EMP Theoretical Notes

Note IX

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Electrode Potentials from Compton Current and Space Charge in Evacuated Cavities

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Abstract:

This note calculates the potentials produced between objects in evacuated cavities and the cavity walls by the Compton current and space charge associated with gamma radiation. Simplified models of the cavity geometry, geometry of the internal object, and impedance connecting the former two are assumed. Three simple geometries are considered: parallel plate, cylindrical, and spherical.

I. Introduction

Although the electric fields and voltages induced in evacuated cavities by the Compton-ejected space charge can be calculated for cavities of simple geometry (as discussed in EMP Theoretical Note V, abbreviated TN V), the calculation of the potential of objects in such cavities with respect to the cavity walls is much more complex. However, such a calculation is needed; since through this mechanism, noise signals can enter circuits and electrical energy can be deposited in circuit elements.

Assuming (as in TN V) that the cavity dimensions are sufficiently small so that the electron transit time across the cavity and the electrical transit time around the cavity walls are both short compared to the times in which the γ radiation changes significantly, then a "steady state" solution may be calculated, simplifying the problem considerably. In general, the calculations will involve an evacuated cavity with an inner electrode, electrically connected to the cavity through some resistance R. The geometries (parallel plate, cylindrical, and spherical) of the electrode-cavity system are taken with a view to simplifying the calculations.

In order to calculate the potentials of the electrodes with respect to cavities it is necessary to consider three processes:

1. The electrode attenuates the γ current. If the potentials in the electrode-cavity system are much less than the mean Compton electron energy (about 0.25 Mev) and if the electrode and cavity walls are made of the same conducting material, then there will be a net Compton electron current into the electrode. This is the characteristic behavior of some Compton diodes (γ radiation detectors).

2. Low energy (a few e.V.) secondary electrons are produced at the electrode and cavity walls by the high energy Compton electrons. Under certain conditions these low energy electrons can flow between the electrode and the cavity walls, partially cancelling the net Compton current into the electrode.

3. The space charge associated with the Compton electrons will set up electric fields which tend to inhibit the flow of the secondary electrons from the electrode to the cavity walls, which partially cancel the net Compton electron current into the electrode. Under certain circumstances this space charge effect will then be a controlling factor, governing the electrode potential.

Certain effects dealing with the secondary electrons will be neglected, such as their interaction with any magnetic fields and the addition of the space charge from the secondary electrons to the Compton space charge. In particular, the latter restriction limits the applicability of the solutions to be developed to cases in which the secondary electron flux in the evacuated cavity is much less than the Compton electron flux. The inclusion of large secondary electron current densities requires a more general solution of the space charge equations.

II. Compton Current Deposited in Center Electrode:

First, consider the Compton current deposited in the center electrode. Since it is assumed that the ratio of the Compton current, J_{c_o}, to the gamma current, $\vec{\gamma}$, (equal in magnitude to the gamma flux, γ, in this case) then, as defined in TN V

$$J_{c_o} = c_J \gamma \qquad \frac{\text{amps}}{\text{meter}^2} \tag{1}$$

where c_J, the constant of proportionality, is approximately

$$c_J \simeq -2 \times 10^{-8} \qquad \frac{\text{coulombs}}{\text{meter}^2\text{-roentgen}} \tag{2}$$

with γ in roentgens per second (air equivalent dose). Then, if the γ-ray mean free path in the electrode (and wall) material is r_γ and the thickness of the electrode (in the direction of J_{c_o}) is t, the fraction of the Compton current density deposited in the electrode, δ, is just

$$\delta \simeq 1 - e^{-t/r_\gamma} \tag{3}$$

and for $t \ll r_\gamma$

$$\delta \simeq \frac{t}{r_\gamma} \tag{4}$$

If the electrode is not of constant thickness to the radiation, then δ must be averaged over the cross-section area of the electrode, A, presented to the radiation and to the Compton current. Thus, the net Compton current into the electrode, I_c, is just

$$I_c = \delta A J_{c_o} = c_J \delta A \gamma \tag{5}$$

or

$$I_c \simeq -2 \times 10^{-8} \delta A \gamma \tag{6}$$

If this Compton current were the only current into the electrode (other than current through the resistor connecting the electrode to the cavity walls), then a determination of the electrode potential could be made. However, there are other currents, specifically the secondary electron currents, which should be considered.

III. Secondary Electron Current from Center Electrode:

As discussed in the previous section, the Compton current into the electrode tends to drive the electrode to a negative potential with respect to the cavity walls. However, neglecting any retarding space charge fields, low energy (a few e.V.) secondary electrons knocked out of the electrode surface by the Compton electrons (both on entering the electrode from one side and on leaving the electrod from the opposite side) tend to cancel the net Compton current deposited in the electrode. Defining the net current of secondary electrons leaving the electrode as I_s, the net current into the electrode, I_{el}, (neglecting displacement current) is

$$I_{el} = I_c - I_s \tag{7}$$

One must be very careful in using the secondary electron current. Actually there are secondary electrons with energies extending up to the Compton electron energies, complicating the picture considerably. A significant fraction of these electrons are grouped in energy between zero and a few e.V. and it is these electrons which are of concern here. However, there will be some effect because of the presence of the higher energy secondary electrons. This effect is difficult to estimate. Only the low energy secondary electrons are included in this analysis.

