EMP Theoretical Notes

Note III

Time Dependent Age Theory

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Abstract:

The problem of an instantaneous point source of neutrons over the boundary between two semi-infinite media is considered. An approximate relation is developed between the Fermi age of a neutron and the time since its emission. This relation is applied to obtain an analytic solution for the spatial, temporal, and spectral distribution of the capture-gamma source in the vicinity of a ground-air interface.

I. Introduction.

The solution of neutron-transport problems in non-critical media to obtain the spatial distribution as a function of neutron energy contains, as an important parameter, the distance between the source and the points of interest. In fact, slowing-down problems can be classified according to the magnitude of this distance. It is natural to introduce a "slowing-down length" which is, roughly speaking, the average crow-flight distance a neutron travels from the source before reaching a given energy. For neutrons of a given energy, the situation will be noticeably different according as the distance from the source is comparable with or much greater than the slowing-down length. The former case corresponds to typical neutrons, and the latter to neutrons which have traveled exceptionally far in reaching the energy in question. The Fermi age is the square of the slowing-down length as calculated on the assumptions of age theory. Age theory is a statistical theory for predicting the distribution of typical neutrons. Therefore, it cannot be expected to predict the correct capture source at distances of many slowing-down lengths from the source.

Another important parameter of a slowing-down problem is the largest fractional decrease in energy that a neutron can undergo in one collision in the medium concerned, and it is called the "collision interval." One assumption of age theory is that the final energy of the neutrons is separated from the initial energy by at least a few collision intervals. This assumption has two important effects on the applicability of age theory to the prediction of secondary gamma sources for the EMP problem. First, the source due to inelastic scattering cannot be predicted because it is due to neutrons that are within one collision interval of the source energy. Second, hydrogenous materials allow scattering to near zero final energy in a single collision, making all final energies violate the assumption that they are several collision intervals from the initial energy. Therefore, a source over water cannot be considered using these methods.

There are still other assumptions made before the Boltzmann transport equation can be reduced to the age equation, but they do not seem to have particular relevance to the EMP problem. A more rigorous and detailed development of these assumptions and their physical implications may be found in a book by Davison[1]. We proceed now to a discussion of the two-media problem that is of direct concern.

Bellman, Marshak, and Wing[2], in the early days of the Manhattan project, developed a solution for the problem of a point source of neutrons over the boundary between two semi-infinite media using the assumptions of age theory. The evaluation of the neutron-induced gamma-ray source of ionization and currents requires, in principle, such a solution of the neutron slowing-down problem. Although Monte Carlo solutions of this problem now allow a closer approach to reality than is possible with age theory, they are difficult to apply directly to the EMP problem. One is tempted to attempt curve fits to the Monte Carlo data. For this purpose it is useful to have some ideas about the functional form of the solutions. Age theory is investigated in this note with the primary motive of seeing whether it will lead to such ideas.

The EMP problem requires that the time dependence of the gamma-ray source be known and age theory is ordinarily considered to be a time independent theory, or one for obtaining the neutron distribution integrated over all time. However, the average time between collisions is known, at least for each medium independently, and the average energy loss per collision is also known. This implies that, for a single-medium problem,

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[1] Davison, B., Neutron Transport Theory (Oxford University Press, London, 1958).

[2] Bellman, R., Marshak, R. E., and Wing, G. M., Philosophical Magazine, Series 7, Vol. 40, p. 297 (March 1949).

the average time required for a neutron to reach a given final energy from a given initial energy can be computed. The age variable of age theory is related to initial and final energy by similar statistical arguments. We are thus led to the conclusion that the distribution of the neutrons in time can be obtained from their distribution in age. The possibility of applying this relationship to the two-media problem of Bellman, Marshak, and Wing is the subject of this note.

The relationship between time and energy for the single-medium problem will be useful for the two-media problem, and so will be derived here. The average time between neutron collisions is given by

$$\Delta t_c = \frac{1}{\sigma_t v} = \frac{1}{\sigma_t}\sqrt{\frac{m}{2E}} ,$$

where σ_t = reciprocal mean free path of the neutrons,

σ_t = macroscopic total cross section,

v = velocity,

E = energy,

m = mass.

