A Compton Diode for Measuring Both the Gamma Flux and One Component of the Gamma Current

I. Introduction

To understand EMP phenomena, it is necessary to know the time history of the gamma radiation, $\gamma(t)$, from two aspects; (1) the Compton current; and (2) the rate of ionization of the air. In the second of these aspects γ is considered as a scalar or isotropic quantity, i.e., it makes no difference as to the direction of travel of the photon. This quantity will be called the gamma flux, the total number of γ rays per second entering a sphere whose cross section is given by the radiation units and given the symbol, γ. However, the first of these aspects requires that γ be considered as a vector quantity because of the strong correlation of the direction of the Compton scattered electrons with the gamma quanta. This vector quantity will be called the gamma current, $\vec{\gamma}$, any component of which can be calculated from the <u>net</u> number of gammas per unit time crossing a unit circle perpendicular to the chosen direction. A particular component of the gamma current will be indicated by a subscript to $\vec{\gamma}$, e.g. γ_x. Note that by these definitions the gamma flux must always be positive but any component of the gamma current can be either positive or negative. Only in the case of unidirectional gammas will $|\vec{\gamma}|$ equal γ and by considering such a case the traditional γ units (roentgens/sec, rads/sec, etc.) can be carried over to $\vec{\gamma}$. Usually, however $|\vec{\gamma}|$ is less than γ.

Typically, gamma detectors have been designed to measure γ (isotropic response) or have used collimation to measure only those γ rays coming from a specified solid angle (usually small), making angular response characteristics of the detector unimportant. The purpose of this note is to describe a gamma detector based on the Compton diode principle which is capable of measuring $\vec{\gamma}$.

Figure 1 (on page 2) is a schematic of the cross section of such a detector consisting of two hemispheres of some dense material, separated from each other and from an external conducting case by some non-hydrogenous dielectric, with each hemisphere connected to one of the center conductors of a twinax cable. An axis of symmetry is drawn through the two hemispheres and the dependence of sensitivity to the gamma rays is determined only by the angle θ with respect to this axis (because of symmetry). For this discussion then, it is necessary to consider only the case of gamma rays coming from one direction and the only important characteristic of this direction is the angle θ. To keep the notation clear then the gamma rays coming from angle θ will be given the symbol $\gamma(\theta)$ and the Compton current associated with these gamma rays will be given the symbol $J_C(\theta)$.

