Subject: EMP Theoretical Note VIII 15 April 1965

TO: Distribution List

1. Part of the following note was first distributed on 16 March 1964 as Los Alamos Document J-13-459. For this note, minor errors in J-13-459 have been corrected and 60 plots of fields generated in the ground by delta-function and step function electric fields at the surface have been added.

2. I am indebted to Lt William R. Graham of the Air Force Weapons Laboratory for additional editing and correction of some errors in the original note. In particular, equation (1-13) was copied incorrectly directly from Wait's referenced work; it was rederived by Lt Graham in his Theoretical Note I.

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EMP Theoretical Notes

Note VIII
15 April 1965

EM Pulse Fields in Dissipative Media

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0.20	4.3000	1.000	81.000	6.000E-09	2.161E-07	29
0.30	0.0200	1.000	16.000	4.000E-09	2.262E-09	30
0.30	4.3000	1.000	81.000	9.000E-09	4.863E-07	31
0.50	4.3000	1.000	81.000	1.500E-08	1.351E-06	32
0.60	0.0200	1.000	16.000	8.000E-09	9.048E-09	33
0.70	4.3000	1.000	81.000	2.100E-08	2.648E-06	34
1.00	0.0200	1.000	16.000	1.333E-08	2.513E-08	35
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2.00	0.0200	1.000	16.000	2.667E-08	1.005E-07	37
2.00	4.3000	1.000	81.000	6.000E-08	2.161E-05	38
3.00	0.0200	1.000	16.000	4.000E-08	2.262E-07	39
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30.00	0.0200	1.000	16.000	4.000E-07	2.262E-05	41
100.00	0.0002	1.000	81.000	3.000E-06	2.513E-06	42

Delta-Function Response, Magnetic Field Parameter vs Time

0.10	4.3000	1.000	81.000	3.000E-09	5.404E-08	43
0.20	4.3000	1.000	81.000	6.000E-09	2.161E-07	44
0.30	0.0200	1.000	16.000	4.000E-09	2.262E-09	45
0.30	4.3000	1.000	81.000	9.000E-09	4.863E-07	46
0.50	4.3000	1.000	81.000	1.500E-08	1.351E-06	47
0.60	0.0200	1.000	16.000	8.000E-09	9.048E-09	48
0.70	4.3000	1.000	81.000	2.100E-08	2.648E-06	49
1.00	0.0200	1.000	16.000	1.333E-08	2.513E-08	50
1.00	4.3000	1.000	81.000	3.000E-08	5.404E-06	51
2.00	0.0200	1.000	16.000	2.667E-08	1.005E-07	52
2.00	4.3000	1.000	81.000	6.000E-08	2.161E-05	53
3.00	0.0200	1.000	16.000	4.000E-08	2.262E-07	54
10.00	0.0200	1.000	16.000	1.333E-07	2.513E-06	55
30.00	0.0200	1.000	16.000	4.000E-07	2.262E-05	56
100.00	0.0002	1.000	81.000	3.000E-06	2.513E-06	57

Step-Function Response, Electric Field Parameter vs Time

0.10	4.3000	1.000	81.000	3.000E-09	5.404E-08	58
0.20	4.3000	1.000	81.000	6.000E-09	2.161E-07	59
0.30	0.0200	1.000	16.000	4.000E-09	2.262E-09	60
0.30	4.3000	1.000	81.000	9.000E-09	4.863E-07	61
0.50	4.3000	1.000	81.000	1.500E-08	1.351E-06	62
0.60	0.0200	1.000	16.000	8.000E-09	9.048E-09	63
0.70	4.3000	1.000	81.000	2.100E-08	2.648E-06	64
1.00	0.0200	1.000	16.000	1.333E-08	2.513E-08	65
1.00	4.3000	1.000	81.000	3.000E-08	5.404E-06	66
2.00	0.0200	1.000	16.000	2.667E-08	1.005E-07	67
2.00	4.3000	1.000	81.000	6.000E-08	2.161E-05	68
3.00	0.0200	1.000	16.000	4.000E-08	2.262E-07	69
10.00	0.0200	1.000	16.000	1.333E-07	2.513E-06	70
30.00	0.0200	1.000	16.000	4.000E-07	2.262E-05	71
100.00	0.0002	1.000	81.000	3.000E-06	2.513E-06	72

Step-Function Response, Magnetic Field Parameter vs Time

I. Plane Wave Incident Upon a Dissipative Half-Drive

There have been several investigations of fields resulting from pulsed dipoles in a conducting homogeneous medium[1-3]; the simpler problem of the fields resulting from an initially prescribed field description over a plane is also of use in many instances and is treated briefly by Wait[1] and Stratton[5].

Taking the initial fields as being zero, the Laplace transform with respect to time of the Maxwell relations are (mks units):

$$\text{curl } \tilde{E} = - s\tilde{B} \tag{1-1}$$

$$\text{curl } \tilde{B} = (\mu\sigma + \epsilon\mu s)\, \tilde{E} \tag{1-2}$$

where the tilde denotes the Laplace transform of the corresponding field quantity. For propagation parallel to a z-axis, the electric and magnetic fields may be taken along the x- and y-axis respectively. The field will be specified on the plane $z = 0$. The resulting transformed equations are:

$$\gamma\tilde{E}_x = s\tilde{B}_y \tag{1-3}$$

$$\gamma\tilde{B}_y = (\mu\sigma + \epsilon\mu s)\, \tilde{E}_x \tag{1-4}$$

with $$\gamma = (\mu\sigma s + \epsilon\mu s^2)^{1/2} \tag{1-5}$$

The resulting fields may then be obtained from the inverse transforms:

$$E_x = L^{-1}\left\{\tilde{E}_x\right\} = L^{-1}\left\{\frac{s\tilde{B}_y}{\gamma}\right\} \tag{1-6}$$

$$B_y = L^{-1}\left\{\tilde{B}_y\right\} = L^{-1}\left\{\frac{\gamma\tilde{E}_x}{s}\right\} \tag{1-7}$$

If the electric field on the plane $z = 0$ is a δ-function* $= E_o\delta(t)$

$$E_x = L^{-1}\left\{E_o e^{-\gamma z}\right\} = E_o L^{-1}\left\{e^{-k\sqrt{s(s+b)}}\right\} \tag{1-8}$$

where $k = \sqrt{\epsilon\mu}\; z$

$b = \sigma/\epsilon$

* Defined by $\delta(x) = 0$ for $x \neq 0$ and $\int_{-\infty}^{\infty} \delta(x)dx = 1$; $u(t - k) = 0$ for $t < k$; $= 1$ for $t > k$.

$\delta(x)$ has dimension x^{-1}

Transform pair 863.1 of Campbell and Foster [4] gives the result (it may be obtained from the transform for $\{e^{-k\sqrt{s(s+b)}}\}/(\sqrt{s(s+b)})\}$, given in most tables of transform pairs, by differentiation with respect to k (see Appendix A-3)):

$$E_x = E_o \left\{ e^{-(bt)/2}\, \delta(t-k) + \frac{bk}{2}\, \frac{e^{-(bt)/2}}{\sqrt{t^2-k^2}}\, I_1\left[\frac{b}{2}\sqrt{t^2-k^2}\right]\right\} u(t-k) \qquad (1\text{-}9)$$

where $I_1(x)$ is the modified Bessel function of the first order, $\delta(x)$ and $u(x)$ are the unit impulse and step functions respectively.

$$B_y = E_o L^{-1}\left\{\frac{\sqrt{\mu\sigma s + \epsilon\mu s^2}}{s}\, e^{-\sqrt{\mu\sigma s + \epsilon\mu s^2}\; z}\right\}$$

$$= \frac{E_o k}{z} L^{-1}\left\{\frac{\sqrt{s(s+b)}}{s}\, e^{-\sqrt{s(s+b)}\; k}\right\} \qquad (1\text{-}10)$$

$$B_y = E_o\sqrt{\epsilon\mu}\left\{\left(\frac{b}{2}+\frac{b^2 k}{8}\right)e^{-(bk)/2} - \frac{b}{2}\int_k^t e^{-(bt)/2}\cdot\frac{I_1\left(\frac{b}{2}\sqrt{t^2-k^2}\right)}{\sqrt{t^2-k^2}}\,dt + \frac{b^4 k^2}{16}\int_k^t e^{-(bt)/2}\left[\frac{I_o\left(\frac{b}{2}\sqrt{t^2-k^2}\right)}{\left(\frac{b}{2}\sqrt{t^2-k^2}\right)^2} - \frac{2I_1\left(\frac{b}{2}\sqrt{t^2-k^2}\right)}{\left(\frac{b}{2}\sqrt{t^2-k^2}\right)^3}\right]dt\right\} u(t-k)$$

$$+ E_o\sqrt{\epsilon\mu}\, e^{-(bk)/2}\,\delta(t-k) \qquad (1\text{-}11)$$

Wait [1] gives the corresponding relations assuming a δ-function B field. For ready access, they are:

$$B_y(t) = B_o\, e^{-(bt)/2}\,\delta(t-k) + B_o\,\frac{bk}{2}\, e^{-(bt)/2}\, \frac{I_1\left(\frac{b}{2}\sqrt{t^2-k^2}\right)}{\sqrt{t^2-k^2}} \qquad (1\text{-}12)$$

$$E_x(t) = \frac{B_o}{\sqrt{\epsilon\mu}} e^{-(bt)/2} \quad \delta(t - k)$$

$$+ \frac{B_o}{\sqrt{\epsilon\mu}} \frac{b}{2} e^{-(bt)/2} \left\{ \frac{t}{\sqrt{t^2 - k^2}} I_1 \left[\frac{b}{2} \sqrt{t^2 - k^2} \right] \right.$$

$$\left. - I_o \left[\frac{b}{2} \sqrt{t^2 - k^2} \right] \right\} u(t - k) \qquad (1\text{-}13)$$

Responses for other driving functions f(t) may be obtained from the impulse response g(t) using the Faltung integral:

$$F(t) = \int_o^t g(\tau) \, f(t - \tau) \, d\tau \qquad (1\text{-}14)$$

In particular, the response to a step function is given as

$$F_u(t) = \int_o^t g(\tau) \, d\tau \qquad (1\text{-}15)$$

These are exact transforms but are rather involved. Since for many problems the times of interest are for times >> ϵ/σ* the displacement current contribution may be neglected with great simplification of the equations. With neglect of displacement currents, $\gamma = \sqrt{\mu\sigma s}$ and the fields are functions of a dimensionless time variable $\tau = t/\sigma\mu z^2$.

For a δ-function E-field, $E(o,t) = E_o \, \delta(t)$; $\tilde{E}_x = E_o e^{-\gamma z}$

$$E_x = \frac{E_o \sqrt{\mu\sigma z^2}}{2 \sqrt{\pi t^3}} \exp\left(- \frac{\mu\sigma z^2}{4t}\right) = \frac{E_o}{2\mu\sigma z^2} \frac{1}{\sqrt{\pi}} \tau^{-3/2} e^{-1/(4\tau)} \qquad (1\text{-}16)$$

$$B_y = \frac{E_o \sqrt{\mu\sigma}}{\sqrt{\pi t}} \exp\left(- \frac{\mu\sigma z^2}{4t}\right) = \frac{E_o}{z} \frac{1}{\sqrt{\pi}} \tau^{-1/2} e^{-1/(4\tau)} \qquad (1\text{-}17)$$

These are plotted in Figures 1 and 2**.

For a δ-function B-field:

$$B_y = \frac{B_o \sqrt{\mu\sigma z^2}}{2 \sqrt{\pi t^3}} \exp\left(- \frac{\mu\sigma z^2}{4t}\right) = \frac{B_o}{2\sigma\mu z^2 \sqrt{\pi}} \tau^{-3/2} e^{-1/(4\tau)} \qquad (1\text{-}18)$$

---

* $\epsilon/\sigma \approx 2 \times 10^{-10}$ sec for sea water, $\approx 10^{-8}$ sec for Nevada Test Site soil

** The graphs do not have high accuracy being slide rule calculations in some cases. Values of the functions were usually obtained from Dwight, Tables of Functions, Dover Publications.

$$E_x = \frac{B_o}{2\sqrt{\pi\mu\sigma t^3}}\left\{\left[\frac{\mu\sigma z^2}{2t} - 1\right]\exp\left[-\frac{\mu\sigma z^2}{4t}\right]\right\}$$

$$= \frac{B_o}{2\mu^2\sigma^2 z^3\sqrt{\pi}}\,\tau^{-3/2}\left[\frac{1}{2\tau} - 1\right]e^{-1/(4\tau)} \qquad (1\text{-}19)$$

For a step function electric field at $z = 0$:

$$E_x = E_1 L^{-1}\left\{\frac{e^{-\gamma z}}{s}\right\}$$

$$B_y = E_1 L^{-1}\left\{\frac{\gamma e^{-\gamma z}}{s^2}\right\}$$

$$E_x = E_1\,\mathrm{erfc}\left(\frac{\sqrt{\mu\sigma z^2}}{2\sqrt{t}}\right) = E_o\,\mathrm{erfc}\,\frac{1}{2\sqrt{\tau}} \qquad (1\text{-}20)$$

$$B_y = E_1\sqrt{\mu\sigma}\left\{2\sqrt{\frac{t}{\pi}}\exp\left[-\frac{\mu\sigma z^2}{4t}\right] - \sqrt{\mu\sigma z^2}\,\mathrm{erfc}\left[\frac{\sqrt{\mu\sigma z^2}}{2\sqrt{t}}\right]\right\}$$

$$= E_1\,\mu\sigma z\left\{\frac{2}{\sqrt{\pi}}\sqrt{\tau}\,e^{-1/(4\tau)} - \mathrm{erfc}\left[\frac{1}{2\sqrt{\tau}}\right]\right\} \qquad (1\text{-}21)$$

These are plotted in Figures 3 and 4.

For a step function magnetic field at $z = 0$:

$$B_o = B_1\,\mathrm{erfc}\left[\frac{\mu\sigma z^2}{2\sqrt{t}}\right] = B_o\,\mathrm{erfc}\left[\frac{1}{2\sqrt{\tau}}\right] \qquad (1\text{-}22)$$

$$E_x = \frac{B_1}{\sqrt{\mu\sigma}}\,\frac{1}{\sqrt{\pi t}}\exp\left[-\frac{\mu\sigma z^2}{4t}\right] = \frac{B_o}{\mu\sigma z\sqrt{\pi}}\,\tau^{-1/2}\exp\left[-\frac{1}{4\tau}\right] \qquad (1\text{-}23)$$

Note: $\mathrm{erf}\,x = \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}dy$

$\mathrm{erfc}\,x = 1 - \mathrm{erf}\,x$

$\frac{d(\mathrm{erf}\,x)}{dx} = \frac{2}{\sqrt{\pi}}e^{-x^2}$

For a square pulse of length T at $z = 0$:

$$E(0,t) = E_1 \qquad 0 < t < T$$

$$E(0,t) = 0 \qquad t > T$$

$$E_x = E_1 L^{-1} \left\{ \frac{1 - Ts}{s} e^{-\gamma z} \right\}$$

$$= E_1 \operatorname{erfc} \frac{1}{2\sqrt{\tau}} \qquad 0 < \tau < \alpha$$

$$= E_1 \left[\operatorname{erfc} \frac{1}{2\sqrt{\tau}} - \operatorname{erfc} \frac{1}{2\sqrt{\tau - \alpha}} \right] \qquad \tau > \alpha \qquad (1\text{-}24)$$

with $\tau = t/(\sigma\mu z^2)$; $\alpha = T/(\sigma\mu z^2)$

$$B_y = E_1 \mu\sigma z \left\{ \frac{2}{\sqrt{\pi}} \sqrt{\tau}\, e^{-1/(4\tau)} - \operatorname{erfc} \frac{1}{2\sqrt{\tau}} \right\} \qquad 0 < \tau < \alpha$$

$$= E_1 \mu\sigma z \left\{ \frac{2}{\sqrt{\pi}} \left[\sqrt{\tau}\, e^{-1/(4\tau)} - \sqrt{\tau - \alpha}\, e^{-1/(4(\tau - \alpha))} \right] - \left[\operatorname{erfc} \frac{1}{2\sqrt{\tau}} - \operatorname{erfc} \frac{1}{2\sqrt{\tau - \alpha}} \right] \right\} \qquad \tau > \alpha \qquad (1\text{-}25)$$

These are also plotted in Figures 3 and 4 for $\alpha = 2$ and 20.

Similarly, the response to a square wave magnetic field at $z = 0$ may be obtained from the corresponding step function response relations.

The Frenchman Flat area of the Nevada Test Site (NTS) has a medium which may be roughly described as having a relative dielectric constant of 16* with an average conductivity to a depth of 6 meters of 0.02 mho/meter and to a depth of 30 meters of 0.03 mho/m. Sea water, at low latitudes, is described approximately by a relative dielectric constant of 80 and a conductivity of 4 mho/meter. With this information Table 1 gives values of times associated with the values of τ.

TABLE 1

z(m)	t(μsec) NTS	t(μsec) SEA WATER
.316	.0025 τ	.5 τ
.6	.009 τ	2. τ
1.	.025 τ	5. τ
2.4	.15 τ	30. τ
30.5	35. τ	500. τ

*This dielectric "constant" may rise to much larger values at low frequencies but indications are that σ is still $>> W\epsilon$

The values of α of Figures 3 and 4 are chosen to correspond to a pulse length of T = 10 μsec with α = 20 for z = 0.316 m, and α = 2 for z = 1 m in sea water.

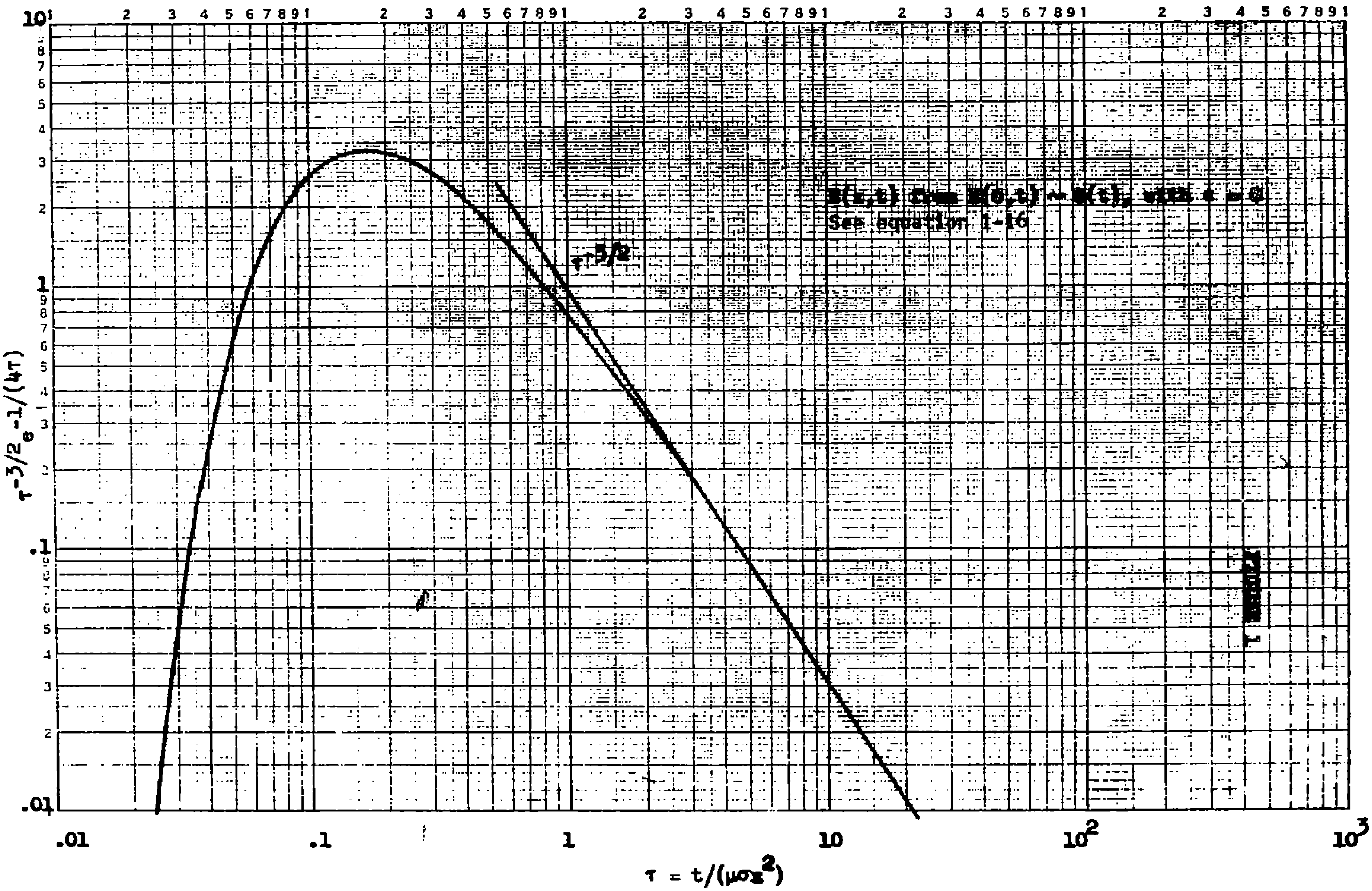
Other values of time are readily obtained from the relation $t = \sigma\mu z^2 \tau$.

To illustrate the error introduced through neglect of the displacement current, two sample problems were calculated using values which were of some practical value. Table 2 gives values of the parameters employed for plotting the figures indicated.

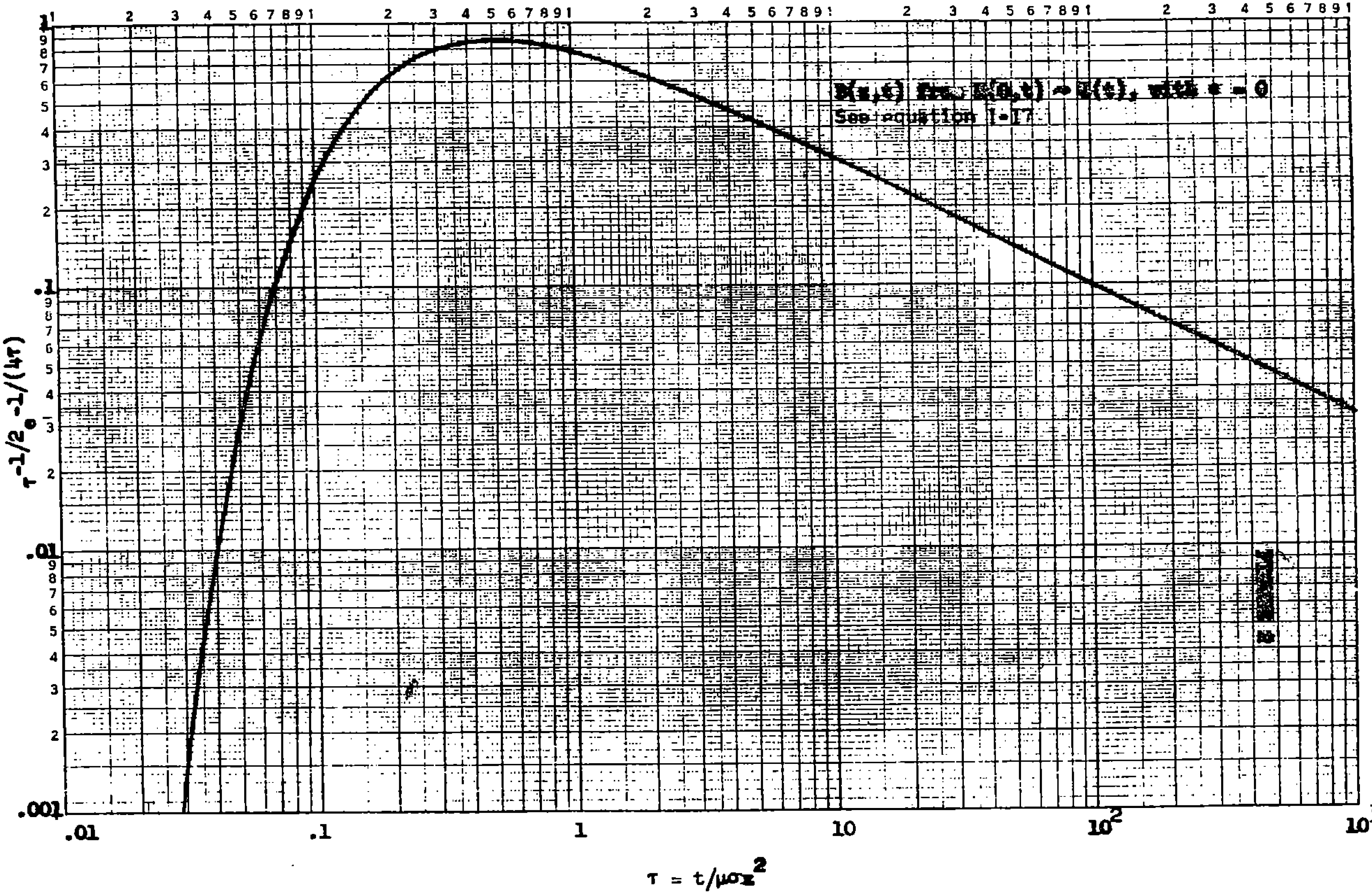
TABLE 2

	Figures 5, 6	Figures 7, 8	Figure 9	Figure 10
σ, mho/m	≈ .02	≈ 4.3	.02	2.0×10^{-4}
$\sigma\mu$, sec/m^2	2.5×10^{-8}	5.4×10^{-6}	2.5×10^{-8}	2.5×10^{-10}
ϵ/ϵ_0	25	81	16	81
Depth z, m	1.2	0.1	0.6	100
$k = \sqrt{\epsilon\mu}\, z$, sec	2.0×10^{-8}	3.0×10^{-9}	8.0×10^{-9}	3.0×10^{-6}
$b = \sigma/\epsilon$, sec^{-1}	0.9×10^{8}	6.0×10^{9}	1.4×10^{8}	2.8×10^{5}
$\mu\sigma z^2$, sec	3.6×10^{-8}	5.4×10^{-8}	9.0×10^{-9}	2.5×10^{-6}

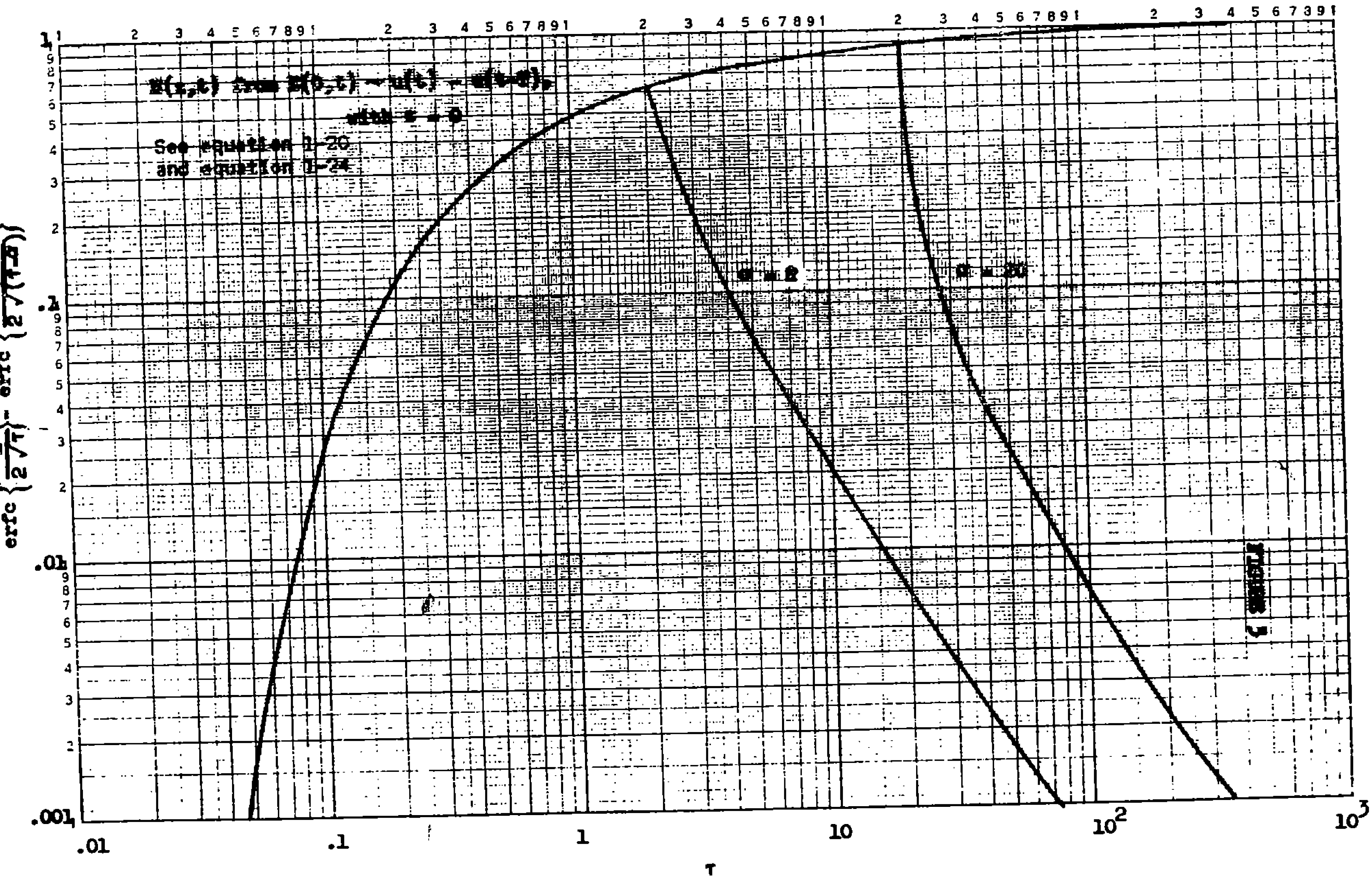
The parameters and depths for Figures 5, 6, and 9 are of use for NTS situations, those of Figures 7 and 8 for ocean water and those of Figure 10 for fresh water (they are the same as used by Stratton[5] in his example). The times used in plotting the figures are for t = 0 at the plane z = 0. For the practical cases, the figures show that neglect of displacement currents have only a small effect upon pulse shapes for media of moderate and high conductivity. Thus Figures 1 through 4 may be used for description of the fields with initial τ given by the propagation time from the plane z = 0 to the distance desired.



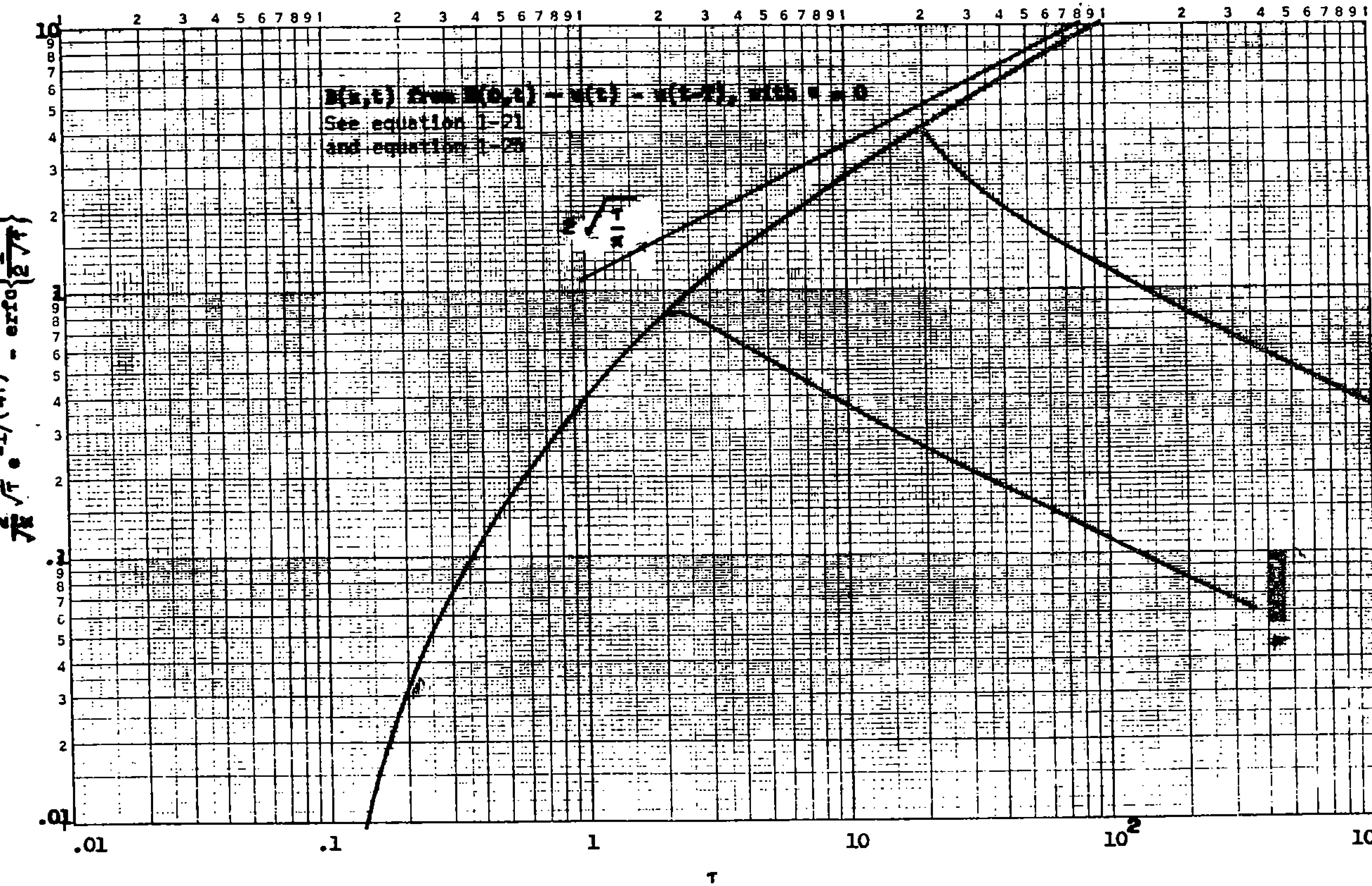
See equation 1-16
τ^{-3/2}
τ^{-3/2} e^{-1/(4τ)}
τ = t/(μσz²)
.01
.1
1
10
10²
10³

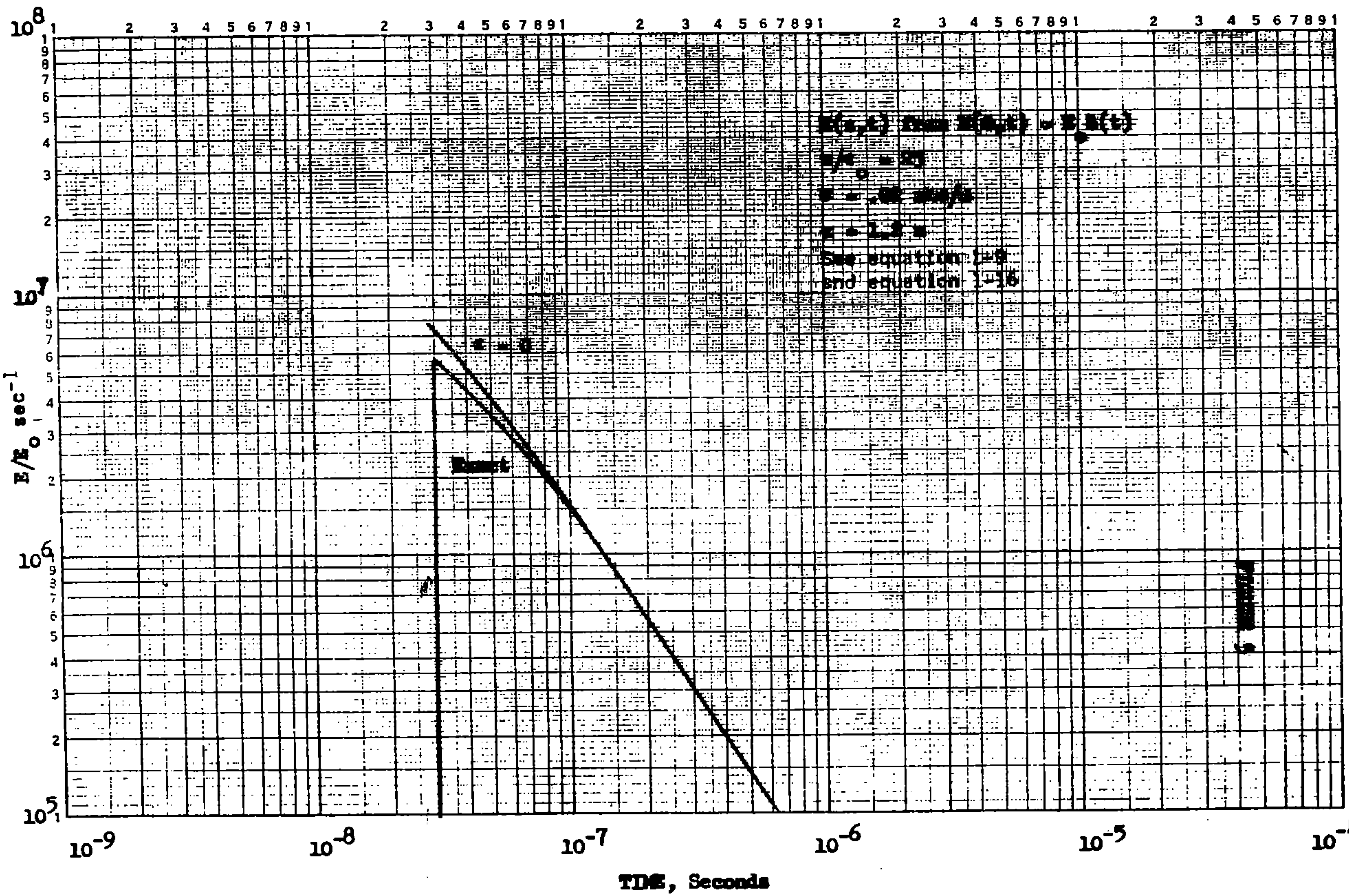


See equation I-17
$\tau^{-1/2}e^{-1/(4\tau)}$
$\tau = t/\mu\sigma z^2$
.01
.1
1
10
10^2
10^3
.001
.01
.1
1