The energy loss per collision due to scattering is expressed as

$$\Delta E_c = E \xi \frac{\sigma_s}{\sigma_t}$$

where ξ = fractional energy loss per scatter,

σ_s = macroscopic scattering cross section.

Thus the average change in energy per unit time is approximated by

$$\frac{dE}{dt} \cong \frac{\Delta E_c}{\Delta t_c} = \sqrt{\frac{2}{m}}\, E^{3/2} \xi \sigma_s .$$

The average time delay between emission of a neutron at energy E_o and its arrival at energy E_f is thus

$$t = \sqrt{\frac{m}{2}} \int_{E_o}^{E_f} \frac{E^{-3/2}\, dE}{\xi \sigma_s} . \tag{1}$$

The change in age between E_0 and E_f is given by the standard expression

$$\tau = \int_{E_0}^{E_f} \frac{E^{-1}\, dE}{3\xi\, \sigma_s^2 (1 - \overline{\cos\theta})} , \tag{2}$$

where $\overline{\cos\theta}$ is the average cosine of the scattering angle.

II. The Time-Independent Solution.

The solution of Bellman, Marshak, and Wing is written in the form

$$\psi_2(\rho, z, \tau) = \frac{e^{-r_1^2/4\tau}}{(4\pi\tau)^{3/2}} - \frac{e^{-r_2^2/4\tau}}{(4\pi\tau)^{3/2}} + C_2(\rho, z, \tau) , \tag{3}$$

where ψ_2 is the slowing-down density in the upper medium (air) and C_2 is a complex function of the cylindrical coordinates, ρ and z. The $z = 0$ plane is the ground plane and the $\rho = 0$ axis is the vertical axis through the source point. The distances r_1, and r_2 are defined by

$$r_1^2 = \rho^2 + (z - z_0)^2 ,$$
$$r_2^2 = \rho^2 + (z + z_0)^2 ,$$

where z_0 is the height of the source point.

The function C_2 is defined in terms of two parameters giving the relationships between the cross sections of the two media. The first reflects the relative density of the two media and is given by

$$D = \sigma_{s1}^2 \xi_1 (1 - \overline{\cos\theta_1}) / \sigma_{s2}^2 \xi_2 (1 - \overline{\cos\theta_2}) ,$$

where the subscript 1 designates parameters of the lower medium and subscript 2 designates parameters of the air. The second parameter reflects the relative importance of scattering events in the two media and is given by

$$\alpha = \frac{\xi_1}{1 - \overline{\cos\theta_1}} \Big/ \frac{\xi_2}{1 - \overline{\cos\theta_2}} .$$

The parameters D and α are assumed to be independent of energy. Since the ground is much more dense than the air, we shall not have to write the general form of the function C_2, but only its limit as D goes to infinity. This is

$$\lim_{D\to\infty} C_2 = \frac{1}{4(\pi\tau)^{3/2}}\left[e^{-r_2^2/4\tau} + \sqrt{\alpha}\int_{r_2}^{\infty}\frac{\left(1-\frac{v^2}{2\tau}\right)e^{-\frac{v^2}{4\tau}}\,dv}{\left[v^2+\alpha z_1^2+2z_1 v\sqrt{\alpha}+\rho^2(\alpha-1)\right]^{1/2}}\right], \tag{4}$$

where $z_1 = z + z_0$.

Examination of the general form of C_2 shows that C_2 goes to zero as D goes to zero. Thus

$$\lim_{D\to 0} \psi_2 = \frac{e^{-r_1^2/4\tau}}{(4\pi\tau)^{3/2}} - \frac{e^{-r_2^2/4\tau}}{(4\pi\tau)^{3/2}} . \tag{5}$$

This limit represents the solution of the problem when the lower medium is a vacuum. One expects the boundary condition at a free surface to be that the slowing-down density vanishes at the extrapolated boundary of a free surface, and that is the case for $z = 0$ in Eq. (5). Therefore, the first two terms in Eq. (3) represent the slowing-down density due to neutrons that have not yet entered the ground and C_2 represents the part of the slowing-down density due to neutrons which have reentered the air from the ground.

