EMP Theoretical Notes

Note VI

The Calculation of Conduction Electron Parameters in Ionized Air

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Abstract:

Increased understanding of the electron mobility in ionized air requires the development of a scheme to calculate this mobility. A technique is developed which has the advantage of calculating the electron temperature and this can be used to evaluate the electron attachment and recombination coefficients.

I. Introduction

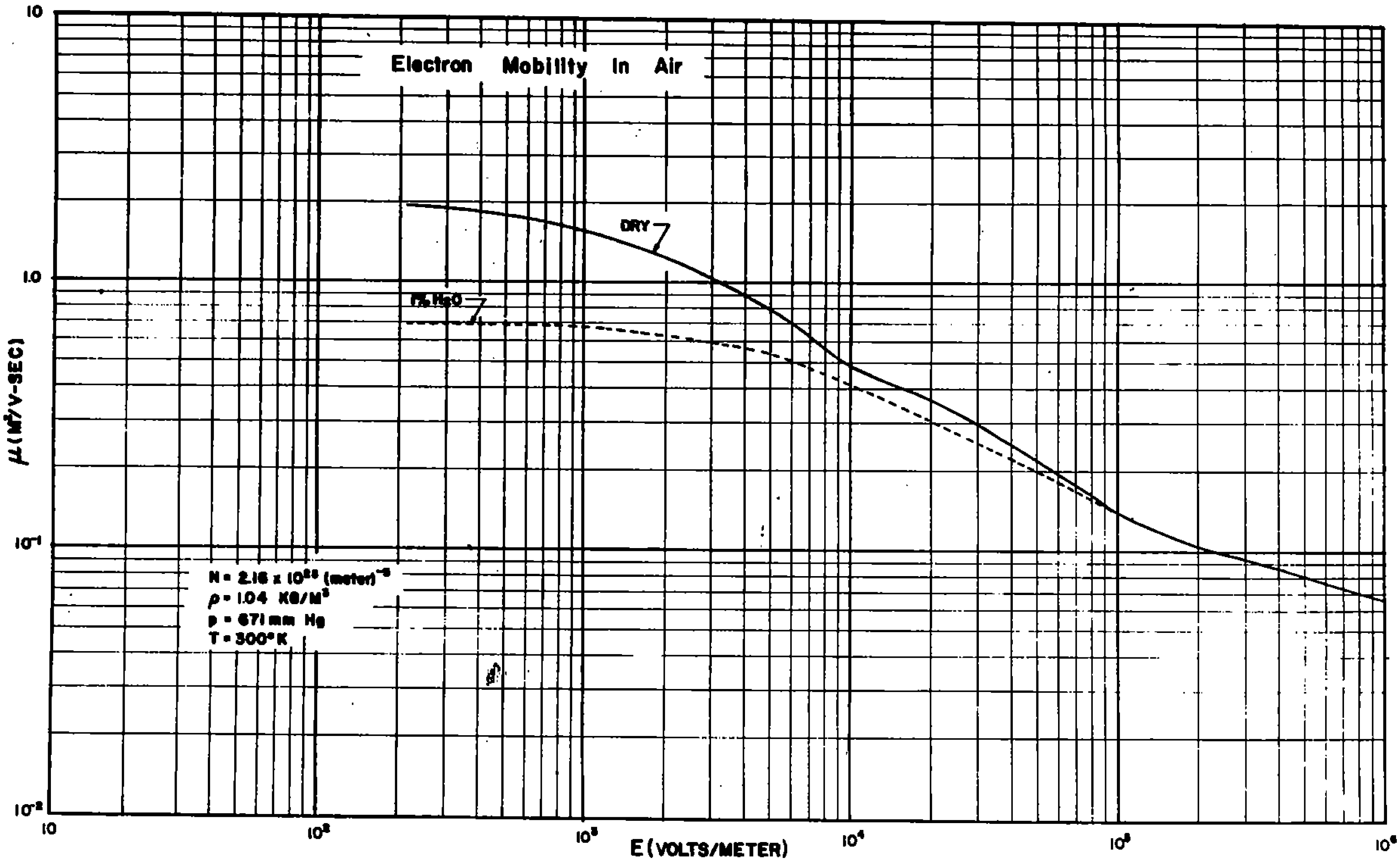
It has been known for some time that some of the conduction electron parameters (mobility, attachment frequency, etc.) are functions of the electron temperature which is generally epithermal in cases of interest to EMP. The effect of the electron temperature on the electron mobility is shown in figure 1, illustrating the dependence of the electron mobility on the electric field (constructed from the data of Pack and Phelps). In addition, this figure shows how the electron mobility is affected by the presence of water vapor (1% molecular fraction in this case).