There will be a maximum number of low energy secondary electrons emitted from each side of the electrode (as referred to the direction of the γ rays), proportional to the number of Compton electrons entering or leaving the electrode surfaces. Defining the secondary current from the side of the electrode toward the γ rays as I_{s_1} and the secondary current from the opposite side as I_{s_2}, then each of these currents will have minimum values (because they are negative)

$$I_{s_1 (min)} = f_{1_{max}} A J_{c_o} \quad \text{amps} \tag{8}$$

and

$$I_{s_2 (min)} = f_{2_{max}} A(1-\delta) J_{c_o} \quad \text{amps} \tag{9}$$

where $f_{1_{max}}$ and $f_{2_{max}}$ are the maximum ratios of the magnitudes of the secondary current to the Compton current for each side of the electrode. These two ratios are usually, but not necessarily, nearly the same. Note the factor of $(1-\delta)$ in equation (9) which expresses the reduction of the Compton current leaving the electrode from that incident on the electrode, $A J_{c_o}$.

The total low energy secondary current is then

$$I_s = I_{s_1} + I_{s_2} \tag{10}$$

where

$$I_{s_1} = f_1 A J_{c_o} \quad (11)$$

and

$$I_{s_2} = f_2 A(1-\delta) J_{c_o} \quad (12)$$

Thus,

$$I_s = A J_{c_o} [f_1 + f_2(1-\delta)] \quad (13)$$

where in general f_1 and f_2 are less than or equal to their maximum values. For the case $\delta \ll 1$ equation (13) reduces to

$$I_s \simeq 2f A J_{c_o} \quad (14)$$

where f is the average of f_1 and f_2.

IV. Compton Space Charge Limitation of Secondary Currents:

To determine the interaction of the Compton and secondary currents so that one may find the net current into the electrode surface, one must consider the electric field and voltage distribution in the cavity-electrode system from the Compton space charge density, ρ_{c_o}. If the electric field is directed away from the electrode surface, no secondary current will flow from this surface since the secondary electrons are assumed to be of zero energy. On the other hand, if the electric field is directed toward the electrode surface, the maximum secondary current will flow from the surface. The region of interest is then where the electric field at the electrode surface is zero because in this region the magnitude of the secondary current can vary from zero to its maximum value, depending on the detailed solution of the space charge equations. If the magnitude of the secondary space charge density, ρ_s, is much less than the magnitude of the Compton space charge density, ρ_{c_o}, i.e.,

$$|\rho_s| \ll |\rho_{c_o}| \quad (15)$$

then for purposes of an approximate calculation ρ_s can be neglected. This criterion will be generally true for "thin" electrodes, i.e., for

$$\delta \ll 1 \quad (16)$$

because in a thin electrode only a small fraction of the Compton current density contributes to the net Compton current into the electrode. No more than this small fraction of the Compton current density then will appear as a secondary current density to cancel the difference in the Compton currents into and out of the electrode. To approximately convert the ratio of secondary to Compton currents to a ratio of space charges, one can multiply this current ratio by the ratio of Compton electron velocity to mean secondary electron velocity. This, in turn, is given approximately by the square root of the ratio of the Compton electron energy (in e.V.) to the magnitude of the voltage between the electrode and cavity walls. It is this last ratio which determines how small the secondary current must be to satisfy the restriction of equation (15).

The criterion of equation (15) will also hold for "thick" electrodes in which the maximum ratio of secondary to Compton electrons (as in equations (8) and (9)) is much less than one, again, with the restrictions as discussed above.

Neglecting the secondary space charge, one can solve for the voltage of the center electrode, V', by the use of Poisson's equation with the boundary condition that the electric field at the center electrode be zero. This voltage will then be the electrode voltage, V_{el}, for the case in which the magnitude of the secondary current is greater than zero but less than its maximum value with the restrictions as discussed above.

A. Parallel Plate Geometry:

In parallel plate geometry (as in figure 1) this parameter (V') can be calculated from Poisson's equation,

$$\nabla^2 V = -\frac{\rho_{c_o}}{\varepsilon_o} \tag{17}$$

With the assumption that the electrode dimensions are much greater than either d_1 or d_2, equation (17) reduces to

$$\frac{\partial^2 V}{\partial z^2} = \frac{-\rho_{c_o}}{\varepsilon_o} \tag{18}$$

where as in TN V

$$\rho_{c_o} = c_\rho \gamma \qquad \frac{\text{coulombs}}{\text{meter}^3} \tag{19}$$

with

$$c_\rho \approx -.9 \times 10^{-16} \ \frac{\text{coulombs}}{\text{meter}^3} \left(\frac{\text{roentgen}}{\text{sec}}\right)^{-1} \tag{20}$$

However, the charge density is ρ_{c_o} only on the side of the electrode toward the gamma source. On the side away from the γ source the Compton space charge density will be multiplied by the same factor, $(1-\delta)$, as for the Compton current density.

