points. Since these time responses are smoothly varying functions of distance, the time-resolved

FIGURE 6

POWER Vs TIME AT 300 METERS FOR AN INITIAL GAMMA RAY AT 2 Mevs (11,000 HISTORIES)

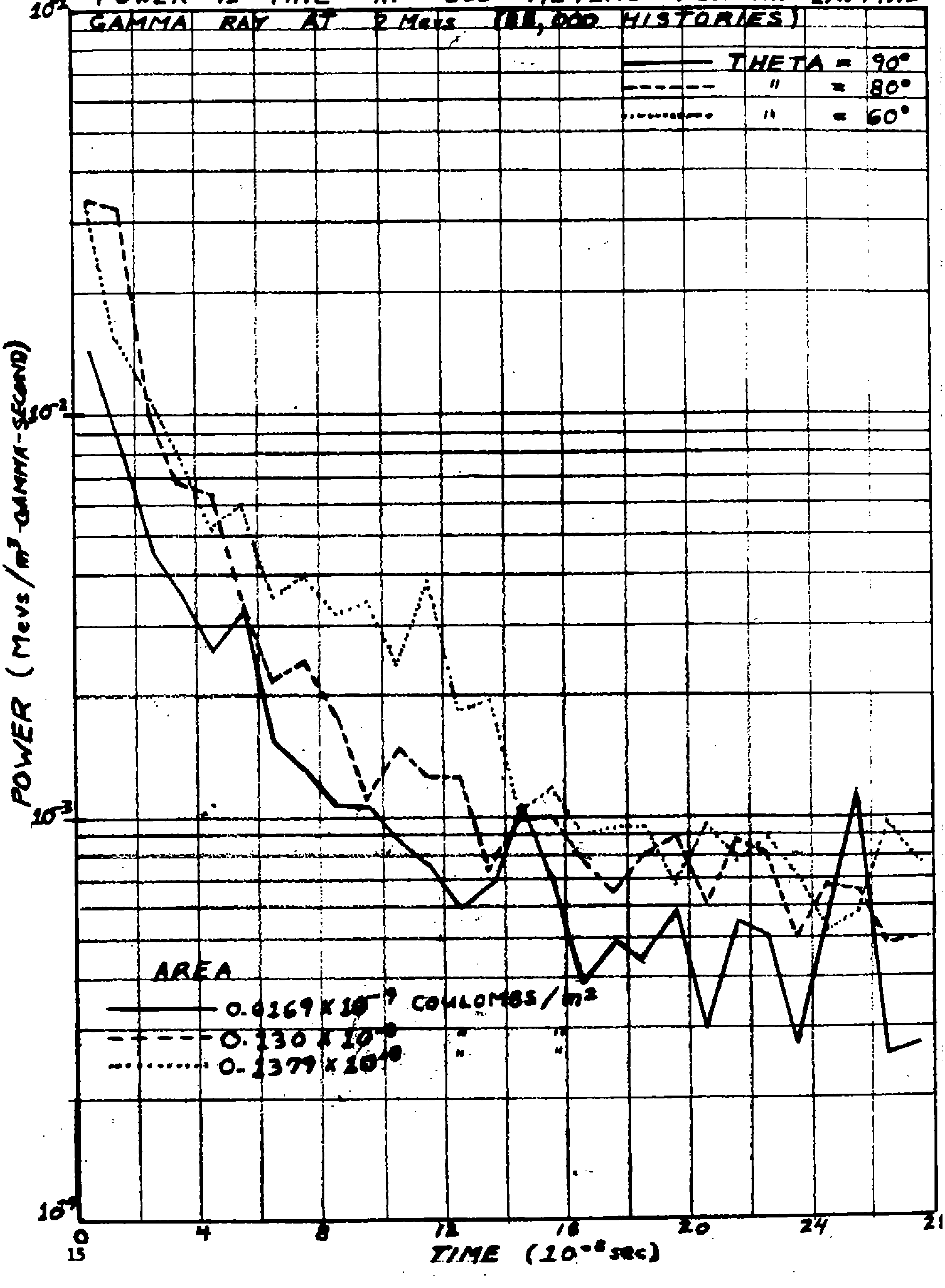


FIGURE 7

RADIAL CURRENT Vs. TIME AT 300 METERS FOR AN INITIAL GAMMA RAY AT 2 Mevs (88,000 HISTORIES)