For cases such as illustrated in figure 1, in which the densities of the various components of the gas (air) remain constant for the time of interest, one can construct a curve of electron mobility vs. electric field. However, if the densities of any of the important constituents of the gas change significantly during the time of interest, then for each time there is a different curve of electron mobility vs. electric field.

As discussed in EMP Theoretical Note II (abbreviated TN II), the presence of ions can significantly affect the electron mobility in some regions. Since the ion densities are changing during the time of interest, it is necessary to use some scheme to calculate the mobility which takes into account the effects of both the electric field and the time changing ion densities. Ideally, the calculation scheme for the electron mobility should be put into a form which would allow for improvement as more measurements are made and new effects are discovered. In addition, since the effect of the electric field on the mobility is related to the electron temperature an extra benefit of such a calculation scheme is the knowledge of the electron temperature, which in turn can be used to improve the calculation of the electron attachment and recombination coefficients. It is the purpose of this note to develop this calculational scheme.

II. Electron Temperature and Mobility

Consider the rate of energy transfer, P_1, from the electric field to the electrons. On the average per electron



Electron Mobility In Air
DRY
1% H2O
N = 2.16 x 10^25 (meter)^-3
ρ = 1.04 KG/M^3
p = 671 mm Hg
T = 300° K
μ (M²/V-SEC)
10
1.0
10^-1
10^-2
10
10^2
10^3
10^4
10^5
10^6
E (VOLTS/METER)

Figure 1

$$P_1 = eEv_d = eE^2\mu_e \tag{1}$$

where e is the electron charge, E is the electric field, v_d is the electron drift velocity, and μ_e is the electron mobility.[1] The mobility can be found from the momentum transfer collision frequency, ν_m, from the definition of ν_m

$$\mu_e = \frac{e}{m}\frac{1}{\nu_m} \tag{2}$$

where m is the electron mass. Thus,

$$P_1 = eE^2\frac{e}{m}\frac{1}{\nu_m} \tag{3}$$

Defining U_e as the average electron temperature (in e.v.) and U_o as the temperature of the air (thermal, .025 e.v.) then the rate of energy exchange, P_2, of the electrons with the other species present in the ionized air defines ν_u, the energy exchange collision frequency, by

$$P_2 = -\left(\frac{3}{2}eU_e - \frac{3e}{2}U_o\right)\nu_u \tag{4}$$

The factor $\frac{3e}{2}$ converts the average electron temperature (in e.v.) to the average electron energy in joules.

One can now write an equation giving the time rate of change of the average electron energy as

$$\frac{3}{2}e\frac{\partial U_e}{\partial t} = P_1 + P_2 \tag{5}$$

or

$$\frac{3}{2}e\frac{\partial U_e}{\partial t} = eE^2\frac{e}{m}\frac{1}{\nu_m} - \left(\frac{3}{2}e\,U_e - \frac{3}{2}e\,U_o\right)\nu_u \tag{6}$$

Thus, the time rate of change of the electron temperature is given by

$$\frac{\partial U_e}{\partial t} = \frac{2}{3}\frac{e}{m}\frac{E^2}{\nu_m} - (U_e - U_o)\,\nu_u \tag{7}$$