III. The Time-Dependent Solution.

For neutrons which have not interacted with the ground the relationships between time, age, and energy given by Eqs. (1) and (2) are valid with $\xi = \xi_2$, $\sigma_s = \sigma_{s2}$, and $\overline{\cos\theta} = \overline{\cos\theta_2}$. Applying these facts to the first two terms of Eq. (3), it is only necessary to find a relationship between τ and t for use in C_2 to have a complete method of associating a time dependence with ψ_2 . Since the time between collisions is very small in the ground compared with the air, we assume that the increase in age due to ground collisions is instantaneous. Thus if we could tell how much of the age increase for given initial and final energies, was due to ground collisions, we could subtract that from the total age change, and then use Eq. (1) on the reduced age change. Thus, for the C_2 term, the time associated with a given age will be

$$t = \frac{\tau - \Delta\tau_g}{\tau} \sqrt{\frac{m}{2}} \int_{E_0}^{E_f} \frac{E^{-3/2}\, dE}{\xi_2 \sigma_{s2}} \tag{6}$$

where $\Delta\tau_g$ is the average age change in the ground. This can be interpreted as reducing the problem to determining $\Delta\tau_g$.

The function ψ_2 is the slowing-down density ignoring capture. To obtain the actual slowing-down density it would be necessary to multiply by an appropriate resonance escape probability. It is therefore legitimate to assume that every neutron that enters the infinitely dense ground must return to the air. Thus $\Delta\tau_g$ is equal to the difference between the average age of the neutrons entering the ground for the first time and the average age of the neutrons leaving the ground. These averages must be computed based on the respective components of the neutron currents normal to the ground plane. Since the $z = 0$ plane has been chosen to be at the extrapolated boundary, the real ground plane is at a positive z, say z_p, equal to the extrapolation length:

$$z_{p2} = \frac{0.7104}{\sigma_{s2}} \tag{7}$$

The normal current of neutrons first entering the ground is given in terms of the slowing-down density of Eq. (5) by

$$J_a = \frac{\partial}{\partial z}\left(\lim_{D\to 0} \psi_2\right)\Big|_{z=z_p} \Big/ \sigma_{s2}^2\, \xi_2 \left(1 - \overline{\cos\theta_2}\right) \tag{8}$$

Likewise, the current of neutrons from the ground is given in terms of Eq. (4) by

$$J_g = \frac{\partial}{\partial z}\left(\lim_{D\to\infty} C_2\right)\Big|_{z=z_p} \Big/ \sigma_{s2}^2\, \xi_2 \left(1 - \overline{\cos\theta_2}\right) \tag{9}$$

The corresponding average ages are

$$\tau_a = \int_{\tau_0}^{\tau_f} \tau \int_0^\infty \rho J_a \, d\rho \, d\tau \Big/ \int_{\tau_0}^{\tau_f} \int_0^\infty \rho J_a \, d\rho \, d\tau \,,$$

$$\tau_g = \int_{\tau_0}^{\tau_f} \tau \int_0^\infty \rho J_g \, d\rho \, d\tau \Big/ \int_{\tau_0}^{\tau_f} \int_0^\infty \rho J_g \, d\rho \, d\tau \,, \tag{10}$$

where τ_0 and τ_f are the ages corresponding to E_0 and E_f, respectively. The desired quantity, $\Delta\tau_g$ is then given by $\Delta\tau_g = \tau_g - \tau_a$.