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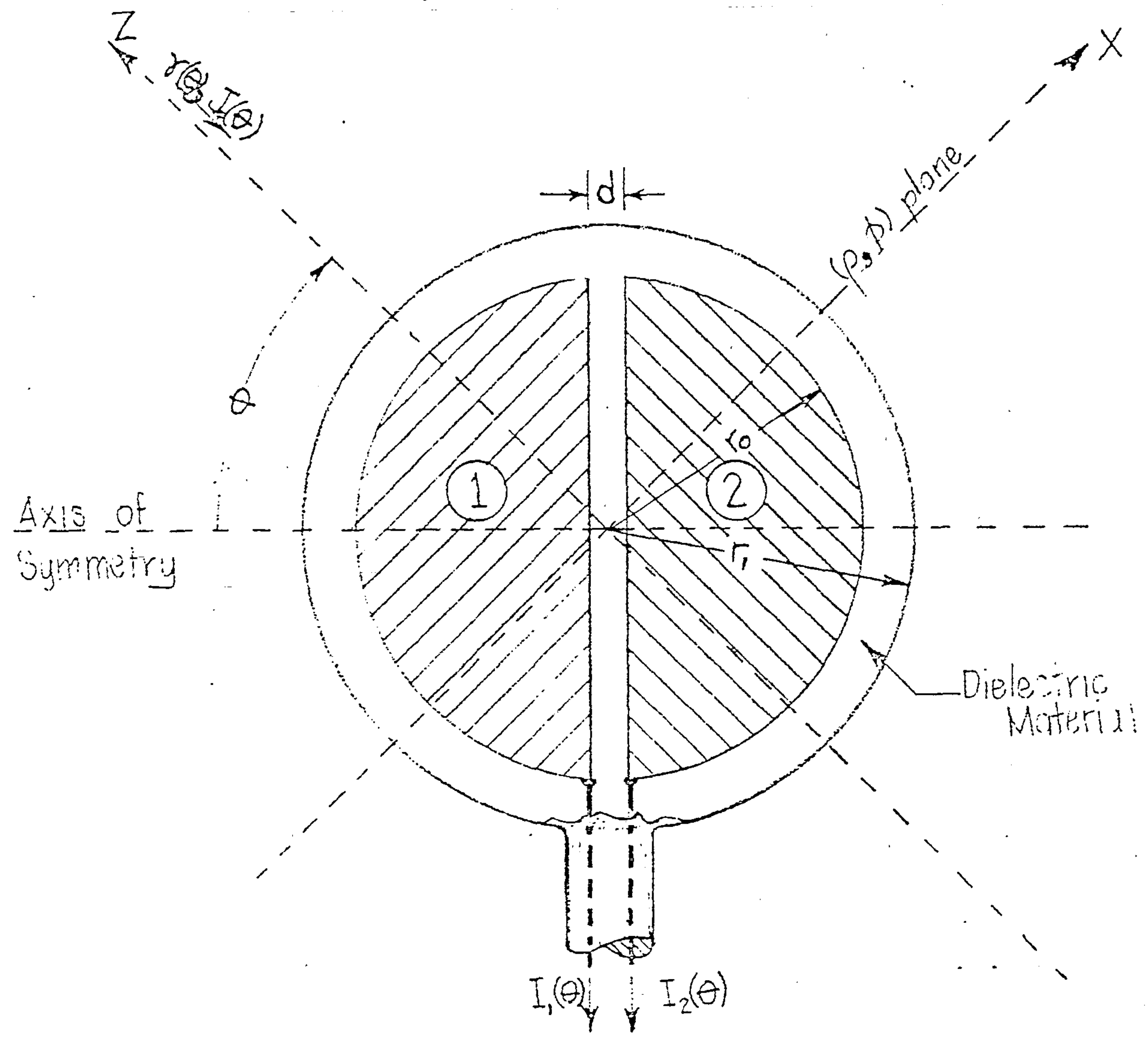

Y is normal to page

Z = differential twinax impedance
Z′ = common mode twinax impedance
K = dielectric constant

Fig. 1 $\overrightarrow{\gamma}$ Compton Diode

Since the detector response is dependent only on θ then all gammas coming from angle θ can be averaged to make $\gamma(\theta)$ in terms of per unit solid angle. Then $\gamma(\theta)$ need be considered from only one direction taken arbitrarily to be the z axis.

The currents, $I_1(\theta)$ and $I_2(\theta)$, arise from the fact that the gamma rays, $\gamma(\theta)$, coming from a particular direction are attenuated in passing through the electrodes. Since the Compton current density, $J_c(\theta)$, is proportional to $\gamma(\theta)$ there will be a net current (negative) deposited in the electrodes. To determine these currents (to first order) will require multiplication of the Compton current density by one minus the gamma attenuation factor, and integration of this quantity over the projected area of the electrode on the (ρ, ϕ) plane (normal to $\gamma(\theta)$). This will later be calculated.

However, certain characteristics of this detector may be determined from some general considerations. Assuming that the distance, d, between the hemispheres is small compared to their radius, r_o, the sum of the currents, $I_1(\theta)$ plus $I_2(\theta)$, clearly is independent of θ for constant $J_c(\theta)$ because in this case there is effectively only one spherical electrode and the response must be isotropic from symmetry considerations. A more important consideration is that if the γ-ray mean free path in the electrode material, r_γ, is much less than the radius, then all the Compton current incident on the sphere can be assumed to be collected with little of $\gamma(\theta)$ passing through the electrode. Thus negligible Compton current leaves a shadowed side of the electrode. This means that the differential current, $I_1(\theta)$ minus $I_2(\theta)$, can be calculated by considering the projected unshadowed areas of the electrodes on the plane normal to $\gamma(\theta)$ (the (ρ, ϕ) plane).

Since the difference in these areas is proportional to cos θ, the differential output is weighted by this factor, or the differential current is proportional to the vector component of $\gamma(\theta)$ in the direction of axial symmetry. Since this is true for all θ then the differential current is just the component of $\vec{\gamma}$ in the direction of axial symmetry. This is the desired characteristic of such a detector.

Now that general features of the detector are established some more detailed calculations will be made on the model of exponential attenuation of $\gamma(\theta)$ and $J_c(\theta)$ passing through the electrodes. Second order scattering will not be considered.