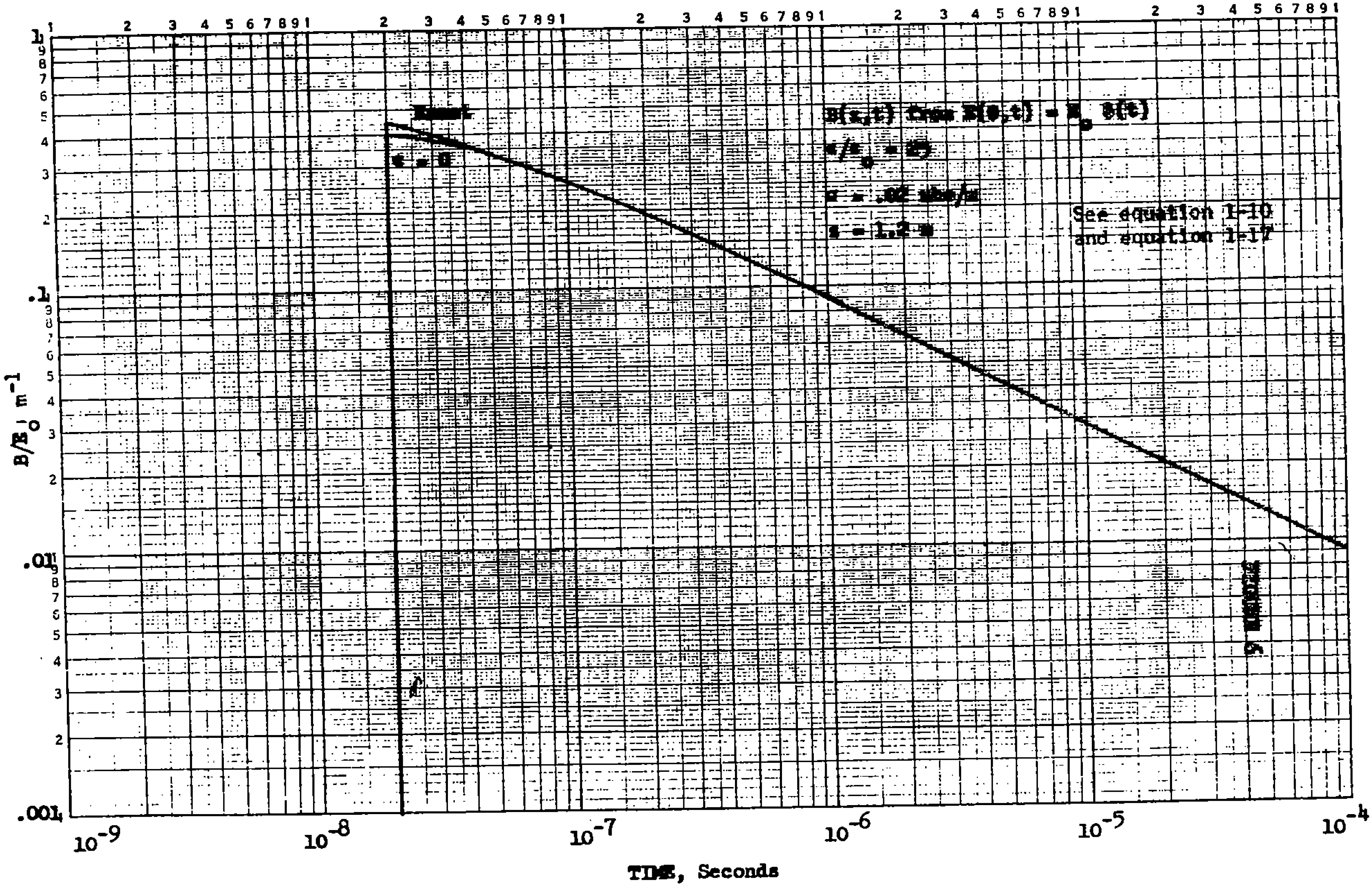


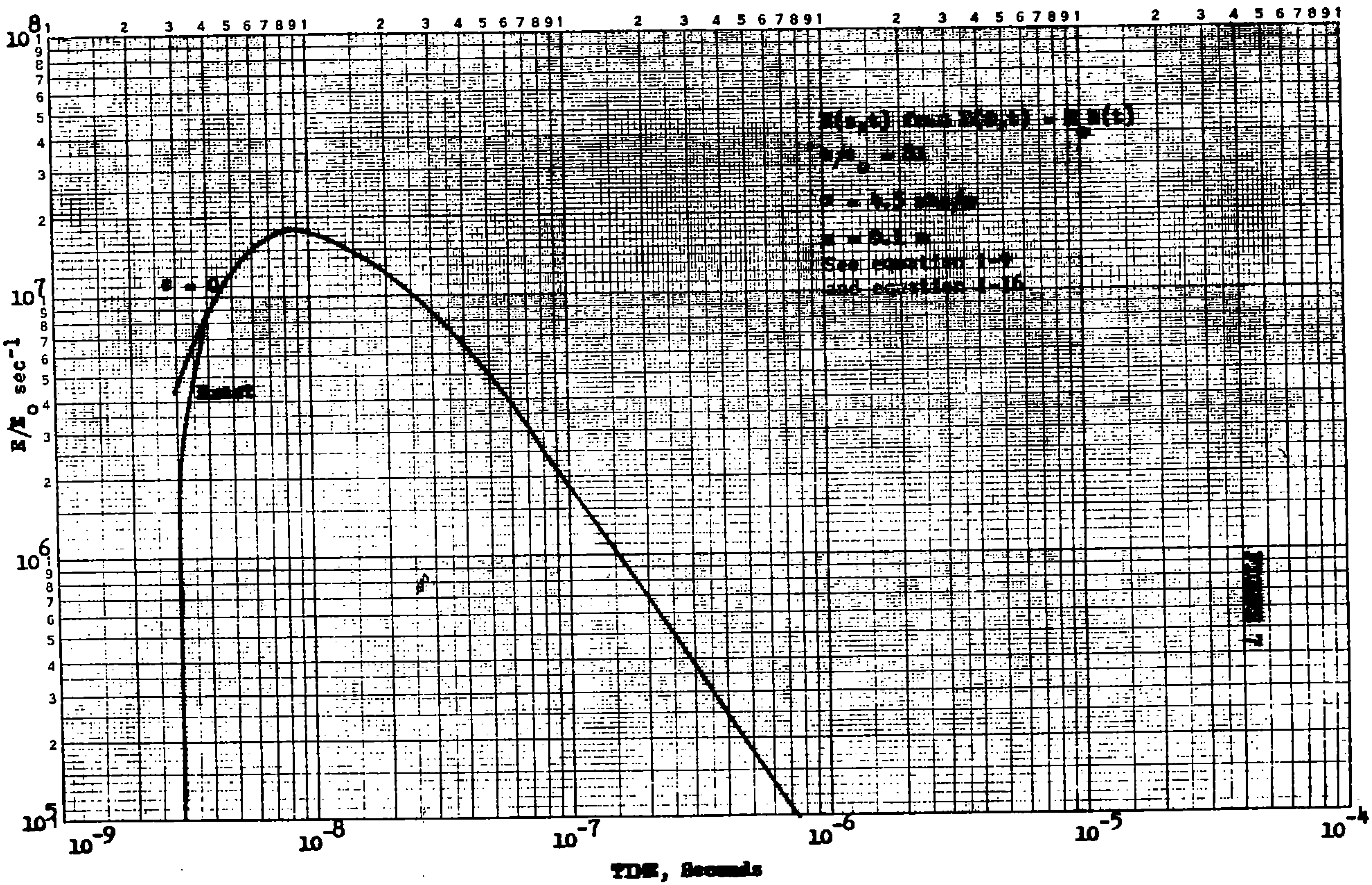
See equation 1-20
and equation 1-24
.01
.1
1
10
10^2
10^3
τ
.001
.01
.1
1
$\text{erfc}\left\{\frac{1}{2\sqrt{\tau}}\right\} - \text{erfc}\left\{\frac{1}{2\sqrt{(\tau-\theta)}}\right\}$


See equation 1-21
$\frac{2}{\sqrt{\pi}}\sqrt{\tau}\, e^{-1/(4\tau)} - \mathrm{erfc}\left\{\frac{1}{2\sqrt{\tau}}\right\}$
τ
.01
.1
1
10
10^2
10^3

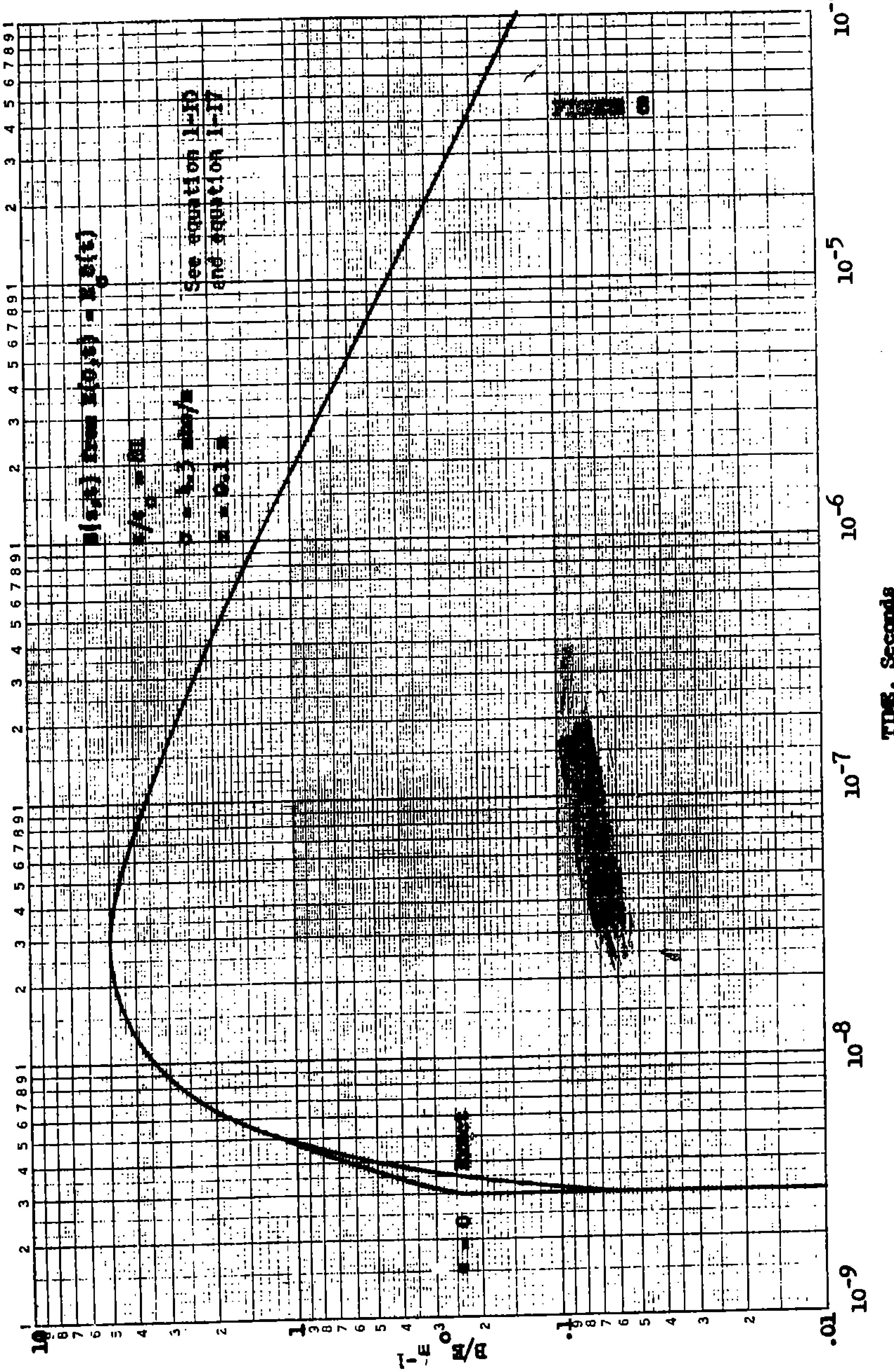


See equation 1-9
and equation 1-16
Exact
E/E_o sec^{-1}
TIME, Seconds


See equation 1-10
and equation 1-17
B/B_o, m^-1
TIME, Seconds
10^-9
10^-8
10^-7
10^-6
10^-5
10^-4
1
.1
.01
.001



10^{8}
10^{7}
10^{6}
10^{5}
E/E_o sec^{-1}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
TIME, Seconds


TIME, Seconds
B/E_o m^-1

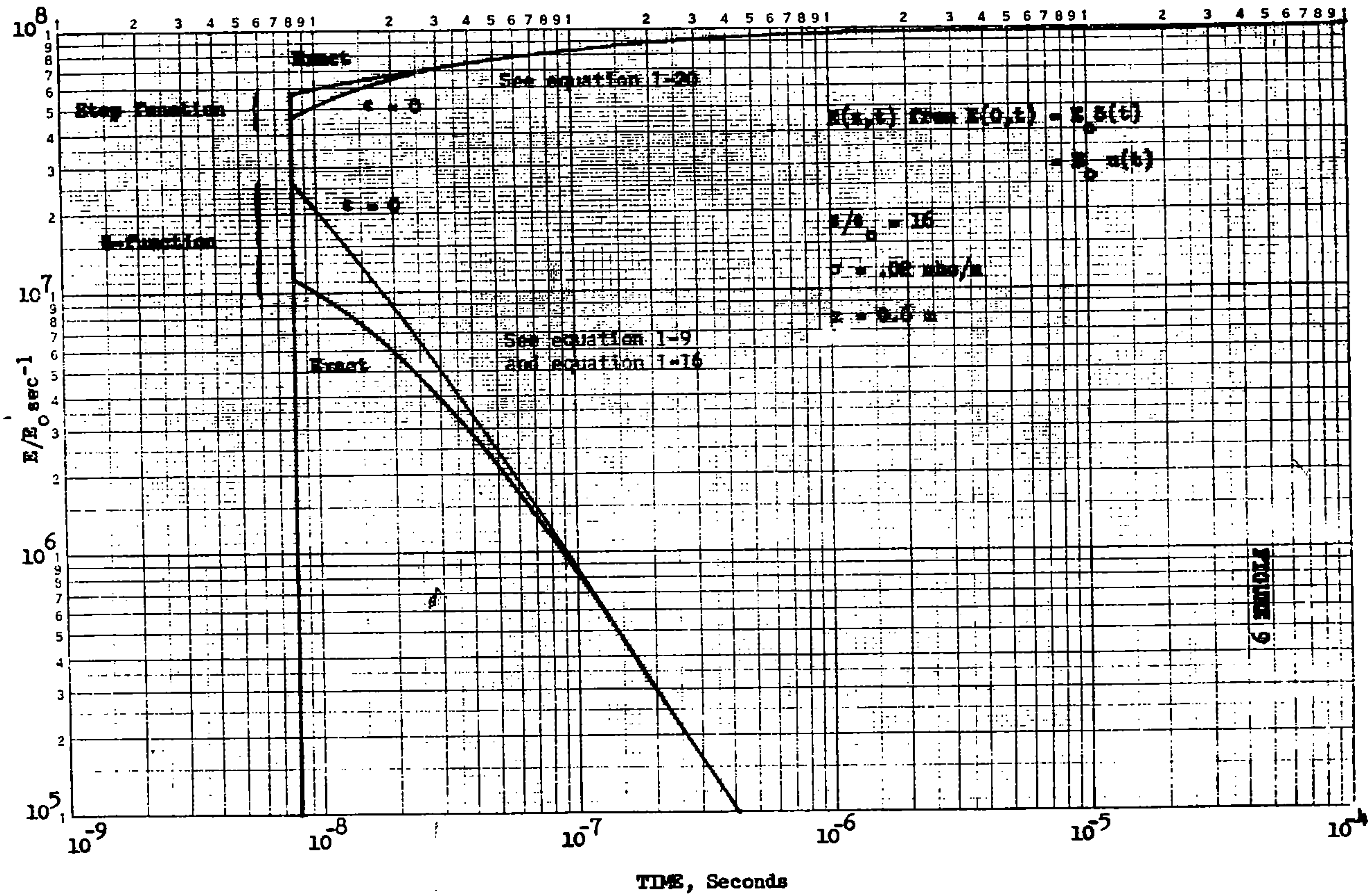


FIGURE 9
Exact
See equation 1-20
Step Function
ε = 0
δ-function
ε = 0
E(x,t) from E(0,t) = E$_o$ δ(t)
= E$_o$ u(t)
ε/ε$_o$ = 16
σ = .02 mho/m
x = 0.6 m
See equation 1-9
and equation 1-16
Exact
E/E$_o$ sec^{-1}
TIME, Seconds

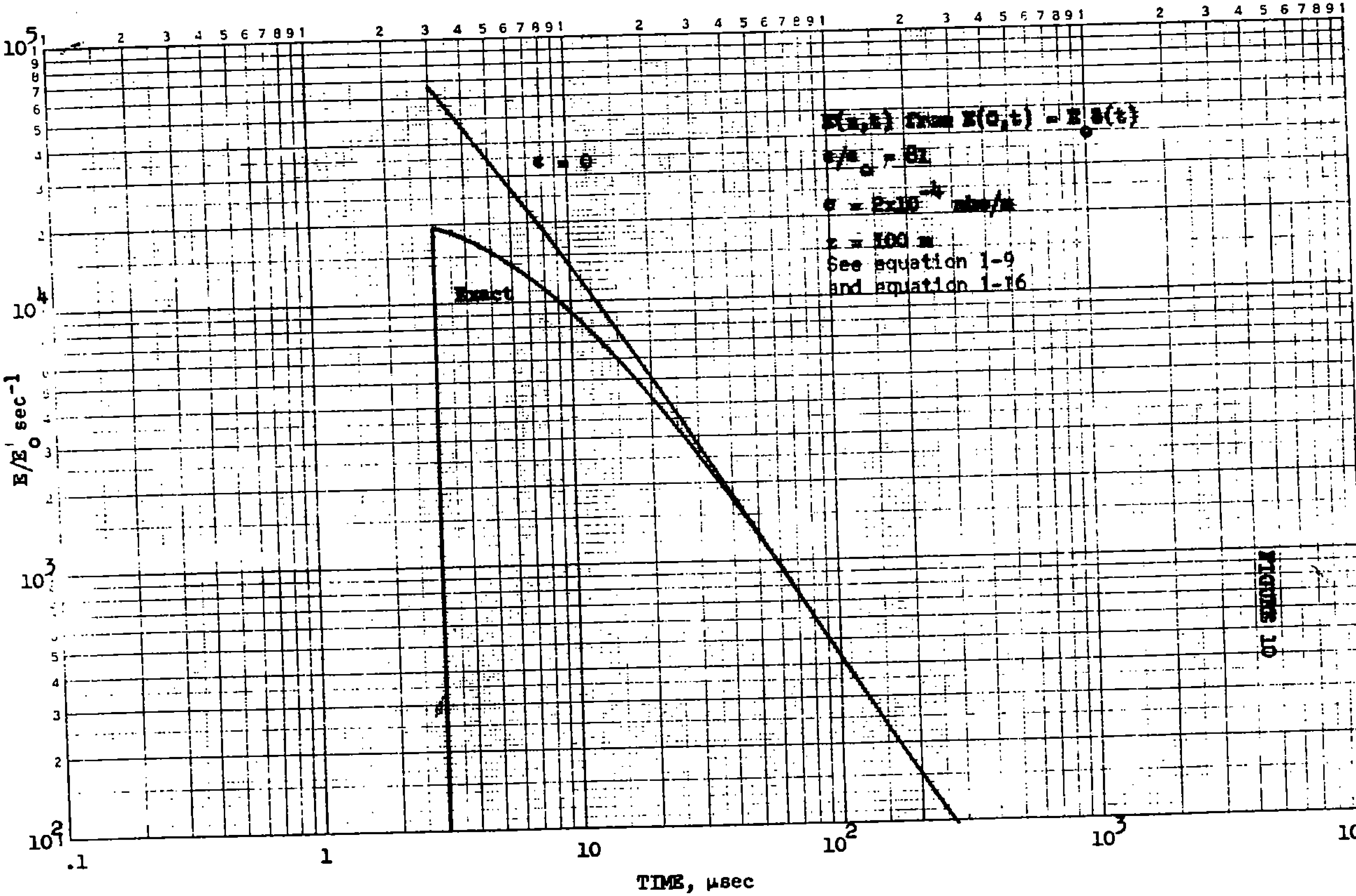


FIGURE 10

REFERENCES

1. J. R. Wait, Appl Sci Res 8B, 213 (1960)
2. B. K. Bhattacharyya, Geophysics, 22, 905 (1957)
3. C. R. Burrows, IEEE AP-11, 286 (1963)
4. Campbell and Foster, Fourier Integrals for Practical Applications, Van Nostrand Co., 1949
5. J. A. Stratton, Electromagnetic Theory, McGraw Hill, 1941

II. Electric and Magnetic Pulsed Field Correlation

Impedance boundary conditions may often be used in place of the exact boundary conditions to relate the tangential component of the fields (or alternately their normal components and normal derivatives) at the surface of a medium which has a high refractive index. These boundary conditions are usually attributed to M. A. Leontovich and are valid for surfaces whose radii of curvature are large compared to the penetration depth and also for inhomogeneous materials whose properties vary slowly compared to the penetration depth. They are accurate to first order in the reciprocal of the index of refraction. The proof of these conditions has been given by T. B. A. Senior[1] who collected the proofs by Leontovich and Fock and made them more readily accessible.

In terms of the parameter η defined by $\eta = \mu/\mu_o N$ where N is the index of refraction, or

$$\eta^{-1} = \sqrt{\frac{\mu_o}{\mu}\left[\frac{\varepsilon}{\epsilon_o} + \frac{\sigma}{i\omega\epsilon_o}\right]} \tag{2-1}$$

the conditions on the fields E and H are:

$$\frac{\partial E_n}{\partial_n} = - ik\eta E_n \tag{2-2}$$

$$\frac{\partial H_n}{\partial_n} = - \frac{ik}{\eta} H_n \tag{2-3}$$

$$\vec{E} - (\vec{n} \times \vec{E}) \vec{E} = + \eta Z \, \vec{n} \times \vec{H} \tag{2-4}$$

where $k = \sqrt{\epsilon_o \mu_o}\, \omega$

$Z = \sqrt{\frac{\mu_o}{\epsilon_o}}$

and n is a coordinate normal to the surface directed outward from the medium. The derivation uses the fact that the tangential gradients of the fields are of order N smaller than the normal gradient and are neglected. The restriction on the smallest radius of curvature is

$$\sqrt{\frac{\mu}{\mu_o} \frac{\sigma}{2\omega\epsilon_o}} \; k\rho \gg 1$$

The inhomogeneity restriction is:

$$\left| \frac{1}{K\eta} \nabla \eta \right| \ll 1$$

Taking the tangential electric field along the x-axis of a rectangular coordinate system with the z-axis directed upward then the tangential condition may be written as

$$E_x = -\eta Z H_y \qquad (2\text{-}5)$$

For the treatment of transient fields it is convenient to use the methods of the Laplace transform, and the Leontovich boundary conditions go over formally to the Laplace transforms with respect to time if $i\omega$ is replaced by the transform variable s. Taking $\gamma = \sqrt{\epsilon\mu s^2 + \mu\sigma s}$, the tangential magnetic field B_y in the medium is given by

$$\tilde{B}_y = -\frac{\gamma}{s}\tilde{E}_x \qquad (2\text{-}6)$$

where the tilde indicates Laplace transform with respect to time. Thus

$$B_y = -\sqrt{\epsilon\mu}\, L^{-1}\left\{ \frac{\sqrt{s(s+b)}}{s}\, \tilde{E}_x \right\} \qquad (2\text{-}7)$$

where $b = \sigma/\epsilon$

If E_x is given by a delta function $= E_0\delta(t)$

$$\tilde{E}_x = E_0 \qquad (2\text{-}8)$$

$$B_y = -\sqrt{\epsilon\mu}\, E_0 L^{-1}\left\{ \frac{\sqrt{s(s+b)}}{s} \right\}$$

$$= -\sqrt{\epsilon\mu}\, E_0 \left\{ \delta(t) + \frac{b}{2}\left[1 - \int_0^t e^{-(bt)/2}\, \frac{I_1\left[\frac{bt}{2}\right]}{t}\, dt \right] \right\} \qquad (2\text{-}9)$$

(These are just equations 1-10 and 1-11 for Z = 0, with propagation toward -Z hence the sign change)

If E_x is given by a step function, $E_x = E_1 u(t)$

$$\tilde{E}_x = E_1/s \qquad (2\text{-}10)$$

$$B_y = -\sqrt{\epsilon\mu}\, E_1\, L^{-1}\left\{ \frac{\sqrt{s(s+b)}}{s^2} \right\}$$

$$= -\sqrt{\epsilon\mu}\, E_1 \left[1 + \frac{bt}{2} - \int_0^{\frac{bt}{2}} \int_0^{\frac{b\xi}{2}} e^{-\zeta}\, \frac{I_1(\zeta)}{\zeta}\, d\zeta\, d\xi \right] \qquad (2\text{-}11)$$

(Which is equation 1-10 with Z = 0)

These are given in Figure 11.

For the important case of high conductivity $\sigma/\epsilon \gg s$ these simplify considerably.

δ-function electric field

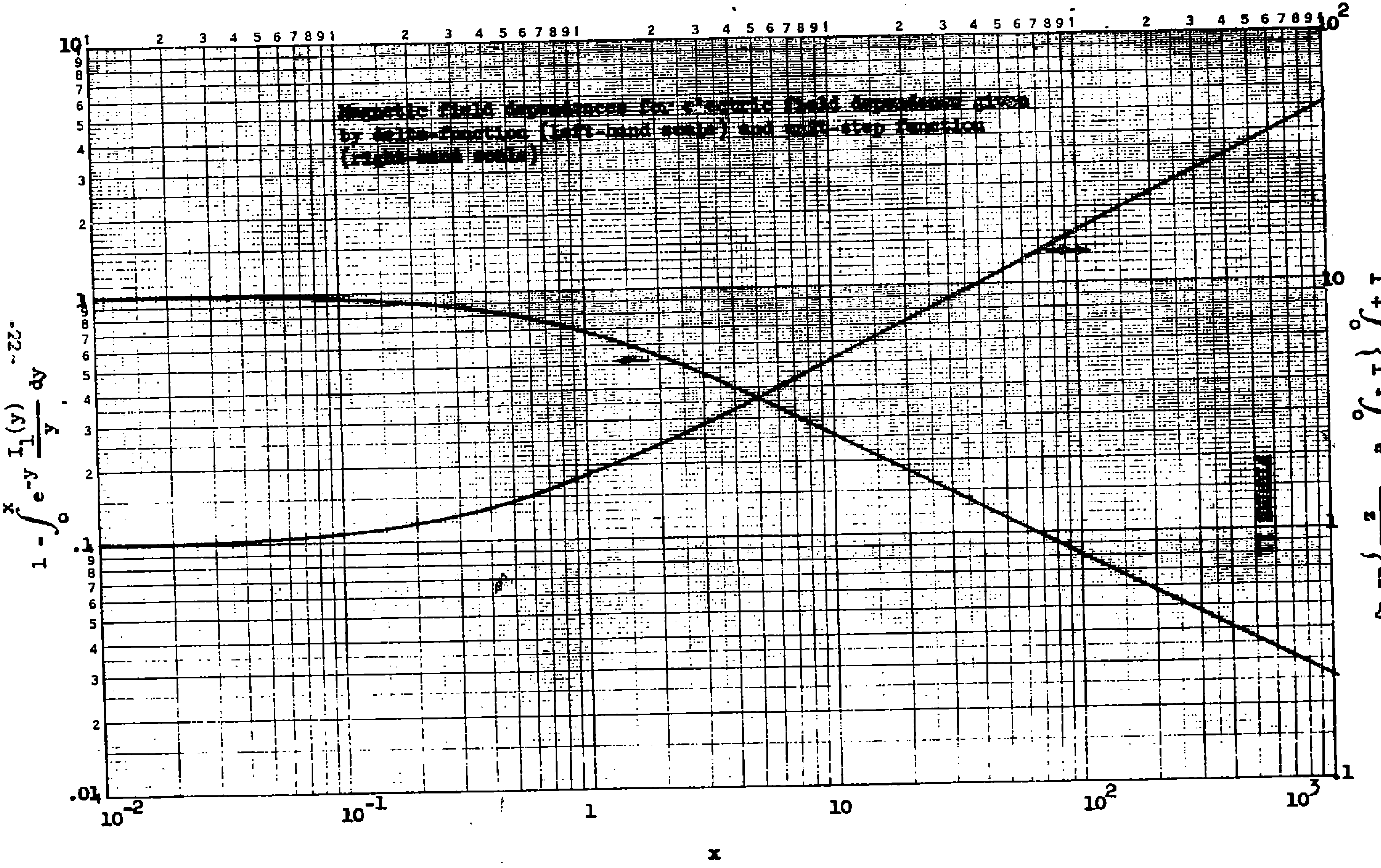
$$B_y = -E_0 \sqrt{\frac{\mu\sigma}{\pi t}} \quad \text{(which is eqn. 1-17 with Z = 0)} \qquad (2\text{-}12)$$

Step function electric field

$$B_y = -2E_1 \sqrt{\frac{\mu\sigma t}{\pi}} \quad \text{(which is eqn. 1-21 with Z = 0)} \qquad (2\text{-}13)$$

REFERENCES

1. T. B. A. Senior, Appli. Sci. Res., B 8, 418 (1960)



Magnetic field dependences for electric field dependence given by delta-function (left-hand scale) and unit-step function (right-hand scale)

APPENDIX

The Laplace transform pairs required to invert the transforms involved are not easily available and in some cases seem incomplete; consequently it seems desirable to present a formal derivation of the transform pairs used. Starting from the well-known transform pair*:

$$L^{-1}\left\{\frac{e^{-k\sqrt{s(s+b)}}}{\sqrt{s(s+b)}}\right\} = e^{-(bt)/2}\, I_o\left[\frac{b}{2}\sqrt{t^2-k^2}\right] u(t-k) \qquad \text{(A-1)}$$

where $I_o(x)$ is the modified Bessel function of order zero and $u(x)$ is the unit step function defined by

$$u(x) = 0 \quad x < 0$$
$$= 1 \quad x > 0$$

Integrating with respect to k from 0 to k

$$L^{-1}\left\{\frac{e^{-k\sqrt{s(s+b)}}}{s(s+b)} - \frac{1}{s(s+b)}\right\}$$

$$= e^{-(bt)/2}\int_o^k I_o\left[\frac{b}{2}\sqrt{t^2-k^2}\right] dk \quad t > k \qquad \text{(A-2)}$$

Differentiating the original pair after k and using $\frac{d}{dx} I_o(x) = I_1(x)$

$$L^{-1}\left\{e^{-k\sqrt{s(s+b)}}\right\} = \frac{bk}{2}\, e^{-(bt)/2}\, \frac{I_1\left[\frac{b}{2}\sqrt{t^2-k^2}\right]}{\sqrt{t^2-k^2}}\, u(t-k)$$

$$+\, e^{-(bt)/2}\,\delta(t-k) \qquad \text{(A-3)}$$

$I_1(x)$ is the modified Bessel function of order one.

Since

$$L\left\{e^{-(bt)/2}\,\delta(t-k)\right\} = e^{-k(s+\frac{b}{2})}$$

the above may be written as

$$L^{-1}\left\{e^{-k\sqrt{s(s+b)}} - e^{-k(s+\frac{b}{2})}\right\}$$
$$= \frac{bk}{2}\, e^{-(bt)/2}\, \frac{I_1\left[\frac{b}{2}\sqrt{t^2-k^2}\right]}{\sqrt{t^2-k^2}} \qquad \text{(A-4)}$$

---

* Pair 88 by R. V. Churchill from the C.R.C. Standard Math. Tables

(This is also the result of using pair 863.1 of Campbell and Foster[1]).

Using the operation

$$L^{-1}\left\{\frac{f(s)}{s}\right\} = \int_0^t F(\tau)d\tau$$

where $f(s) = L\left\{F(t)\right\}$

$$L^{-1}\left\{\frac{e^{-k\sqrt{s(s+b)}}}{s} - \frac{e^{-k(s+\frac{b}{2})}}{s}\right\} = \frac{bk}{2}\int_0^t e^{-(bt)/2}\,\frac{I_1\left[\frac{b}{2}\sqrt{t^2-k^2}\right]}{\sqrt{t^2-k^2}}\,dt \qquad t > k \qquad \text{(A-5)}$$

Differentiating this after k and using

$$\frac{\partial}{\partial x}\int_x^t f(x,\tau)\,d\tau = \int_x^t \frac{\partial f(x,\tau)}{\partial x}\,d\tau - f(x,x)$$

$$L^{-1}\left\{\frac{\sqrt{s(s+b)}}{s}\,e^{-k\sqrt{s(s+b)}} - \frac{(s+\frac{b}{2})\,e^{-k(s+\frac{b}{2})}}{s}\right\}$$

$$= -\frac{b}{2}\int_k^t e^{-(bt)/2}\,\frac{I_1\left[\frac{b}{2}\sqrt{t^2-k^2}\right]}{\sqrt{t^2-k^2}}\,dt$$

$$-\frac{bk}{2}\int_k^t e^{-(bt)/2}\,\frac{\partial}{\partial k}\left[\frac{I_1\left(\frac{b}{2}\sqrt{t^2-k^2}\right)}{\sqrt{t^2-k^2}}\right]dt$$

$$+\frac{bk}{2}\,e^{-(bt)/2}\,\frac{I_1\left(\frac{b}{2}\sqrt{t^2-k^2}\right)}{\sqrt{t^2-k^2}}\Bigg|_{t=k} \qquad t > k \qquad \text{(A-6)}$$

$$= \frac{b^2k}{8}\,e^{-(bk)/2} - \frac{b}{2}\int_k^t e^{-(bt)/2}\,\frac{I_1\left(\frac{b}{2}\sqrt{t^2-k^2}\right)}{\sqrt{t^2-k^2}}\,dt$$

$$+\frac{b^4k^2}{16}\int_k^t e^{-(bt)/2}\left[\frac{I_o\left(\frac{b}{2}\sqrt{t^2-k^2}\right)}{\left(\frac{b}{2}\sqrt{t^2-k^2}\right)^2} - \frac{2I_1\left(\frac{b}{2}\sqrt{t^2-k^2}\right)}{\left(\frac{b}{2}\sqrt{t^2-k^2}\right)^3}\right]dt \qquad \text{(A-7)}$$

But

$$L^{-1}\left\{\left(1 + \frac{b}{2s}\right) e^{-k(s + \frac{b}{2})}\right\} = e^{-(bk)/2}\, \delta(t - k) + \frac{b}{2} e^{-(bk)/2}\, u(t - k)$$

Hence

$$L^{-1}\left\{\frac{\sqrt{s(s + b)}}{s} e^{-k\sqrt{s(s + b)}}\right\} = e^{-(bk)/2}\, \delta(t - k)$$

$$+ \left\{\left(\frac{b}{2} + \frac{b^2 k}{8}\right) e^{-(bk)/2} - \frac{b}{2}\int_k^t e^{-(bt)/2} \frac{I_1\left(\frac{b}{2}\sqrt{t^2 - k^2}\right)}{\sqrt{t^2 - k^2}}\, dt\right.$$

$$+ \frac{b^4 k^2}{16}\int_k^t e^{-(bt)/2}\left[\frac{I_0\left(\frac{b}{2}\sqrt{t^2 - k^2}\right)}{\left(\frac{b}{2}\sqrt{t^2 - k^2}\right)^2}\right.$$

$$\left.\left. - \frac{2I_1\left(\frac{b}{2}\sqrt{t^2 - k^2}\right)}{\left(\frac{b}{2}\sqrt{t^2 - k^2}\right)^3}\right] dt\right\} u(t - k) \qquad \text{(A-8)}$$

or with k = 0

$$L^{-1}\left\{\frac{\sqrt{s(s + b)}}{s}\right\} = \delta(t) + \frac{b}{2}\left[1 - \int_0^t e^{-(bt)/2} \frac{I_1\left[\frac{bt}{2}\right]}{t}\, dt\right] u(t) \qquad \text{(A-9)}$$

$$L^{-1}\left\{\frac{\sqrt{s(s + b)}}{s^2}\right\} = \left[1 + \frac{bt}{2} - \int_0^{(bt)/2} dx \int_0^{(bx)/2} e^{-y} \frac{I_1(y)}{y}\, dy\right] u(t) \qquad \text{(A-10)}$$

REFERENCES

1. Campbell and Foster, Fourier Integrals for Practical Applications, Van Nostrand Co., 1949

2. C.R.C. Standard Mathematical Tables, page 326, Laplace Transforms, taken from 'Modern Operational Mathematics in Engineering' by R. V. Churchill

III. Plots of the Electric and Magnetic Fields Generated by a Delta Function and Step Function Electric Field, in Dimensionless Form.

The ordinates for the plots are:

Plot	Ordinate	
δ-Function Electric Field		
E_x	$E_x \tau / E_o$	(see equation 1-9)
B_y	$B_y z / E_o$	(see equation 1-11)
Step-Function Electric Field		
E_x	E_x / E_1	(see equation 1-20)
B_y	$B_y z / \tau E_1$	(see equation 1-21)

Key to the variables in the following plots

E_o is Impluse Field in volt-sec/meter

E_1 is value of step field in volts/meter

$b = \sigma/\varepsilon$

$k = \sqrt{\varepsilon\mu}\; z$

$a = bk = \sqrt{\frac{\mu}{\varepsilon}}\; \sigma z$

$T = bk^2 = \mu\sigma z^2$

$w = t/k = at/\tau$

Summary page for the graphs.