By integrating the current over ρ in performing the averaging without any additional weighting factor, it has been implicitly assumed that either $\Delta\tau_g$ is independent of ρ, or that sources at all radii in the ground plane result in equal probabilities of reaching the detector with a given final energy. Also, by setting the upper limit on the τ integration to τ_f, which refers to an energy of arrival at the detector, we have assumed that $\Delta\tau_g$ is independent of detector position. Although it is conceptually possible to avoid making these assumptions, the method of integrating over the ground plane then becomes too ponderous for practical application. It seems reasonable that a neutron entering the ground anywhere in the ground plane will emerge at approximately the same location with an energy loss approximately independent of position in the plane. Whether the stronger assumptions actually made are justified could be verified from Monte Carlo data.

Once $\Delta\tau_g$ has been determined as a function of initial and final energy, one can proceed to find the time-dependent neutron flux using Eq. (3). The time integrated flux is

$$n_2(\rho, z, E_0, E_f) = \psi_2(\rho, z, \tau(E_0, E_f)) / \xi_2 \sigma_{s2} E_f \,, \tag{11}$$

where $\tau(E_0, E_f)$ is obtained from Eq. (3). According to the present approximate theory, this is made up of multiples of two delta functions in time in the following manner

$$n_2(\rho, z, E_0, E_\xi, t) = \frac{\left[\left(\lim_{D\to\infty} \psi_2\right)\delta(t-t_2) + \left(\lim_{D\to\infty} C_2\right)\delta(t-t_1)\right]}{\xi_2 \sigma_{s2} E_\xi} \, , \tag{12}$$

where t_2 is evaluated from Eq. (1) and t_1, is evaluated from Eq. (6). Inserting Eqs. (4) and (5) the neutron flux distribution ignoring capture is

$$n_2(\rho, z, E_0, E_\xi, t) = \frac{1}{\xi_2 \sigma_{s2} E_\xi} \left[\left\{ \frac{e^{-r_1^2/4\tau}}{(4\pi\tau)^{3/2}} - \frac{e^{-r_2^2/4\tau}}{(4\pi\tau)^{3/2}} \right\} \delta(t-t_2) \right. \tag{13}$$
$$\left. + \frac{1}{4(\pi\tau)^{3/2}} \left\{ e^{-r_2^2/4\tau} + \sqrt{\alpha} \int_{r_2}^{\infty} \frac{\left(1 - \frac{u^2}{2\tau}\right) e^{-\frac{u^2}{4\tau}} du}{\left[u^2 + \alpha z_1^2 + 2 z_1 u \sqrt{\alpha} + \rho^2(\alpha - 1)\right]^{1/2}} \right\} \delta(t-t_1) \right] .$$

IV. Capture Effects.

In single-medium problems, the slowing-down density including capture, ψ_c, is obtained from the slowing-down density ignoring capture, ψ, by multiplying by the resonance escape probability, P, as follows:

$$\psi_c(\rho, z, \tau(E_0, E_\xi)) = P(E_0, E_\xi)\, \psi(\rho, z, \tau(E_0, E_\xi)) \tag{14}$$

The resonance escape probability is given by

$$P(E_0, E_\xi) = \exp\left[- \int_{E_\xi}^{E_0} \frac{\sigma_c \, dE}{\xi \sigma_s E} \right] \tag{15}$$

The problem of extending this method to the two-media problem is essentially similar to that of associating a time with an age in the two-media problem. For those neutrons that have never been in the ground, the correct resonance escape probability is obviously given by Eq. (15) with $\sigma_c = \sigma_{c2}$, $\xi = \xi_2$, and $\sigma_{s2} = \sigma_s$. We call this probability P_2 . There is a different escape probability, P_1 , for those neutrons which have reentered the air from the ground. Going through the steps analogous to Eqs. (11), (12), and (13) again, replacing ψ_2 by ψ_{2c} , the slowing-down density in air including capture; one obtains for the neutron flux distribution including capture