On the side of the electrode toward the γ rays, then, one can integrate equation (17) once, giving

$$\frac{\partial V}{\partial z} = \frac{-\rho_{c_o}}{\varepsilon_o}(z-d_1) \qquad \frac{\text{volts}}{\text{meter}} \tag{21}$$

where z = 0 is taken as the cavity wall and $\frac{\partial V}{\partial z}$ is zero at the electrode wall.

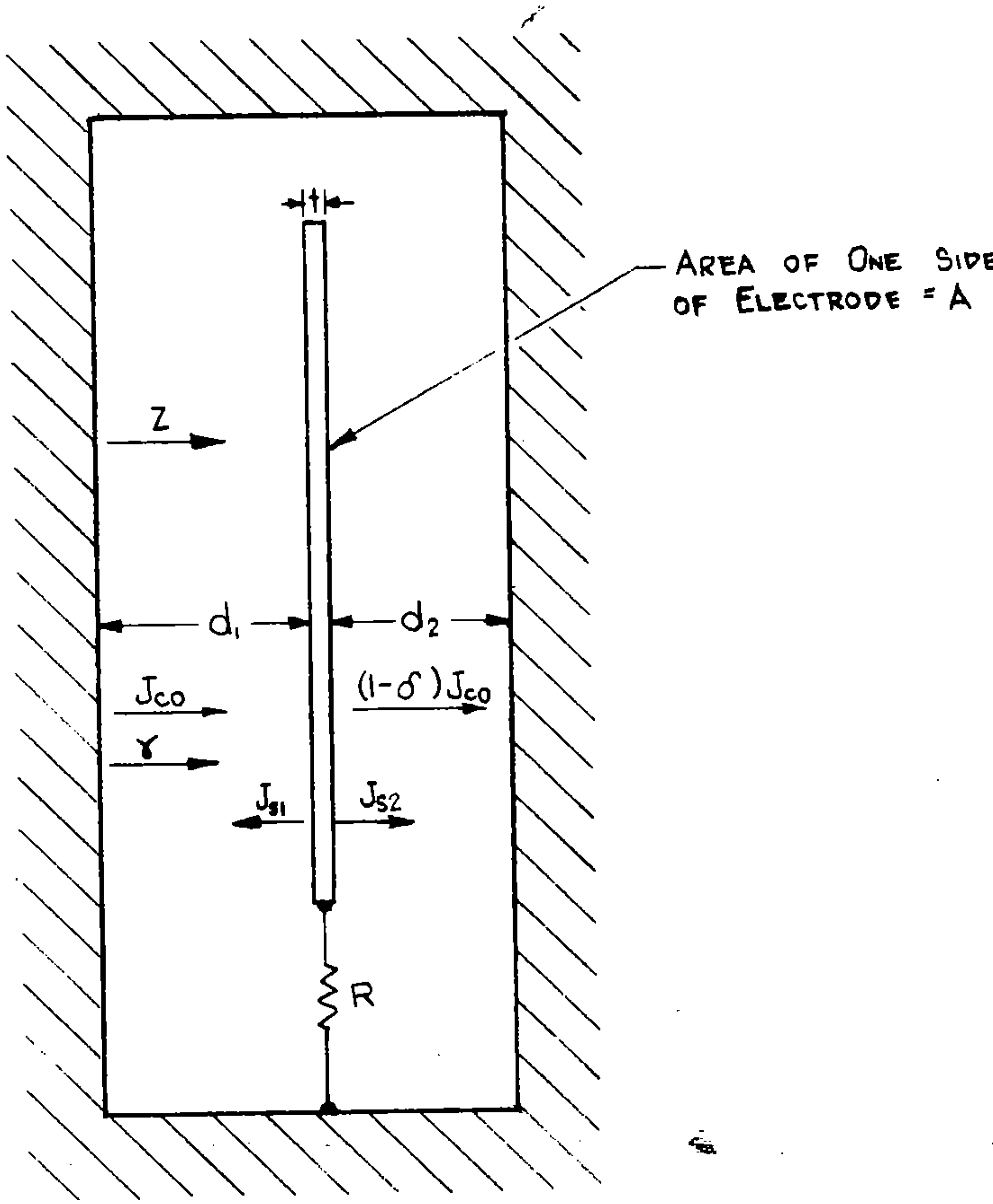

Fig. 1 PARALLEL PLATE GEOMETRY

By integrating equation (21) one can obtain the electrode voltage, V_1', as

$$V_1' = \frac{\rho_{c_o}}{2\varepsilon_o} d_1^2 \quad \text{volts} \tag{22}$$

or

$$V_1' = \frac{c_\rho}{2\varepsilon_o} \gamma \quad d_1^2 \quad \text{volts} \tag{23}$$