II. Detector Geometry

For convenience in analyzing this detector define a cylindrical coordinate system, (ρ, ϕ, z), based on the (x, y, z) system (right handed) indicated in figure 1. The (x, y) plane (normal to $\gamma(\theta)$) transforms into the (ρ, ϕ) plane as shown in figure 2. The (ρ, ϕ) coordinates are normalized to the circle of intersection of the electrodes with the (ρ, ϕ) plane giving the transformation equations as

$$
\begin{aligned}
x &= r_0 \cos \phi \\
y &= r_0 \sin \phi \\
z &= Z
\end{aligned}
\tag{1}
$$

Using these relations some important distances and boundaries can be expressed in the chosen coordinate system. First the height, z_1, of a point on the spherical electrode surfaces from this reference plane is

$$z_1 = \pm r_0 \sqrt{1 - \rho^2} \tag{2}$$

Second the height, z_2, of the plane dividing the hemispheres from the (ρ, ϕ) plane is given by

$$z_2 / x = \tan \theta$$

or

$$z_2 = r_0 \rho \cos \phi \tan \theta \tag{3}$$

These two heights are needed to calculate the gamma attenuation. It is assumed for these calculations (and those to follow) that the distance between the electrodes is small compared to r_o and can be neglected.

Finally the circle which defines the boundary of the hemispherical electrode surfaces projects onto the (ρ, ϕ) plane as an ellipse as shown in figure 2 and is defined by the parameters ρ_1 and ϕ_1 related by equating z_1 and z_2 from eqns. (2) and (3) giving

$$\rho_1 = (1 + \cos^2 \phi_1 \tan^2 \theta)^{-\frac{1}{2}} \tag{4}$$

This is the equation of an ellipse with major axis 1.0, minor axis, $\cos\theta$, and area, $\pi \cos\theta$. This boundary divides the projection of the hemispherical electrodes into three regions. This division is needed to set up the surface integrals to be used later. Regions A_1 and A_2 apply to only electrodes 1 and 2 (as in figure 1) respectively while region A_3 applies to both electrodes with recognition that electrode 1 is shadowing electrode 2.