$$n_{c2}(\rho, z, E_0, E_f, t) = \frac{1}{\xi_2 \sigma_{s2} E_f}\left[P_2\left\{\frac{e^{-r_1^2/4\tau}}{(4\pi\tau)^{3/2}} - \frac{e^{-r_2^2/4\tau}}{(4\pi\tau)^{3/2}}\right\}\delta(t-t_2)\right.$$
$$\left. + \frac{P_1}{4(\pi\tau)^{3/2}}\left\{e^{-r_3^2/4\tau} + \sqrt{\alpha}\int_{r_2}^{\infty}\frac{\left(1-\frac{U^2}{4\tau}\right)e^{-\frac{U^2}{4\tau}}\,dU}{\left[U^2+\alpha z_1^2+2z_1U\sqrt{\alpha}+\rho^2(\alpha-1)\right]^{1/2}}\right.\right. , \qquad (16)$$

To determine P_1 , we note that the part of the age loss, $\Delta\tau_g$, that occurred in the ground has already been computed for those neutrons that have been in both media. For that part of the age loss, the appropriate escape probability is found by using $\sigma_c = \sigma_{c1}$, $\xi = \xi_1$, and $\sigma_s = \sigma_{s1}$ in Eq. (15). For the part of the age loss that occurred in air, the appropriate cross sections are those for air. Thus one obtains

$$P_1 = \frac{\tau-\Delta\tau_g}{\tau}\exp\left[-\int_{E_f}^{E_0}\frac{\sigma_{c2}\,dE}{\xi_2\sigma_{s2}E}\right] + \frac{\Delta\tau_g}{\tau}\exp\left[-\int_{E_f}^{E_0}\frac{\sigma_{c1}\,dE}{\xi_1\sigma_{s1}E}\right] . \qquad (17)$$

If it is desired to determine the spatial, temporal, and spectral distribution of air-capture gamma rays, it is necessary to fold the resulting neutron distribution into the probability for generating a gamma of a given energy for a neutron of energy E_f . This probability is, in general, designated by a cross section differential in gamma energy, but because capture gamma spectra are ordinarily considered to be independent of neutron energy, we will write it as follows:

$$\frac{d\sigma_{c2}}{dE_\gamma} = \sigma_{c2}(E_f)\, f_2(E_\gamma) \quad , \tag{18}$$

where E_γ is the gamma ray energy and $f(E_\gamma)\,dE_\gamma$ is the probability of a neutron capture resulting in a gamma ray between E_γ and $E_\gamma + dE_\gamma$. The gamma ray source distribution is then

$$n_{\gamma 2}(\rho, z, E_0, E_f, t) = f_2(E_\gamma) \int_0^{E_0} \sigma_{c2}(E_f)\, n_{c2}(\rho, z, E_0, E_f, t)\, dE_f \quad . \tag{19}$$

For a given E_0, t_2 and t_1, are functions of E_f. For a given t, the delta functions will make the integral in Eq. (19) be equal to the integrand (less the delta function) evaluated at the E_f corresponding to t. We call the E_f that one obtains from the equation $t = t_2$ E_{f2} and the one from $t = t_1$, E_{f1}. The τ corresponding to E_{f2} will be τ_2 and the τ corresponding to E_{f1} will be τ_1. Then one obtains

$$n_{\gamma 2}(\rho, z, E_0, E_\gamma, t) = f_2(E_\gamma) \Bigg[\frac{P_2(E_{f2})\, \sigma_{c2}(E_{f2})}{\xi_2(E_{f2})\, \sigma_{s2}(E_{f2})\, E_{f2}} \left\{ \frac{e^{-r_1^2/4\tau_2}}{(4\pi\tau_2)^{3/2}} - \frac{e^{-r_2^2/4\tau_2}}{(4\pi\tau_2)^{3/2}} \right\} \tag{20}$$

$$+ \frac{P_1(E_{f1})\, \sigma_{c2}(E_{f1})}{\xi_2(E_{f1})\, \sigma_{s2}(E_{f1})\, E_{f1}\, 4(\pi\tau_1)^{3/2}} \left\{ e^{-r_2^2/4\tau_1} + \sqrt{\alpha} \int_{r_2}^{\infty} \frac{\left(1 - \frac{U^2}{4\tau_1}\right) e^{-\frac{U^2}{4\tau_1}}\, dU}{\left[U^2 + \alpha z_1^2 + 2 z_1 U \sqrt{\alpha} + \rho^2(\alpha - 1)\right]^{1/2}} \right\} \Bigg] .$$