For δ-function electric field at z=0

$$\frac{E_x \tau}{E_o} = a e^{-a/2} \; \delta(x-1)$$

$$+ \frac{a^2}{2} \frac{e^{-ax/2}}{\sqrt{x^2-1}} \; I_1 \left(\frac{a}{2} \sqrt{x^2-1}\right) \; \mu(x-1)$$

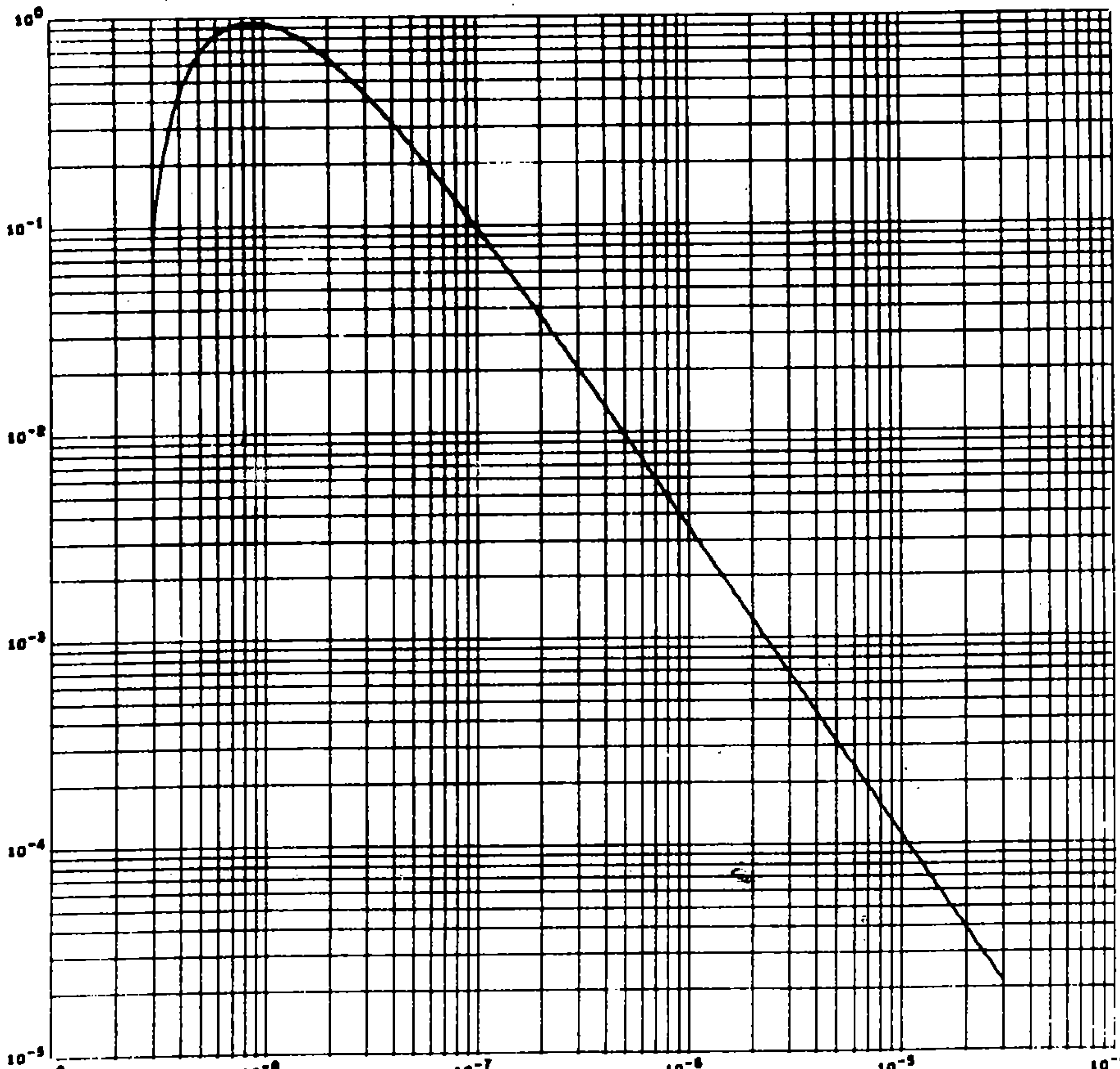
$$\frac{B_y z}{E_o} = a e^{-a/2} \; \delta(x-1)$$

$$+ \left\{ \frac{a}{2} \left(1 + \frac{a}{4}\right) e^{-a/2} + \left(\frac{a}{2}\right)^4 \int_1^x d_y \left[\frac{I_o \left(\frac{a}{2} \sqrt{y^2-1}\right)}{\left(\frac{a}{2} \sqrt{y^2-1}\right)^2} - \frac{(y^2+1) I_1 \left(\frac{a}{2} \sqrt{y^2-1}\right)}{\left(\frac{a}{2} \sqrt{y^2-1}\right)^3} \right] \right\} \mu(x-1)$$

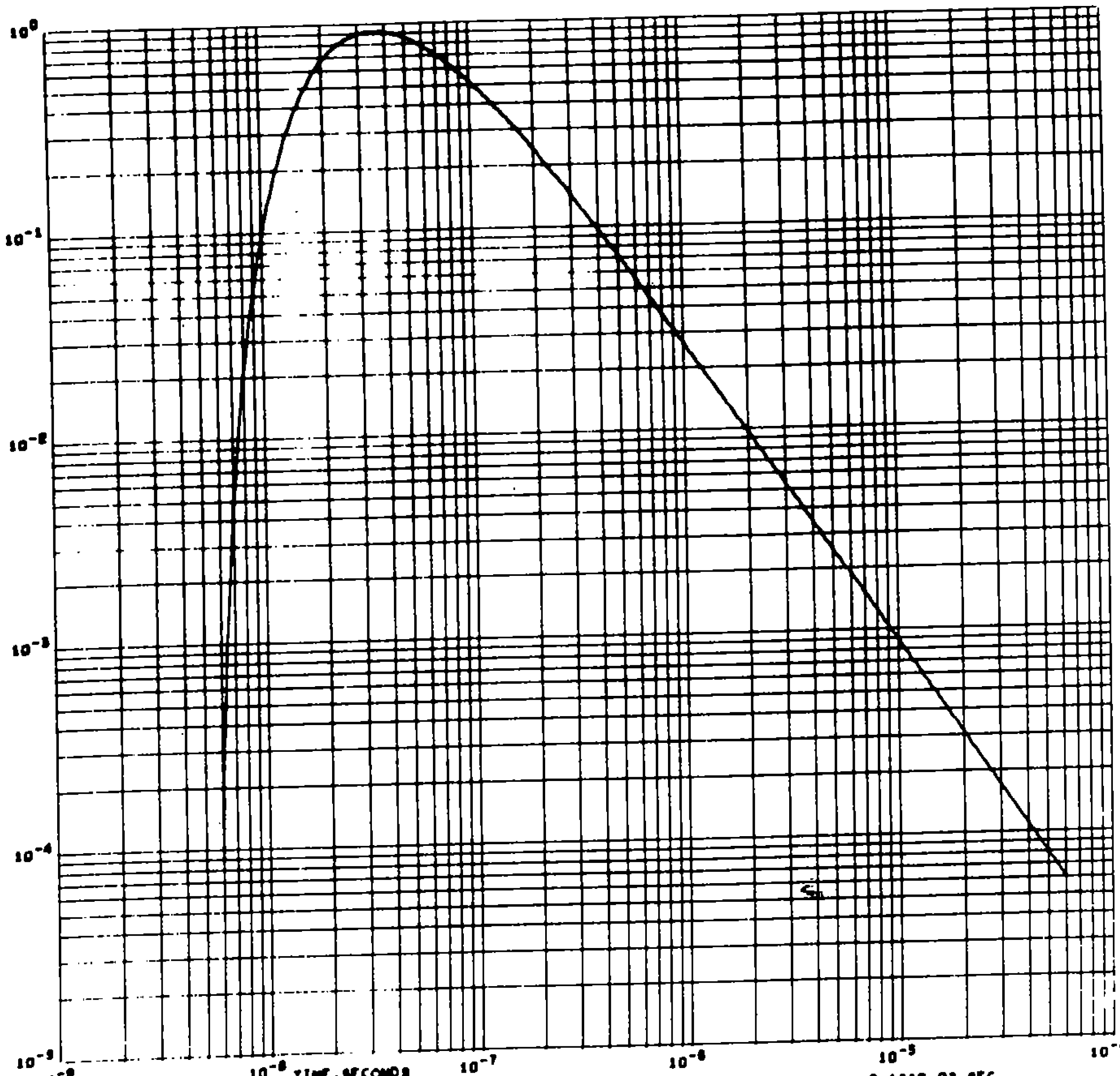
For large w

$$\frac{E_x \tau}{E_o} = \frac{1}{2\sqrt{\pi}} \frac{e^{-T/4t}}{(t/T)^{3/2}}$$

$$\frac{B_y z}{E_o} = \frac{1}{\sqrt{\pi}} \frac{e^{-T/4t}}{(t/T)^{1/2}}$$

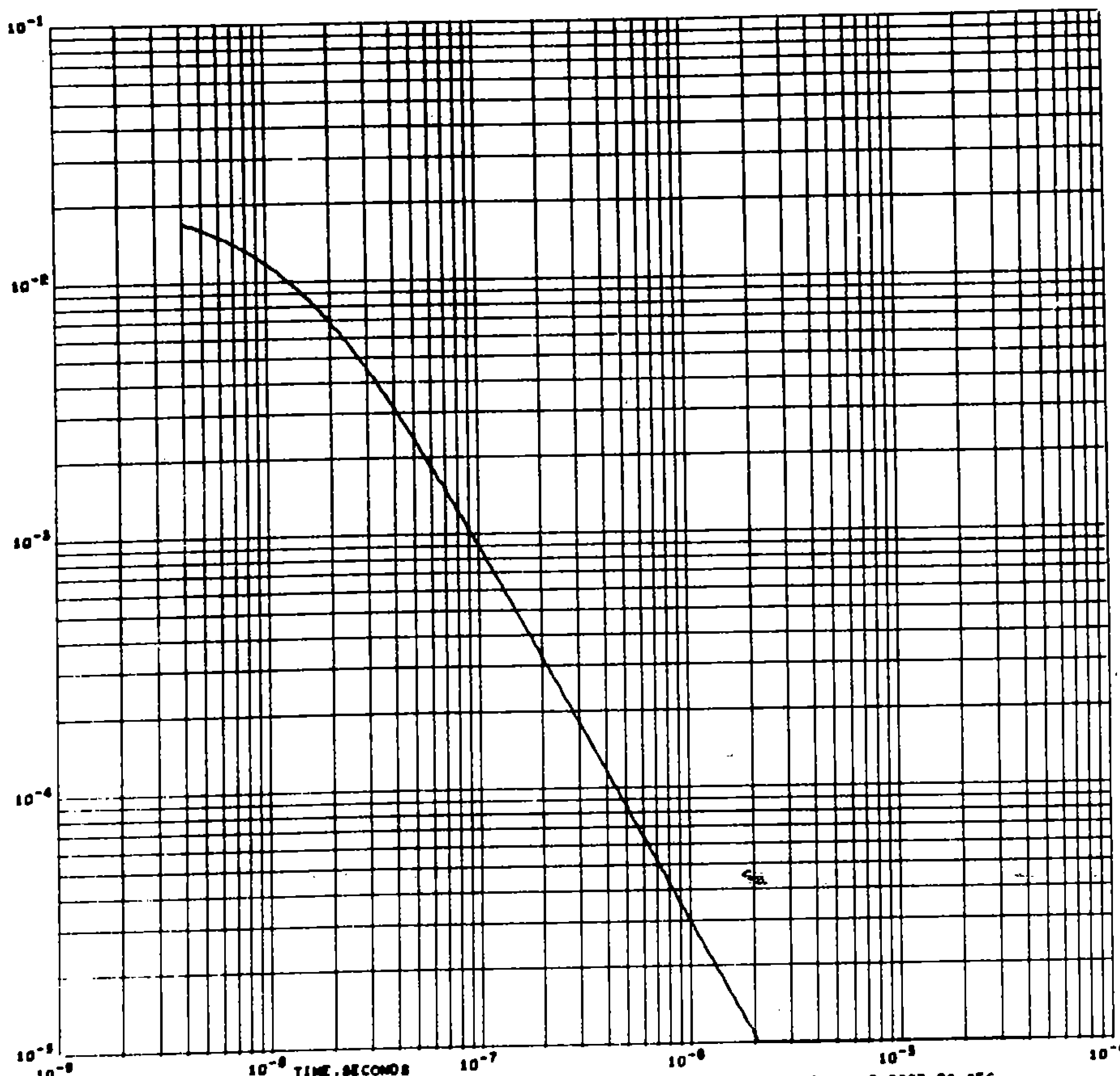


EMP PROPAGATION, (J-13-459)
Z = 0.10 METERS SIGMA = 4.3000 MHOS/METER
K = 3.000E-08 SEC MU = 1.000
TAU = 5.404E-08 SEC EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T



K = 6.000E-09 SEC
MU = 1.000
TAU = 2.161E-07 SEC
EPSILON = 81.000

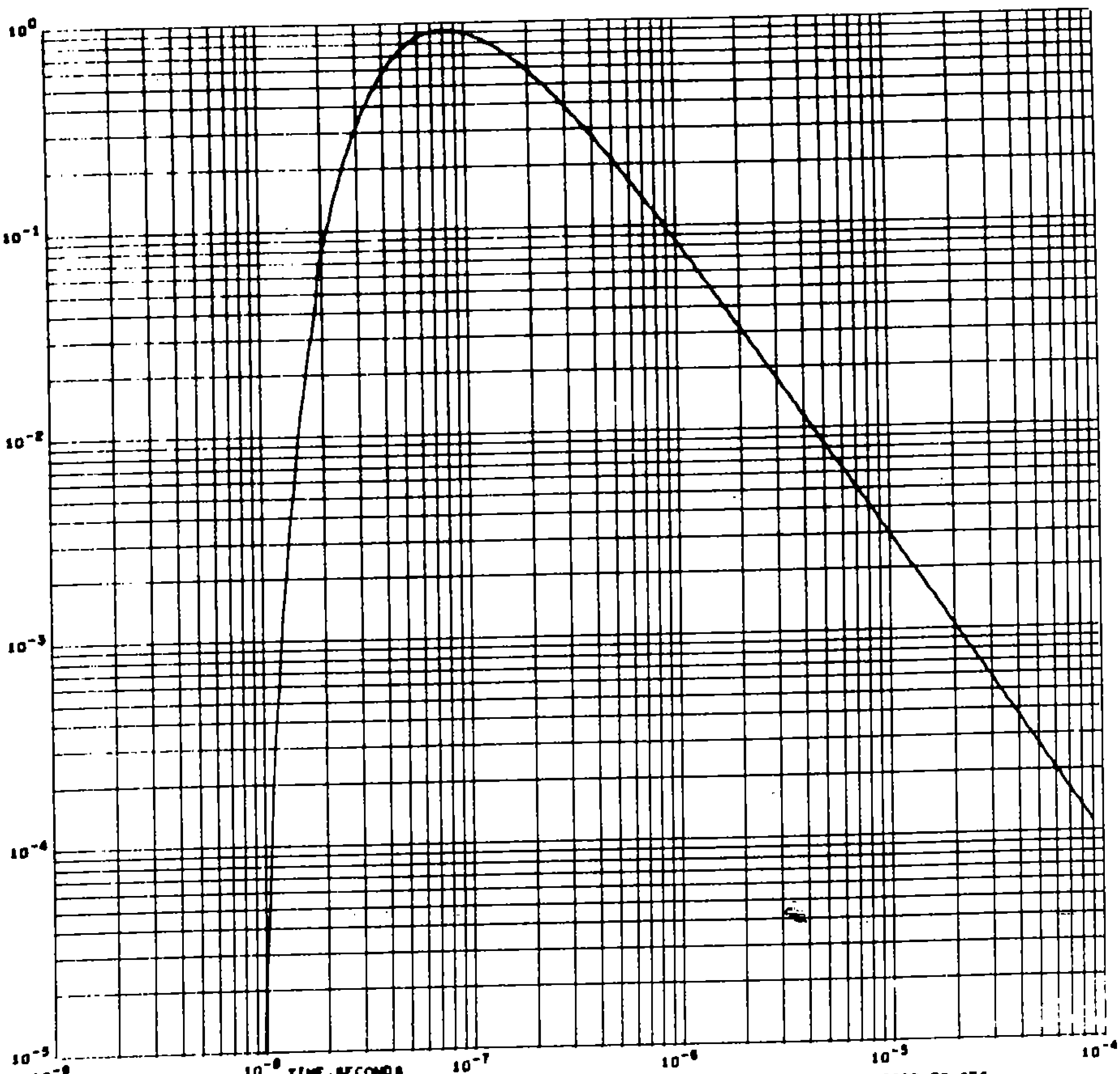
EMP PROPAGATION, (J-13-458)
Z = 0.20 METERS
SIGMA = 4.3000 MHOS/METER
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T



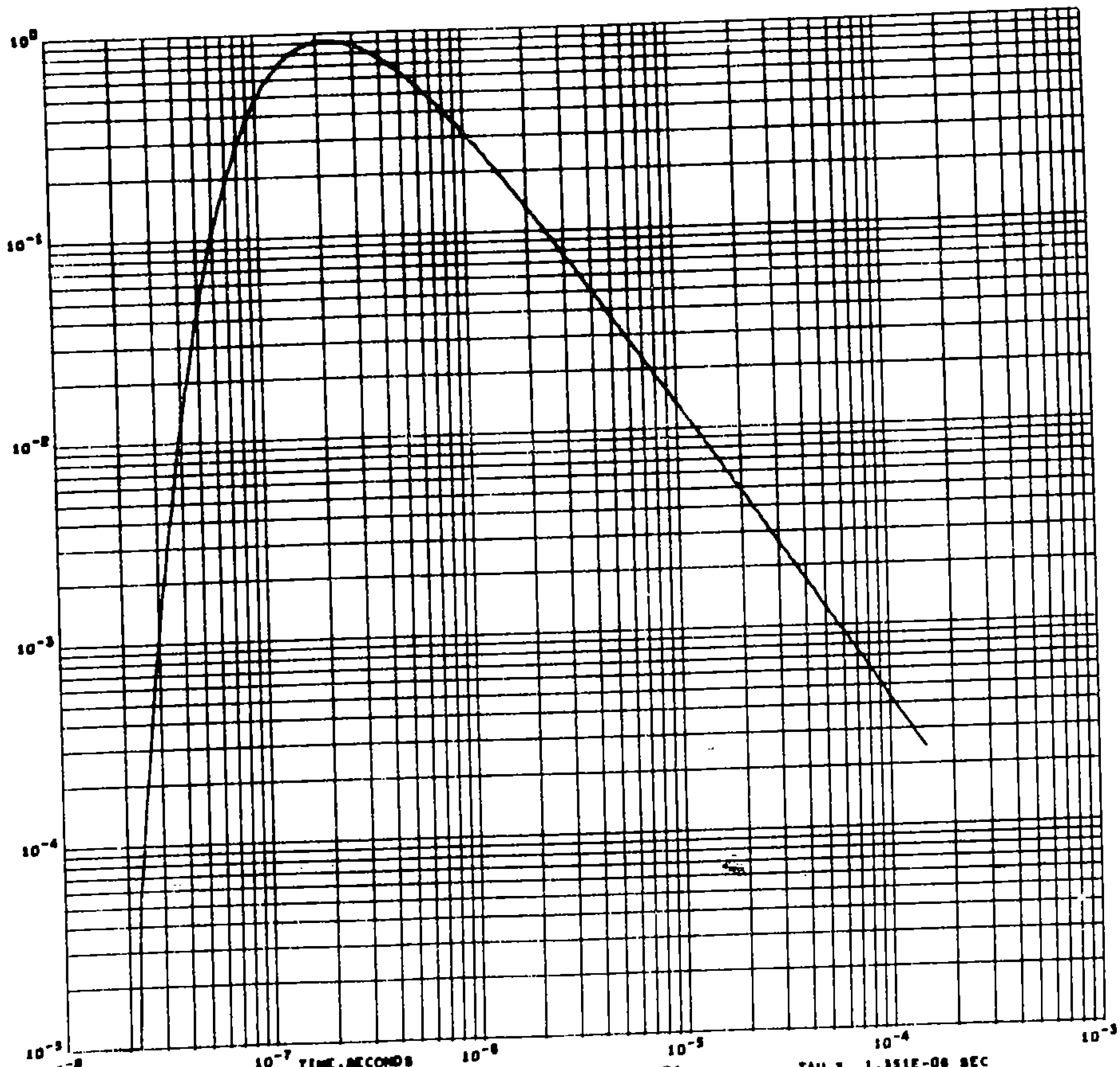
EMP PROPAGATION, (J-13-459)
Z = 0.30 METERS SIGMA = 0.0200 MHOS/METER
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T

K = 4.000E-09 SEC
MU = 1.000

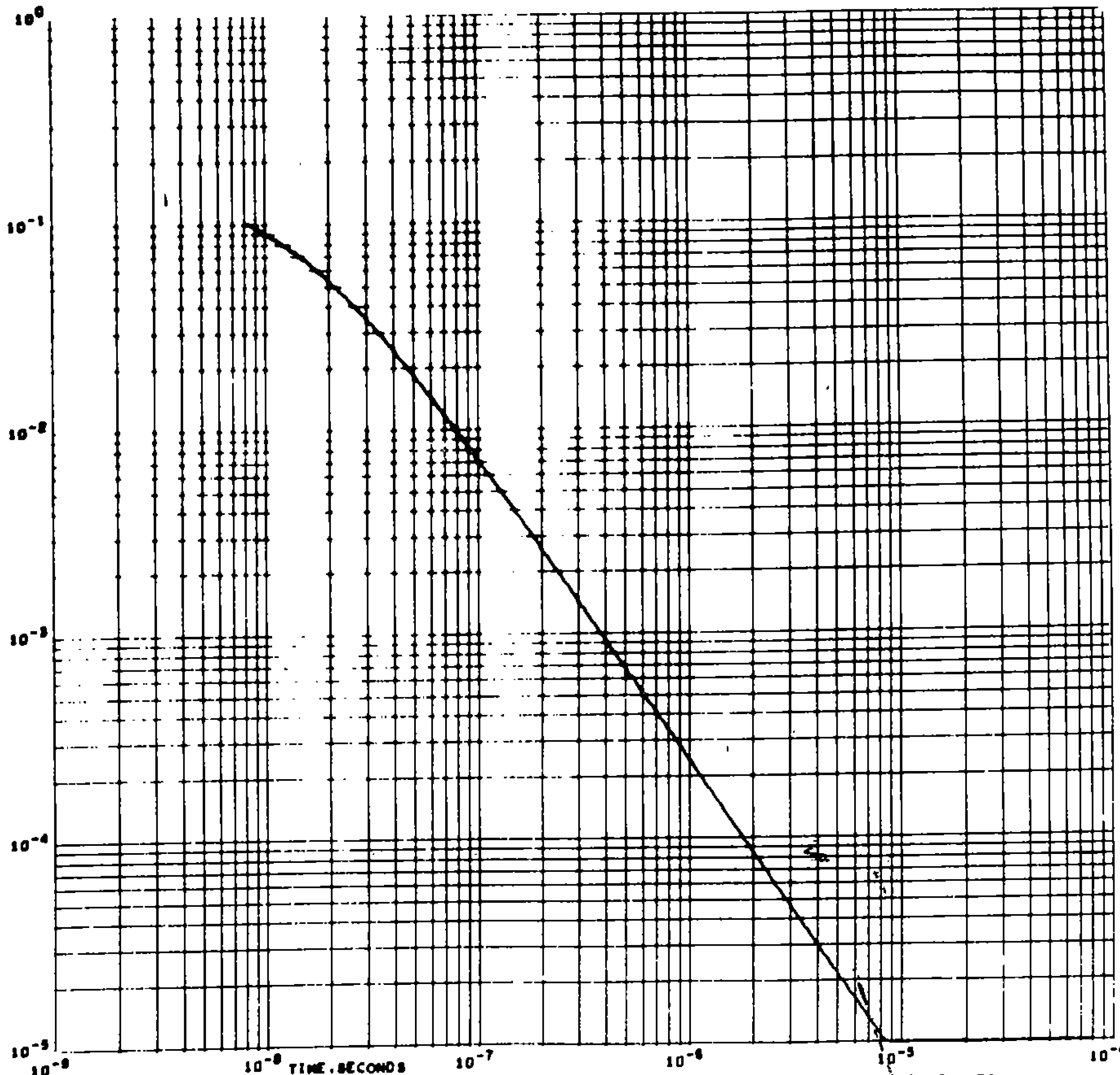
TAU = 2.262E-09 SEC
EPSILON = 16.000



EMP PROPAGATION, (J-13-459)
Z = 0.30 METERS SIGMA = 4.3000 MHOS/METER
K = 9.000E-09 SEC
MU = 1.000
TAU = 4.863E-07 SEC
EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T



EMP PROPAGATION, (J-13-459)
Z = 0.50 METERS
SIGMA = 4.3000 MHOS/METER
K = 1.300E-08 SEC
MU = 1.000
TAU = 1.351E-06 SEC
EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T



EMP PROPAGATION, (J-13-459)

Z = 0.00 METERS SIGMA = 0.0200 MHOS/METER

K = 8.000E-09 SEC

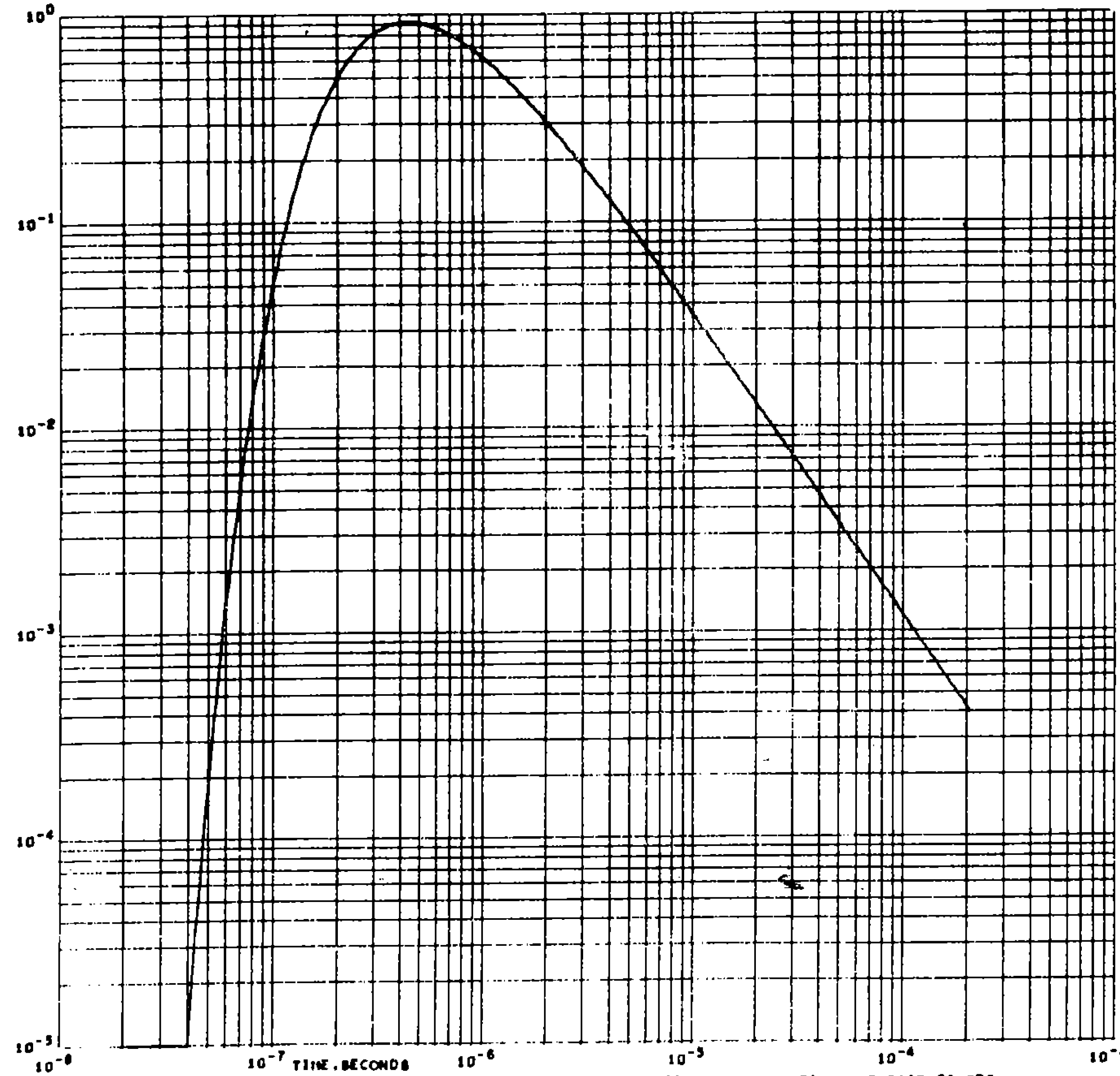
MU = 1.000

TAU = 9.046E-09 SEC

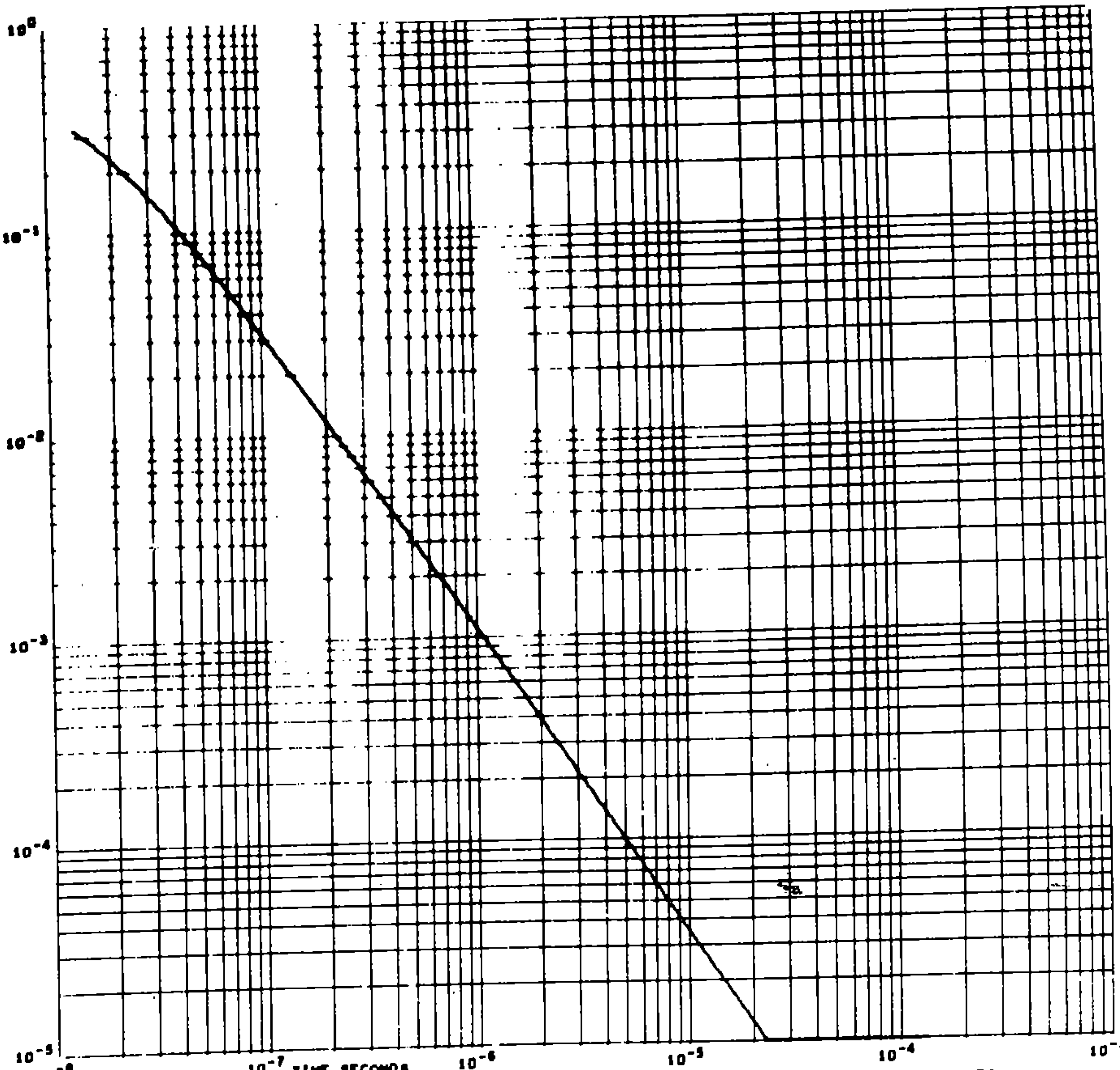
EPSILON = 16.000

DELTA FUNCTION ELECTRIC FIELD AT Z=0

E VS T



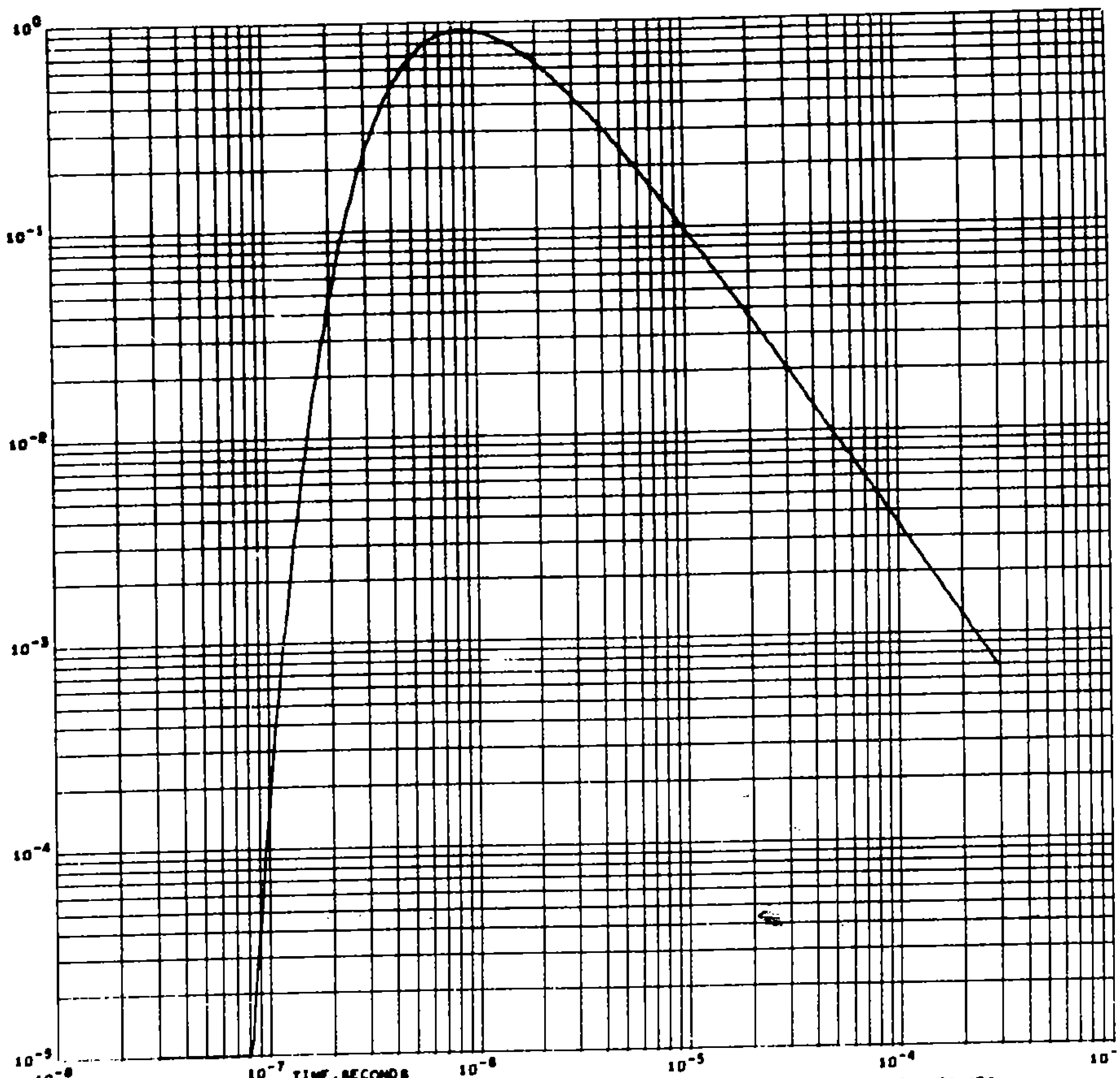
EMP PROPAGATION. (J-15-459)
Z = 0.70 METERS SIGMA = 4.3000 MHOS/METER K = 2.100E-08 SEC TAU = 2.640E-06 SEC
MU = 1.000 EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T



EMP PROPAGATION, (J-13-498)
Z = 1.00 METERS   SIGMA = 0.0200 MHOS/METER
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T

K = 1.333E-08 SEC
MU = 1.000

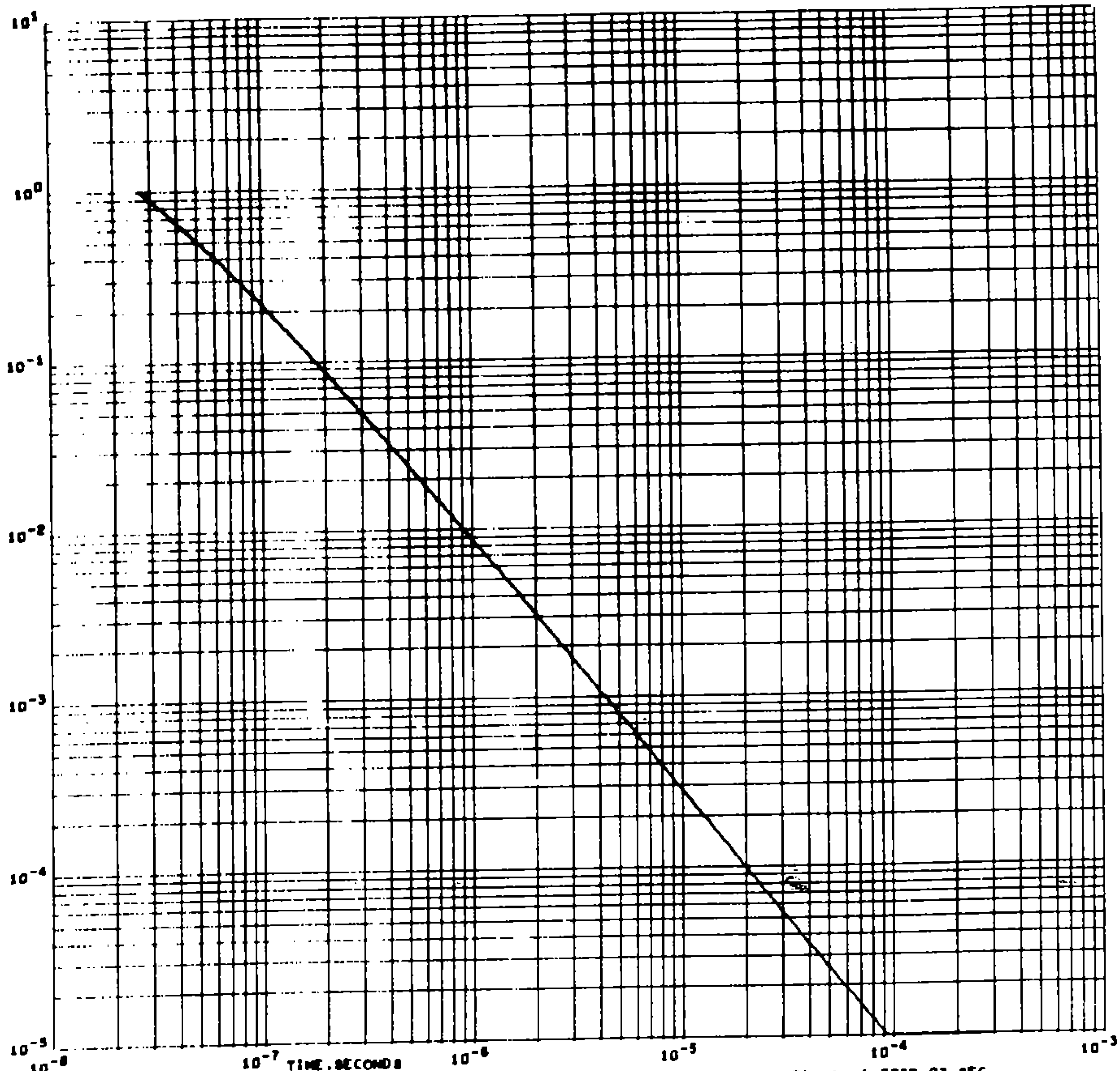
TAU = 2.513E-08 SEC
EPSILON = 16.000



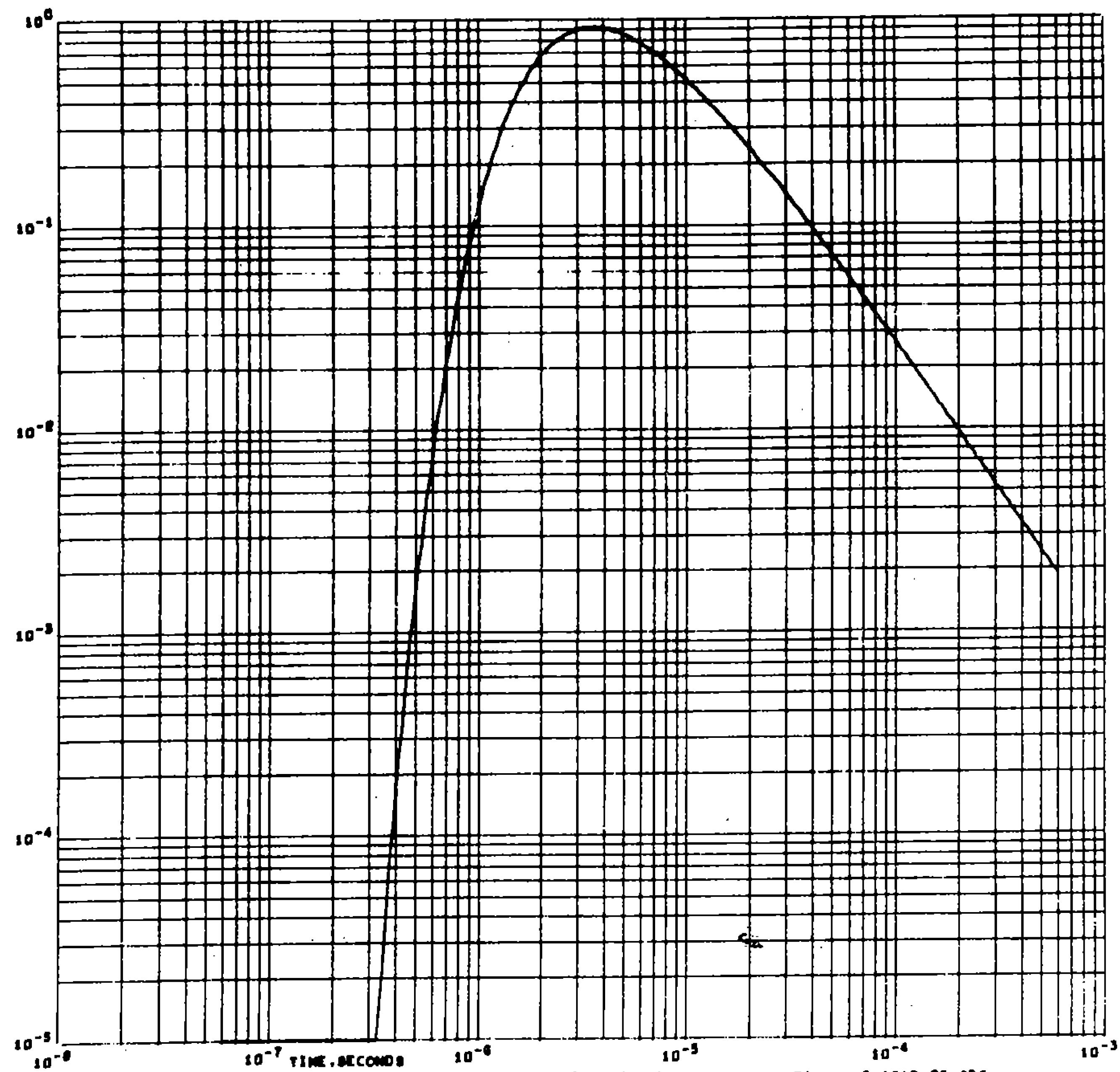
EMP PROPAGATION. (J-13-459)
Z = 1.00 METERS SIGMA = 4.3000 MHOS/METER
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T