Assuming that the time of interest for the EMP is long compared with the time required to stabilize the electron temperature, then the time derivative of the electron temperature can be set to zero, simplifying the calculations considerably.[2] Thus, from equation (7)

$$\frac{2}{3}\frac{e}{m}\frac{E^2}{\nu_m} = (U_e - U_o)\,\nu_u \tag{8}$$

1. All units are taken in the rationalized m.k.s. system unless otherwise specified.
2. The problem of the electron thermalization time is planned to be treated in another Theoretical Note.

or

$$\nu_m \nu_u = \frac{2}{3} \frac{e}{m} \frac{E^2}{U_e - U_o} \tag{9}$$

Let j denote the various ionic and neutral species in the ionized air. Then the collision frequencies, ν_m, and ν_u, can be expressed in terms of the magnitude of the electron velocity, v_e, the densities of the particles with which the electrons collide, N_j, and the momentum transfer collision cross sections, σ_{m_j}, and energy exchange collision cross sections, σ_{u_j}, for collisions of electrons with the various species in the ionized air. Thus,

$$\nu_m = \sum_j \nu_{m_j} = v_e \sum_j N_j \sigma_{m_j} \tag{10}$$

and

$$\nu_u = \sum_j \nu_{u_j} = v_e \sum_j N_j \sigma_{u_j} \tag{11}$$

For simplicity v_e is defined arbitrarily from the average electron temperature (measured or calculated) by the relationship

$$U_e = \frac{1}{2} \frac{m}{e} v_e^2 \tag{12}$$

This in turn defines the values of the cross sections of equations (10) and (11). It is assumed here that the actual distribution of electron energies has little influence on the average collision frequencies, i.e., that for practical purposes the average cross sections depend only on the average electron temperature. Thus, it is presumed that the average parameters (electron temperature, mobility, etc.), as measured in various experiments involving one dominant scattering species, can be combined, for a case in which there are more than one important scattering species, by the techniques outlined in this note. No attempt is made to average the various parameters over any assumed electron temperature distribution function.

Equation (9) can now be rewritten as

$$v_e^2 \left[\sum_j N_j \sigma_{m_j}\right] \left[\sum_j N_j \sigma_{u_j}\right] = \frac{2}{3} \frac{e}{m} \frac{E^2}{U_e - U_o} \tag{13}$$

or

$$\left[\sum_j N_j \sigma_{m_j}\right] \left[\sum_j N_j \sigma_{u_j}\right] = \frac{2}{3} \frac{e}{m v_e^2} \frac{E^2}{U_e - U_o} \tag{14}$$

Using equation (12) to replace v_e with U_e one has

$$\left[\sum_j N_j \sigma_{m_j}\right] \left[\sum_j N_j \sigma_{u_j}\right] = \frac{1}{3} \frac{E^2}{U_e (U_e - U_o)} \tag{15}$$

Given the densities N_j, and the electric field E, then knowing the cross sections, σ_{m_j} and σ_{u_j}, as a function of the electron temperature, equation (15) can be solved for the electron temperature.

With the electron temperature known the momentum transfer collision cross sections can be calculated and thus the electron mobility can be calculated from

$$\mu_e = \frac{e}{mv_e \sum_j N_j \sigma_{m_j}} = \left[\left(2\frac{m}{e} U_e\right)^{1/2} \sum_j N_j \sigma_{m_j} \right]^{-1} \qquad (16)$$

Using the scheme outlined in this section, and knowing the cross sections as a function of electron temperature, the densities of the electron scatterers, and the electric field, one can solve for the electron mobility. This procedure should be particularly useful in a computer solution for the fields in which the continuous evaluation of the electron mobility and temperature can be easily made by the computer.