or

$$V_1' \approx -.5 \times 10^{-5} \gamma d_1^2 \text{ volts} \tag{24}$$

On the side of the electrode away from the γ source one can calculate the corresponding voltage, V_2', by including the factor $(1-\delta)$ and changing d_1 to d_2 in equation (22). Thus

$$V_2' = (1-\delta)\frac{\rho_{c_o}}{2\varepsilon_o} d_2^2 \quad \text{volts} \tag{25}$$

or

$$V_2' = \frac{c_\rho}{2\varepsilon_o} \gamma \ (1-\delta) d_2^2 \quad \text{volts} \tag{26}$$

or

$$V_2' \approx -.5 \times 10^{-5} \gamma (1-\delta) d_2^2 \text{ volts} \tag{27}$$

For parallel plate geometry, then, one can consider two voltages (corresponding to each side of the electrode) which determine the flow of the secondary currents. However, in the cases of cylindrical and spherical geometry, for simplicity of solution, it will be necessary to assume that the Compton space charge density is the same on both sides of the electrode. For comparison with these cases one can consider the special case of parallel plate geometry in which

$$d_1 = d_2 = d$$

$$\delta \ll 1$$

$$V_1' \approx V_2' = V' \tag{28}$$

and thus

$$V' = \frac{\rho_{c_o}}{2\varepsilon_o} d^2 \quad \text{volts} \tag{29}$$

or

$$V' = \frac{c_\rho}{2\varepsilon_o} \gamma d^2 \quad \text{volts} \tag{30}$$

or

$$V' \approx -.5 \times 10^{-5} \gamma d^2 \text{ volts} \tag{31}$$

In this special case there is only one voltage which determines the flow of secondary current. Since V' is negative, electrode voltages less than V' mean that all the secondary current is flowing, while electrode voltages greater than V' mean that no secondary current is flowing (again under the restrictions discussed in the first part of this section).

B. Cylindrical Geometry:

In cylindrical geometry (as in figure 2) with the assumption that the length is much greater than the diameter, i.e.,

$$l >> b \tag{32}$$

Poisson's equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial V}{\partial r}\right) = \frac{-\rho_{c_o}}{\varepsilon_o} \tag{33}$$

assuming that ρ_{c_o} is a constant, i.e.,

$$\delta << 1 \tag{34}$$

Setting $\frac{\partial V}{\partial r}$ at r = a equal to zero, one can multiply equation (33) by r and integrate to obtain

$$r\frac{\partial V}{\partial r} = -\int_a^r r\frac{\rho_{c_o}}{\varepsilon_o}\,dr = \frac{-\rho_{c_o}}{2\varepsilon_o}(r^2 - a^2) \tag{35}$$

and, therefore,

$$\frac{\partial V}{\partial r} = \frac{-\rho_{c_o}}{2\varepsilon_o}\left(r - \frac{a^2}{r}\right) \quad \frac{\text{volts}}{\text{meter}} \tag{36}$$

Integrating equation (36) to obtain V', one has

$$V' = \frac{-\rho_{c_o}}{2\varepsilon_o}\int_b^a \left(r - \frac{a^2}{r}\right) dr = \frac{\rho_{c_o}}{2\varepsilon_o}\left[\frac{b^2 - a^2}{2} - a^2 \ln\left(\frac{b}{a}\right)\right] \text{volts} \tag{37}$$

or

$$V' = \frac{c_{\rho}}{2\varepsilon_o}\gamma\left[\frac{b^2 - a^2}{2} - a^2 \ln\left(\frac{b}{a}\right)\right] \quad \text{volts} \tag{38}$$

or

$$V' \approx -.5 \times 10^{-5}\,\gamma\left[\frac{b^2 - a^2}{2} - a^2 \ln\left(\frac{b}{a}\right)\right] \quad \text{volts} \tag{39}$$

This last equation gives the relation, similar to equation (31) for parallel plate geometry, determining the secondary current flow in cylindrical geometry.