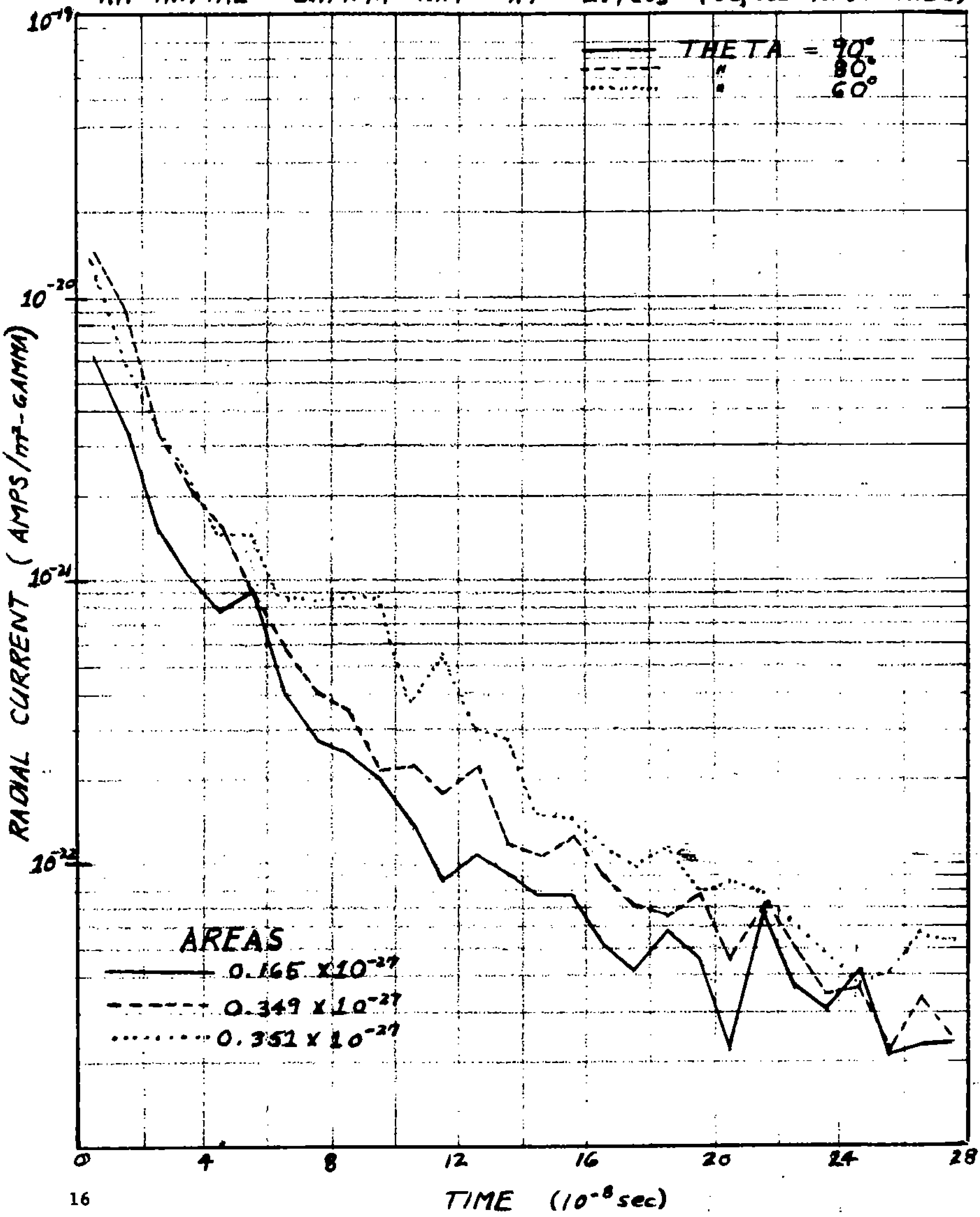


FIGURE 8

THETA CURRENT Vs TIME AT 300 METERS FOR AN INITIAL GAMMA RAY AT 2 Mevs ([illegible],000 HISTORIES)