The next step in a practical problem would be to fold this result into the source spectrum. In doing this it should be recognized that P_1, P_2, τ_1, τ_2, E_{f1}, and E_{f2} are all functions of E_0. Thus we simply write for a source distribution represented by $S(E_0)$

$$n(\rho, z, E_\gamma, t) = f(E_\gamma) \int_0^{E_{max}} S(E_0)\, \frac{n(\rho, z, E_0, E_\gamma, t)}{f(E_\gamma)}\, dE_0 \quad , \tag{21}$$

where E_{max} is the maximum neutron energy of interest.

Similar techniques could be developed for predicting the gamma ray source due to capture in the ground, but it seems more appropriate to view ground capture as a semi-static process by virtue of the rapidity of the collision process relative to the speed of the delivery from the air. Further, if a small area of the ground plane is considered, the neutrons can be considered as a plane source at the ground plane. Thus, for a particular energy component of this source, say E_g, the slowing-down density neglecting capture can be written as

$$\psi_1(\rho, z, E_o, E_g, E_f, t) = \varphi_{1\rho}(\rho, z_{p2}, E_o, E_g, t)\, \psi_{1z}(z, \tau_3) \tag{22}$$

where $$\tau_3 = \int_{E_g}^{E_f} \frac{E^{-1}\, dE}{3\xi_1 \sigma_{s1}^2 (1 - \overline{\cos\theta_1})}$$

The solution for a plane source at z_s in an infinite medium is

$$\psi = \frac{e^{-\frac{(z-z_s)^2}{4\tau}}}{(4\pi\tau)^{1/2}} \quad . \tag{23}$$

We assume that the air is approximately a vacuum when considering transport in the ground, so that the slowing-down density should vanish at the extrapolated boundary at a distance from the ground plane of

$$z_{p1} = \frac{0.7104}{\sigma_{s1}} \quad . \tag{24}$$

This can be accomplished by putting a negative image source at a distance $2z_{p1}$ above the ground plane. Thus we obtain

$$\psi_{1z} = \frac{e^{-\frac{(z_{p2}-z)^2}{4\tau_3}}}{(4\pi\tau_3)^{1/2}} - \frac{e^{-\frac{(z_{p2}+2z_{p1}-z)^2}{4\tau_3}}}{(4\pi\tau_3)^{1/2}} \tag{25}$$

This function describes the variation in z completely, so if it were normalized to be unity at the surface, it could simply be multiplied onto the known flux at the surface to obtain the flux at any depth in the ground. Defining the normalized function as

$$\psi_{1zN} = \frac{e^{-\frac{(z_{p2}-z)^2}{4\tau_3}} - e^{-\frac{(z_{p2}+2z_{p1}-z)^2}{4\tau_3}}}{1 - e^{-z_{p1}/\tau_3}} , \tag{26}$$

then the flux distribution including capture below the ground is given by

$$n_{c1}(\rho, z, E_0, E_f, t) = n_{c2}(\rho, z_{p2}, E_0, E_g, t)\, \psi_{1zN}(z, \tau_3). \tag{27}$$

Applying the same reasoning that led to Eq. (20) again, the capture gamma distribution is