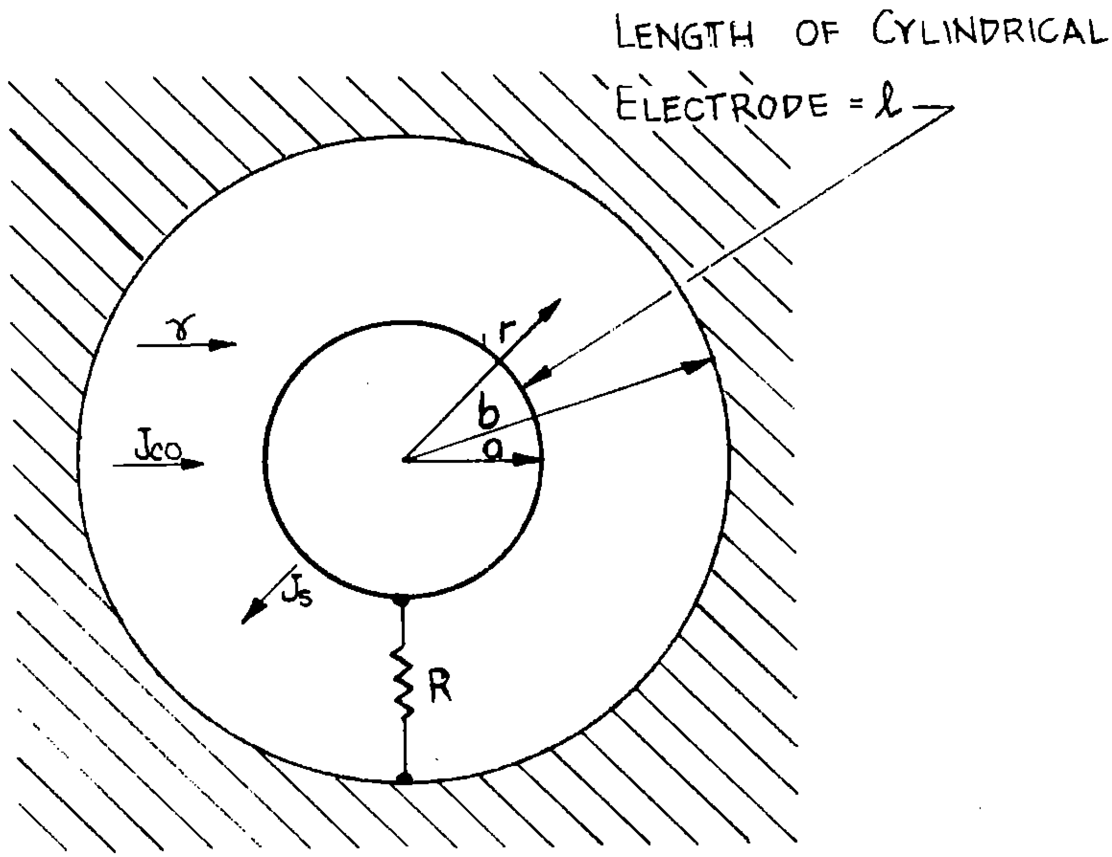

Fig. 2 CYLINDRICAL GEOMETRY

C. Spherical Geometry:

In spherical geometry (as in figure 3) with the restriction of equation (34), Poisson's equation reduces to

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = -\frac{\rho_{c_o}}{\varepsilon_o} \qquad (40)$$

Again setting $\frac{\partial V}{\partial r}$ at r = a equal to zero and multiplying equation (40) by r^2 one can integrate this equation and obtain

$$r^2 \frac{\partial V}{\partial r} = -\frac{\rho_{c_o}}{\varepsilon_o} \int_a^r r^2 dr = \frac{-\rho_{c_o}}{3\varepsilon_o} \left(r^3 - a^3\right) \qquad (41)$$

and, therefore,

$$\frac{\partial V}{\partial r} = -\frac{\rho_{c_o}}{3\varepsilon_o} \left(r \frac{-a^3}{r^2}\right) \quad \frac{\text{volts}}{\text{meter}} \qquad (42)$$

Integrating equation (42) to obtain V' one has

$$V' = -\frac{\rho_{c_o}}{3\varepsilon_o} \int_b^a \left(r \frac{-a^3}{r^2}\right) dr = \frac{\rho_{c_o}}{3\varepsilon_o} \left[\frac{b^2-a^2}{2} + a^3 \left(\frac{1}{b} - \frac{1}{a}\right)\right] \text{volts} \qquad (43)$$

or

$$V' = \frac{c_\rho}{3\varepsilon_o} \gamma \left[\frac{b^2-a^2}{2} + a^3 \left(\frac{1}{b} - \frac{1}{a}\right)\right] \text{volts} \qquad (44)$$

or

$$V' = -.34 \times 10^{-5} \gamma \left[\frac{b^2-a^2}{2} + a^3 \left(\frac{1}{b} - \frac{1}{a}\right)\right] \text{volts} \qquad (45)$$

With this last equation one now has the relation between V' and γ for the three simple geometries: parallel plate, cylindrical, and spherical.

One can now consider the interaction of the Compton current, the secondary current, the Compton space charge, and the resistance between the electrode and the cavity walls. This interaction, which will determine the electrode potential, can be considered for two general cases:

1. thin, symmetrically-placed electrodes,

2. electrodes in parallel plate geometry.

These considerations follow in the next two sections.