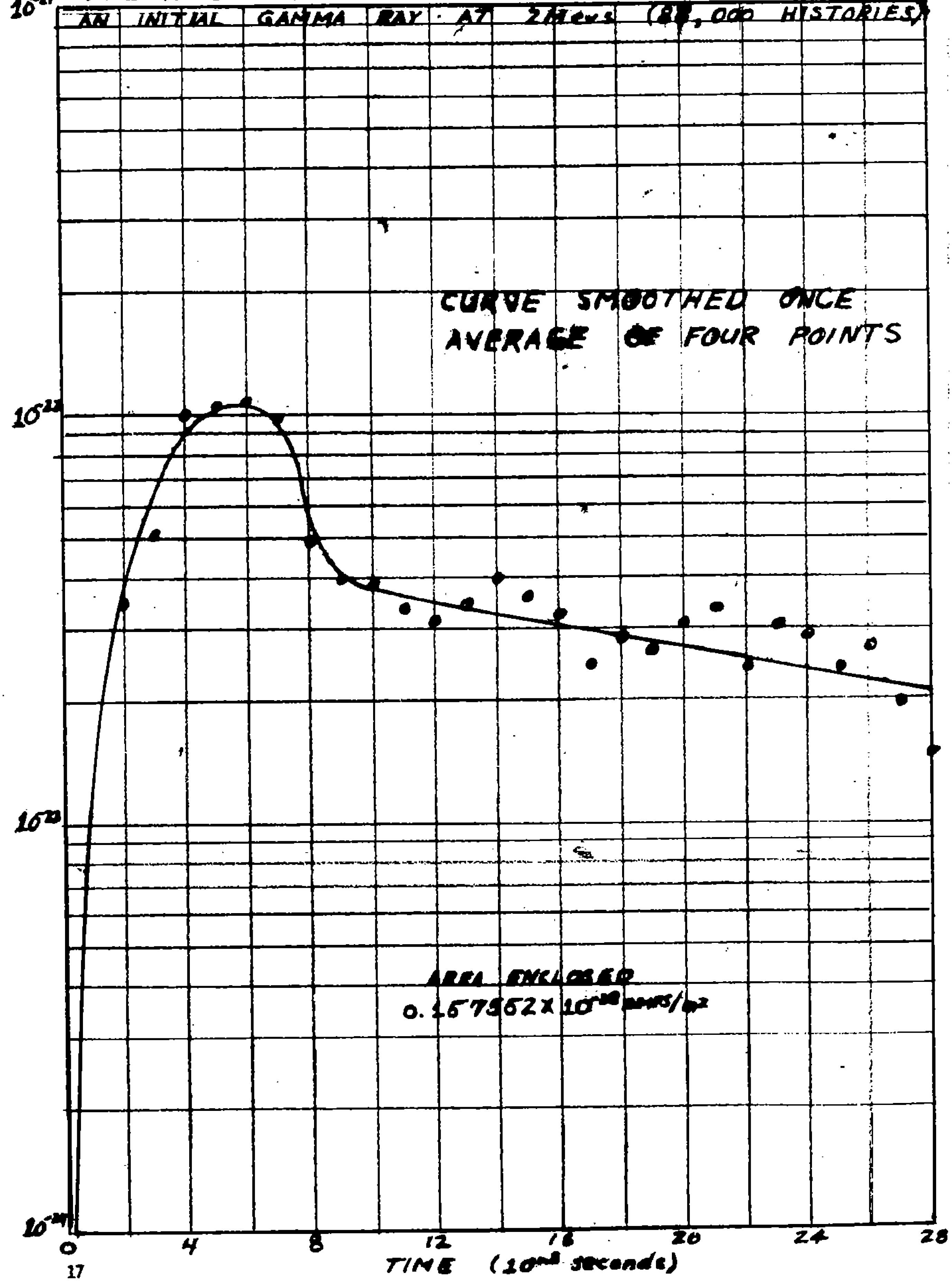


FIGURE 9

POWER Vs. TIME AT 100 METERS FOR AN INITIAL GAMMA RAY AT 2 Mev (88,000 HISTORIES)