R = 3.000E-08 SEC
MU = 1.000

TAU = 5.404E-06 SEC
EPSILON = 81.000



EMP PROPAGATION. (J-13-458)
Z = 2.00 METERS
SIGMA = 0.0200 MHOS/METER
K = 2.667E-08 SEC
MU = 1.000
TAU = 1.005E-07 SEC
EPSILON = 16.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T



EMP PROPAGATION, (J-13-459)
Z = 2.00 METERS SIGMA = 4.3000 MHOS/METER
K = 8.000E-08 SEC
MU = 1.000
TAU = 2.161E-05 SEC
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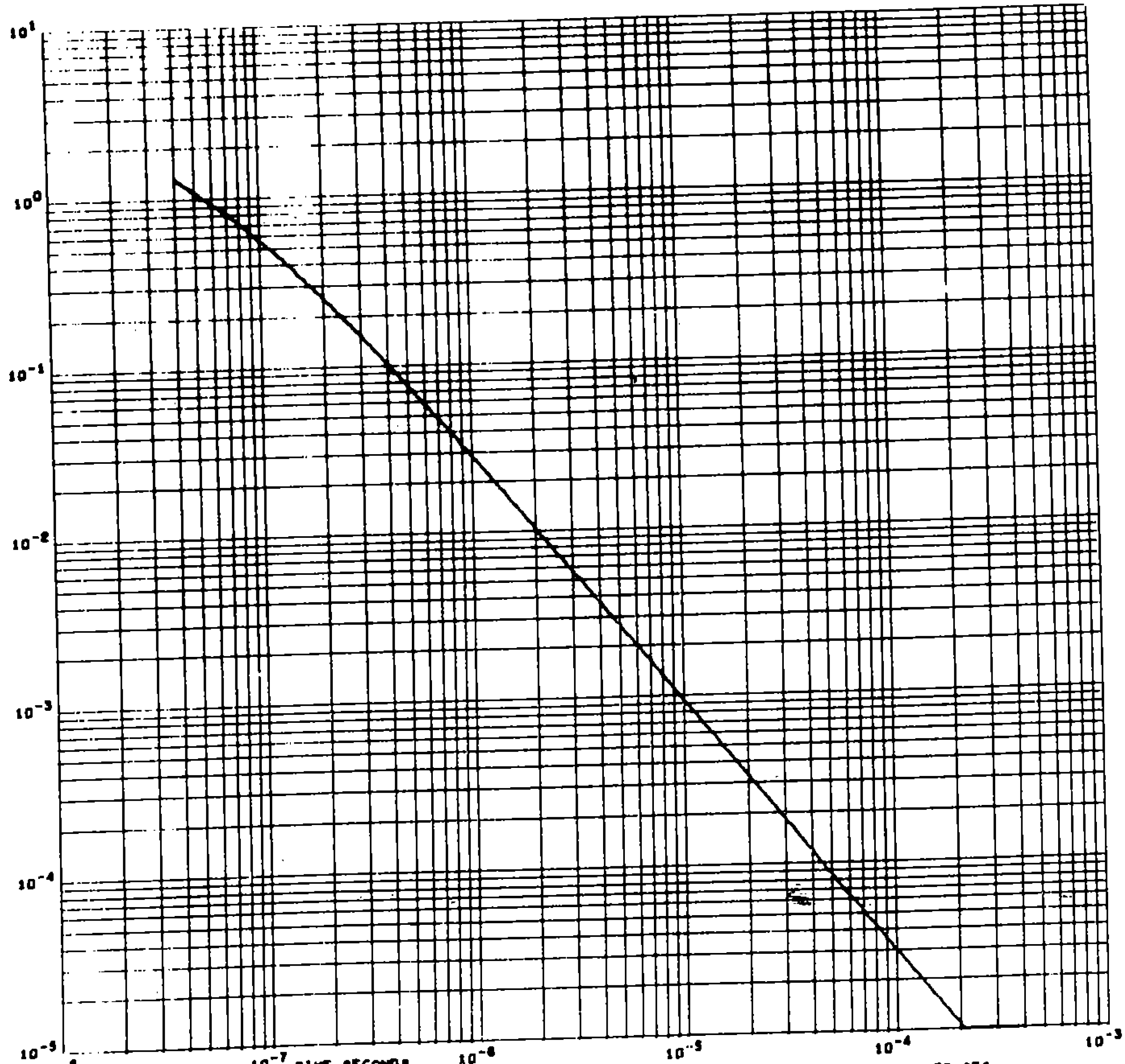
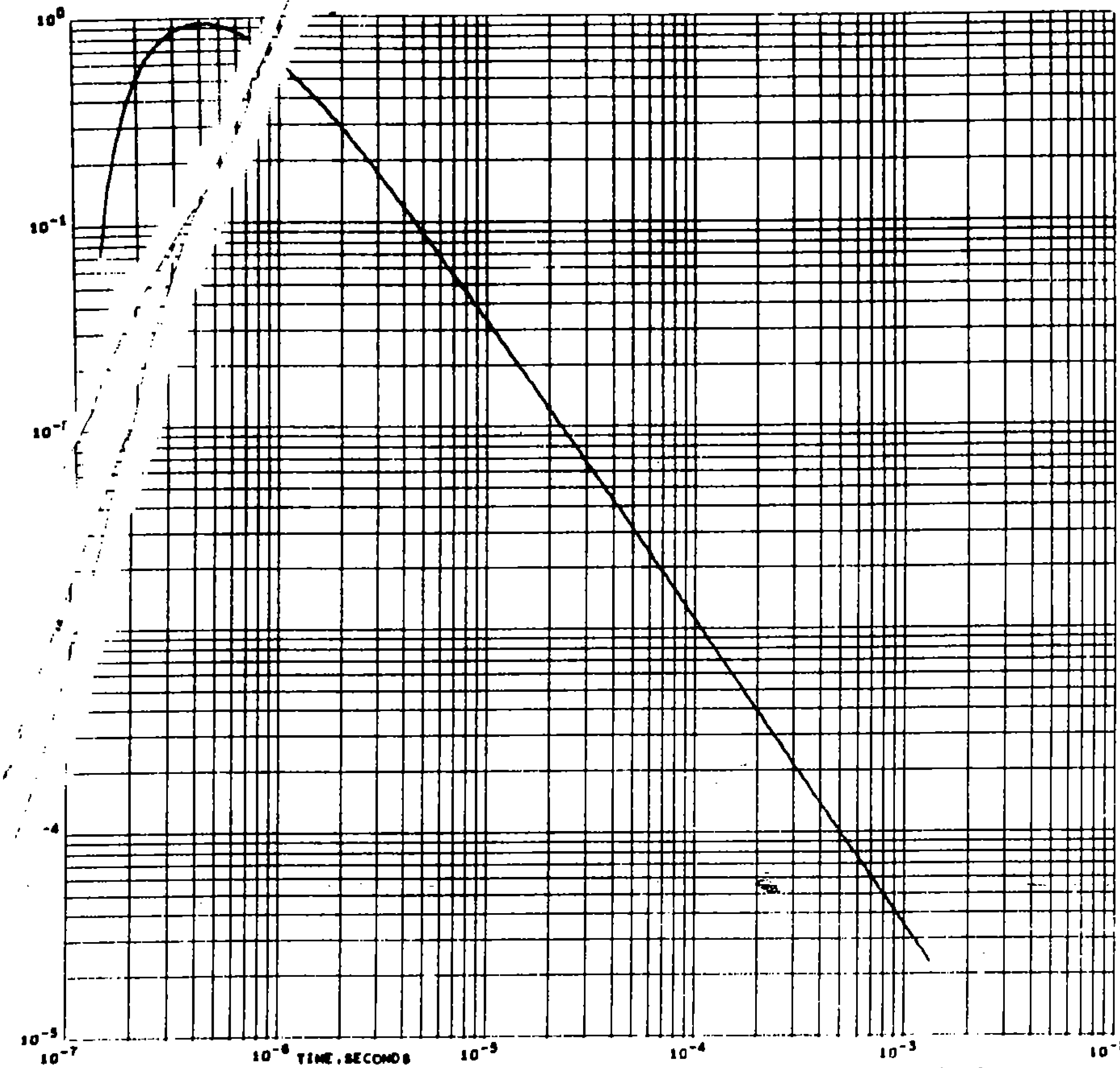
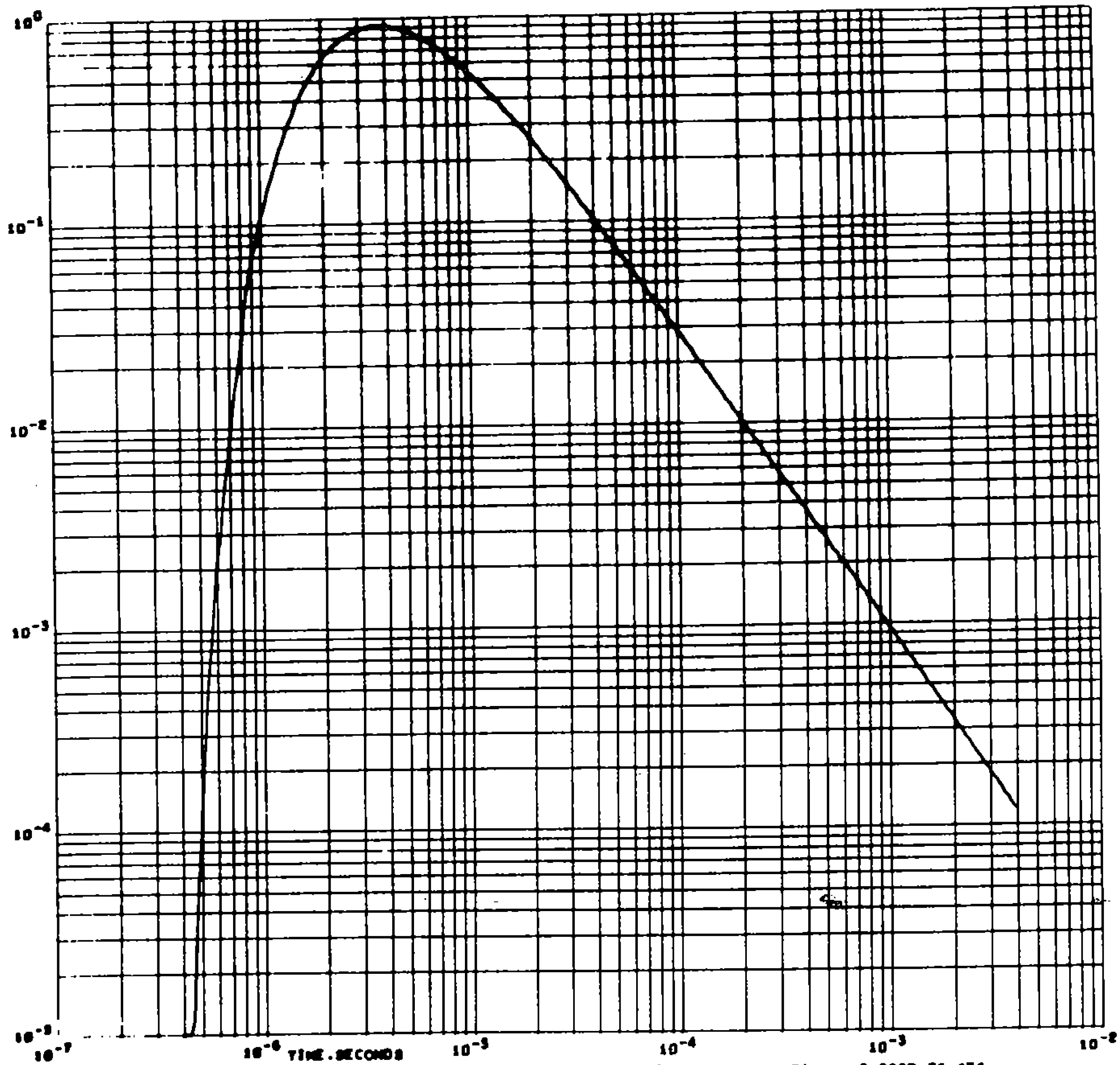
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T



K = 4.000E-08 SEC
TAU = 2.262E-07 SEC
EMP PROPAGATION, (J-13-459)
Z = 3.00 METERS SIGMA = 0.0200 MHOS/METER MU = 1.000 EPSILON = 16.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T



ɪᴘ PROPAGATION. (J-13-458)
= 10.00 METERS SIGMA = 0.0200 MHOS/METER
K = 1.333E-07 SEC
MU = 1.000
TAU = 2.513E-06 SEC
EPSILON = 16.000
JELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T

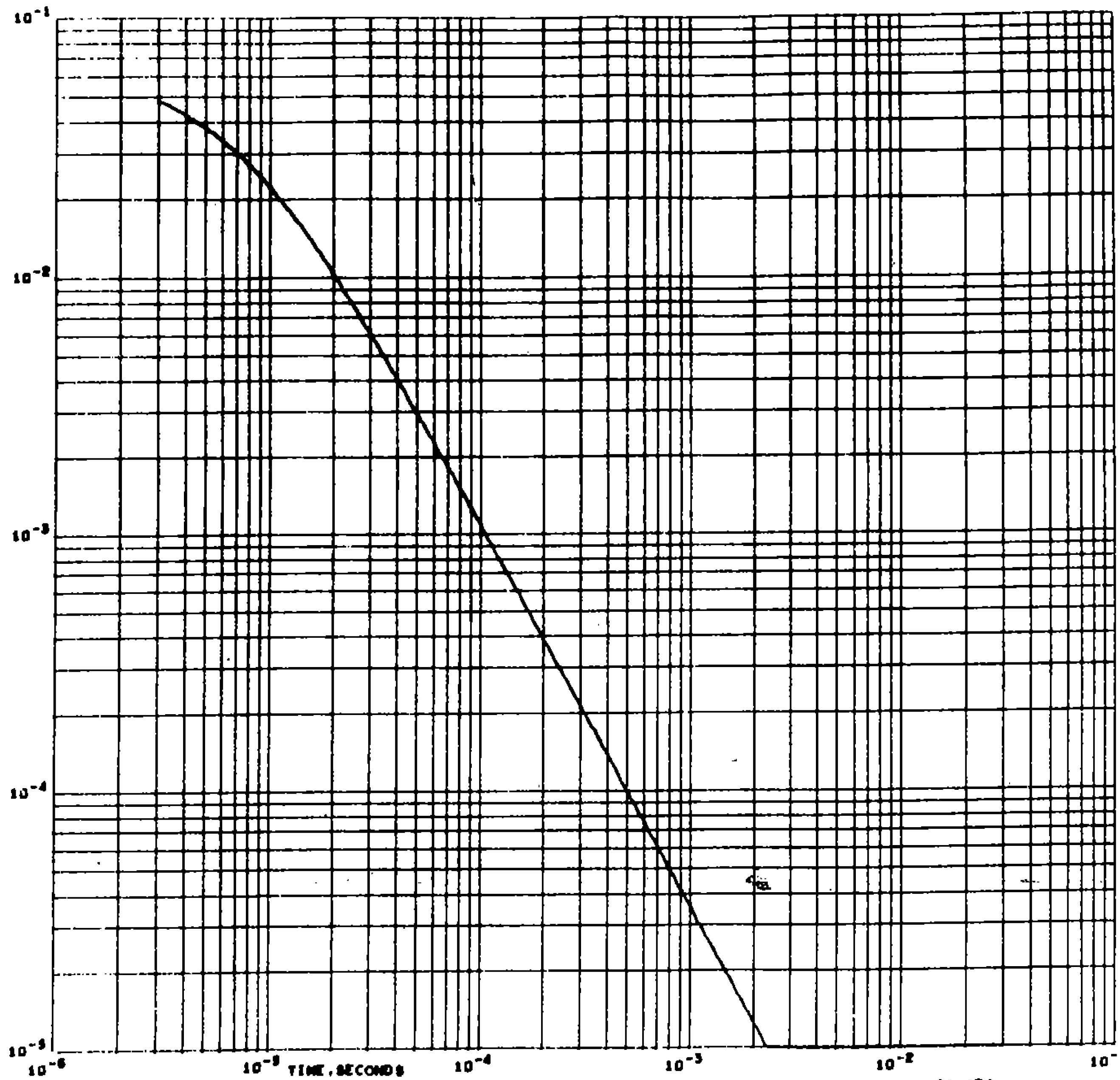


MP PROPAGATION. (J-13-450) K = 4.000E-07 SEC TAU = 2.262E-05 SEC

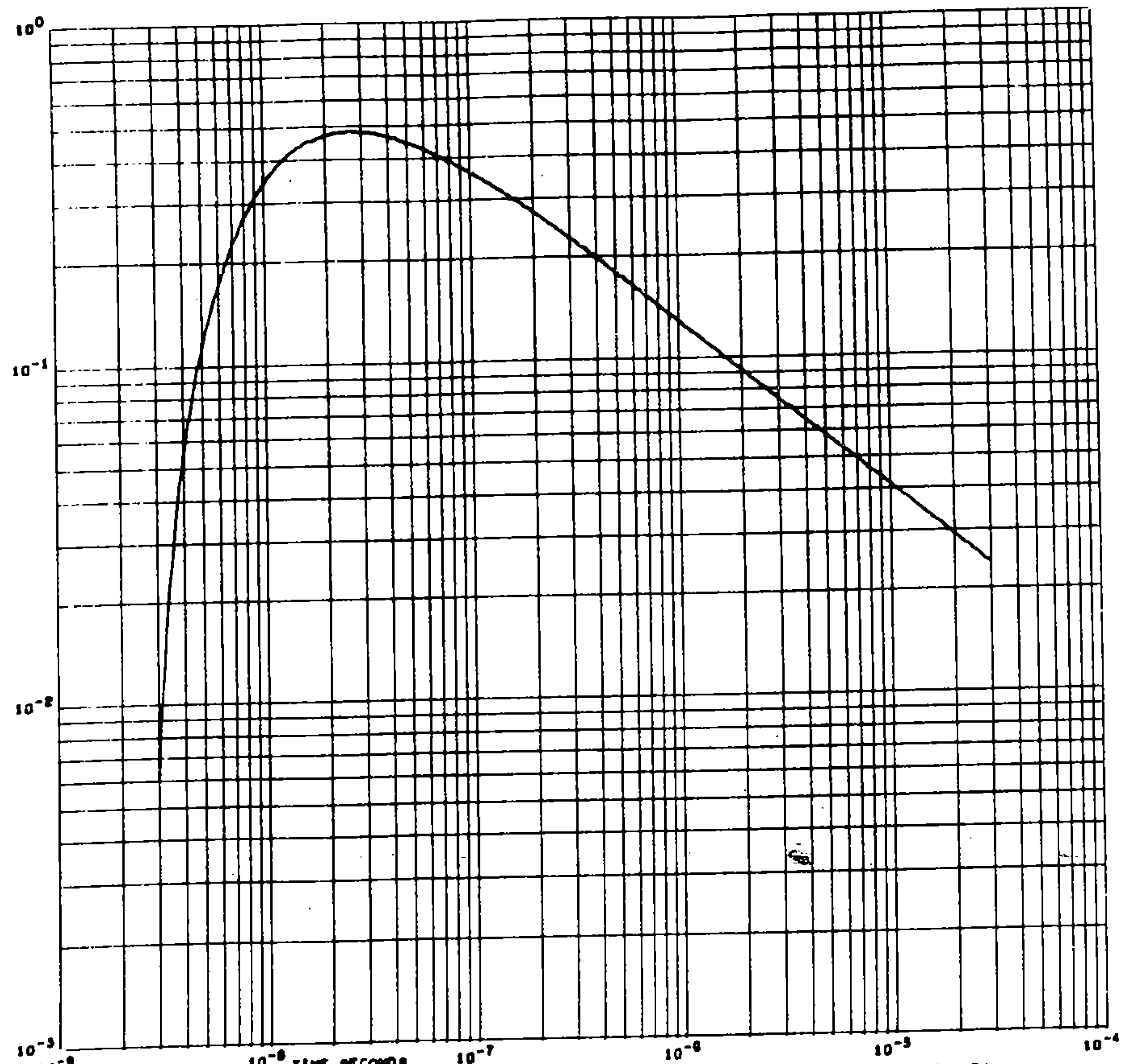
= 30.00 METERS SIGMA = 0.0200 MHOS/METER MU = 1.000 EPSILON = 16.000

ELTA FUNCTION ELECTRIC FIELD AT Z=0

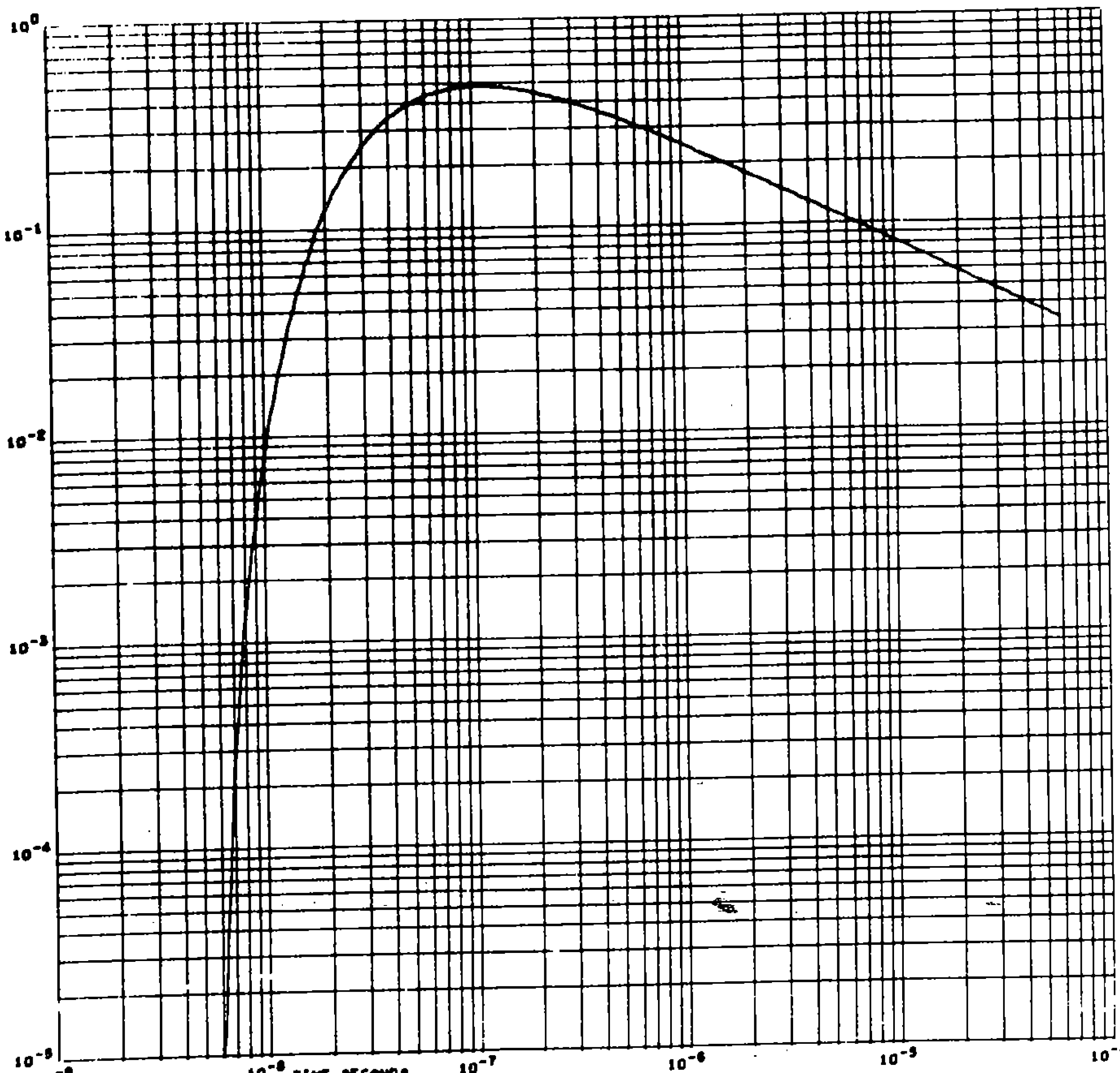
VS T



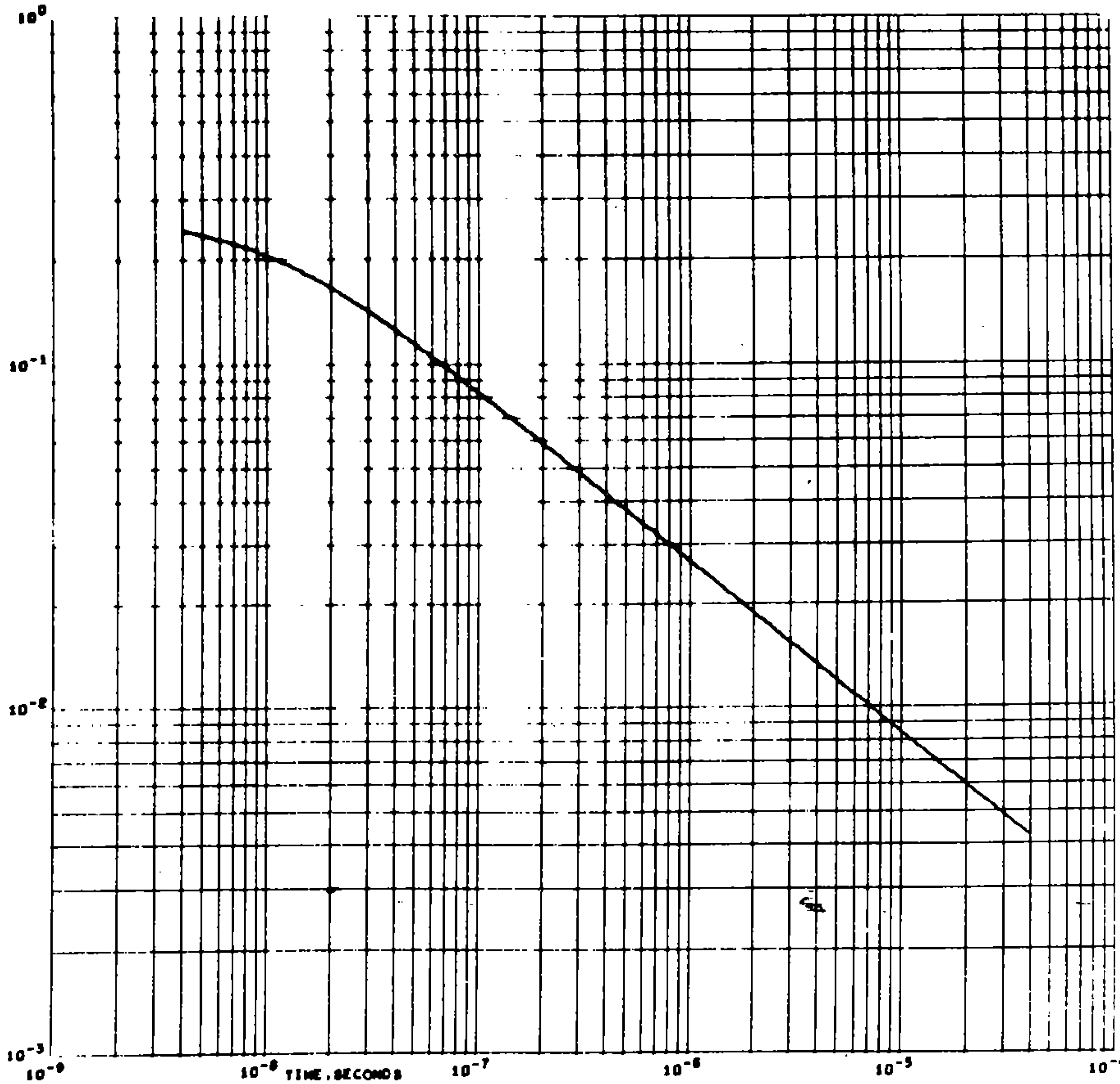
EMP PROPAGATION, (J-13-459)
Z = 100.00 METERS   SIGMA = 0.0002 MHOS/METER   K = 3.000E-06 SEC   TAU = 2.513E-06 SEC
MU = 1.000   EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
E VS T



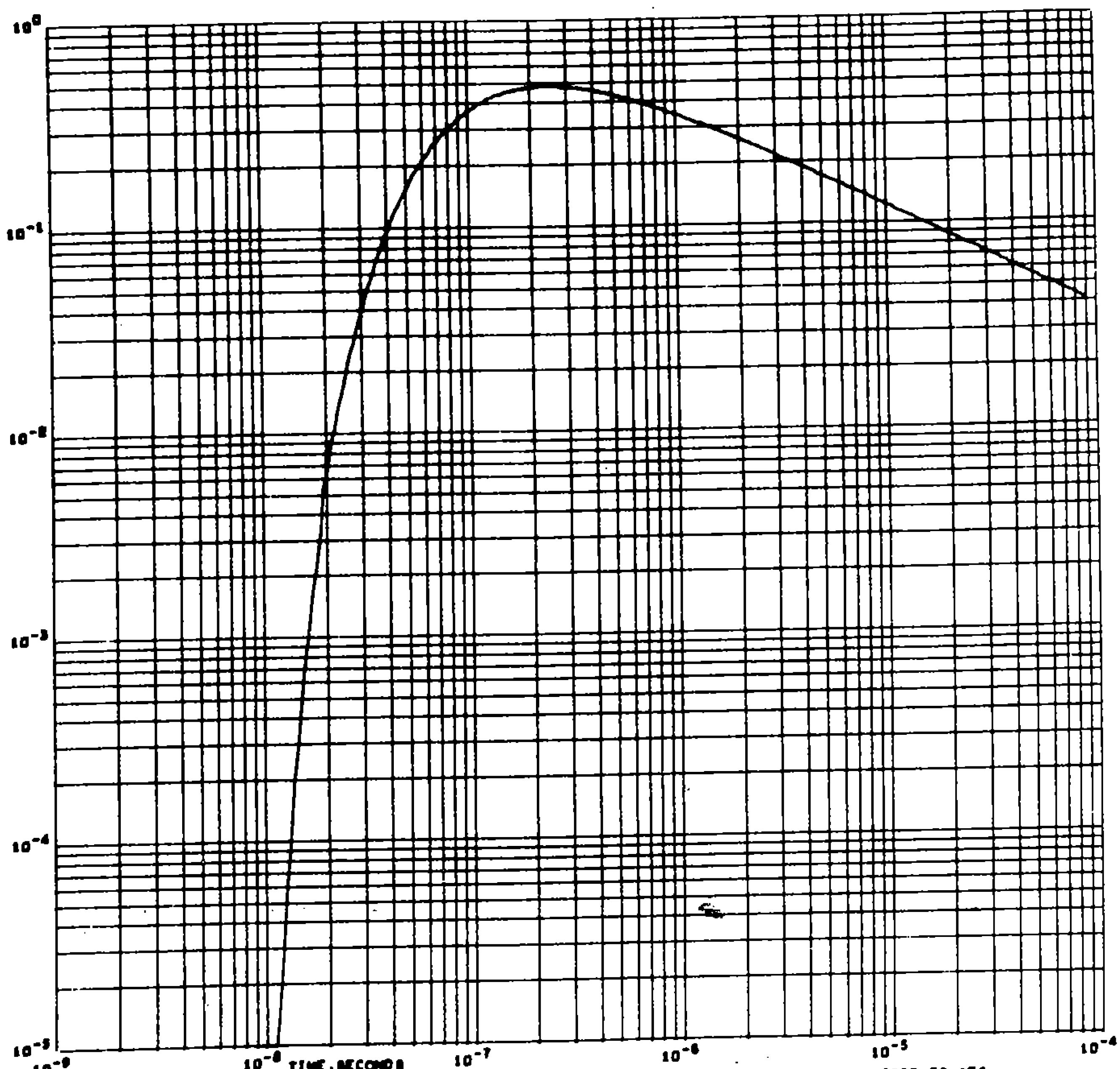
EMP PROPAGATION. (J-13-459)
Z = 0.10 METERS SIGMA = 4.3000 MHOS/METER
K = 3.000E-09 SEC
MU = 1.000
TAU = 5.404E-08 SEC
EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



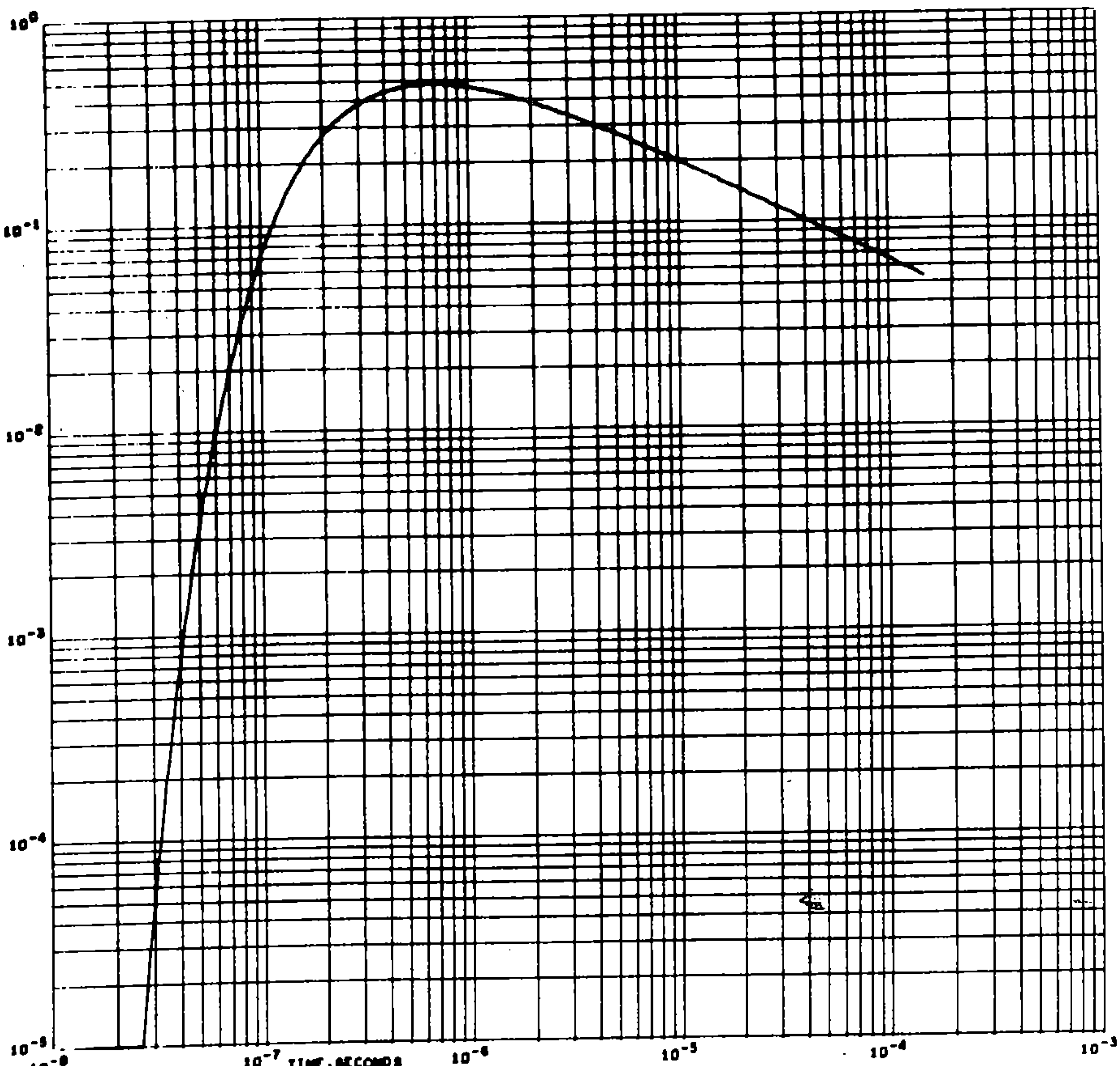
EMP PROPAGATION, (J-13-459)
Z = 0.20 METERS SIGMA = 4.3000 MHOS/METER
K = 6.000E-09 SEC
MU = 1.000
TAU = 2.161E-07 SEC
EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



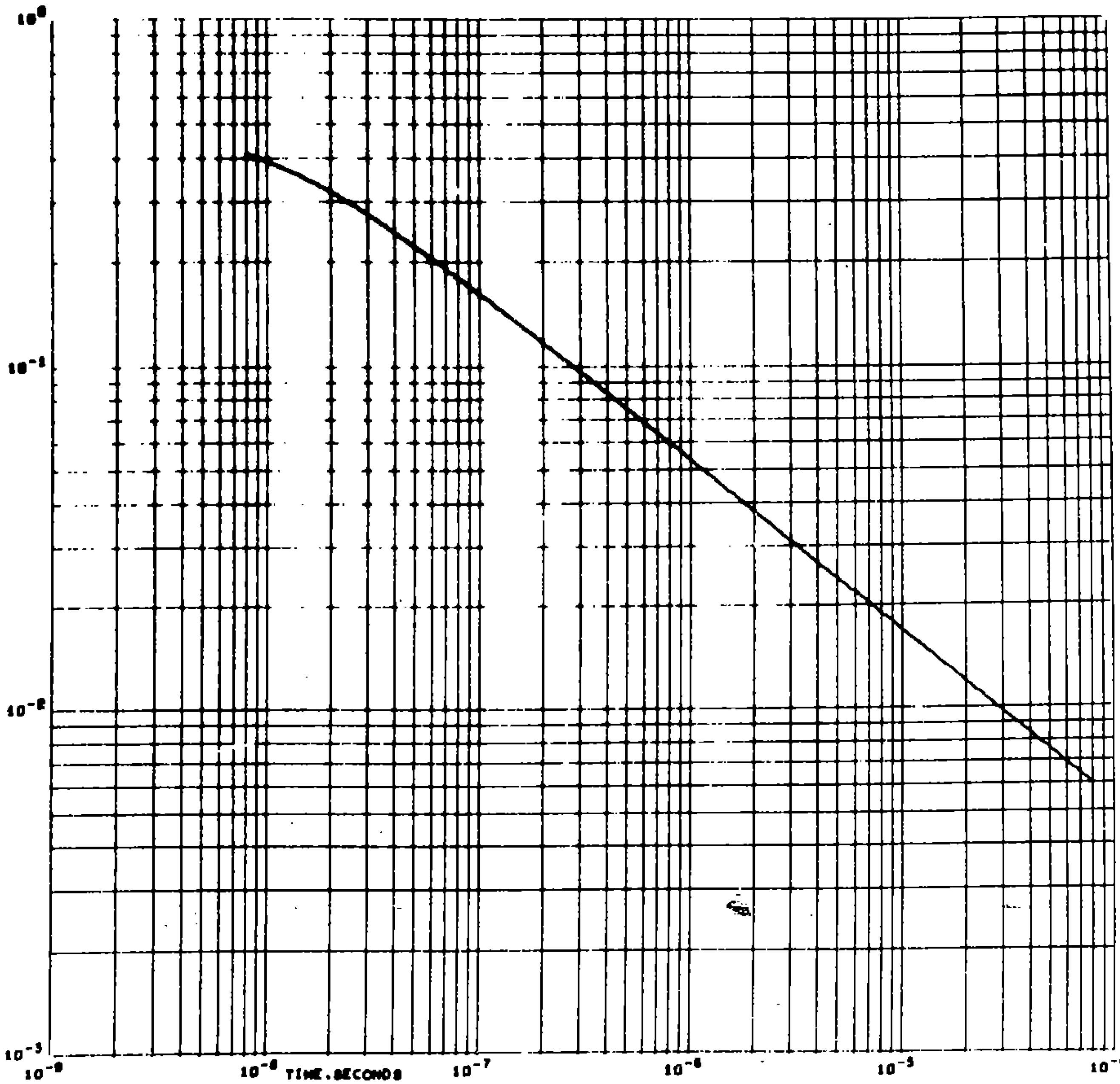
EMP PROPAGATION, (J-13-459)
Z = 0.30 METERS SIGMA = 0.0200 MHOS/METER
K = 4.000E-09 SEC MU = 1.000
TAU = 2.262E-09 SEC EPSILON = 16.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