$$n_{\gamma 1}(\rho, z, E_0, E_\gamma, t) = \psi_{1zN}(z, \tau_3)\, f_1(E_\gamma) \Bigg[\frac{\rho_2(E_{f2})\,\sigma_{c1}(E_{f2})}{\xi_2(E_{f2})\,\sigma_{s2}(E_{f2})\,E_{f2}} \left\{ \frac{e^{-\frac{r_1^2(z_{p2})}{4\tau_2}} - e^{-\frac{r_2^2(z_{p2})}{4\tau_2}}}{(4\pi\tau_2)^{3/2}} \right\}$$
$$+ \frac{\rho_1(E_{f1})\,\sigma_{c1}(E_{f1})}{\xi_2(E_{f1})\,\sigma_{s2}(E_{f1})\,E_{f1}\,4(\pi\tau_1)^{3/2}} \Bigg\{ e^{-\frac{r_2^2(z_{p2})}{4\tau_1}}$$
$$+ \sqrt{\alpha} \int_{r_2(z_{p2})}^{\infty} \frac{\left(1 - \frac{U^2}{4\tau_1}\right) e^{-\frac{U^2}{4\tau_1}}\, dU}{\left[U^2 + \alpha(z_0 + z_{p2})^2 + 2(z_0 + z_{p2})U\sqrt{\alpha} + \rho^2(\alpha - 1)\right]^{1/2}} \Bigg\} \Bigg] ,$$

where $f_1(E_\gamma)$ and $\sigma_{c1}(E_{f2})$ define the capture cross section of the ground.

V. Summary.

We will now attempt to demonstrate the significance of these formulii in curve fitting Monte Carlo data. Dr. John Malik has kindly supplied the results of a Monte Carlo solution for the total gamma source in the ground and in the air including those from inelastic scattering, as a function of all five variables.

This solution, despite a relatively coarse grid on the five variables, requires more than 250 pages of computer output. Partial separation of the variables and curve fitting seems to be the only practical way of using this input to predict ionization rates and Compton currents.

Cross plots of this data indicate that during the time span of interest for the EMP problem, air capture can be neglected, but that air inelastics, ground inelastics, and ground capture all have regions of the five dimensional space where they are important. Because age theory is only applicable to the capture component, it tells us nothing about what to expect as a gamma ray source in air during this time regime. We, therefore, confine our attention to the source in the ground.

Further examination of the cross plots reveals that the total gamma ray source in the ground can be approximated as the combination of six functions of not more than three variables, as follows:

$$n_{\gamma T1}(E_n, E_\gamma, r, z, t) = n_{\gamma I1}(E_n, E_\gamma, r, z, t) + n_{\gamma C1}(E_n, E_\gamma, r, z, t) \ ,$$

where

$$n_{\gamma I1}(E_n, E_\gamma, r, z, t) = S_{EI}(E_\gamma, E_n)\, S_{rI}(E_n, r, t)\, S_{zI}(E_n, z, t) \ ,$$

and (29)

$$n_{\gamma C1}(E_n, E_\gamma, r, z, t) = S_{EC}(E_\gamma)\, S_{rC}(E_n, r, t)\, S_{zC}(E_n, z, t) \ .$$

Obviously, $n_{\gamma I1}$ represents the contribution to the source due to inelastics, and age theory is not applicable to it. However, we already have some intuitive knowledge of the functional forms to be expected for the three factors involved in it. The flux of unscattered neutrons at the ground surface is proportional to

$$f_0(E_n, r, t) = \frac{e^{-\sigma_{s2}(E_n)\, r}}{r^2}\ \delta\left(t - \sqrt{\frac{m}{2E_n}}\, r\right) \ . \tag{30}$$

The terms due to multiple scattering that result in ground inelastics will be expected to have a similar form, since the high energy elastic scattering is primarily forward and results in relatively little energy loss. By similar reasoning, the attenuation of the neutrons capable of inelastic scattering in the ground would be expected to be represented by a function of the form

$$S_{zI} = e^{-\frac{z}{\lambda}} \tag{31}$$

where λ is the effective relaxation length for neutrons entering the ground with a given energy determined from E_n and t. The function S_{EI} represents the average spectrum of inelastics generated in the ground from neutrons of initial energy E_n and can be determined empirically. Similar reasoning can be applied to obtaining appropriate distribution functions for air inelastics.