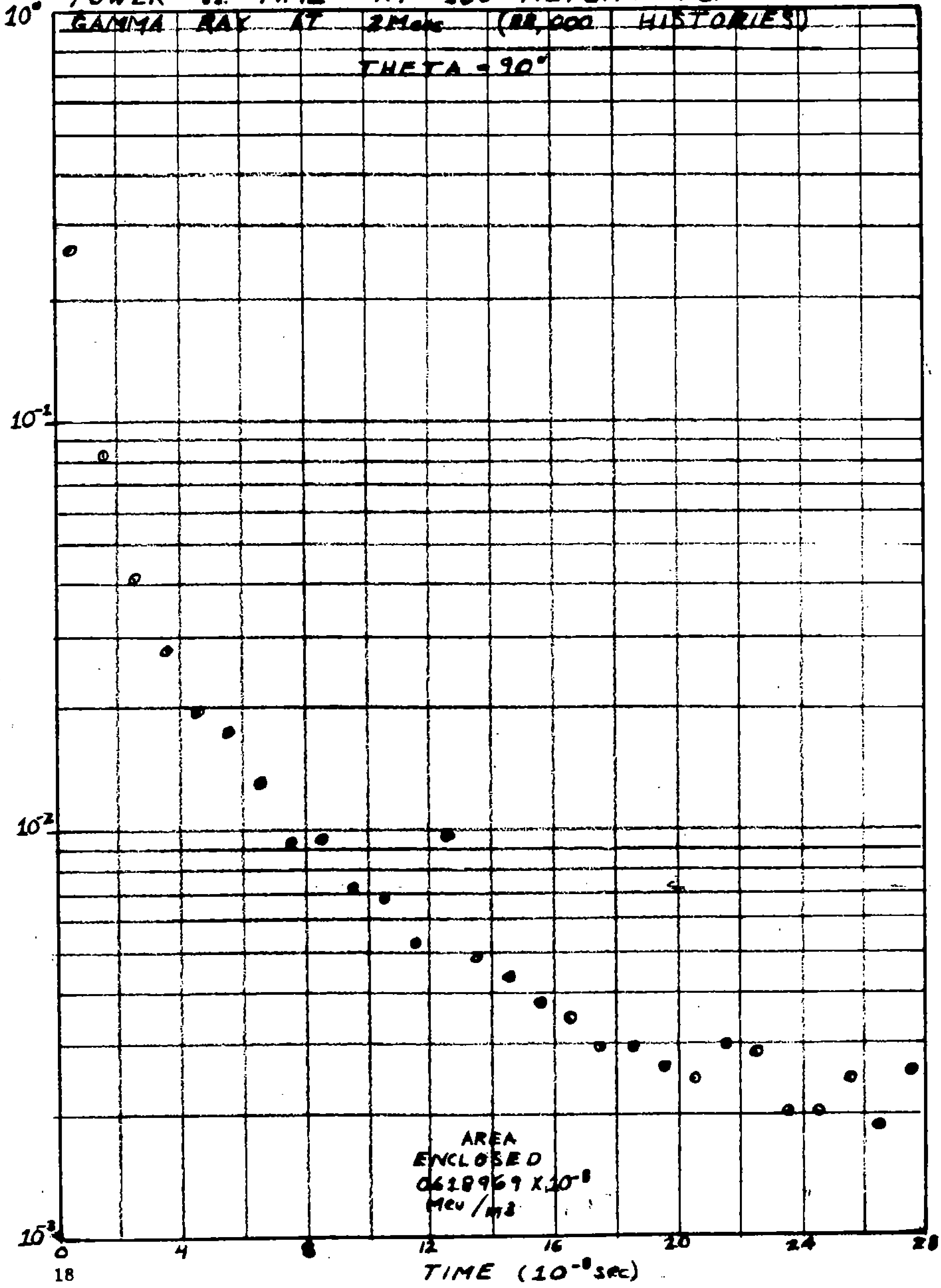


FIGURE 10

RADIAL CURRENT Vs. TIME AT 100 METERS FOR AN INITIAL GAMMA RAY AT 2 Mev

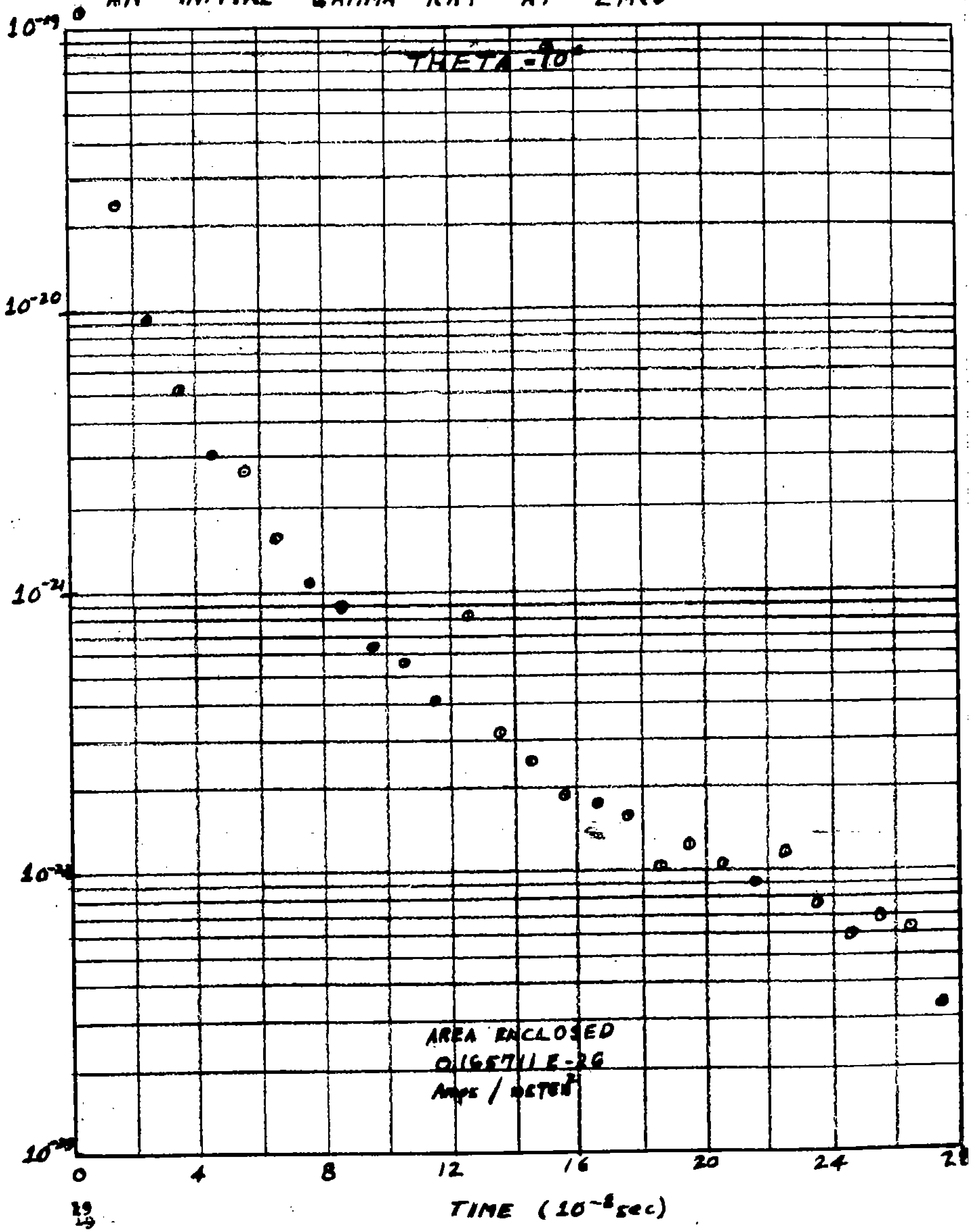
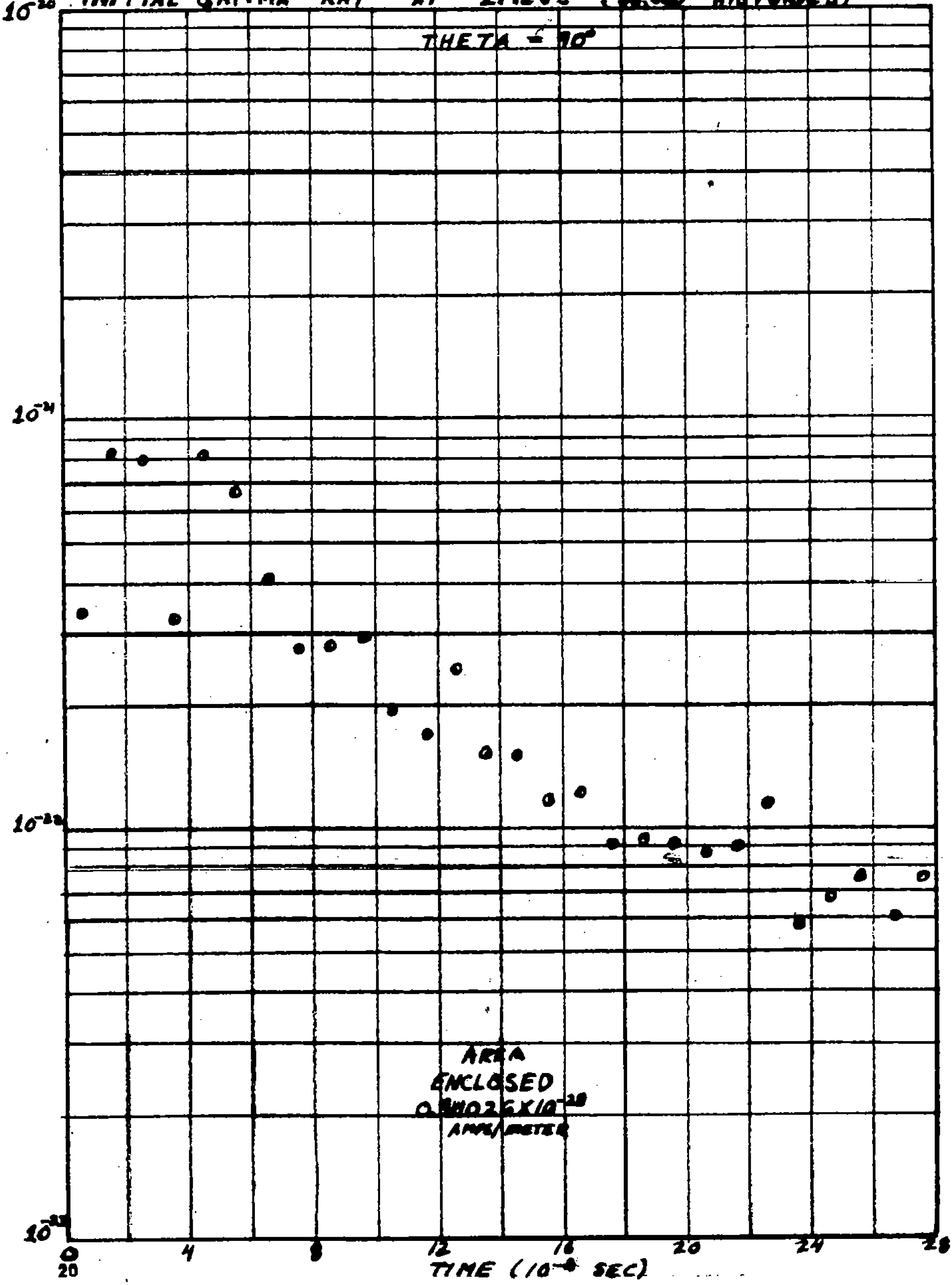


FIGURE 11

THETA CURRENT Vs. TIME AT 100 METERS FOR AN INITIAL GAMMA RAY AT 2MEVS (88,000 HISTORIES)



responses can be inferred over all space. Then, with the shapes of the response curves and the area contained under these curves being known, we can synthesize the actual ionization and Compton current responses to a delta function, point source of monoenergetic gamma rays at all points in space as functions of time. From these, the expected current (or ionization rate) from any shaped source, $S(t)$, of gamma rays per second can be calculated using equation (2).

The results from this analysis are intended for insertion into a computer solution of Maxwell's equations such as in reference AFWL TR 64-153.