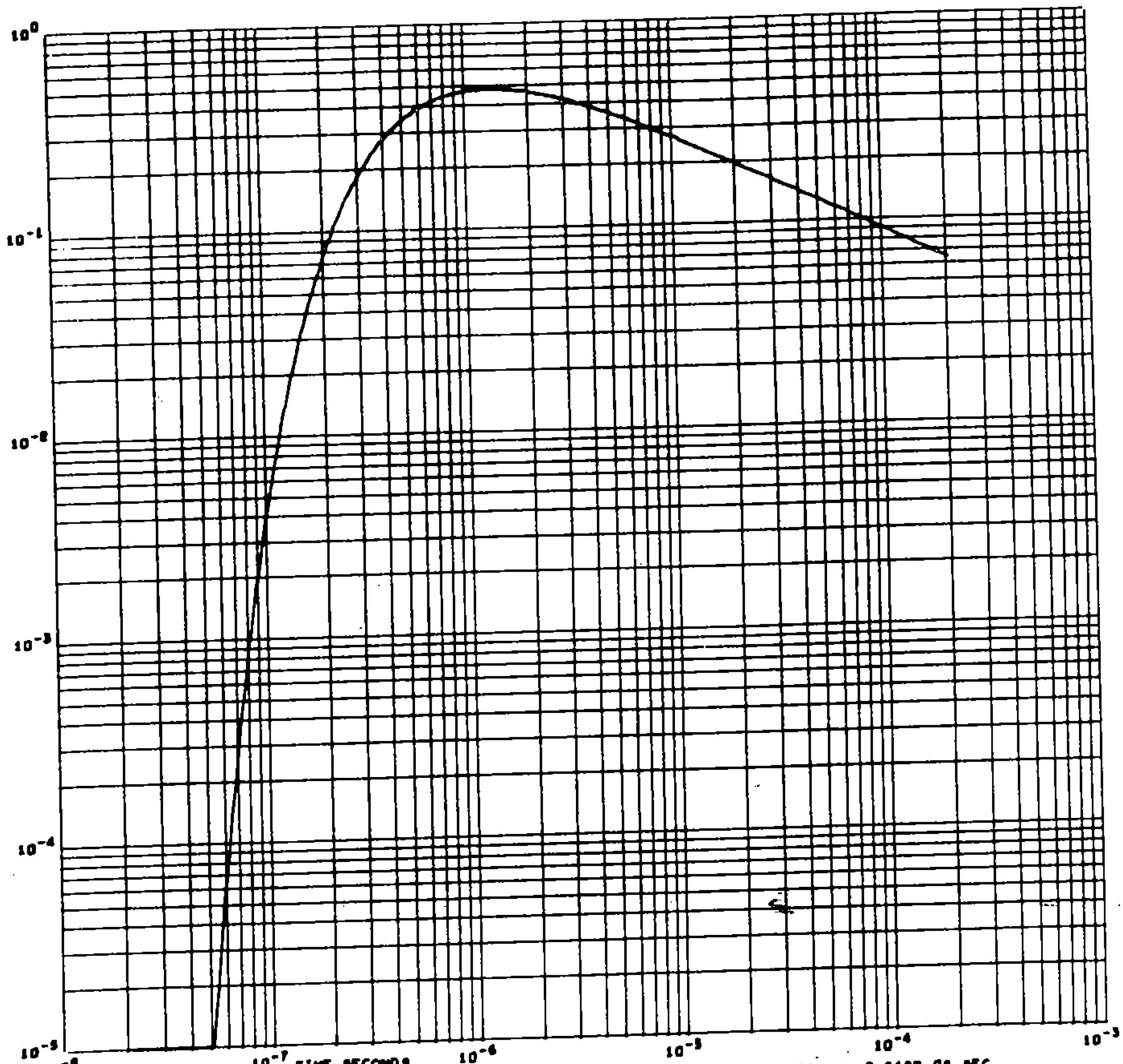
EMP PROPAGATION, (J-13-499)
Z = 0.30 METERS SIGMA = 4.3000 MHOS/METER
K = 9.000E-09 SEC
MU = 1.000
TAU = 4.863E-07 SEC
EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



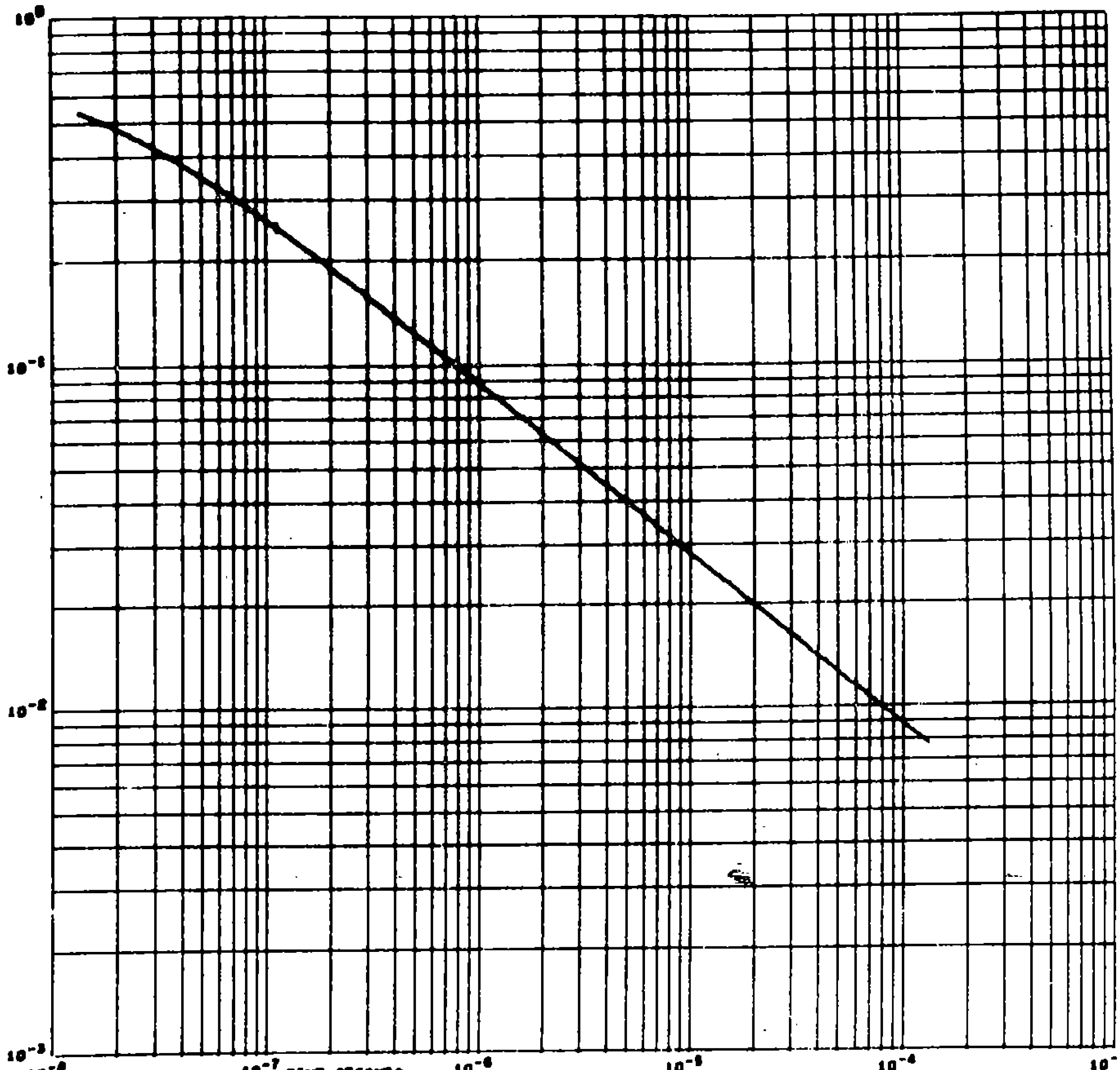
EMP PROPAGATION, (J-13-458)
Z = 0.90 METERS SIGMA = 4.3000 MHOS/METER K = 1.500E-08 SEC MU = 1.000 TAU = 1.351E-08 SEC EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



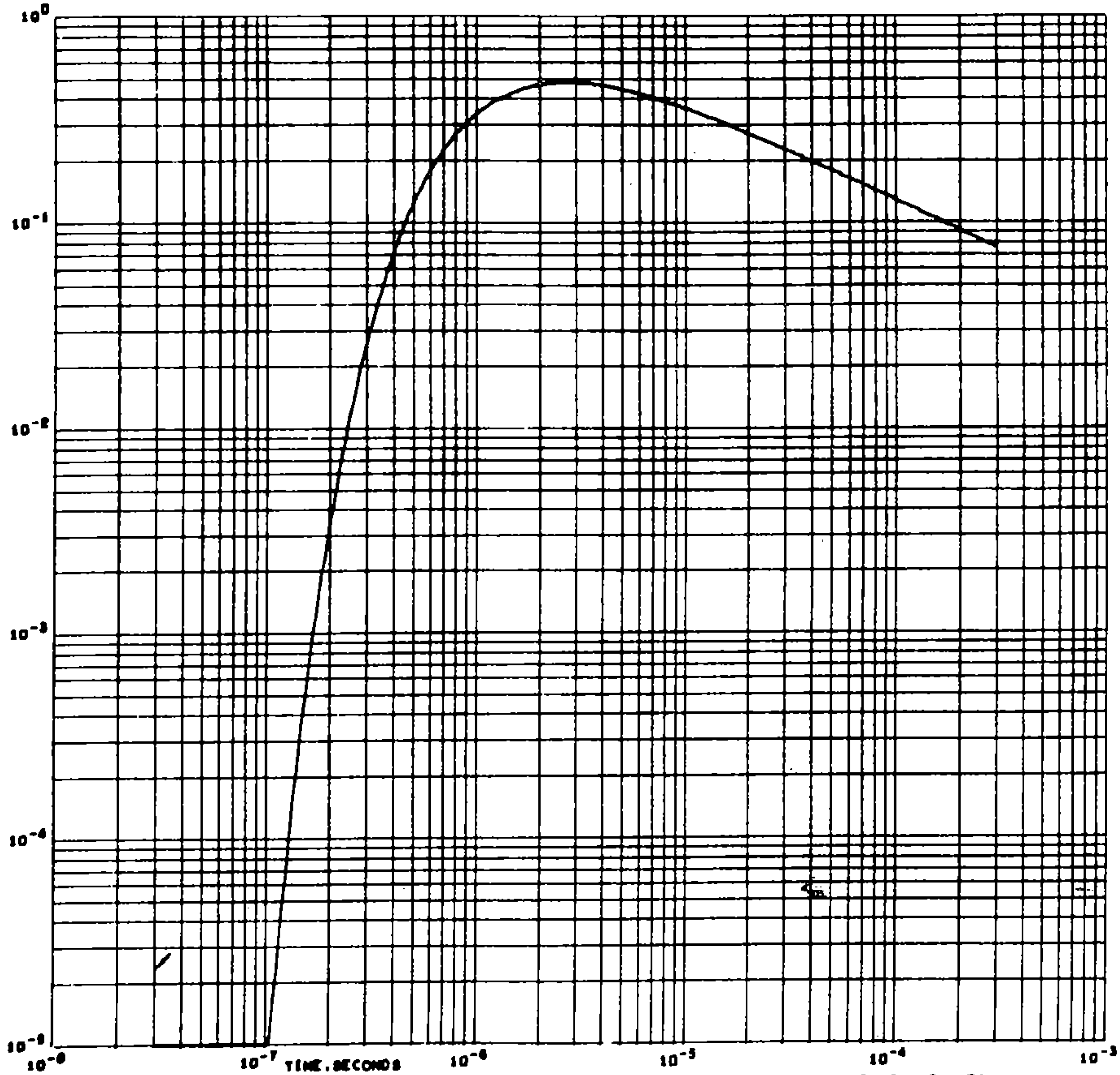
EMP PROPAGATION. (J-13-459)
Z = 0.60 METERS   SIGMA = 0.0200 MHOS/METER   K = 8.000E-09 SEC   MU = 1.000   TAU = 8.048E-09 SEC   EPSILON = 16.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



EMP PROPAGATION. (J-13-459)
Z = 0.70 METERS
SIGMA = 4.3000 MHOS/METER
K = 2.100E-08 SEC
MU = 1.000
TAU = 2.648E-08 SEC
EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



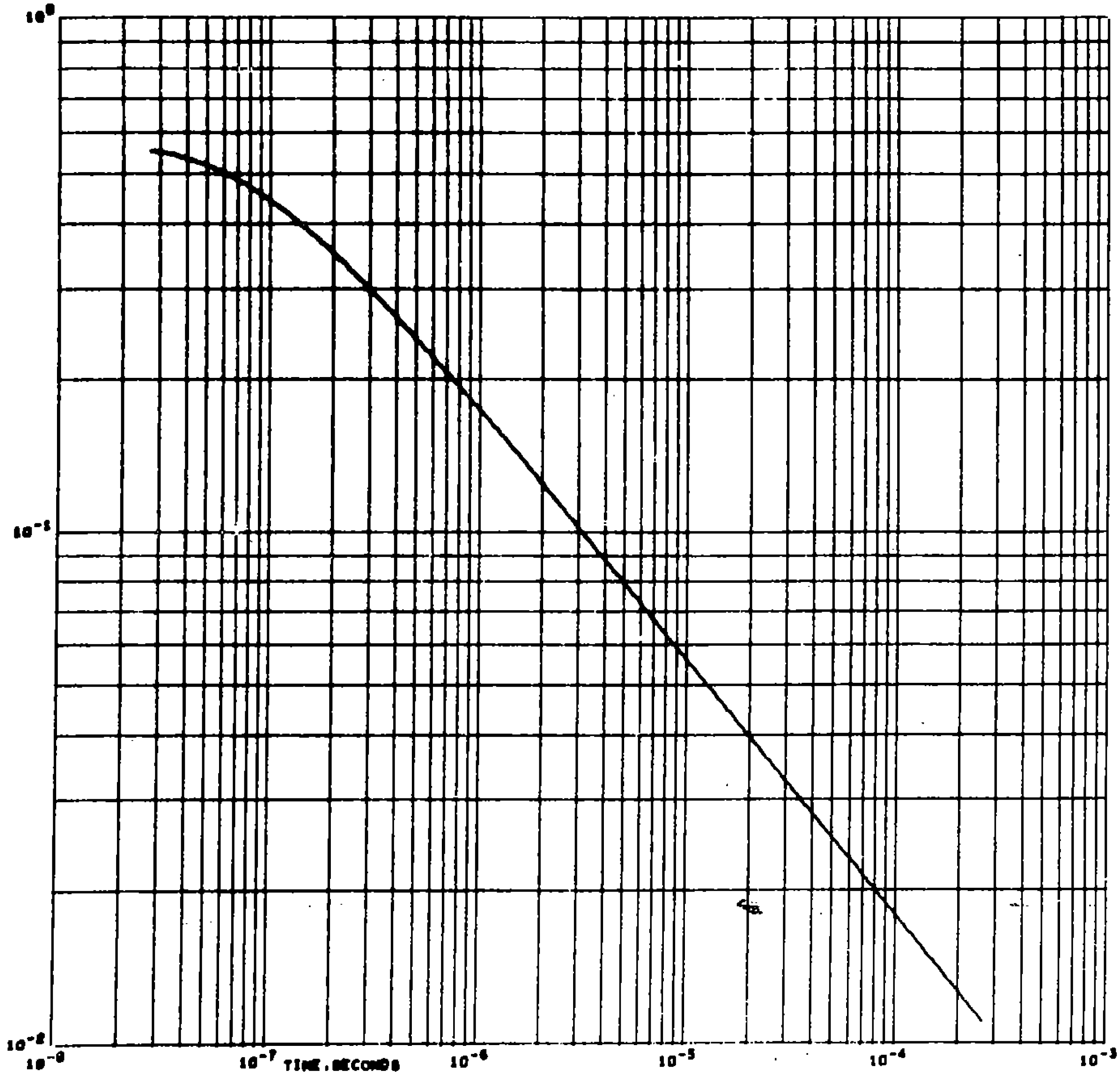
EMP PROPAGATION, (J-13-459)
Z = 1.00 METERS   SIGMA = 0.0200 MHOS/METER   K = 1.333E-08 SEC   TAU = 2.513E-08 SEC
MU = 1.000   EPSILON = 16.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
S VS T



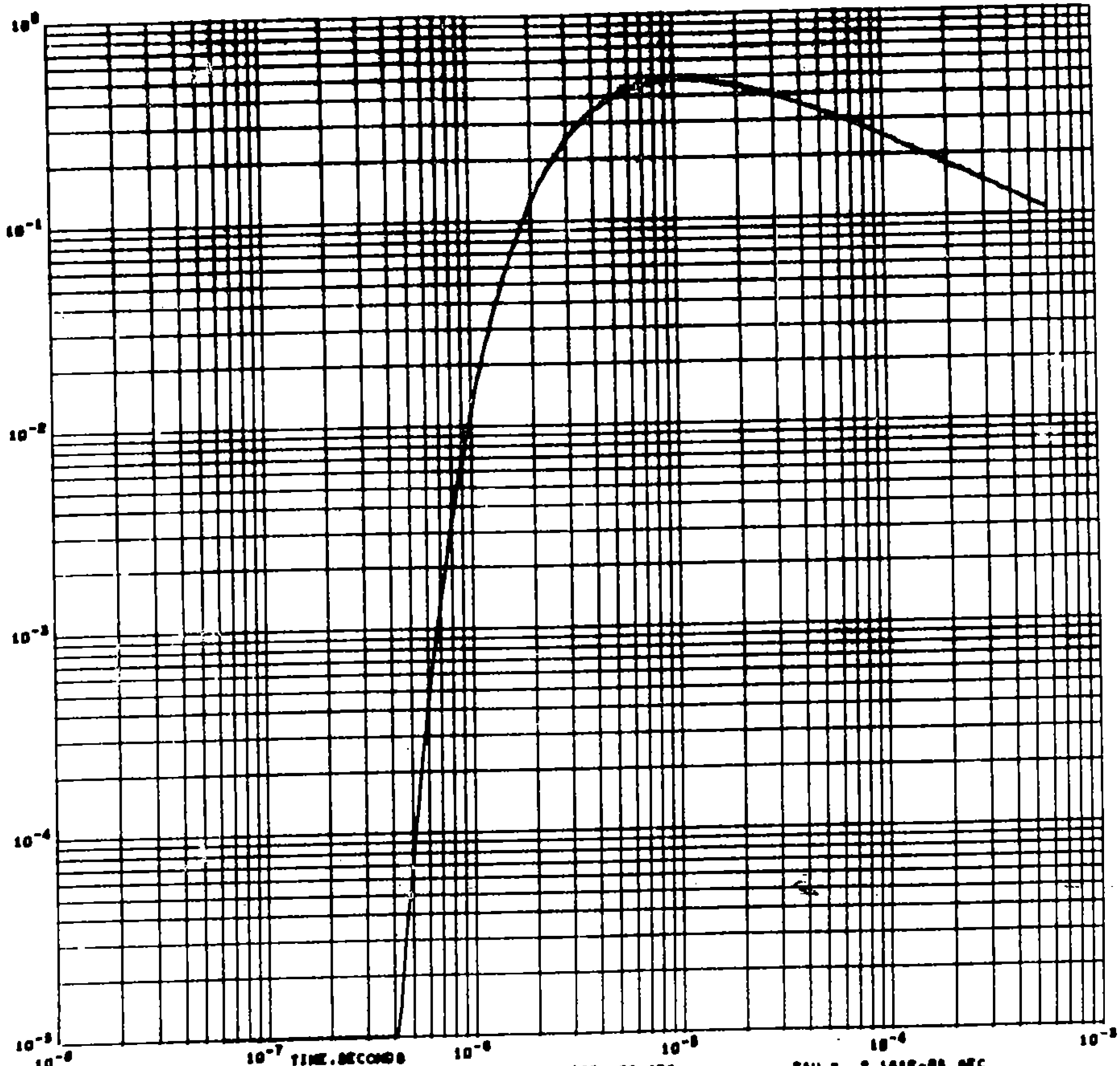
EMP PROPAGATION, (J-13-459)
Z = 1.00 METERS SIGMA = 4.3000 MHOS/METER
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T

K = 3.000E-08 SEC
MU = 1.000

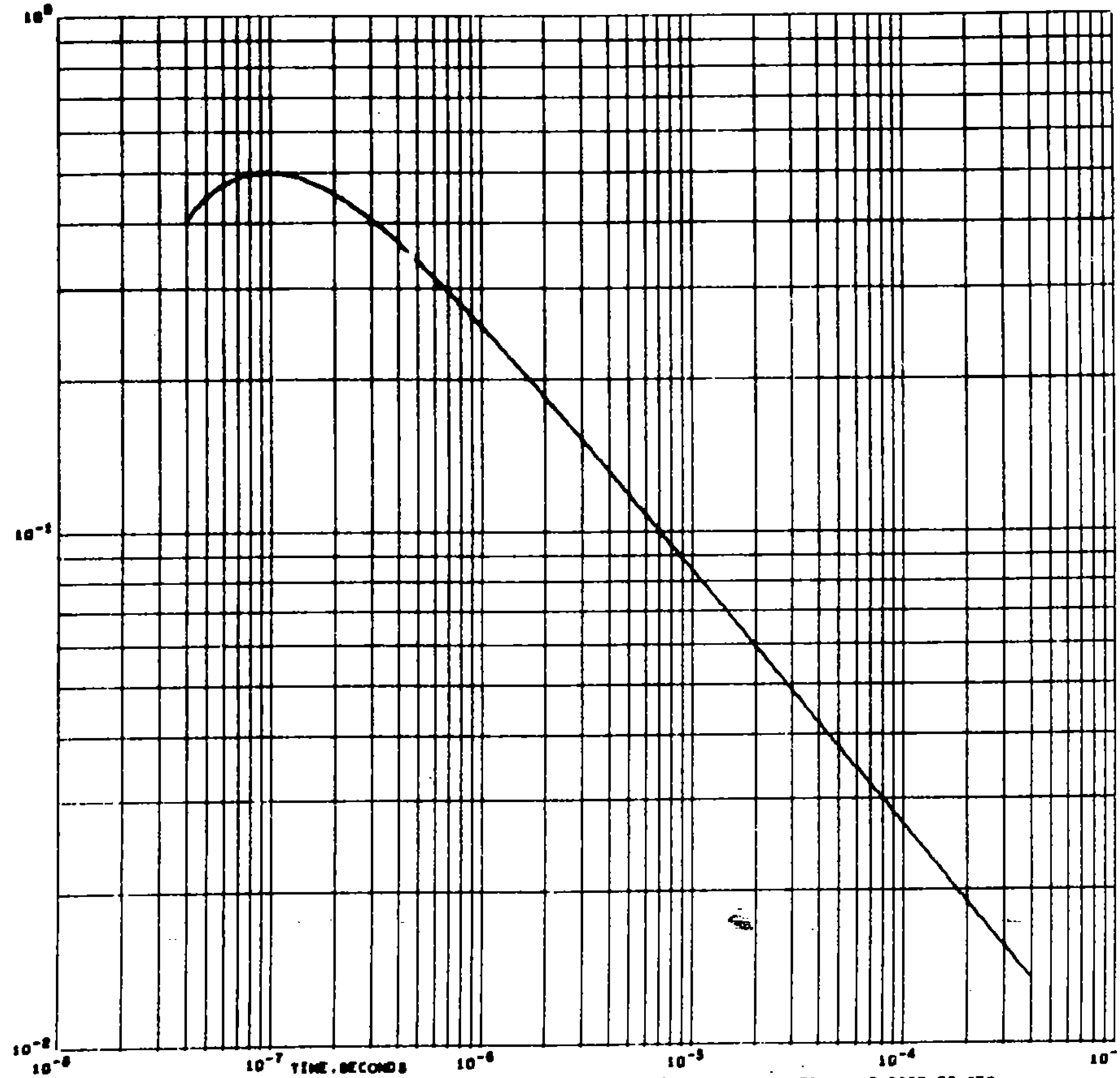
TAU = 3.404E-06 SEC
EPSILON = 81.000



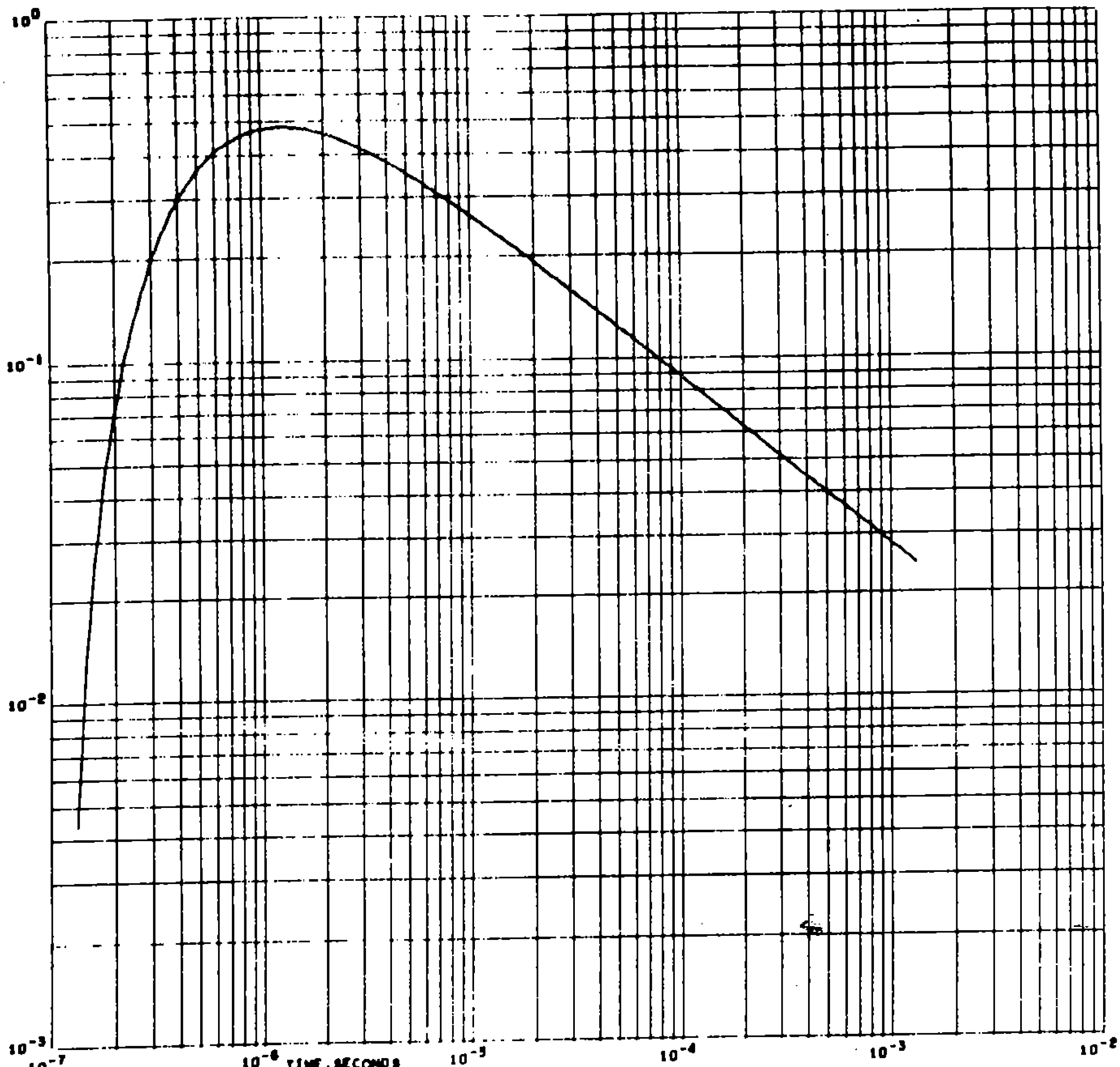
EMP PROPAGATION, (J-13-480)
Z = 2.00 METERS SIGMA = 0.0200 MHOS/METER
K = 2.667E-08 SEC
MU = 1.000
TAU = 1.005E-07 SEC
EPSILON = 16.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



EMP PROPAGATION. (J-13-459)
Z = 2.00 METERS
SIGMA = 4.3000 MHOS/METER
K = 6.000E-08 SEC
MU = 1.000
TAU = 2.181E-08 SEC
EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



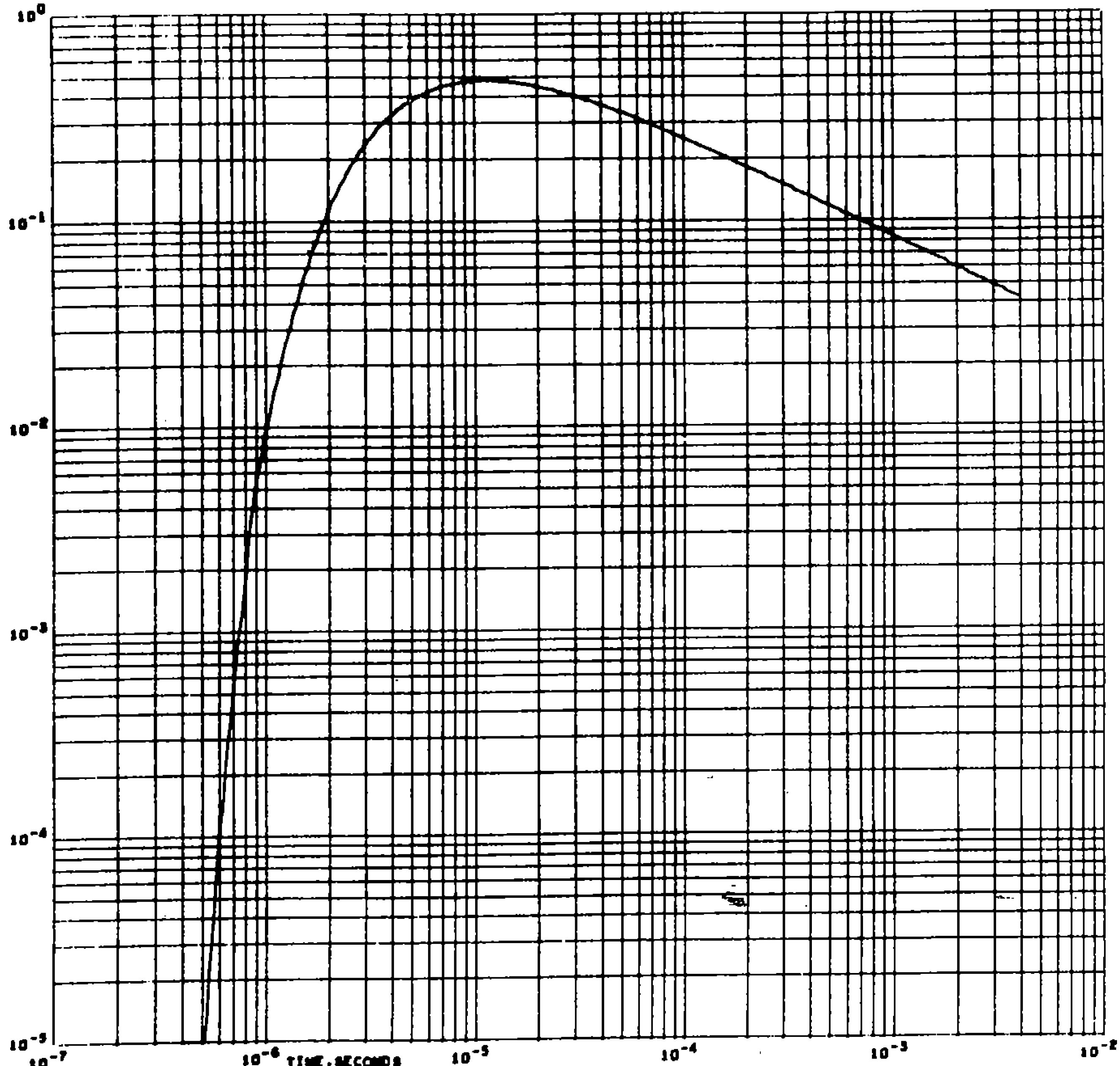
EMP PROPAGATION, (J-13-488)
Z = 3.00 METERS   SIGMA = 0.0200 MHOS/METER   K = 4.000E-08 SEC   TAU = 2.262E-07 SEC
NU = 1.000   EPSILON = 16.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



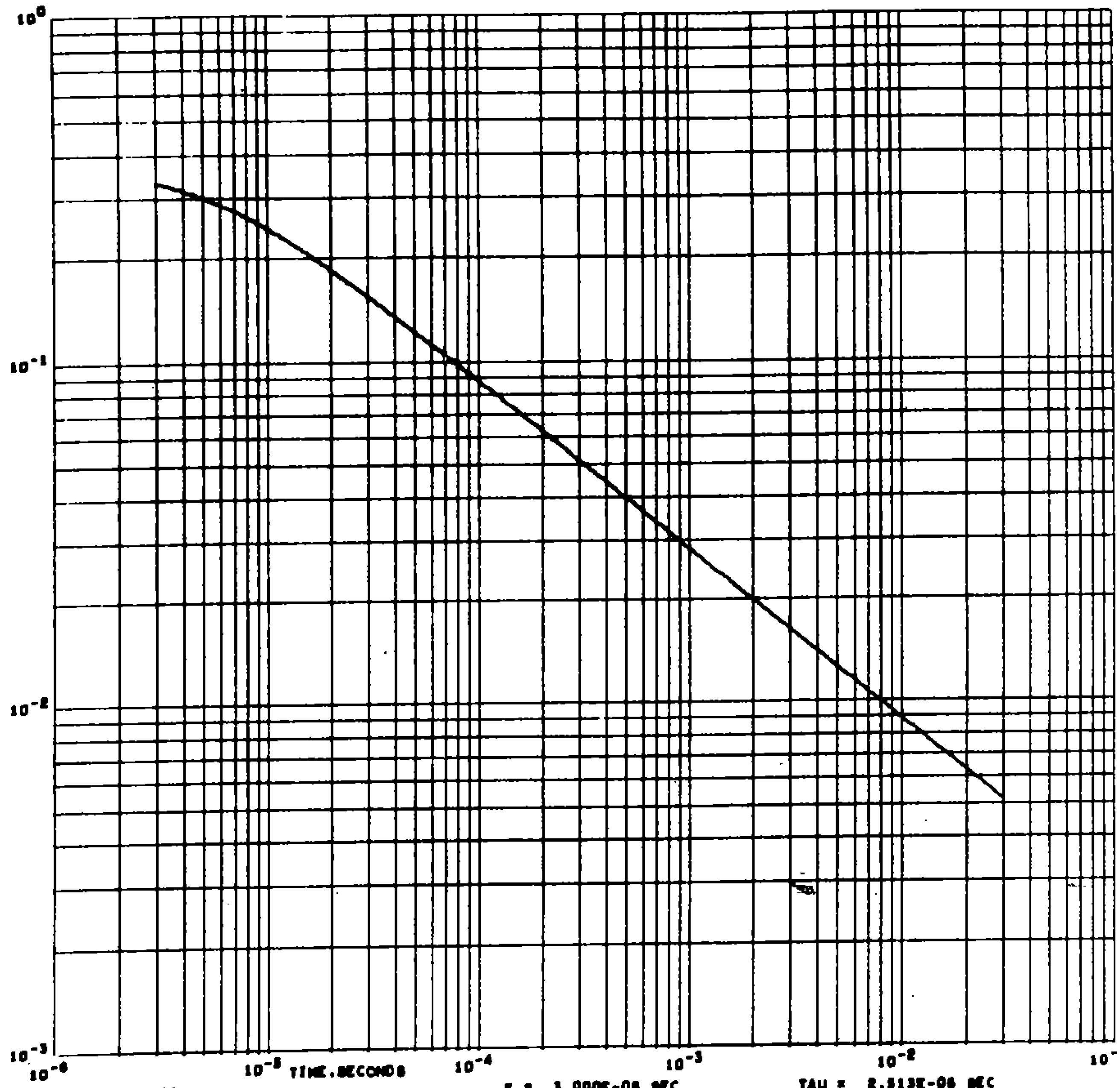
EMP PROPAGATION. (J-13-459)
Z = 10.00 METERS SIGMA = 0.0200 MHOS/METER
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T