III. Cross Sections vs. Electron Temperature

As noted in the preceeding section, to calculate the electron mobility in air one needs the electron momentum transfer and energy exchange collision cross sections as functions of electron temperature. These cross sections can be obtained from experiments and/or theoretical calculations. At the present there are three species in the air which are known to be significant in determining the electron mobility:

(1) oxygen and nitrogen molecules (dry air)

(2) water vapor molecules

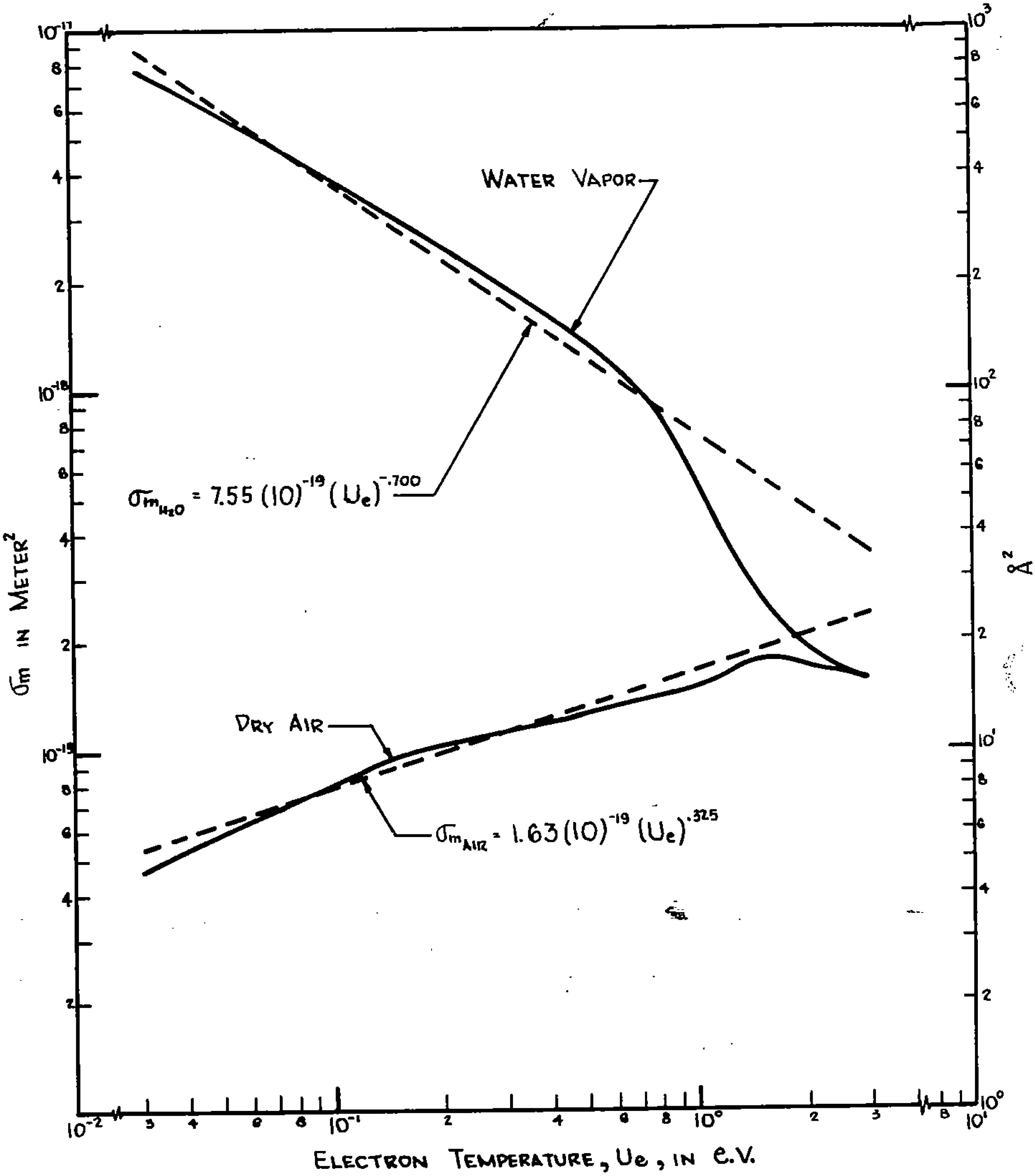
(3) ions (both positive and negative)(see TN II)