V. Voltages and Currents for Thin, Symmetrically-Placed Electrodes:

A first case to consider is that in which the restriction of equation (34) (thin electrode) holds. This will be applicable to all three geometries, considered with the additional restriction that the electrode

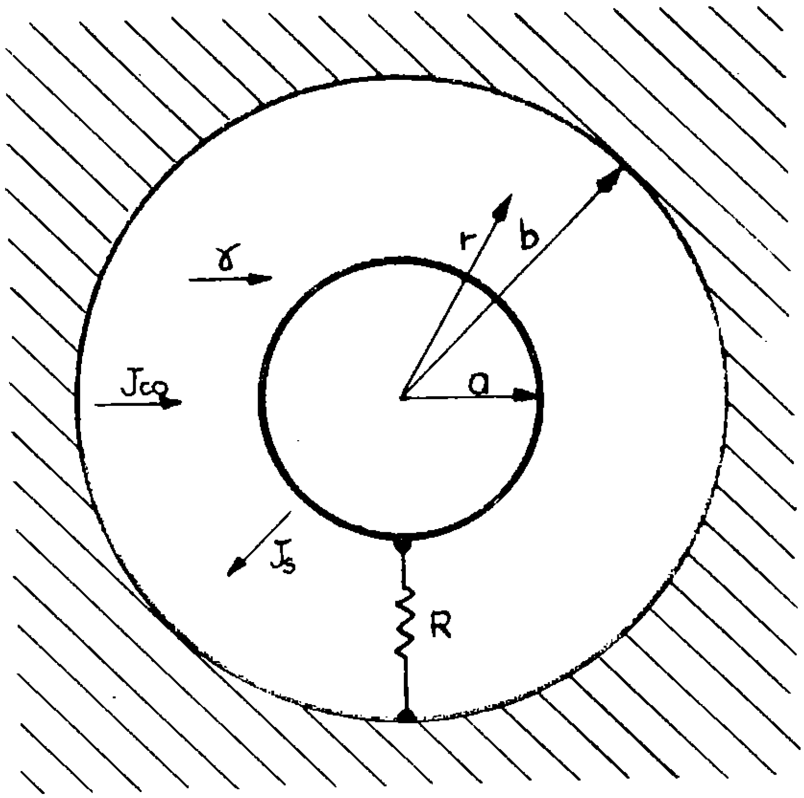

Fig. 3 SPHERICAL GEOMETRY

in parallel plate geometry is centrally placed. Then there will be a single V' which determines the flow of secondary current. This is illustrated in figure 4, in which the electrode voltage, V_{el}, as a function of the resistance, R, which connects the electrode to the cavity walls, is divided into three regions. In region 1, V_{el} is greater than V' and is given from the Compton current calculations in equation (5) as

$$V_{el} = I_c R = R\delta A \, J_{c_o} \quad \text{volts} \tag{46}$$

or

$$V_{el} = c_J R\delta A\gamma \quad \text{volts} \tag{47}$$

or

$$V_{el} = -2 \times 10^{-8} \, R\delta A\gamma \quad \text{volts} \tag{48}$$

where δ and A are as considered in Section II, δ being the average γ attenuation factor for the γ rays and A being the cross section area of the center electrode. For cylindrical geometry

$$A = 2 \, al \quad \text{meter}^2 \tag{49}$$

and for spherical geometry

$$A = \pi a^2 \quad \text{meter}^2 \tag{50}$$

There is no necessity for restricting the dimensions in parallel plate geometry to a specific shape.

As R is increased, V_{el} will approach V', and in region 2

$$V_{el} = V' \tag{51}$$

where V' has been calculated for the three geometries in equations (29), (37), and (43). In this region the current through R will be given by

$$I_{el} = \frac{V'}{R} = I_c - I_s \tag{52}$$

and thus

$$I_s = I_c - \frac{V'}{R} \tag{53}$$

The ratio of secondary to net Compton current is

$$\frac{I_s}{I_c} = 2f = 1 - \frac{V'}{I_c R} \tag{54}$$

Since V' and I_c are both proportional to γ, this ratio is independent of γ (within the range of V' as discussed in Section III).

Finally, if the maximum ratio of secondary to Compton current is small enough, then as R is increased V_{el} will again decrease. This is shown in region 3 of figure 4. For this to occur, it is necessary that