R = 1.333E-07 SEC
MU = 1.000

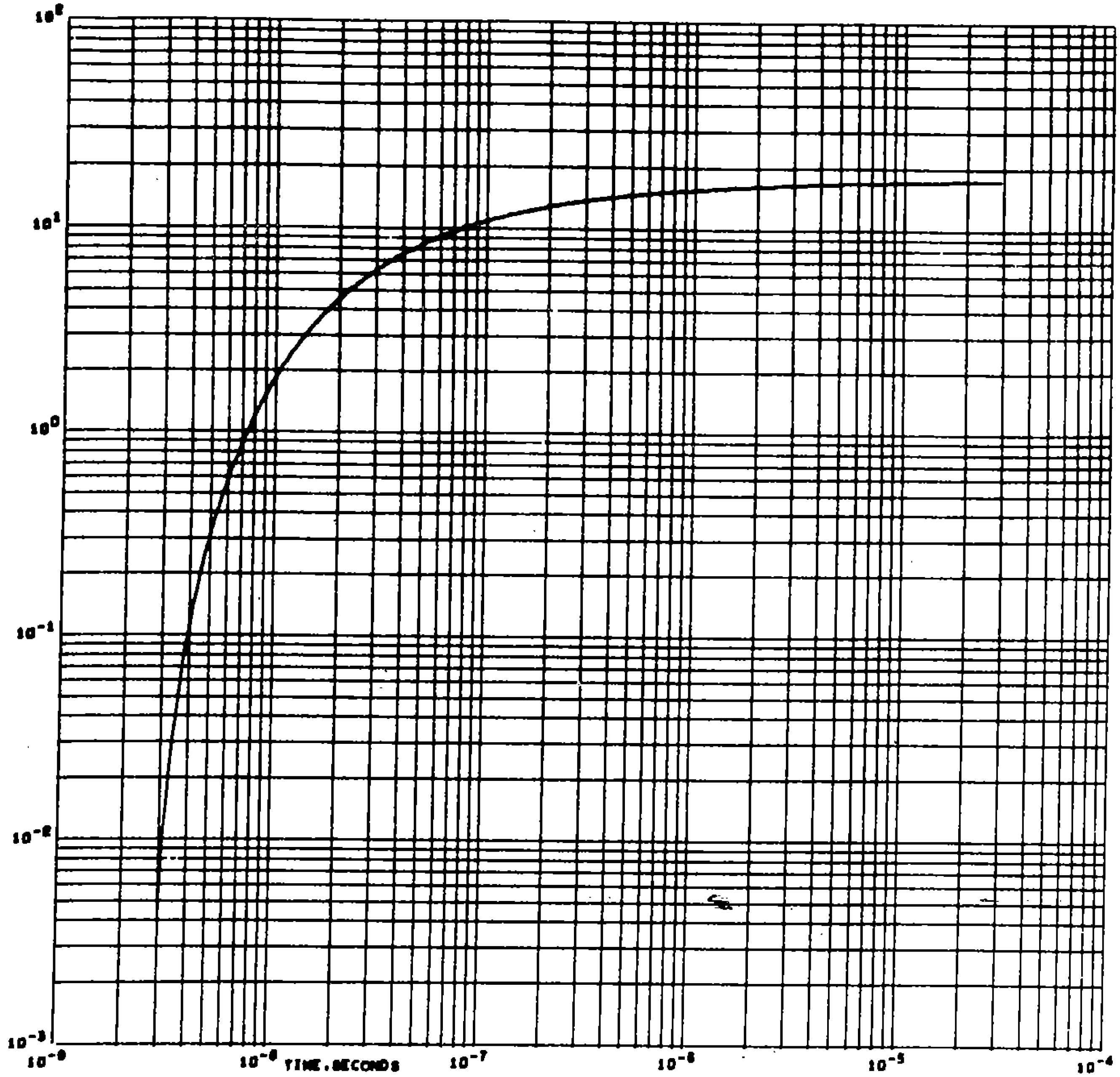
TAU = 2.513E-06 SEC
EPSILON = 16.000



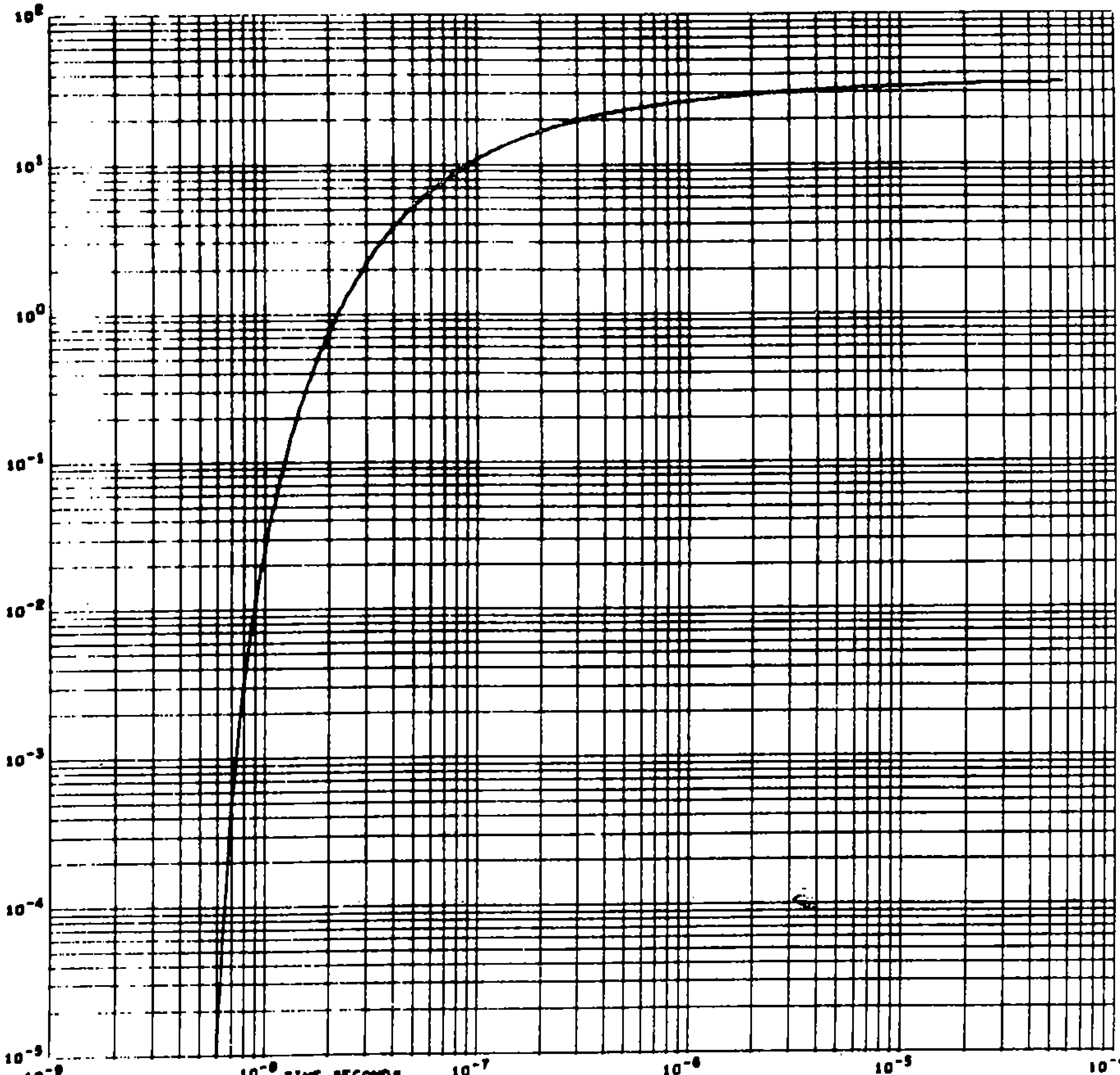
EMP PROPAGATION, (J-13-459)
Z = 30.00 METERS SIGMA = 0.0200 MHOS/METER K = 4.000E-07 SEC TAU = 2.262E-05 SEC
MU = 1.000 EPSILON = 16.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



EMP PROPAGATION. (J-13-459)
Z = 100.00 METERS   SIGMA = 0.0002 MHOS/METER   K = 3.000E-08 SEC   NU = 1.000   TAU = 2.513E-06 SEC   EPSILON = 81.000
DELTA FUNCTION ELECTRIC FIELD AT Z=0
B VS T



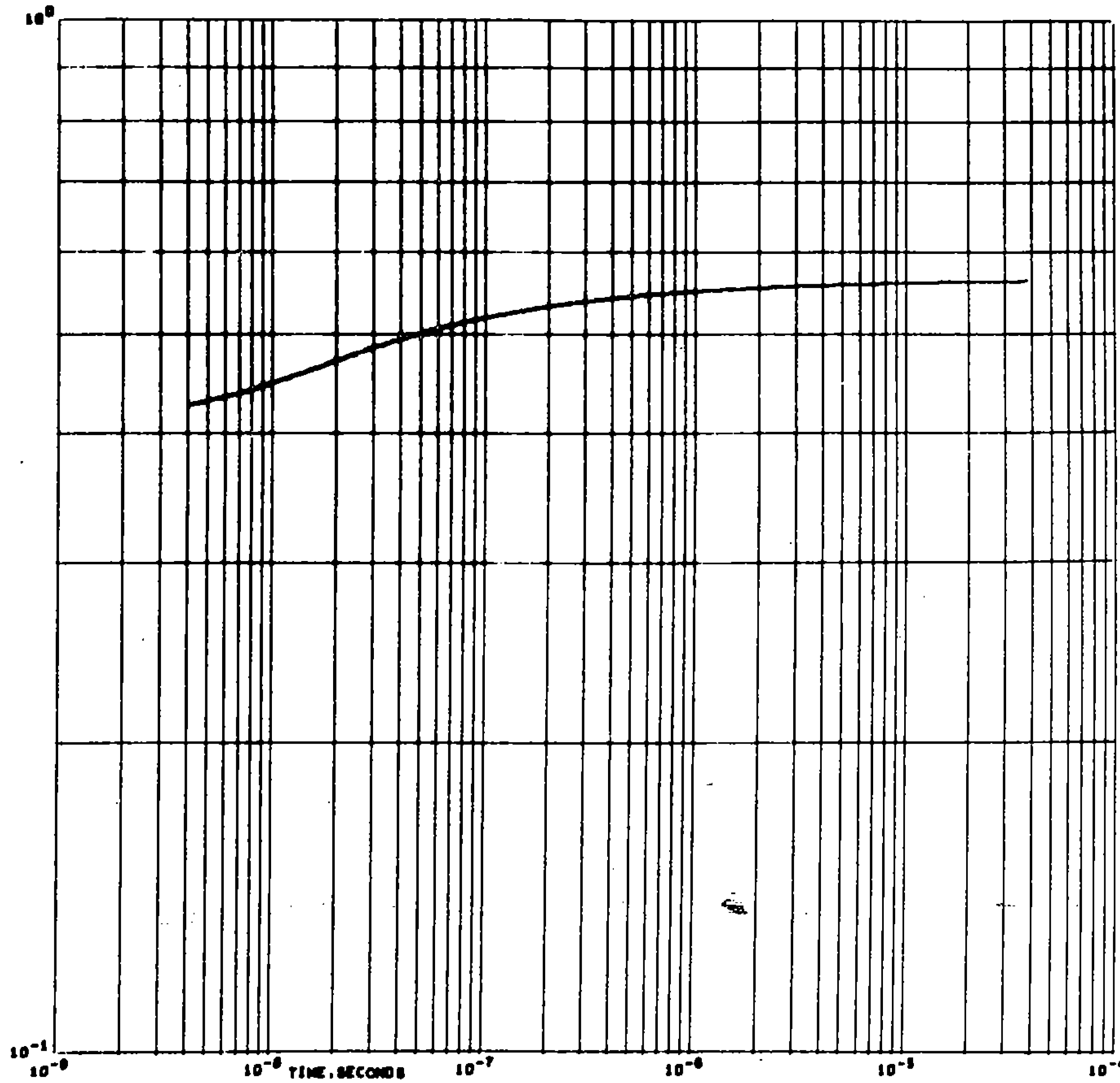
EMP PROPAGATION, (J-13-459) K = 3.000E-09 SEC TAU = 5.404E-08 SEC
Z = 0.10 METERS SIGMA = 4.3000 MHOS/METER MU = 1.000 EPSILON = 81.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T



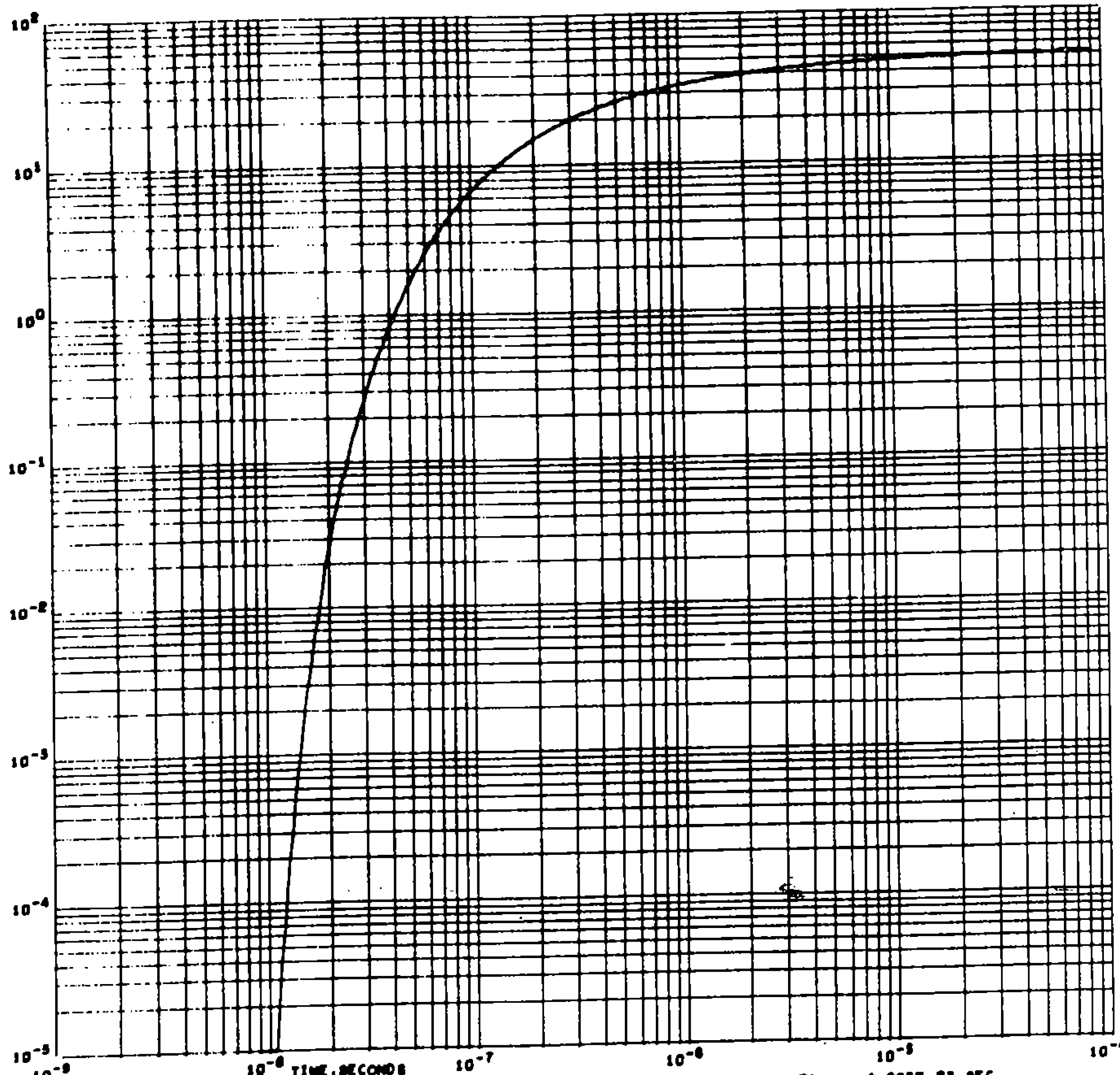
EMP PROPAGATION. (J-13-459)
Z = 0.20 METERS SIGMA = 4.3000 MHOS/METER
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T

K = 6.000E-08 SEC
MU = 1.000

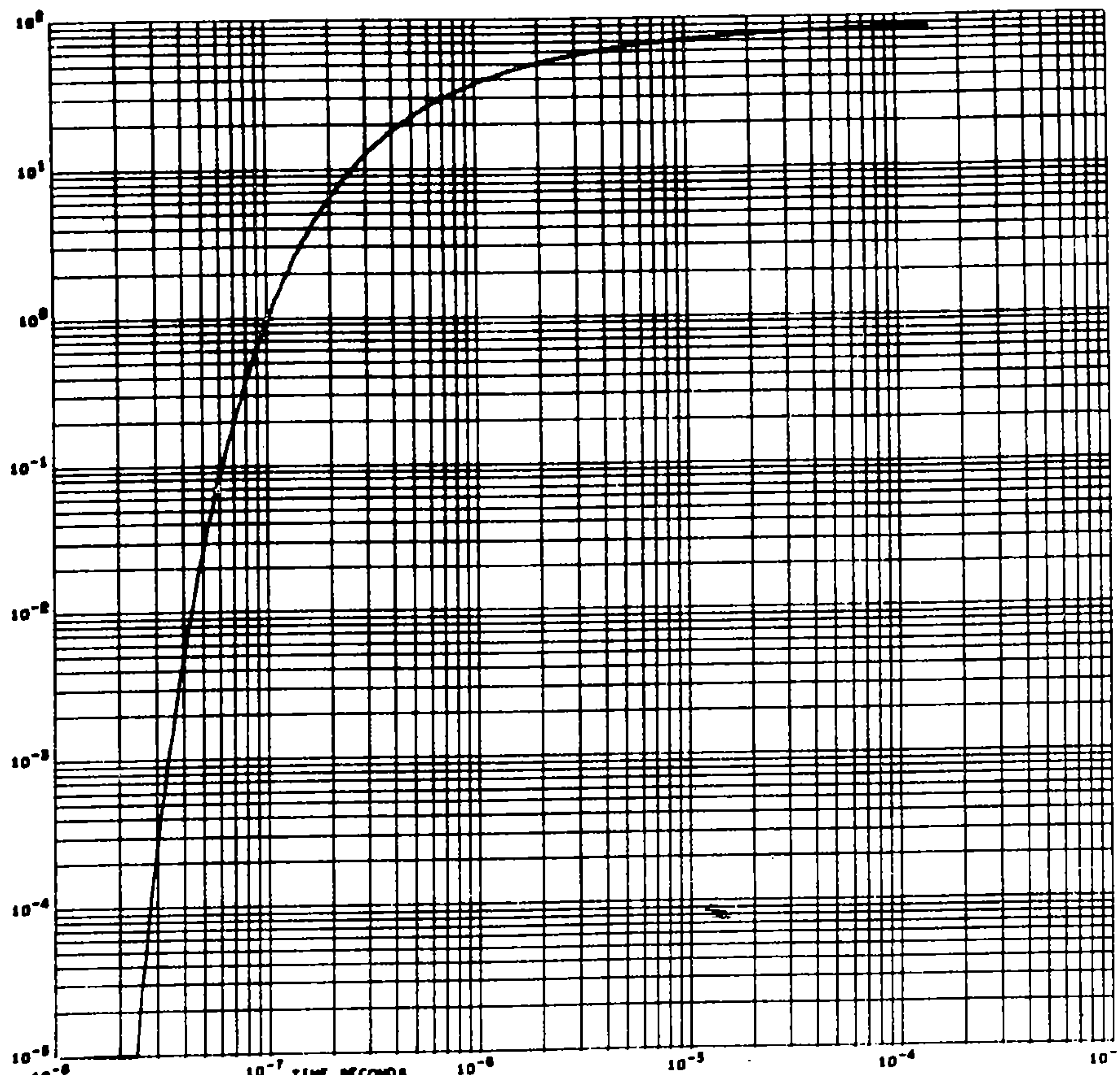
TAU = 2.161E-07 SEC
EPSILON = 81.000



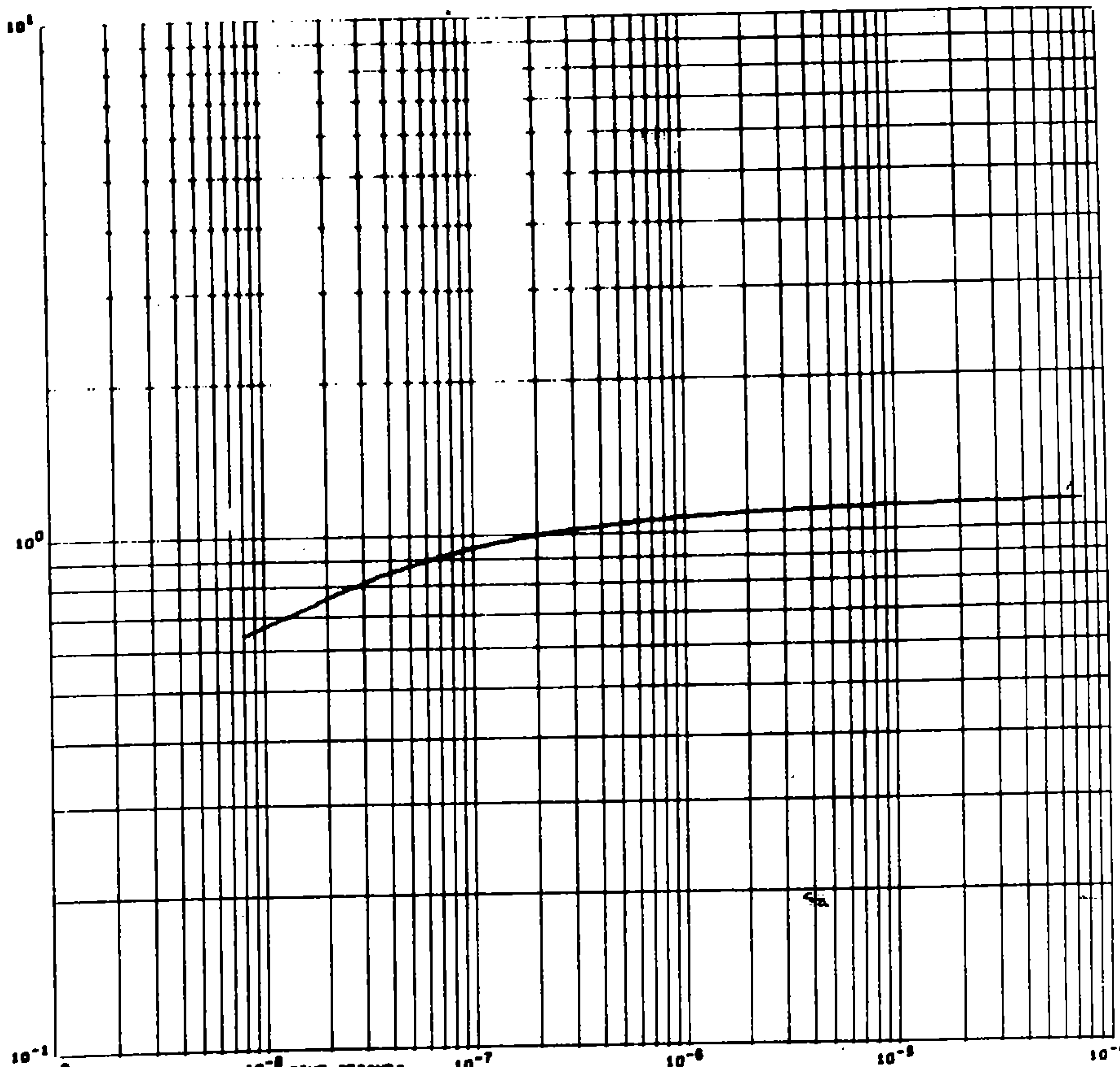
EMP PROPAGATION, (J-13-480)
Z = 0.30 METERS   SIGMA = 0.0200 MHOS/METER   K = 4.000E-09 SEC   TAU = 2.282E-09 SEC
MU = 1.000   EPSILON = 16.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T



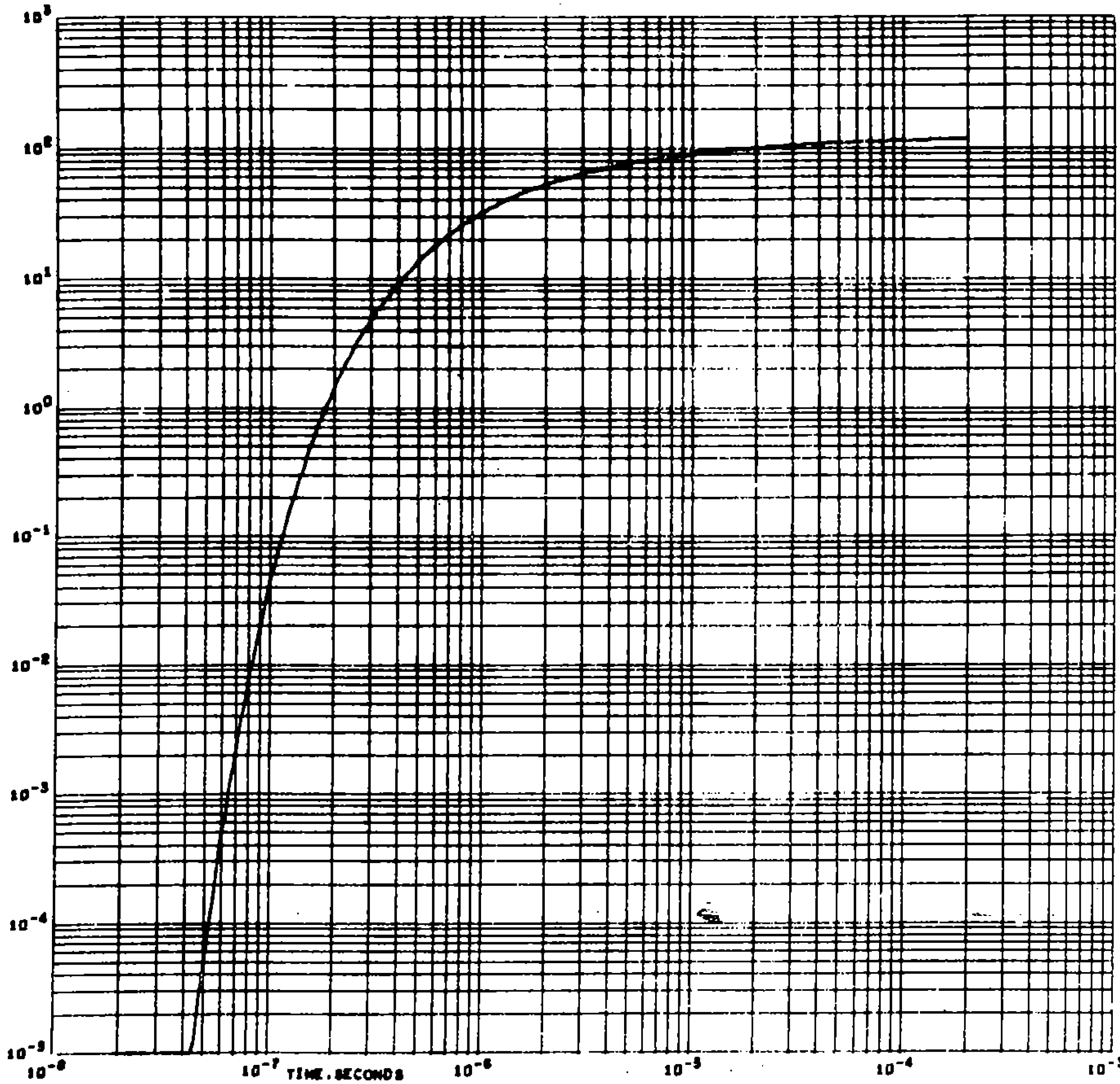
EMP PROPAGATION, (J-13-459)
Z = 0.30 METERS
SIGMA = 4.3000 MHOS/METER
R = 9.000E-09 SEC
MU = 1.000
TAU = 4.883E-07 SEC
EPSILON = 81.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T



EMP PROPAGATION, (J-13-459)
Z = 0.50 METERS SIGMA = 4.3000 MHOS/METER
K = 1.500E-08 SEC
MU = 1.000
TAU = 1.351E-06 SEC
EPSILON = 81.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T



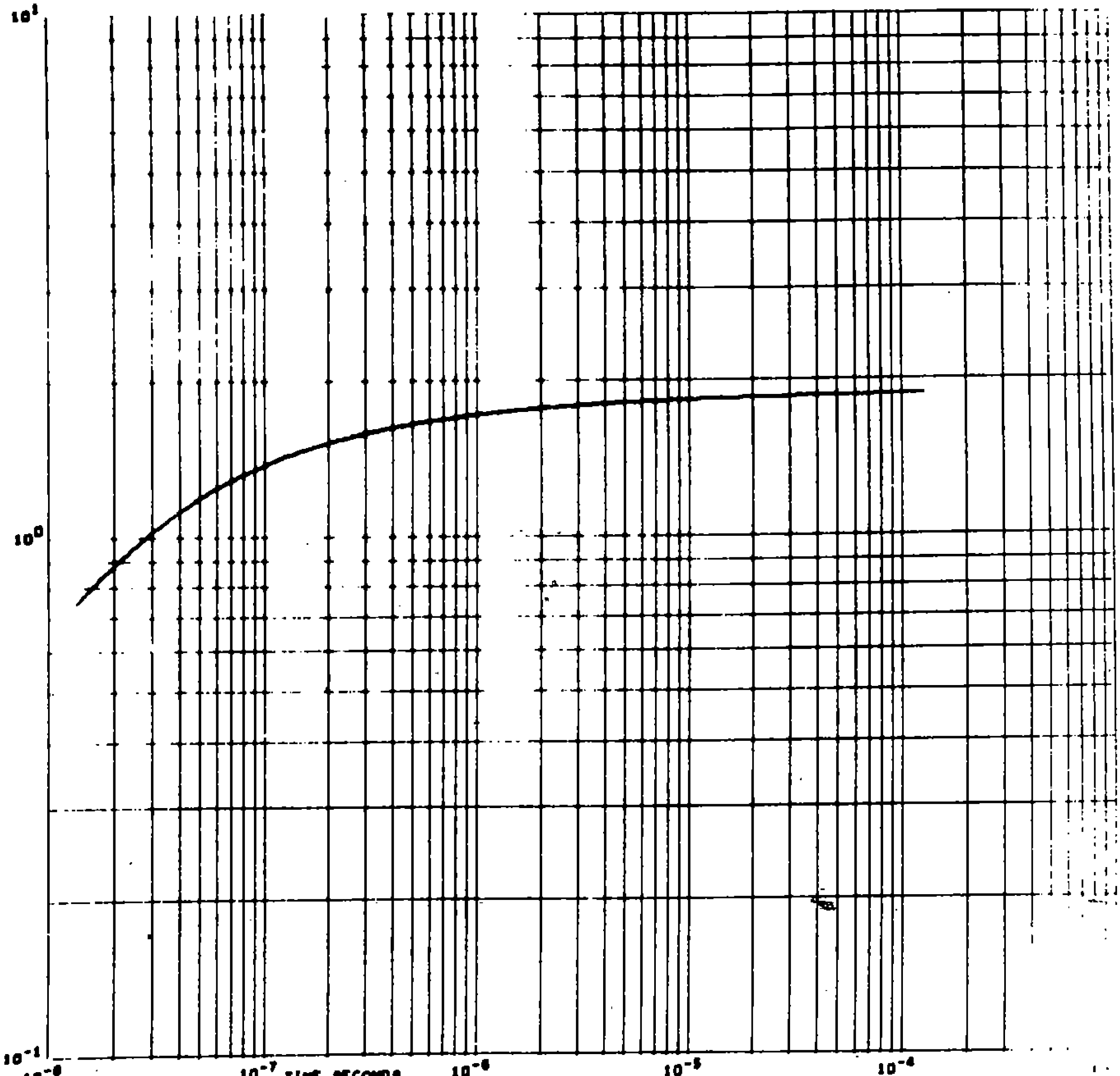
EMP PROPAGATION, (J-13-459)
Z = 0.00 METERS
SIGMA = 0.0200 MHOS/METER
K = 8.000E-09 SEC
MU = 1.000
TAU = 9.048E-09 SEC
EPSILON = 16.000
STEP FUNCTION ELECTRIC FIELD A, Z=0
E VS T



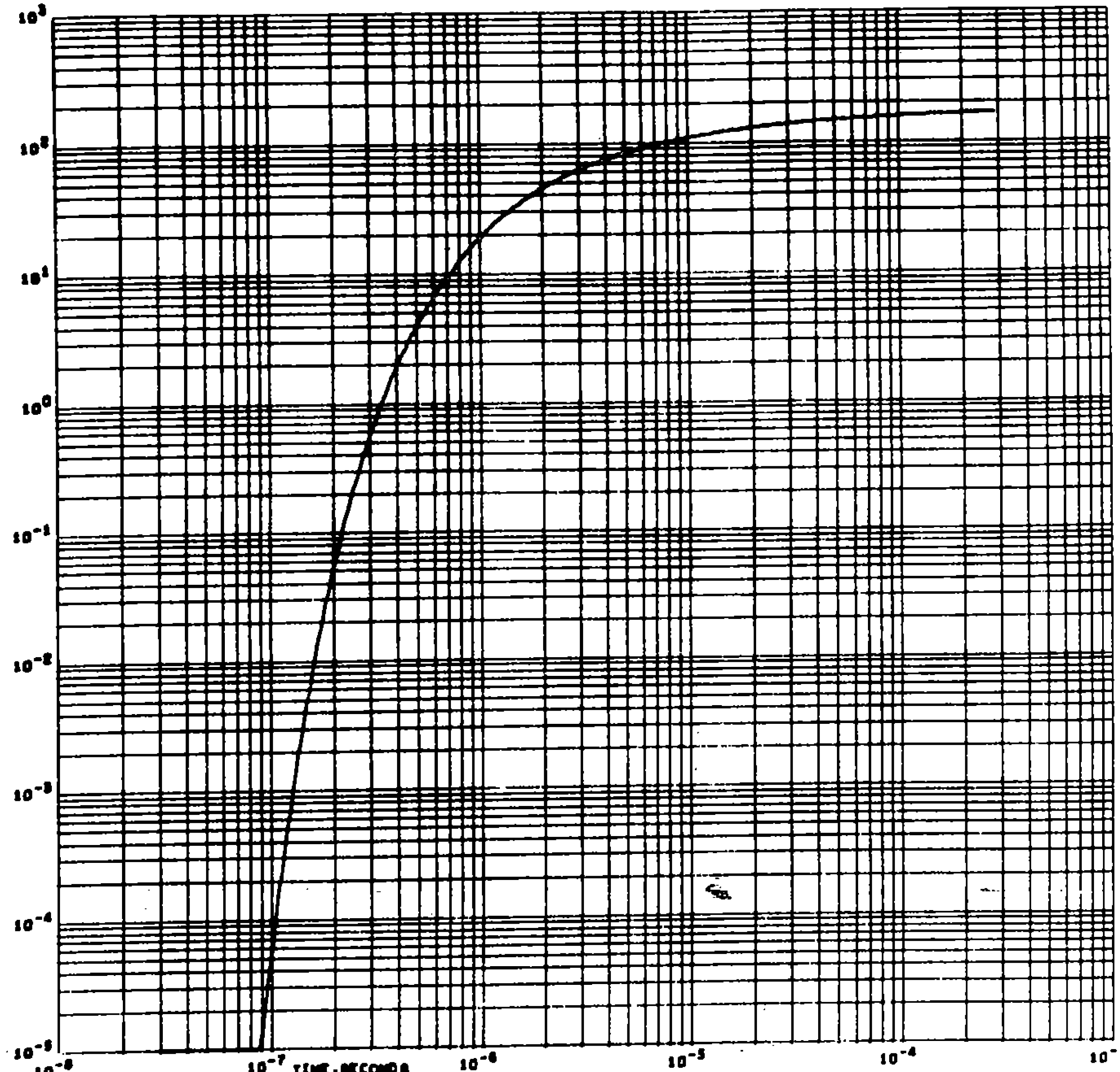
EMP PROPAGATION, (J-13-459)
Z = 0.70 METERS SIGMA = 4.3000 MHOS/METER
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T

K = 2.100E-08 SEC
MU = 1.000

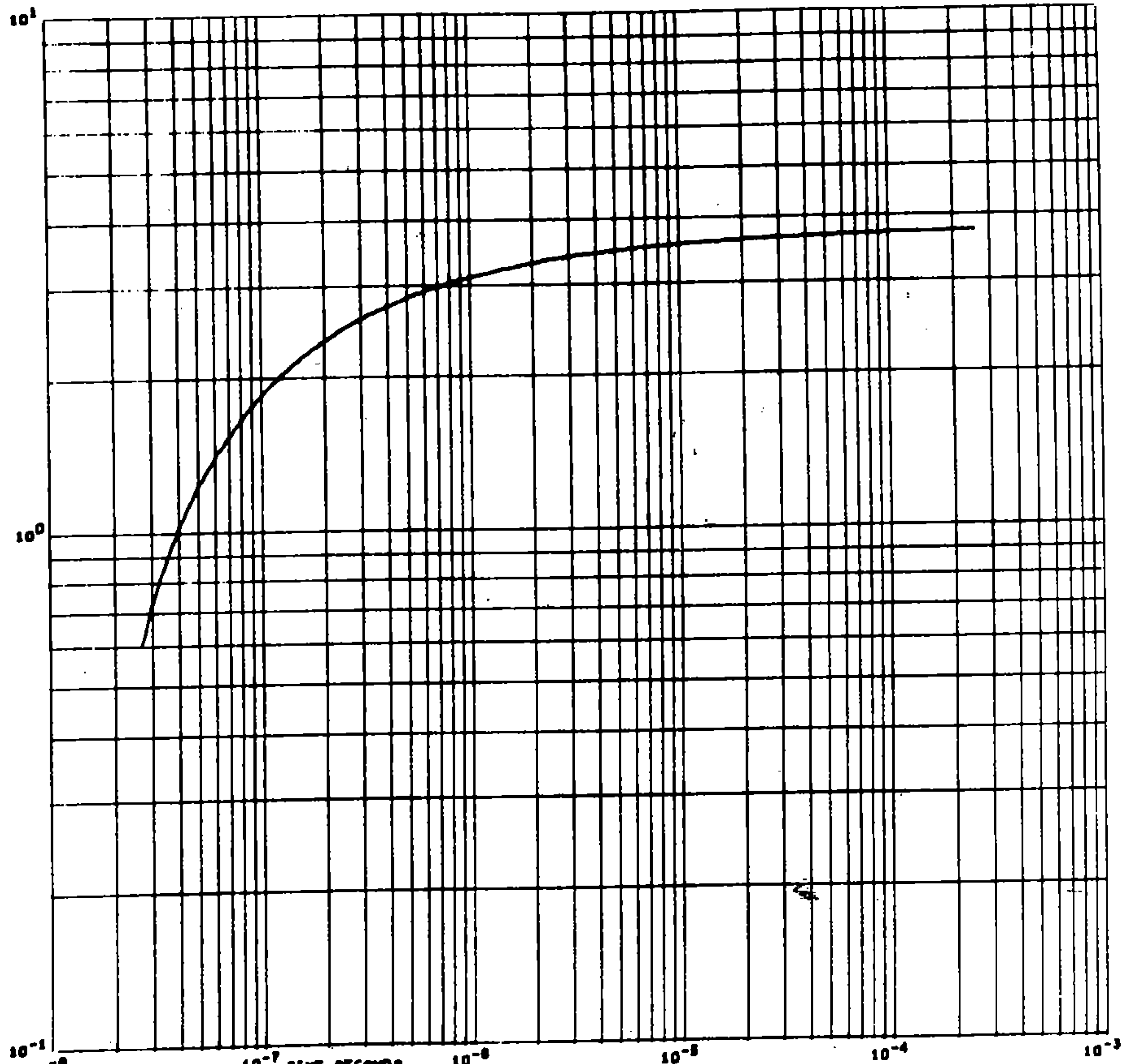
TAU = 2.648E-06 SEC
EPSILON = 81.000



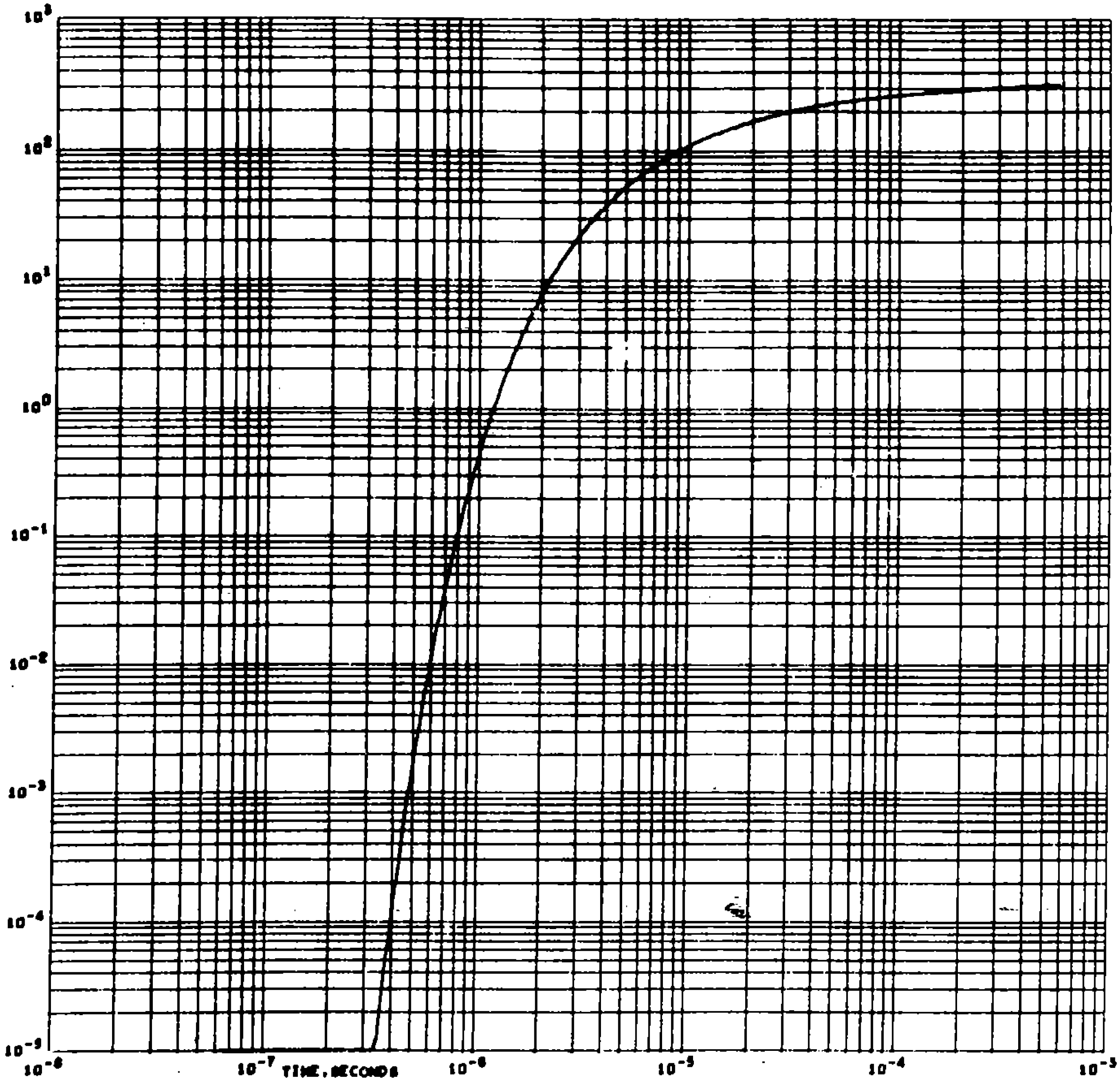
EMP PROPAGATION, (J-13-490)
Z = 1.00 METERS SIGMA = 0.0200 MHOS/METER
R = 1.333E-08 SEC
MU = 1.000
TAU = 2.213E-08 SEC
EPSILON = 16.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T