Measurements of electron mobility and temperature in essentially neutral oxygen, nitrogen, and water vapor have been made by Frost, Pack, Phelps, and Voshall. The measurements in nitrogen and water vapor are reported in Phys. Rev. 121, 3 (Feb. 1, 1961), 127, 5 (Sept. 1, 1962), and 127, 6 (Sept. 15, 1962).[3] The momentum transfer collision cross sections for dry air and for water vapor, calculated from these measurements, are plotted against the electron temperature in figure 2. Empirical approximations to these curves are also shown.

Figure 3 shows the energy exchange collision cross sections for the same gases. It must be remembered that for these measurements the electron temperature is not the temperature of all the free electrons but only the average temperature which was found by first measuring the electron diffusion coefficient, D, and then using the Einstein relationship

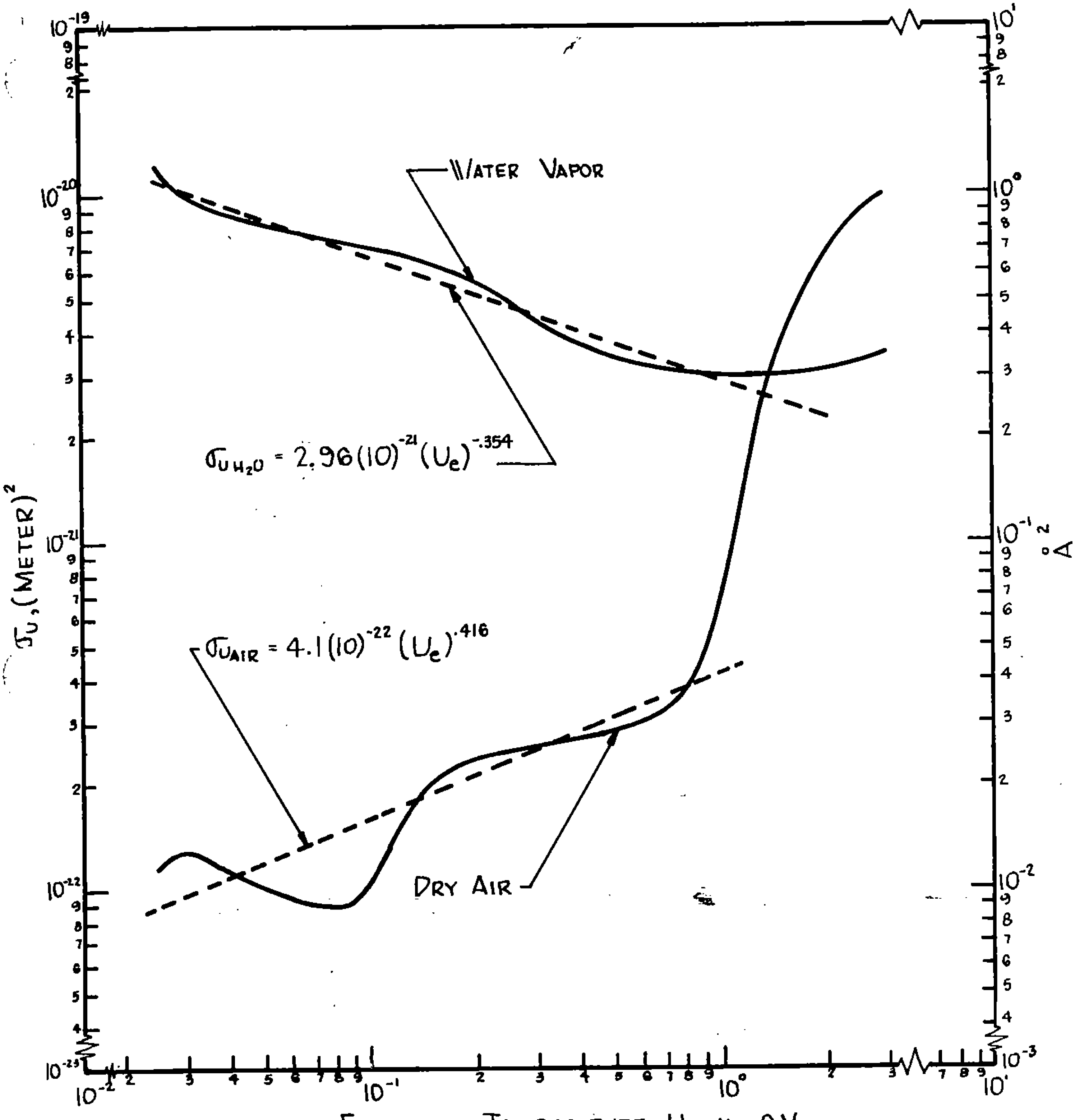
$$U_e = \frac{D}{\mu_e} \qquad (17)$$

3. The original curves for all these measurements have been generously supplied to the author by Dr. Phelps of Westinghouse.