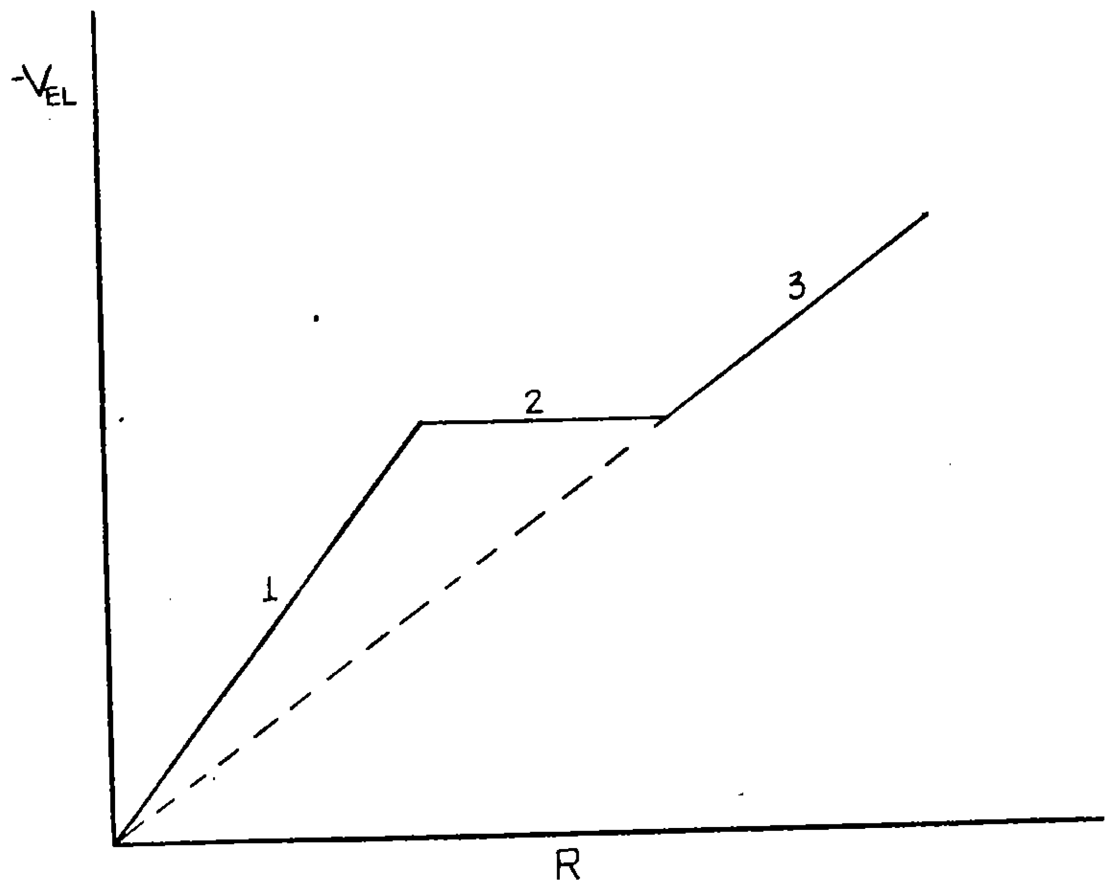

Fig. 4 ELECTRODE VOLTAGE FOR THIN, SYMMETRICALLY-PLACED ELECTRODES

$$I_{s_{min}} > I_c \qquad (55)$$

(remembering that these currents are negative). When this is true, then in region 3

$$V_{el} = (I_c - I_{s_{min}})R = A\, J_{c_o}\, (\delta-2f_{max})R \text{ volts} \qquad (56)$$

or

$$V_{el} = c_J\, A\, (\delta-2f_{max})\gamma\, R \quad \text{volts} \qquad (57)$$

or

$$V_{el} = -2 \times 10^{-8}\, A\, (\delta-2f_{max})\, \gamma\, R \text{ volts} \qquad (58)$$

If the restriction of equation (55) does not hold, then region 3 (in figure 4) does not exist and V_{el} is limited to V'.

Thus, for thin, symmetrically-placed electrodes in evacuated cavities, one can, by this approximate analysis, arrive at the dependence of the electrode potential on the electrode-cavity parameters and the γ radiation intensity. As illustrated in figure 4, however, this dependence is not described by one simple function, but broken into several functional dependences for different ranges of the various parameters.

VI. Voltages and Currents for Electrodes in Parallel-Plate Geometry:

The results of Section V can be generalized to some extent for parallel plate geometry since neither is it necessary to assume that the electrode is centrally placed nor that the fractional attenuation of the γ rays by the electrode is small. However, in this treatment it is still necessary to assume that the secondary current is small compared to the Compton current. In this case, one can consider two V' 's as in equations (22) and (25), as well as two secondary currents as in equations (8), (9), and (10).

The results of this analysis are illustrated in figure 5, which divides the dependence of V_{el} on R into five regions. Region 1 is the same as in figure 4, i.e., no secondary current is flowing and the results are the same as in equation (46). Unless V'_1 and V'_2 from equations (22) and (25) are equal (in which case the results are as in figure 4), then V_{el} is given for region 2 as the largest (because V_{el} is negative) of V'_1 and V'_2.