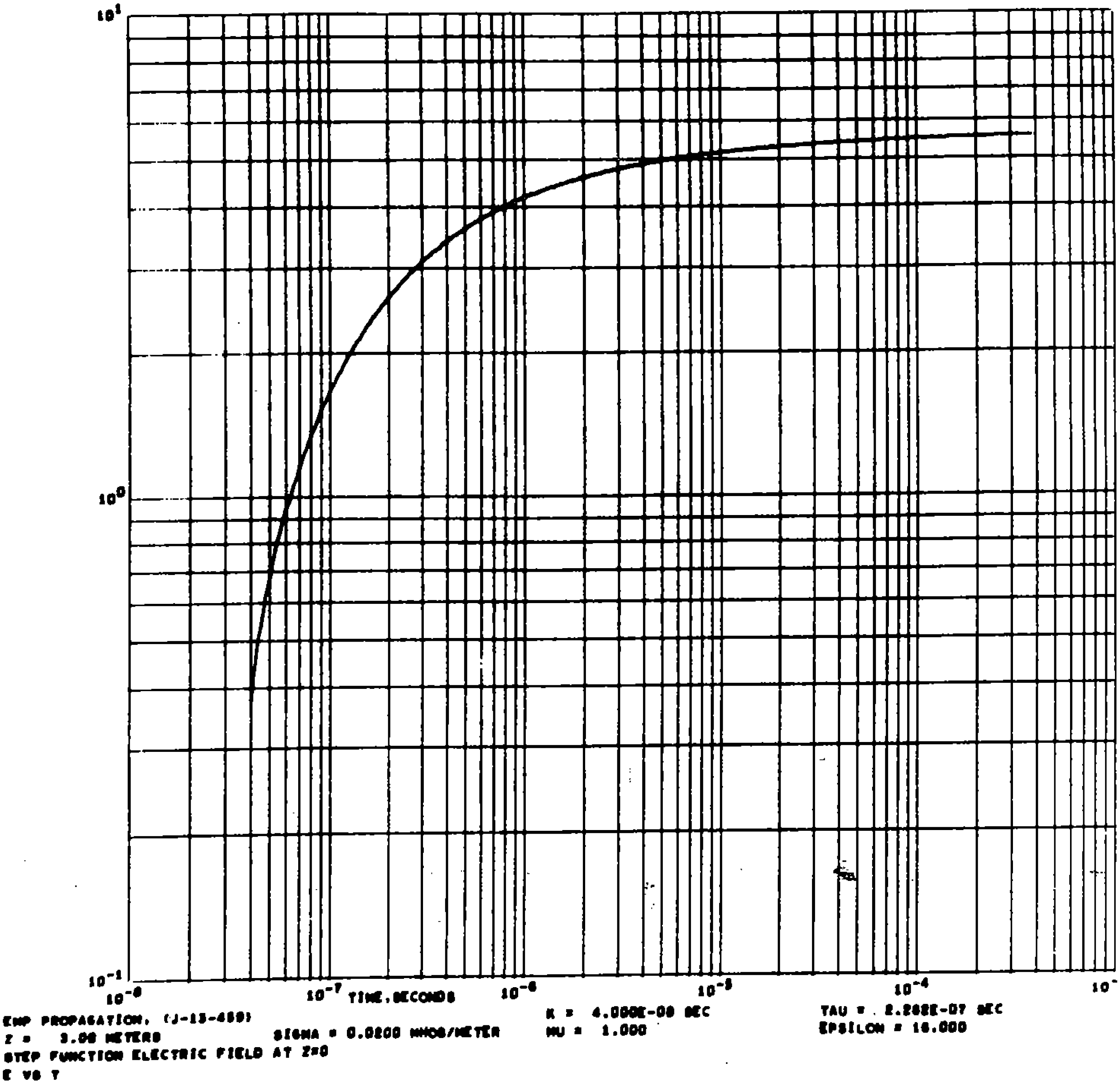
EMP PROPAGATION, (J-13-480)
Z = 1.00 METERS
SIGMA = 4.3000 MHOS/METER
K = 3.000E-08 SEC
MU = 1.000
TAU = 3.404E-08 SEC
EPSILON = 81.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T



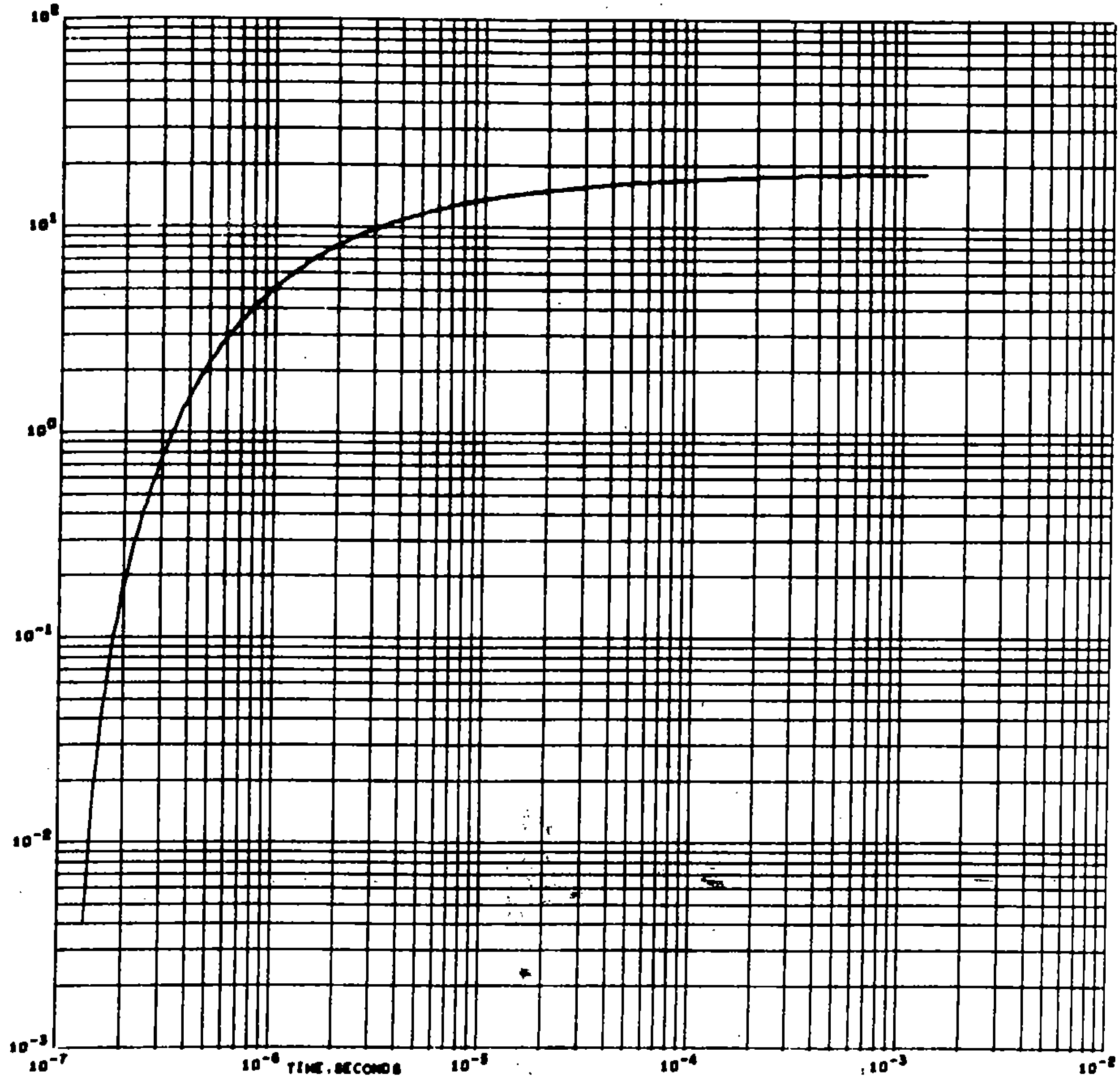
EMP PROPAGATION, (J-13-459)
Z = 2.00 METERS SIGMA = 0.0200 MHOS/METER
K = 2.667E-08 SEC
MU = 1.000
TAU = 1.005E-07 SEC
EPSILON = 16.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T



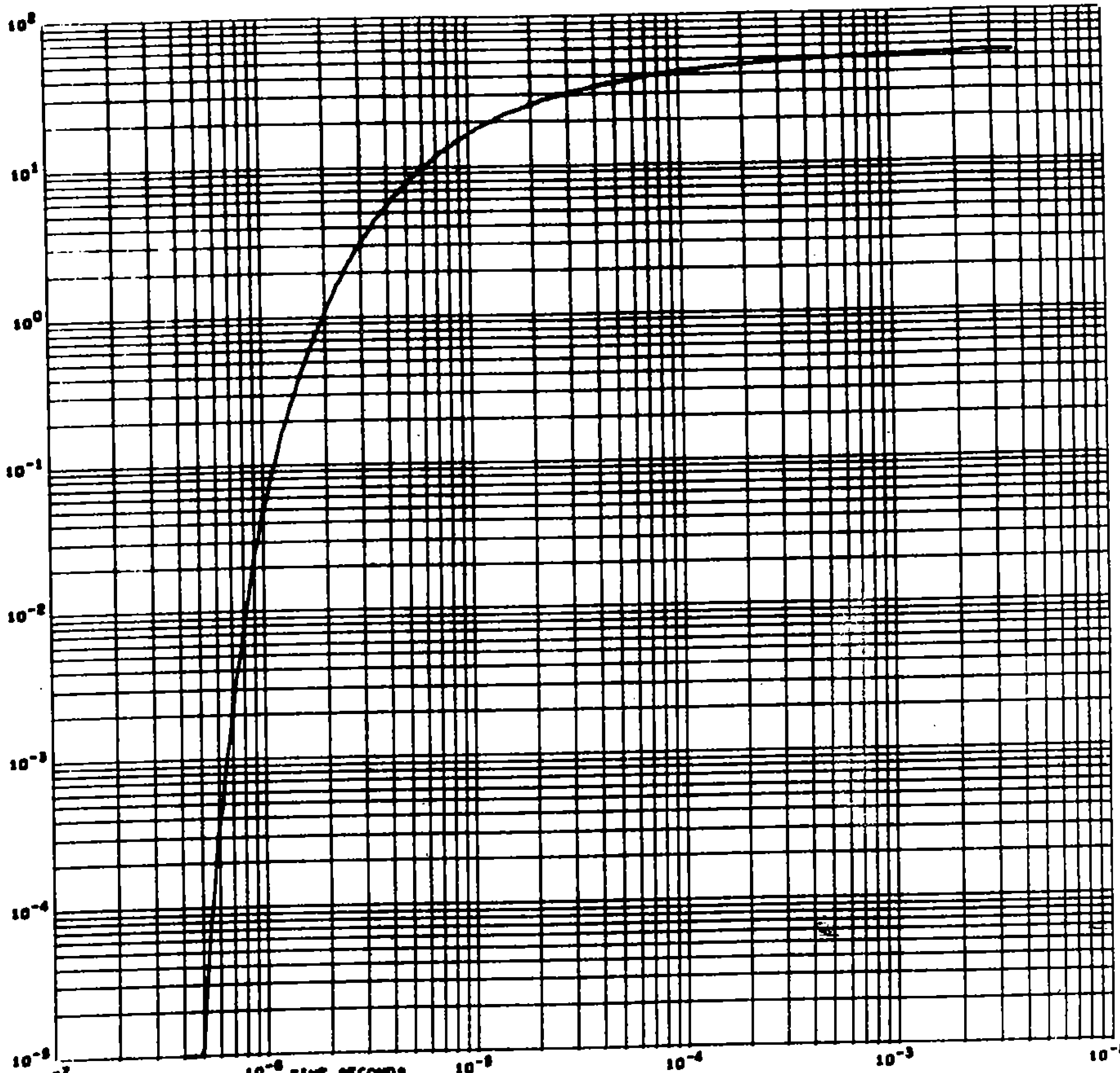
EMP PROPAGATION, (J-13-450)
Z = 2.00 METERS   SIGMA = 4.3000 MHOS/METER   K = 6.000E-08 SEC   MU = 1.000   TAU = 2.161E-05 SEC   EPSILON = 81.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T



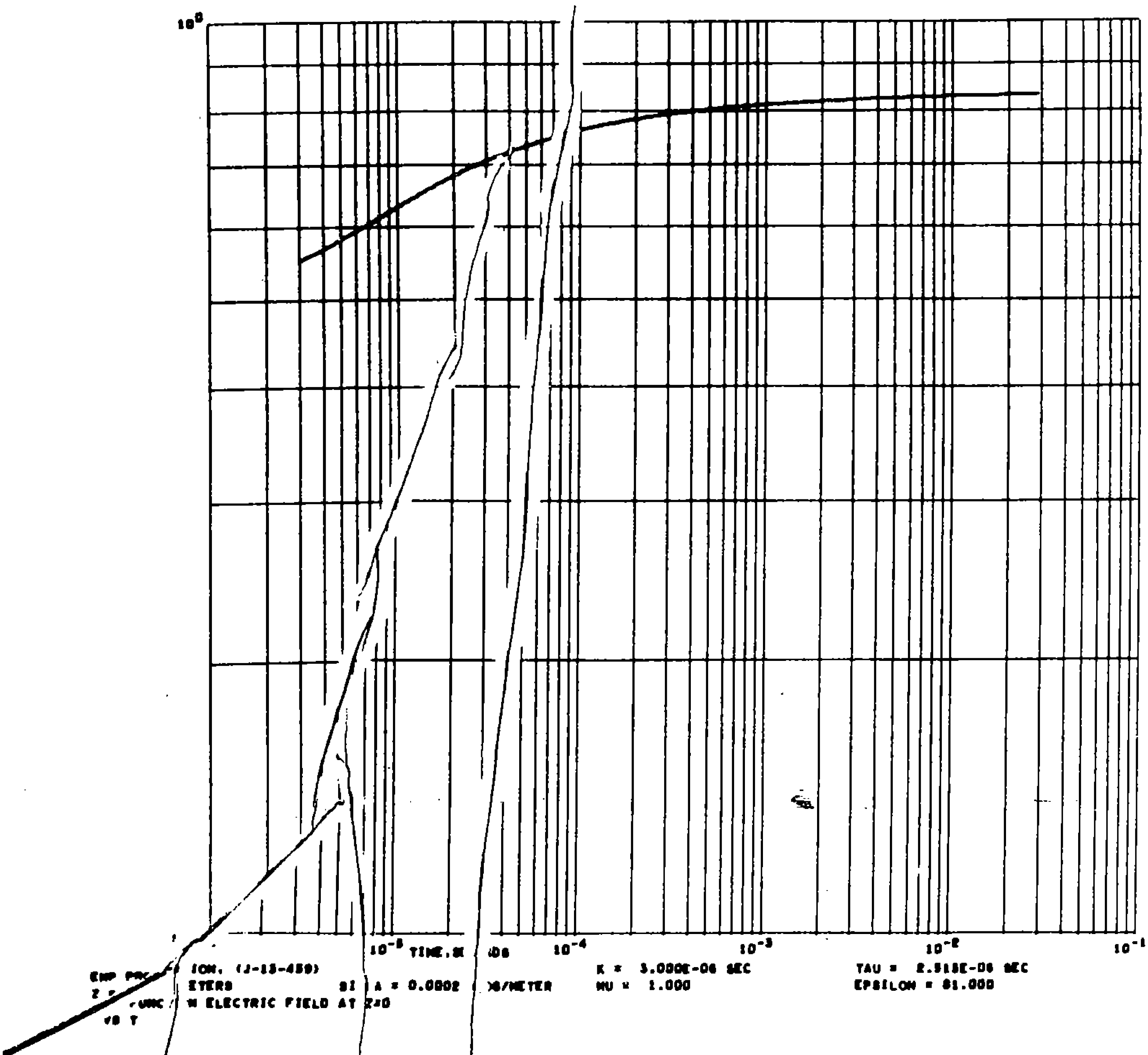
10^1
10^0
10^-1
10^-8
10^-7
TIME,SECONDS
10^-6
10^-5
10^-4
10^-3
EMP PROPAGATION, (J-13-459)
Z = 3.00 METERS
SIGMA = 0.0200 MHOS/METER
K = 4.000E-08 SEC
MU = 1.000
TAU = 2.282E-07 SEC
EPSILON = 16.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T

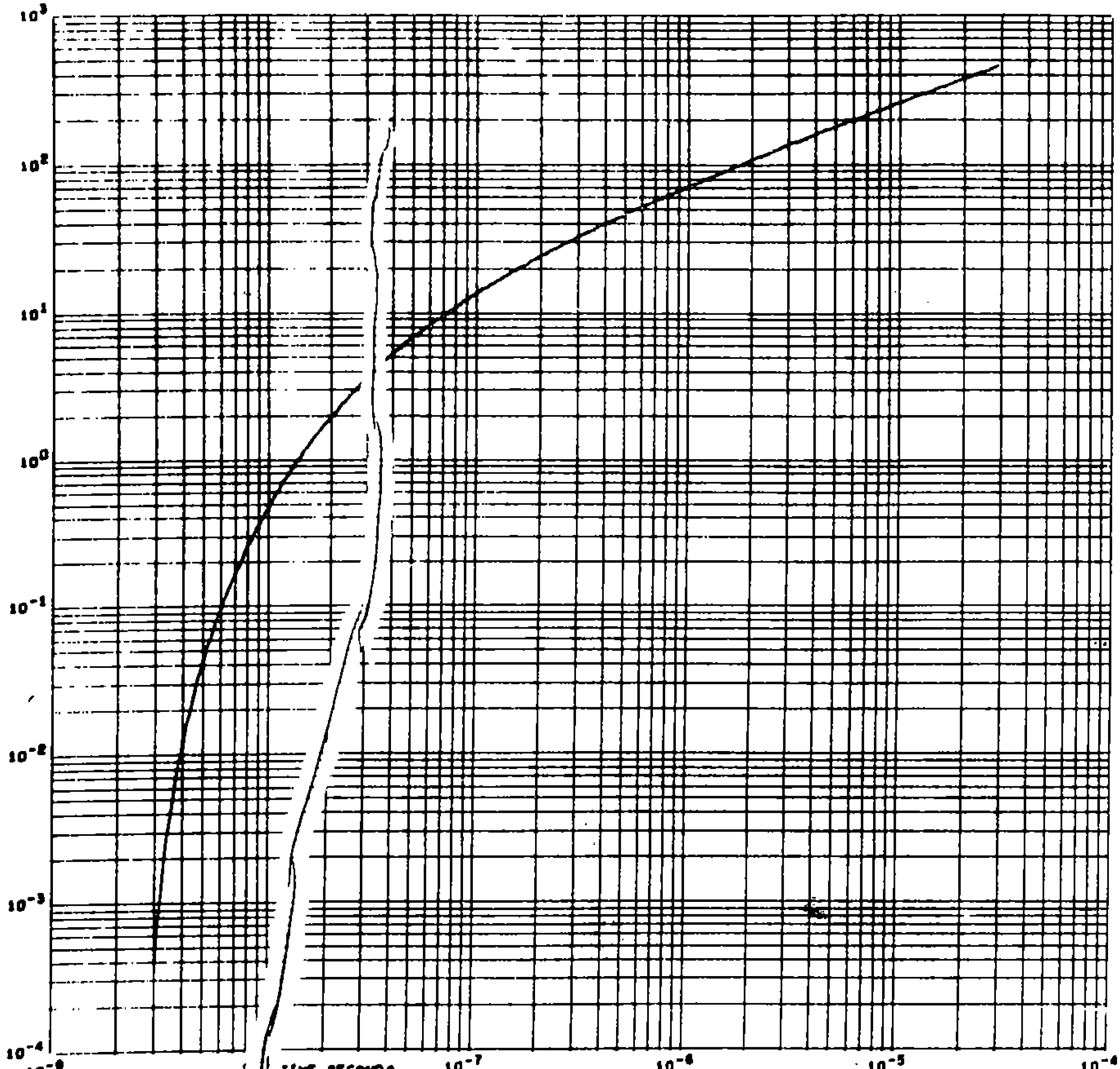


EMP PROPAGATION, (J-13-459)
Z = 10.00 METERS   SIGMA = 0.0200 MHOS/METER   K = 1.333E-07 SEC   TAU = 2.513E-08 SEC
MU = 1.000   EPSILON = 16.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T



EMP PROPAGATION, (J-13-49B)
Z = 30.00 METERS
SIGMA = 0.0200 MHOS/METER
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T
K = 4.000E-07 SEC
MU = 1.000
TAU = 2.262E-05 SEC
EPSILON = 16.000

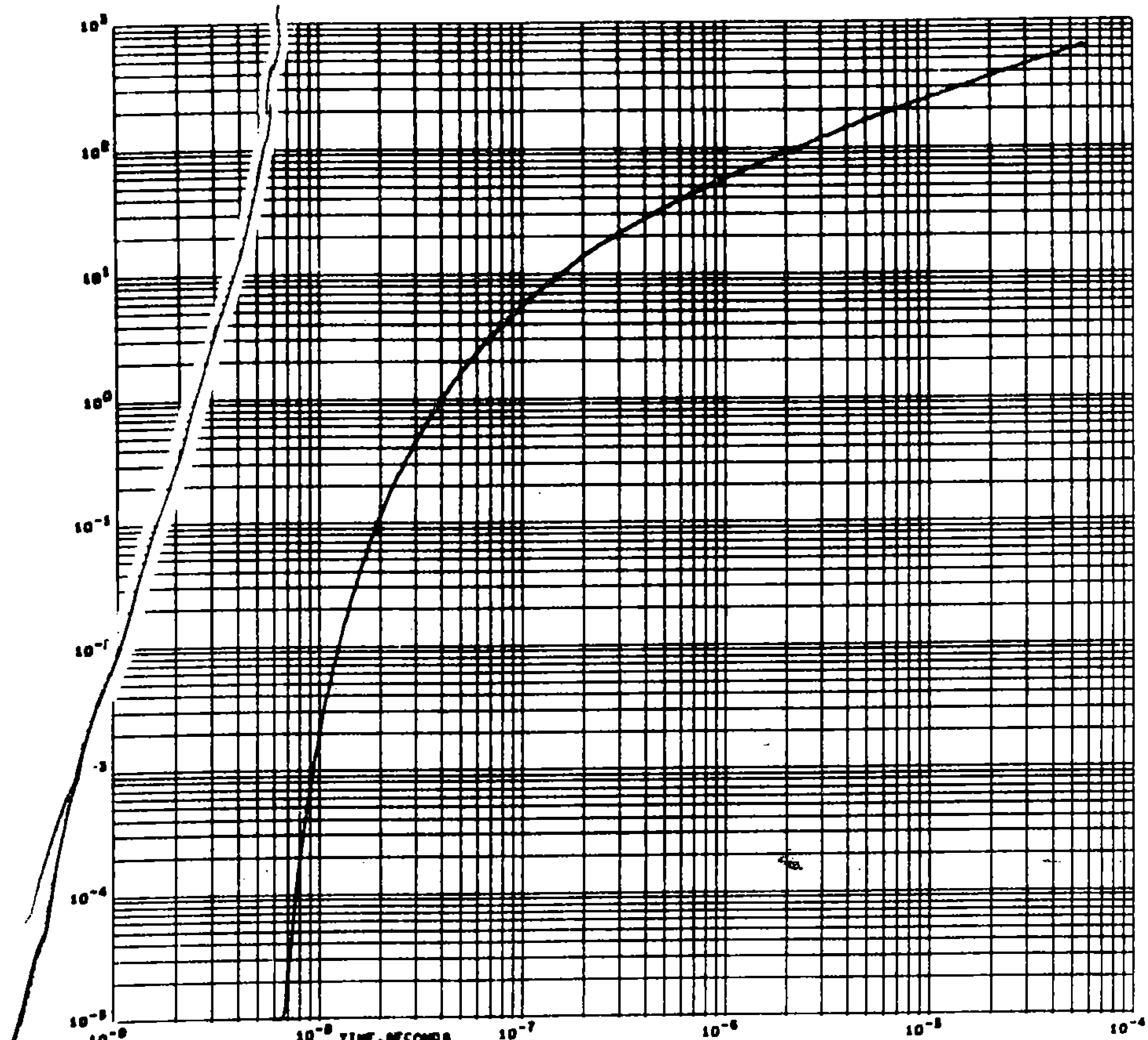

10^0
10^-5 TIME,SE   DS
10^-4
10^-3
10^-2
10^-1
K = 3.000E-06 SEC
MU = 1.000
TAU = 2.518E-06 SEC
EPSILON = 81.000
ELECTRIC FIELD AT Z=0
A = 0.0002



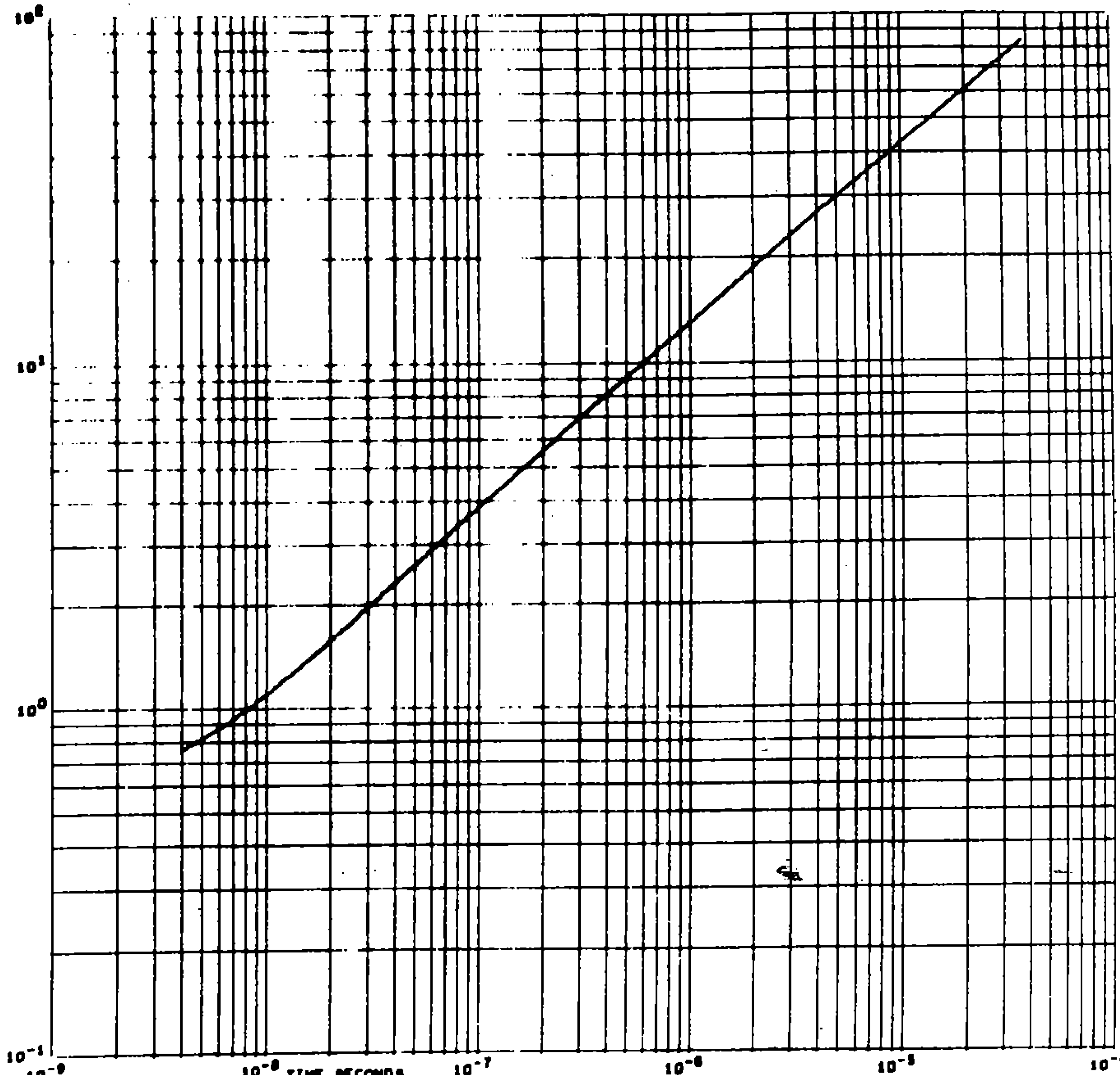
EMP PROPAGATION. (J-13-459)
Z = 0.10 METERS SIGM .3000 MHOS/METER
STEP FUNCTION ELECTRIC FIELD AT 1
B VS T

K = 3.000E-08 SEC
MU = 1.000

TAU = 5.404E-08 SEC
EPSILON = 81.000



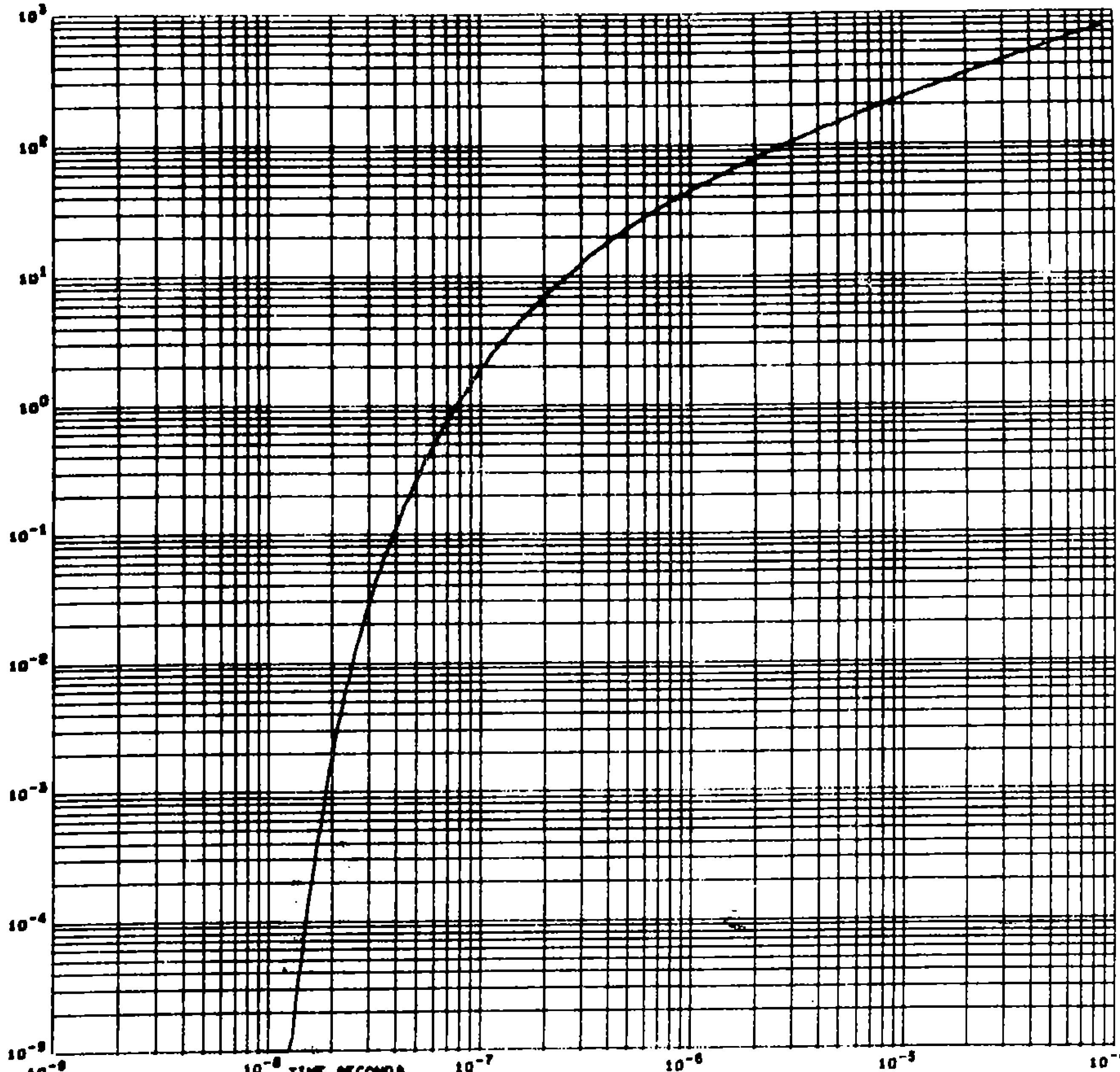
PROPAGATION, (J-13-459)
0.20 METERS
SIGMA = 4.3000 MHOS/METER
K = 6.000E-09 SEC
NU = 1.000
TAU = 2.161E-07 SEC
EPSILON = 81.000
P FUNCTION ELECTRIC FIELD AT Z=0
'S T