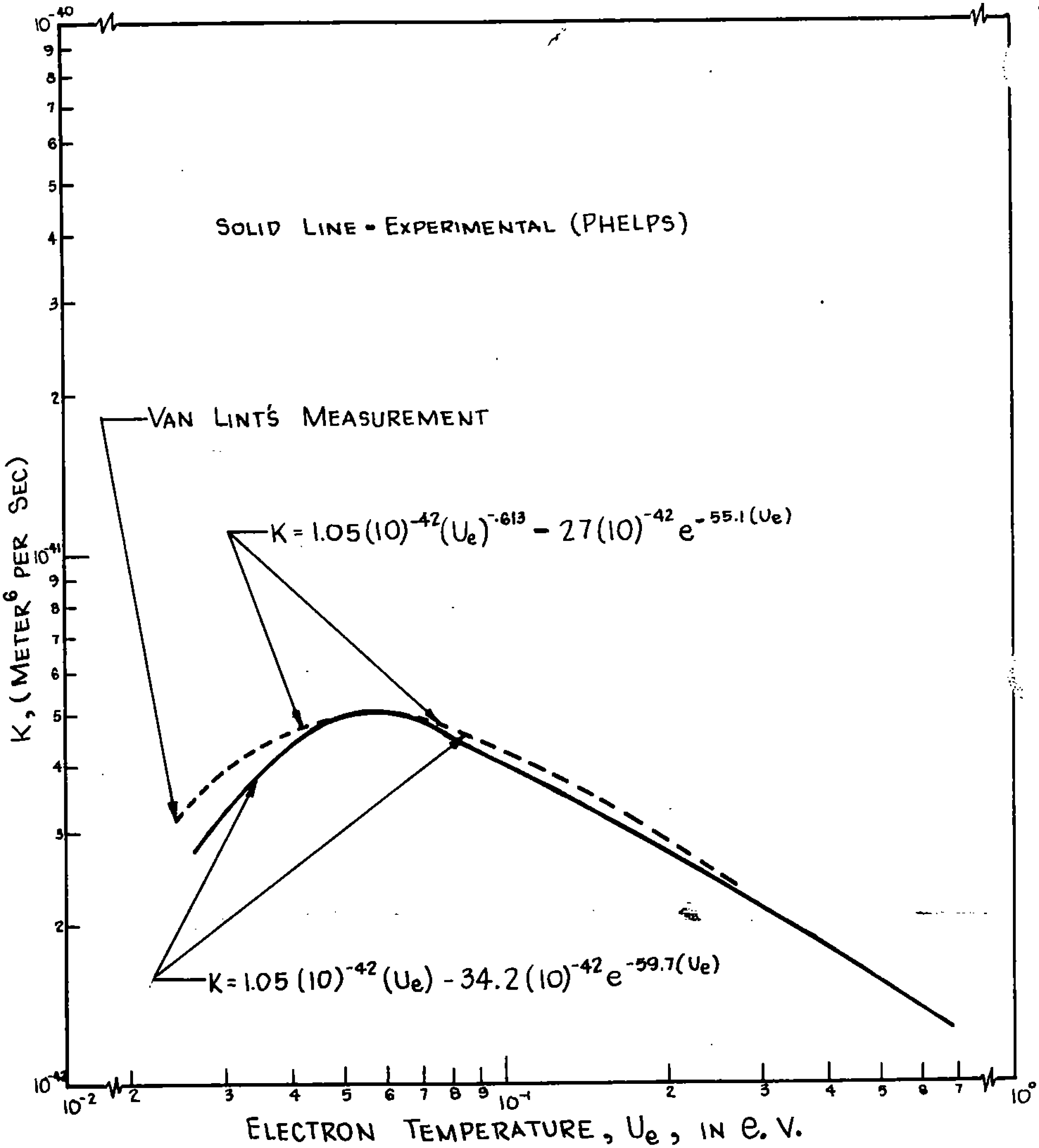
MOMENTUM TRANSFER COLLISION CROSS SECTIONS

FIG. 2



Energy Exchange Collision Cross Sections

Fig. 3



ELECTRON ATTACHMENT COEFFICIENT (TO O_2)

FIG. 4

for electrons to neutral oxygen molecules as a function of electron temperature has been reported by Chanin, Phelps, and Biondi in Phys. Rev. 128, 1 (Oct. 1, 1962). This is illustrated in figure 4. The attachment frequency ν_a, is related to the attachment coefficient, K, by

$$\nu_a = K N^2 f_{O_2} (f_{O_2} + \xi f_{N_2}) \qquad (21)$$

where N is the molecular density of the air, f_{O_2} and f_{N_2} are respectively the molecular fractions of oxygen and nitrogen, and ξ is the efficiency of nitrogen as a third body in the attachment of electrons to oxygen molecules.

Also indicated in figure 4 is Van Lint's value for the electron attachment coefficient which may be more reliable for low electron temperatures because it accounts for the fact that the conduction electrons must pass through the peak of the attachment curve in the process of thermalizing from their initial energy at production (by the radiation).

The attachment coefficient appears in turn in the equation for the electron density, n_e,

$$\frac{\partial n_e}{\partial t} = S - \nu_a n_e - \alpha_r N_+ n_e \qquad (22)$$

where S is the rate of electron generation, N_+ is the positive ion density, and α_r is the electron-ion recombination coefficient. If ξ is taken as

$$\xi = .031 \qquad (23)$$

from the data of Van Lint reported in RTD-TDR-63-3076 (December 1963), then equation (21) can be reduced to