Similarly, region 3 will occur as in figure 4 except that in this case

$$I_{s_{1,2}\,(min)} > I_c \qquad (59)$$

where the secondary current is the one associated with the larger V'. Region 4 is then given by the minimum of the two V' 's, i.e.

$$V_{el} = V'_{1,2_{min}} \qquad (60)$$

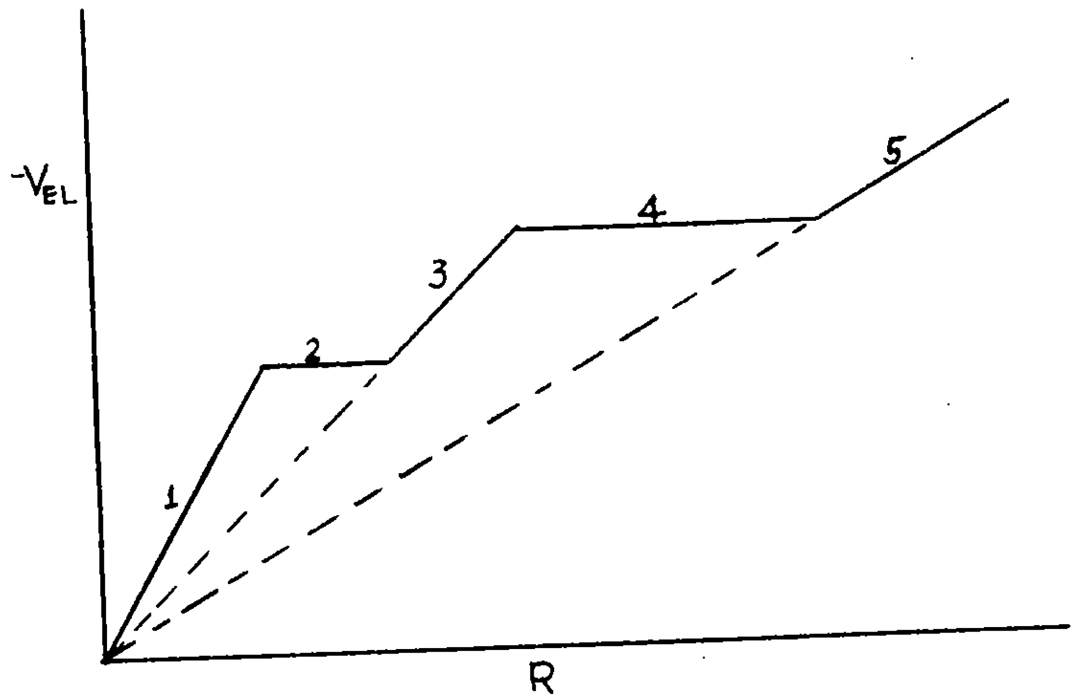

Fig. 5 ELECTRODE VOLTAGE IN PARALLEL PLATE GEOMETRY

Finally, region 5 occurs if

$$I_{s_1 (min)} + I_{s_2 (min)} > I_c \qquad (61)$$

so that

$$V_{el} = (I_c - \left[I_{s_1} + I_{s_2}\right] min) \, R \text{ volts} \qquad (62)$$

or

$$V_{el} = AJ_{c_o} \, (\delta - f_{1_{max}} - (1-\delta) f_{2_{max}}) R \text{ volts} \qquad (63)$$

or

$$V_{el} = c_J \left[AJ_{c_o} (\delta - f_{1_{max}} - (1-\delta) f_{2_{max}}) \right] R \, \gamma \text{ volts} \qquad (64)$$

or $$V_{el} \approx -2 \times 10^{-8} \left[AJ_{c_o} (\delta - f_{1_{max}} - (1-\delta) f_{2_{max}}) \right] R \, \gamma \text{ volts} \qquad (65)$$

Depending on δ, $f_{1_{max}}$, and $f_{2_{max}}$, any regions beyond region 2 may not exist.

Thus, for the more general case allowable in parallel plate geometry, the electrode potential can be related to the various electrode-cavity parameters and the γ radiation intensity in much the same manner as for the previous case involving thin, symmetrically-placed electrodes. However, as illustrated in figure 5, this dependence is more complex.

VIII. Summary:

By the techniques outlined in the previous sections, one can, under certain restrictions, obtain a steady-state solution for the potentials of objects in evacuated cavities (in simple geometries) by considering the resistance connecting the object to the cavity walls, together with three phenomena:

1. The Compton current deposited in the object.

2. The secondary electrons emitted from the object by the Compton electrons.

3. The Compton space charge voltage which determines the interaction of the previous two phenomena.

The analysis carried out in the previous sections can be extended to include reactive impedances between the objects and the cavity walls, but, in this case, the time history of the radiation will have to be considered, since at any given time the interaction of the Compton and secondary currents will be strongly influenced by the previous radiation time history.

However, from the analysis presented in this Note one can see how the parameters associated with the Compton current (e.g., δA), the secondary current (e.g., f_{max}) and the Compton space charge voltages (as determined by the cavity and electrode dimensions) determine the voltage and currents associated with objects in evacuated cavities. It must be remembered that these results apply only to the cases in which the secondary current density is much less than the Compton current density in magnitude and in which the electrode potentials are significantly larger (in magnitude) than the secondary electron energy (in e.V.) and correspondingly smaller than the Compton electron energy.