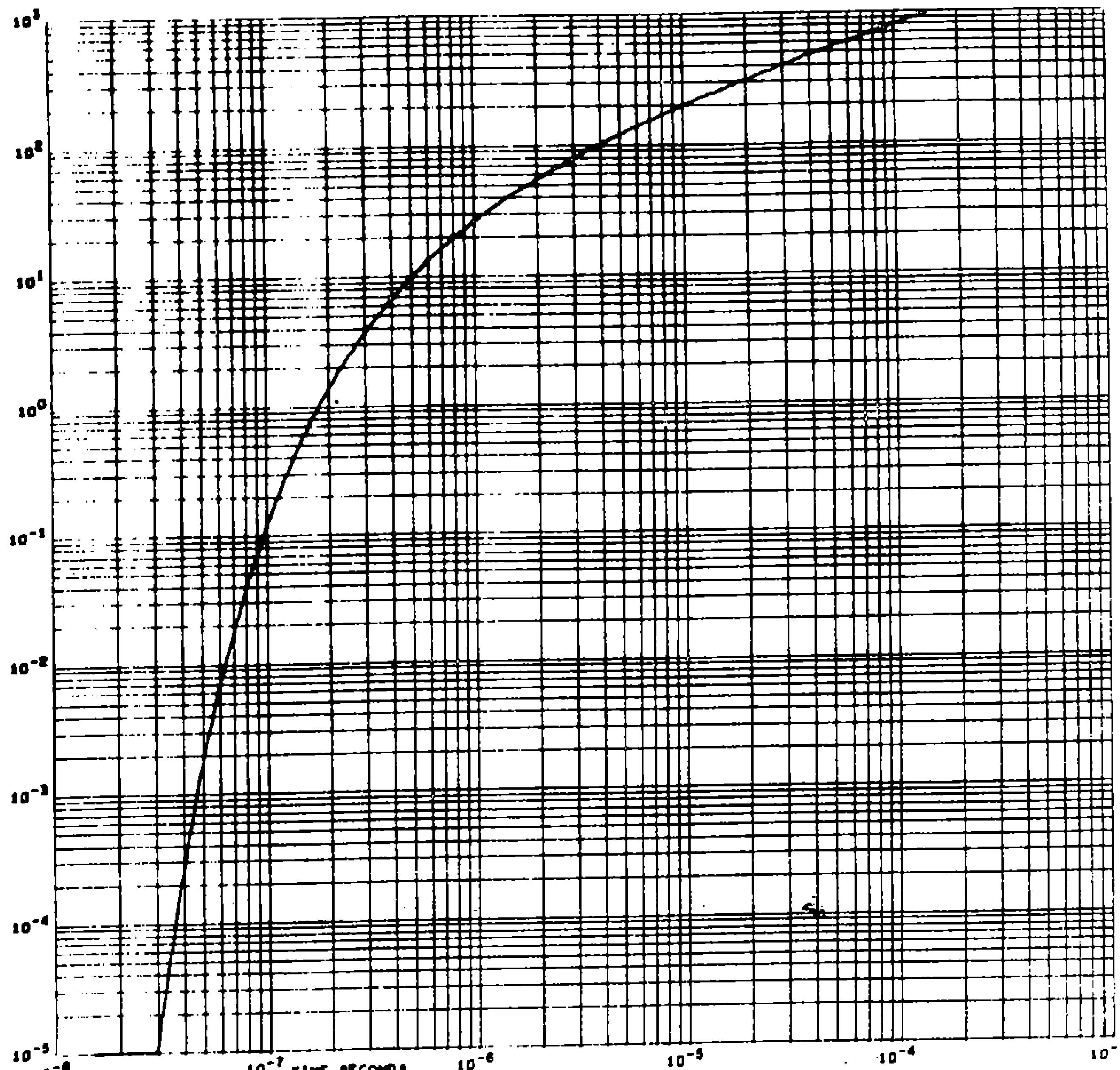


EMP PROPAGATION, (J-13-459)
= 0.30 METERS SIGMA = 0.0200 MHOS/METER
FUNCTION ELECTRIC FIELD AT Z=0

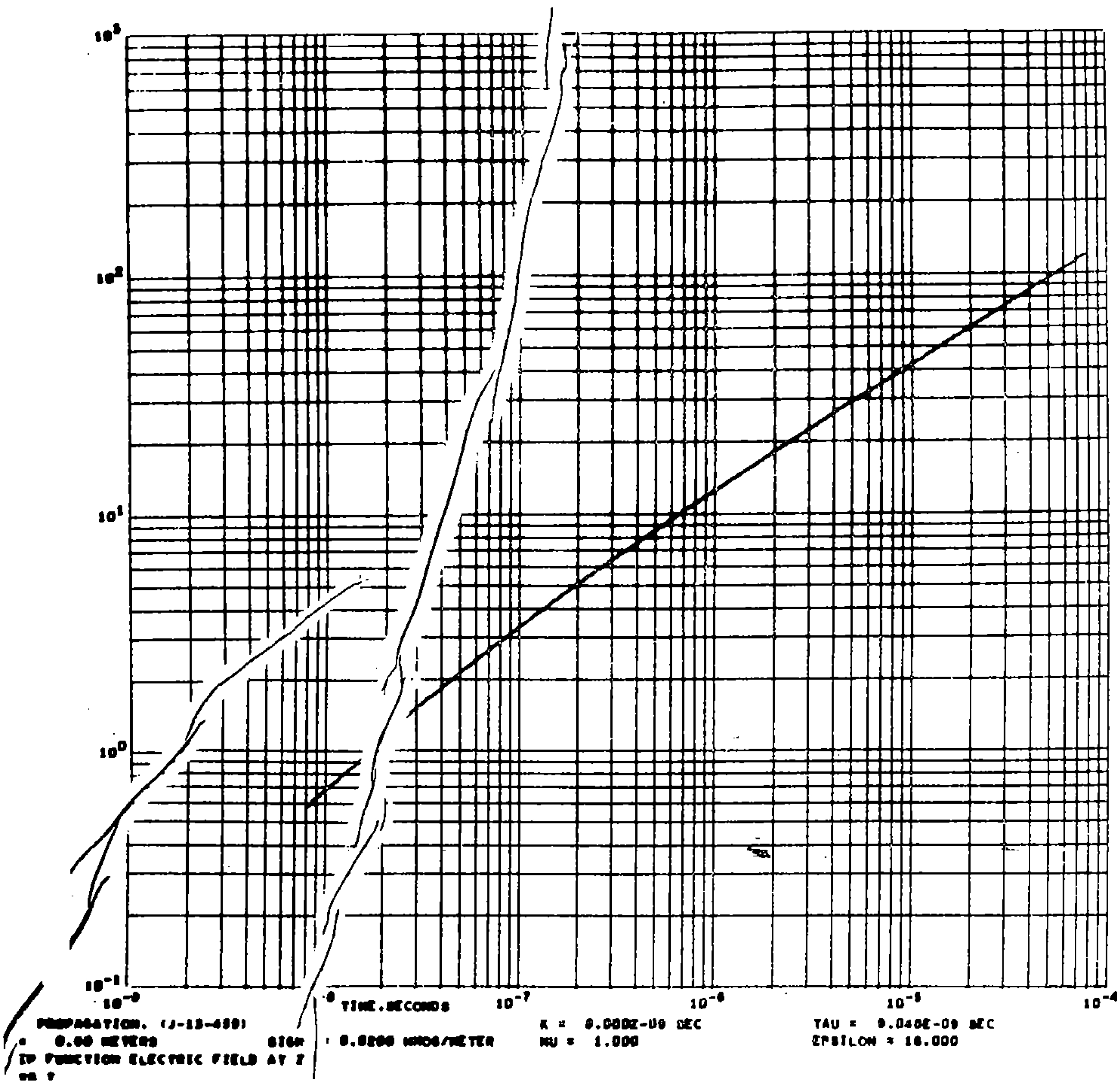
K = 4.000E-08 SEC
MU = 1.000

TAU = 2.262E-09 SEC
EPSILON = 16.000

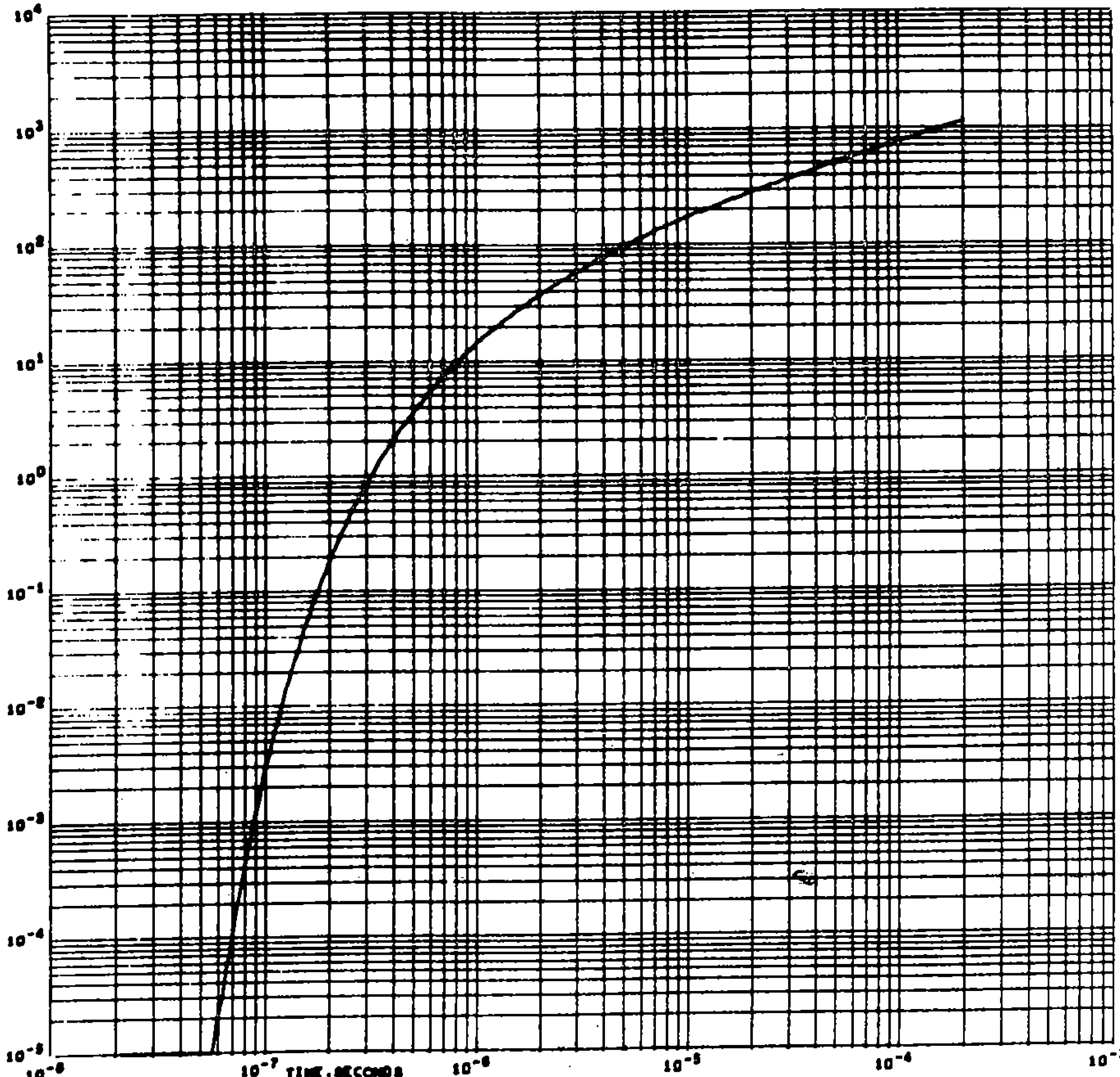

TIME, SECONDS
K = 9.000E-09 SEC
MU = 1.000
EPSILON = 81.000
Z = 0.30 METERS
STEP FUNCTION ELECTRIC FIELD AT Z=0



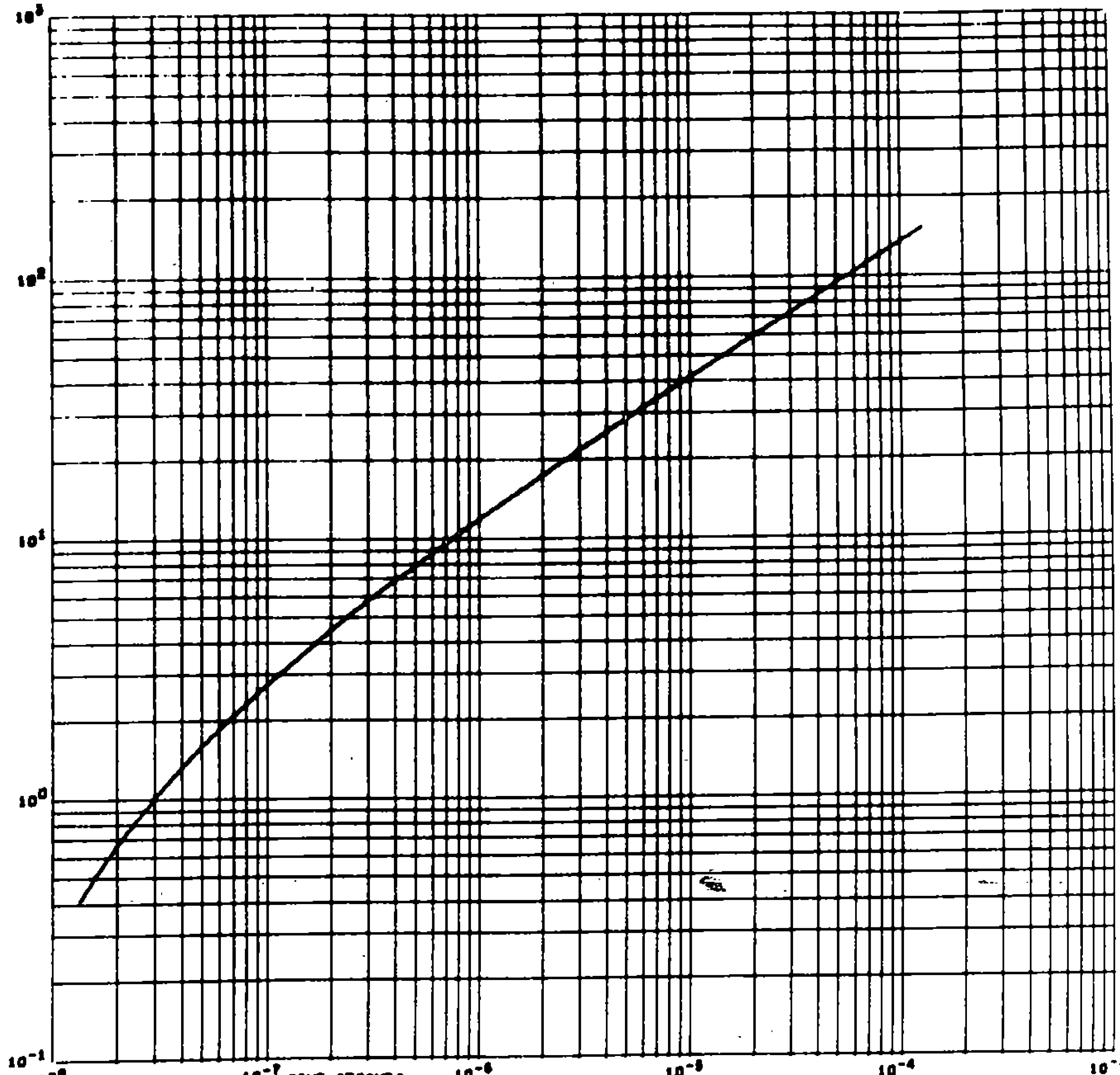
EMP PROPAGATION, (J-13-459)
Z = 0.50 METERS SIGMA = 4.3000 MHOS/METER
STEP FUNCTION ELECTRIC FIELD AT Z=0
K = 1.500E-08 SEC
MU = 1.000
TAU = 1.391E-06 SEC
EPSILON = 81.000



10^3
10^2
10^1
10^0
10^{-1}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
TIME,SECONDS
MU = 1.000
EPSILON = 16.000



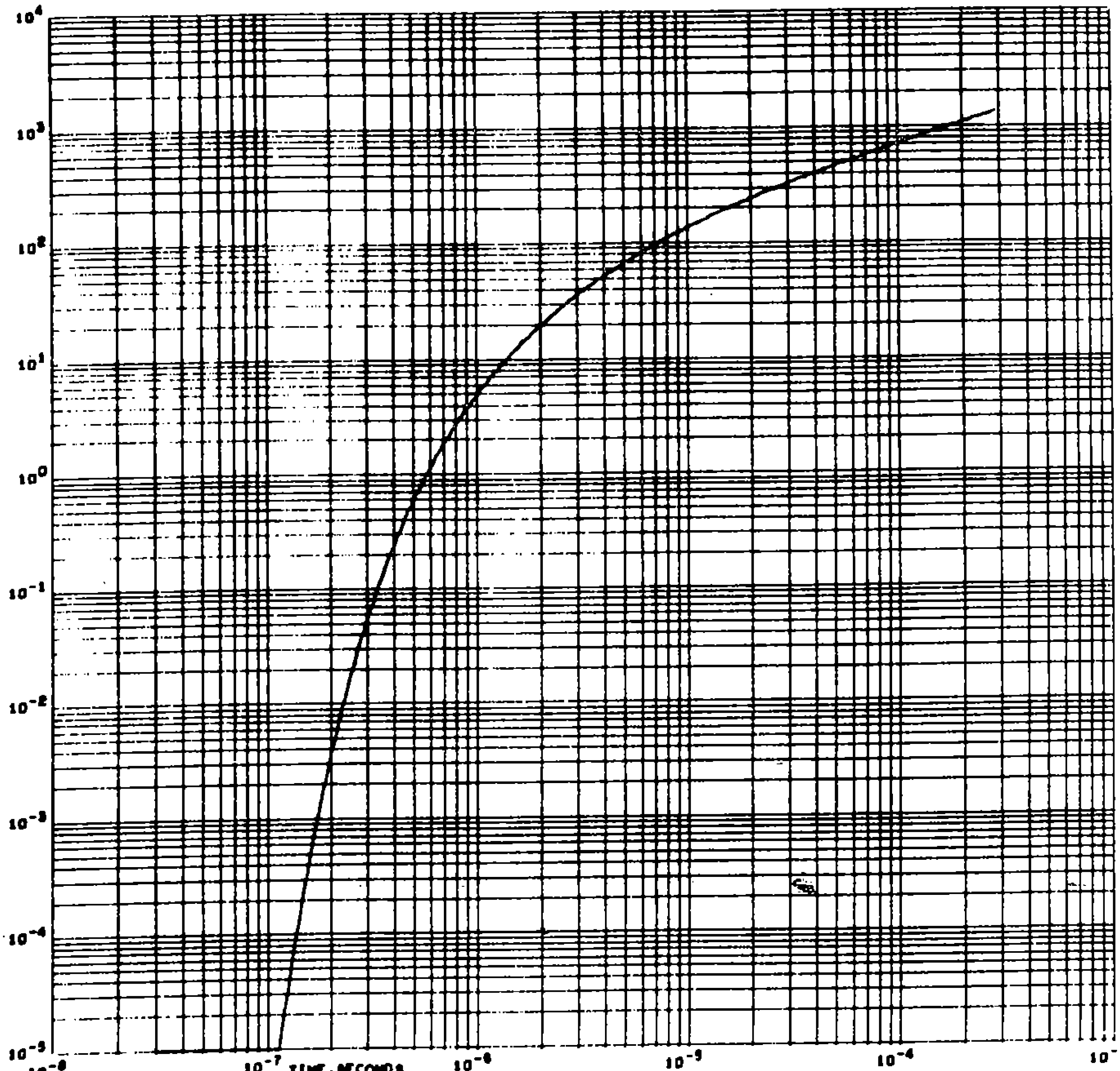
EMP PROPAGATION, (J-13-489)
Z = 0.70 METERS SIGMA = 4.3000 MHOS/METER
K = 2.100E-08 SEC MU = 1.000
TAU = 2.648E-06 SEC EPSILON = 81.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
B VS T



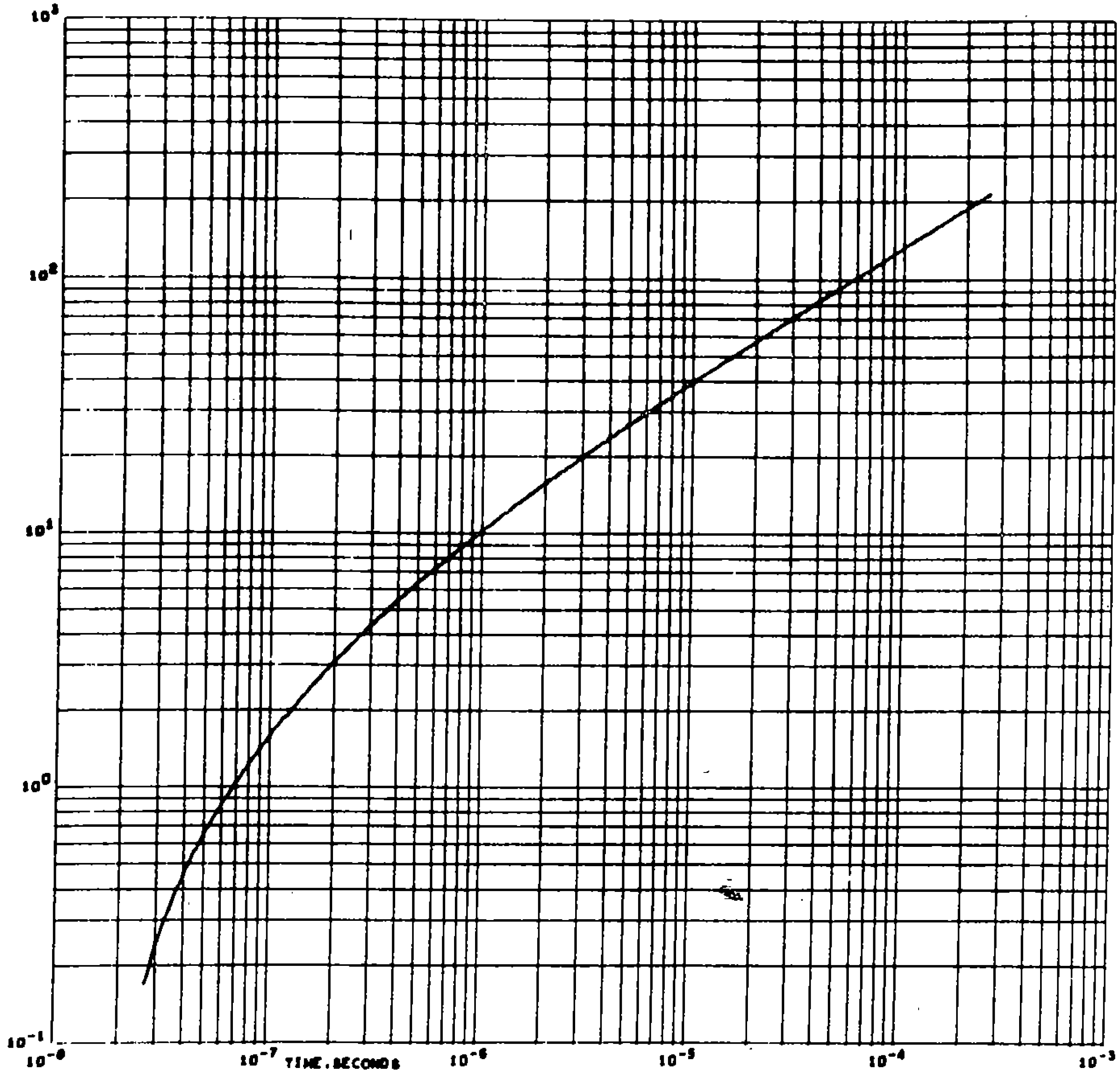
EMP PROPAGATION, (J-13-459)
Z = 1.00 METERS SIGMA = 0.0200 MHOS/METER
STEP FUNCTION ELECTRIC FIELD AT Z=0
B VS T

K = 1.333E-08 SEC
MU = 1.000

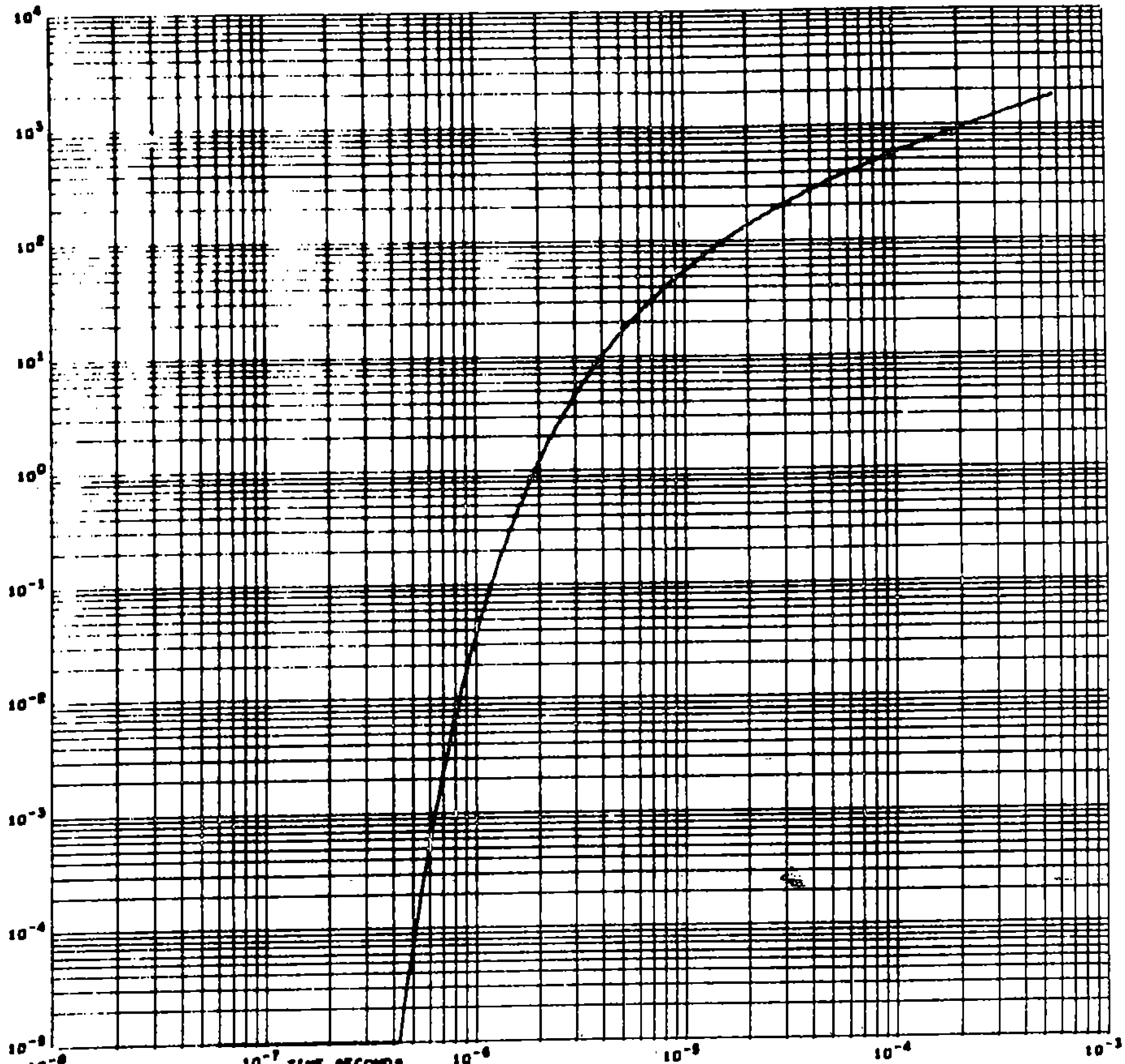
TAU = 2.513E-08 SEC
EPSILON = 16.000



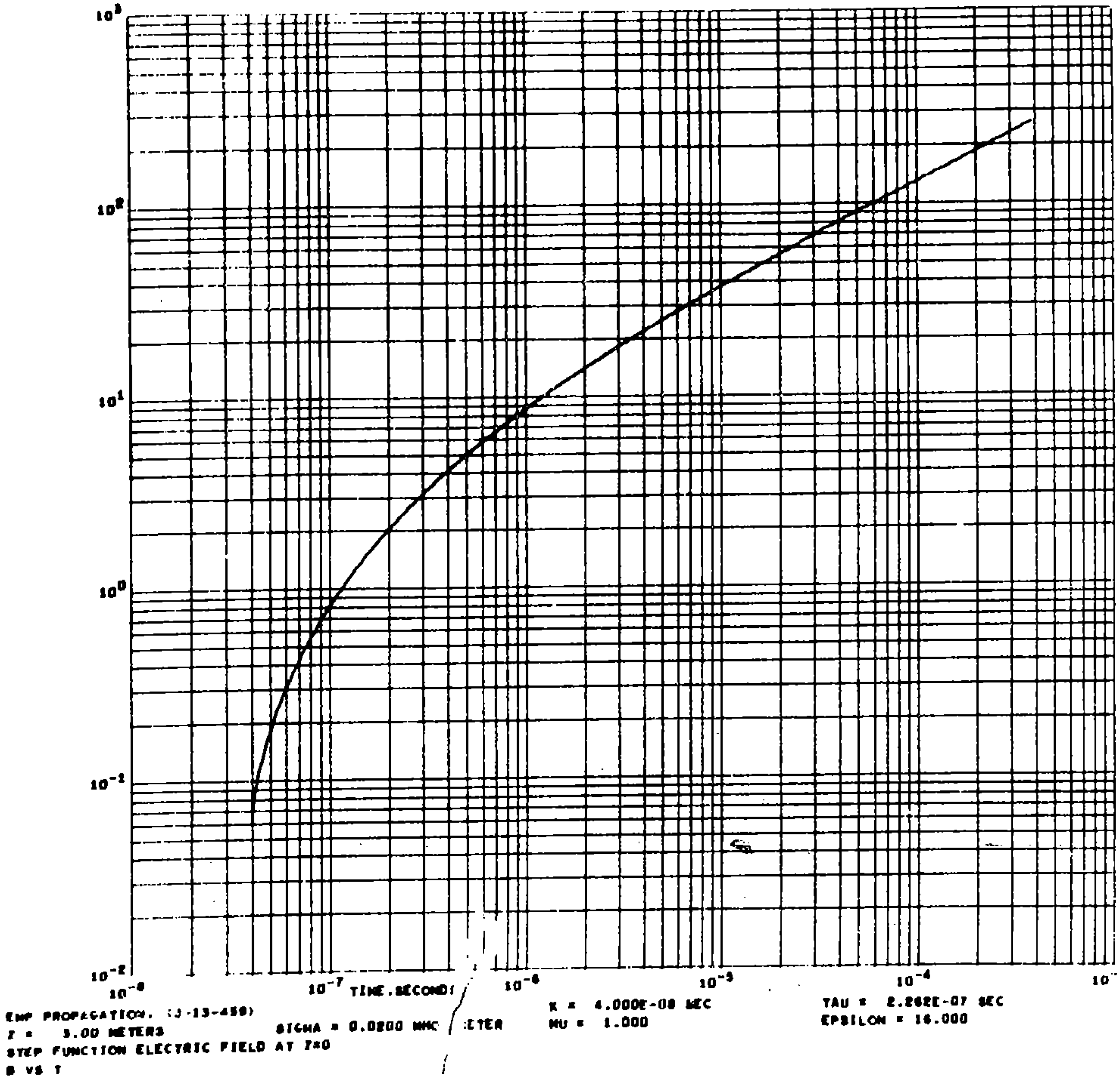
EMP PROPAGATION. (J-13-480)
Z = 1.00 METERS
SIGMA = 4.3000 MHOS/METER
K = 3.000E-08 SEC
MU = 1.000
TAU = 5.404E-06 SEC
EPSILON = 81.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
B VS T

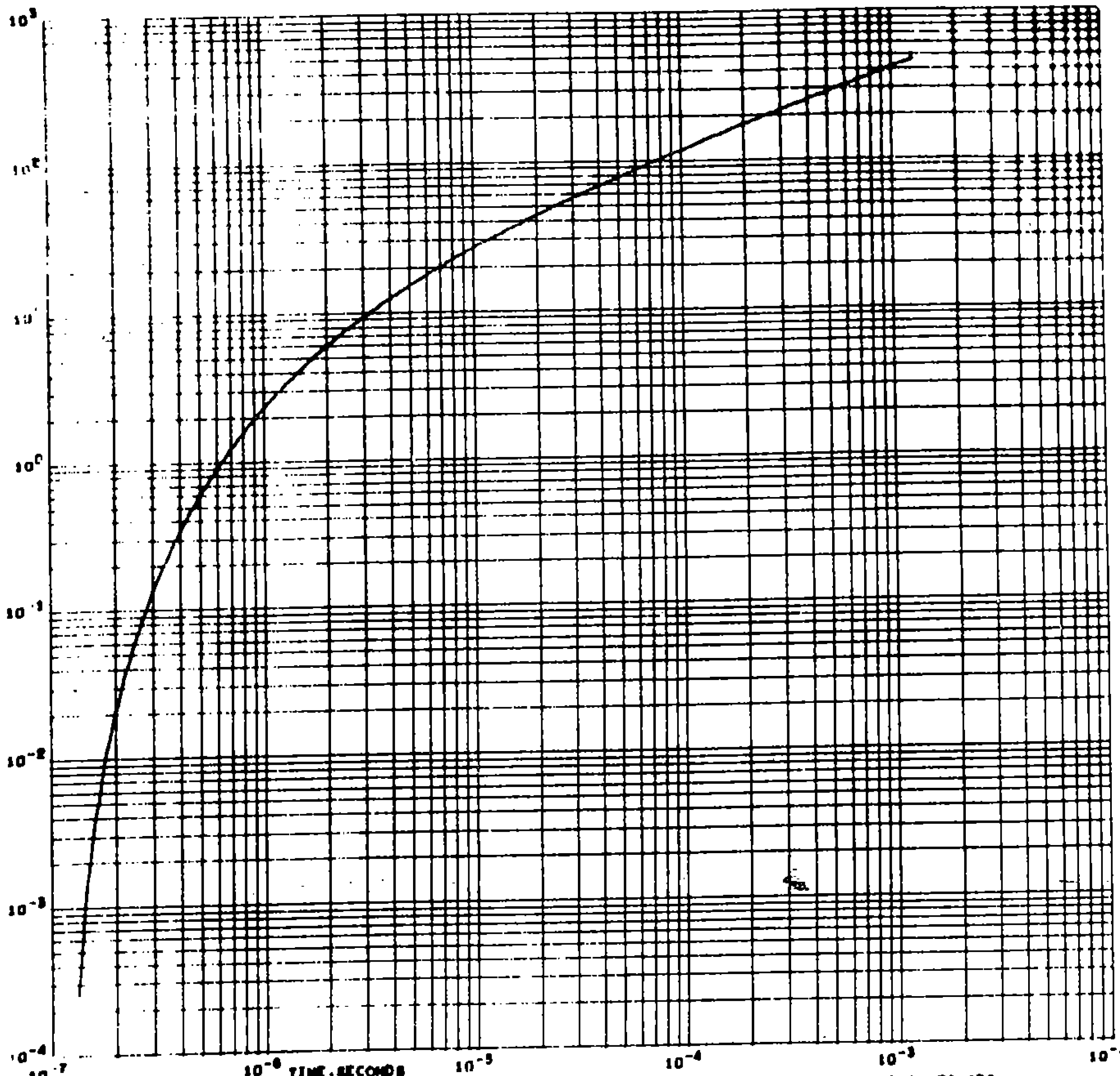


EMP PROPAGATION. (J-13-459)
Z = 8.00 METERS   SIGMA = 0.0200 MHOS/METER
K = 2.667E-08 SEC   MU = 1.000
TAU = 1.005E-07 SEC   EPSILON = 16.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
E VS T

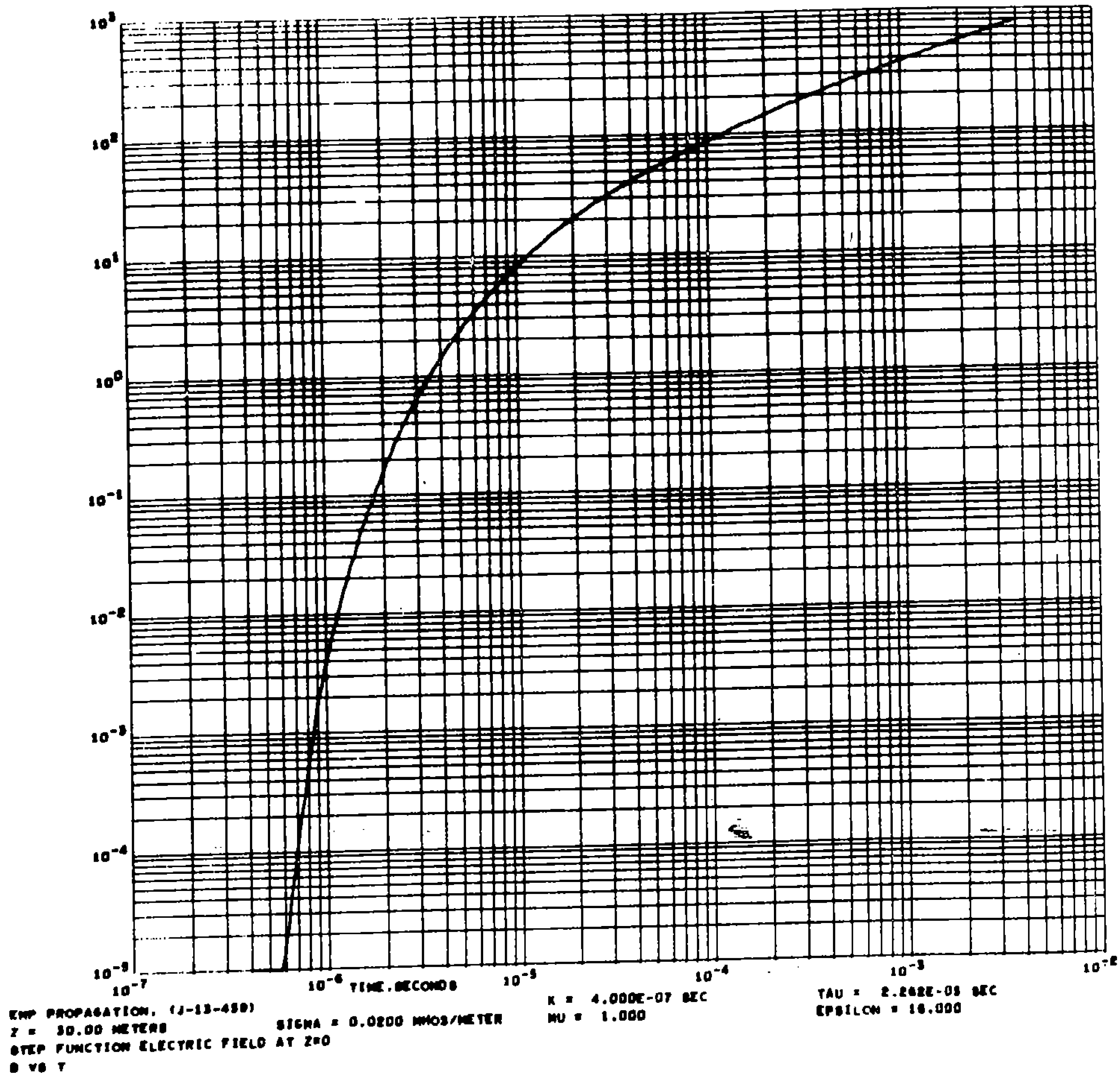


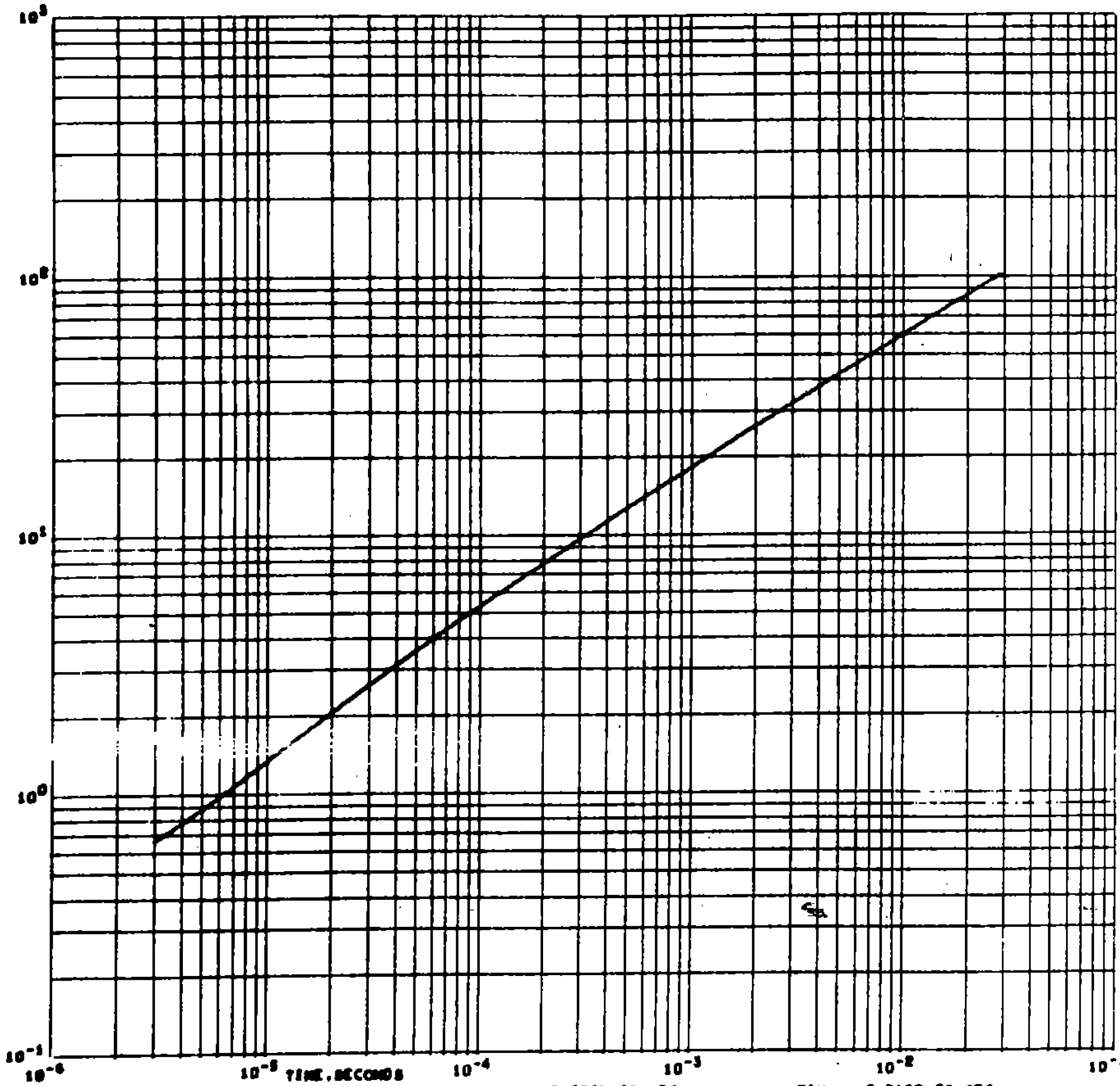
EMP PROPAGATION. (J-13-480)
Z = 2.00 METERS   SIGMA = 4.3000 MHOS/METER   K = 6.000E-08 SEC   MU = 1.000   TAU = 2.161E-05 SEC   EPSILON = 81.000
STEP FUNCTION ELECTRIC FIELD AT Z=0


10^3
10^2
10^1
10^0
10^-1
10^-2
10^-8
10^-7
TIME, SECONDS
10^-6
10^-5
10^-4
10^-3
EMP PROPAGATION, (J-13-459)
Z = 3.00 METERS
SIGMA = 0.0200 MHO/METER
STEP FUNCTION ELECTRIC FIELD AT Z=0
B VS T
K = 4.000E-08 SEC
MU = 1.000
TAU = 2.262E-07 SEC
EPSILON = 16.000



EMP PROPAGATION, (J-13-459)
Z = 10.00 METERS
SIGMA = 0.0200 MHOS/METER
R = 1.333E-07 SEC
MU = 1.000
TAU = 2.313E-08 SEC
EPSILON = 16.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
B VS T


10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-7
10^-6
TIME, SECONDS
10^-5
10^-4
10^-3
10^-2
EMP PROPAGATION, (J-13-450)
Z = 30.00 METERS
SIGMA = 0.0200 MHOS/METER
STEP FUNCTION ELECTRIC FIELD AT Z=0
B VS T
K = 4.000E-07 SEC
MU = 1.000
TAU = 2.262E-05 SEC
EPSILON = 16.000



EMP PROPAGATION, (J-13-459)
Z = 100.00 METERS
SIGMA = 0.0002 MHOS/METER
K = 3.000E-06 SEC
MU = 1.000
TAU = 2.513E-06 SEC
EPSILON = 81.000
STEP FUNCTION ELECTRIC FIELD AT Z=0
B VS T