Returning to age theory, we can reasonably expect it to be useful in assigning functional forms to the three factors contained in the ground-capture source, $n_{\gamma c1}$. The factor $S_{Ec}(E_\gamma)$ of Eq. (29) is obviously the factor $f_1(E_\gamma)$ of Eq. (28). The factor $S_{zc}(E_n, z, t)$ may be associated with the factor $\psi_{1zn}(z, \tau_3)$ of Eqs. (26) through (28), since both contain the entire z dependence of $n_{\gamma c_1}$. This implies that τ_3 is some function of E_n and t. In Eq. (22), τ_3 was given as a function of E_o, which corresponds to E_n, and E_g, the energy at which the neutrons enter the ground. To develop an equation for τ_3 as a function of E_n and t, we would need, in addition to Eq. (22), a relation between t, E_n, and E_g, so that the variable E_g could be eliminated. This relation is provided by Eq. (1) with $E_o = E_n$, $E_f = E_g$, $\xi = \xi_2$, and $\sigma_s = \sigma_{s2}$. However, from the curve-fitting view point, it is probably better to use Eq. (26) with τ_3 to be determined from the Monte Carlo data as a function of E_n and t.

The remaining factor of $n_{\gamma c1}$, namely $S_{rc}(E_n, r, t)$ must be the factor in the square bracket of Eq. (28).

$$S_{rc}(E_n, r, t) = \frac{P_2(E_{f2})\,\sigma_{c_i}(E_{f2})}{\xi_2(E_{f2})\,\sigma_{s2}(E_{f2})\,E_{f2}} \left\{ \frac{e^{-\frac{r_1^2}{4\tau_2}}}{(4\pi\tau_2)^{3/2}} - \frac{e^{-\frac{r_2^2}{4\tau_2}}}{(4\pi\tau_2)^{3/2}} \right\} \tag{32}$$

$$+ \frac{P_1(E_{f1})\,\sigma_{c_1}(E_{f1})}{\xi_2(E_{f1})\,\sigma_{s2}(E_{f1})\,4(\pi\tau_1)^{3/2}} \left\{ e^{-\frac{r_2^2}{4\tau_1}} + f_2\left(\frac{r_2^2}{4\tau_1}\right) \right\}$$

where

$$f_2\left(\frac{r_2^2}{4\tau_1}\right) \approx \sqrt{\alpha} \int_{r_2}^{\infty} \frac{\left(1 - \frac{U^2}{4\tau_1}\right) e^{-\frac{U^2}{4\tau_1}}\, dU}{\left[U^2 + \alpha(z_o + z_{p2})^2 + 2(z_o + z_{p2})U\sqrt{\alpha} + \rho^2(\alpha - 1)\right]^{1/2}}$$

Since α is considered to be one number for each combination of media, say air and ground, f_2 need only be evaluated once for each value of $r_2^2/4\tau_1$. This is probably still too complex a function to be curve fitted directly. Therefore, we note that E_{f1}, E_{f2}, τ_1, and τ_2 are all functions of E_n and t, and we write:

$$S_{rc}(E_n, r, t) = f_3(E_n, t)\left\{e^{-\frac{r_1^2}{4\tau_2}} - e^{-\frac{r_2^2}{4\tau_2}}\right\} + f_4(E_n, t)\left\{e^{-\frac{r_2^2}{4\tau_1}} + f_2\left(\frac{r_2^2}{4\tau_1}\right)\right\} \tag{33}$$

Then we regard $f_3(E_n, t)$, $f_4(E_n, t)$, $\tau_1(E_n, t)$, and $\tau_2(E_n, t)$ as four functions of two variables which must be determined from the Monte Carlo data.

If the above theory is satisfactorily accurate, the problem of evaluating the ground capture gamma source has been reduced from that of a general function of five variables to evaluating five functions of E_n and t, and combining them in a specific way to find the desired source term. At present, only the validity of Eqs. (29) has been established. The next step is to evaluate the validity of Eqs. (26) and (33). Similar techniques for applying Eqs. (30) and (31) to the inelastic source problem must then be developed. When this has been accomplished, the problem of neutron-induced gammas may at least be reduced to manageable form for use in subsequent calculations.