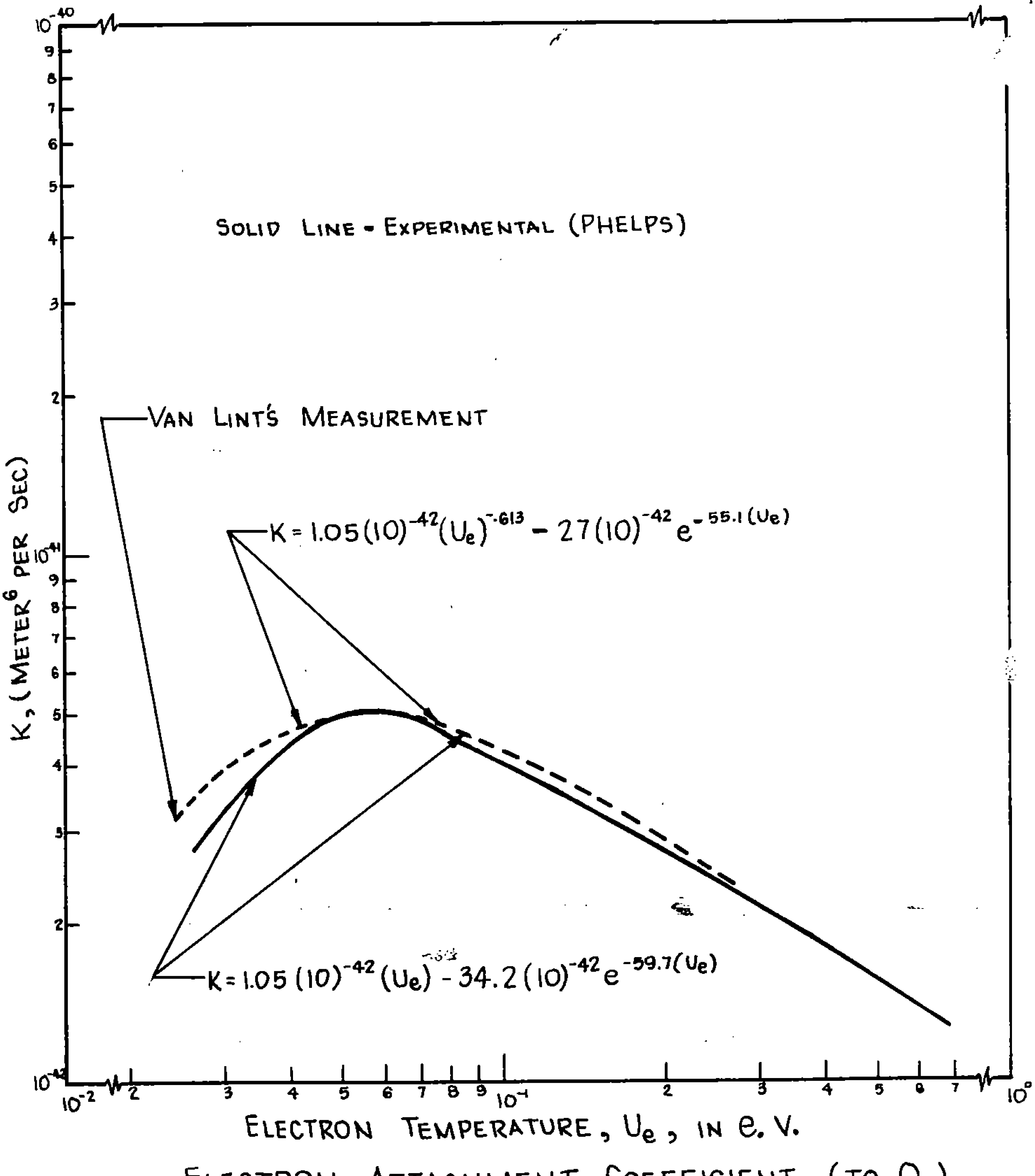
$$\nu_a = .049 N^2 K \qquad (24)$$

However, ξ may also be a function of the electron temperature but the author is unaware of any such measurements.

The other parameter, α_r, in the equation for the electron density has been measured by several people. These measurements indicate that this parameter is a function of pressure with the value at atmospheric pressure being much larger than the value at millimeter pressures. The most reliable value of α_r at atmospheric pressure is that of Van Lint,

$$\alpha_r = 4.5 \times 10^{-12} \text{ meter}^3/\text{sec} \qquad (25)$$

However, it would seem reasonable to expect that this parameter, as well as ν_a, is a function of the electron temperature, but at the present the author is not aware of any applicable measurements. If and when such measurements are made the results can also be incorporated into this calculational scheme.



ELECTRON ATTACHMENT COEFFICIENT (TO O_2)
FIG. 4

V. Conclusions

In summary, by considering the energy balance equations for the conduction electrons together with the various cross sections as a function of electron temperature one can

(1) solve for the electron mobility and temperature, given the electric field and density of electron scatterers

(2) use this electron temperature to better calculate the electron attachment and recombination coefficients.

This technique is a very general one which is capable of being updated as new effects are discovered and/or known effects are refined.

The author would like to thank Mr. Richard Sutton for his calculational assistance in the preparation of this note.