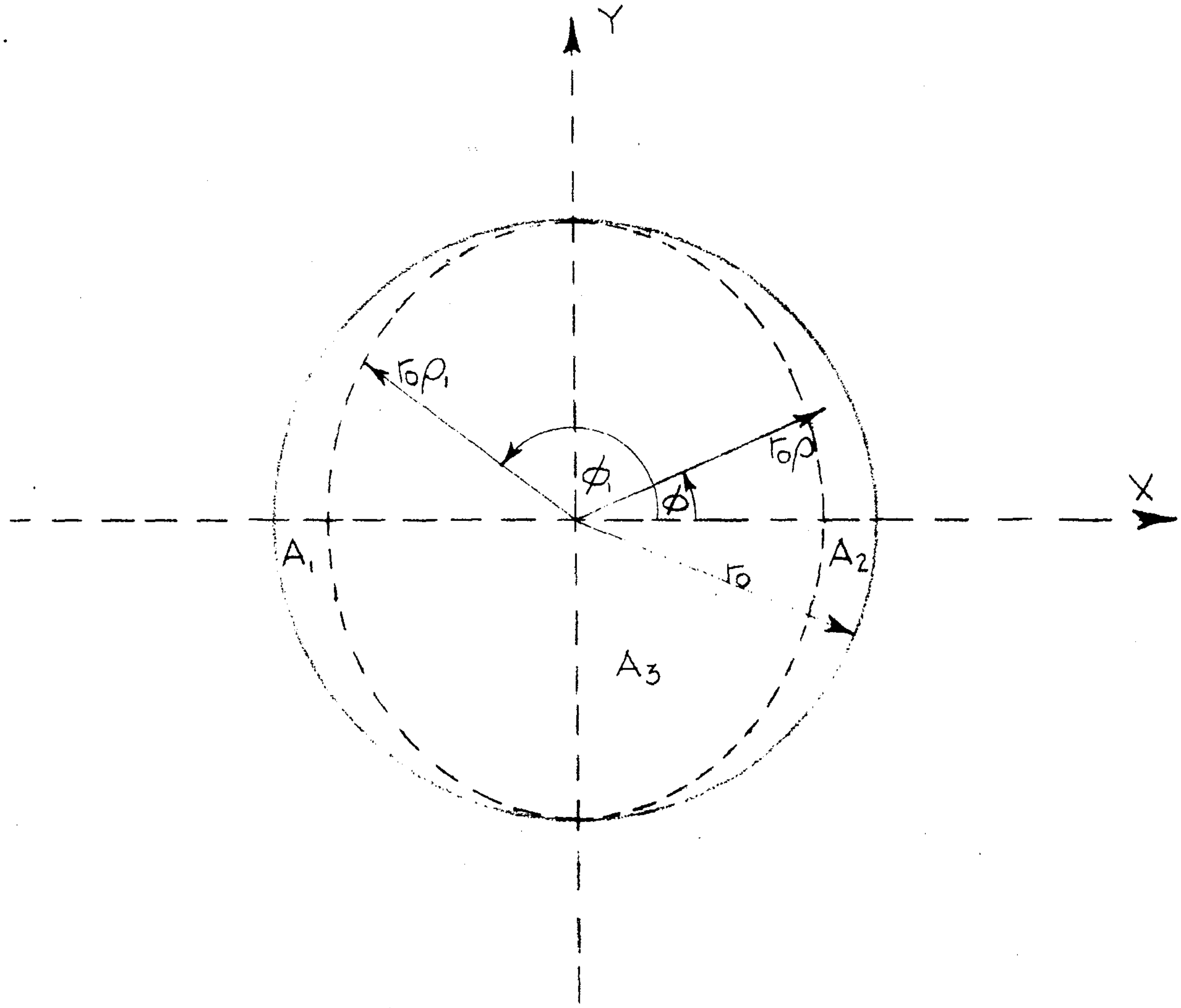

Fig. 2 Normalized Cylindrical Coordinate System.

When the surface integrals for the currents in the electrodes are set up z_1 and z_2 will be used for computing the thicknesses of the electrodes in the direction of the gamma rays. The incremental area, dA, for these integrals is

$$dA = r_0^2 \rho \, d\rho \, d\phi \tag{5}$$

Equations (2) to (5) together with the boundary lines of the regions in the (ρ, ϕ) plane give the required geometric relationships to set up the integrals for the detector sensitivity.

III. Detector Sensitivity

A. Compton Current Calculations

For the purpose of these calculations it will be assumed that for any projected incremental electrode area, dA, (normal to the gamma flux from angle θ) the incremental current, $J(\rho, \phi)$ dA, into the electrode is

$$J(\rho,\phi)\,dA = J_c(\rho,\phi)\left(1 - e^{-\frac{\Delta z(\rho,\phi)}{r_\gamma}}\right)dA \tag{6}$$

where $J_c(\rho, \phi)$ dA is the incremental Compton current incident on the electrode (referred to the projected incremental electrode area), $\Delta z(\rho,\phi)$ is the electrode thickness in the direction of the gamma rays, and r_γ is the gamma mean free path in the electrode material. $J_c(\rho,\phi)$ is assumed to be in equilibrium with the gamma flux which for a given photon energy is generally independent of material because of the predominance of Compton scattering. The ratio of Compton current to gamma flux, then, will be the same in the dielectric surrounding the electrodes as in air, but will change somewhat in a high atomic mass electrode material, implying that the current leaving the side of the electrode away from the gamma flux (represented by the exponential in eqn. (6)) may be a little inaccurate. However eqn. (6) will provide a good approximation, particularly as $\Delta z(\rho, \phi)$ becomes much larger than r_γ, making the Compton current leaving the electrode much less than that entering.

Therefore the expressions to evaluate to determine the currents in the electrodes, I, will be of the form

$$I = \int_A J_c(\rho,\phi)\left(1 - e^{-\frac{\Delta z(\rho,\phi)}{r_\gamma}}\right) dA \tag{7}$$

where the area of integration is the part of the (ρ, ϕ) plane on which a particular electrode projects.

B. Electrode Currents

Eqn. (7) can now be used to compute I_1 (θ) and I_2 (θ), the electrode currents. To compute I_1 (θ) one must first use eqns. (2) and (3) to evaluate the electrode thickness, $(\Delta z)_1$.

$$(\Delta z)_1 = \begin{cases} r_o\left(\sqrt{1-\rho^2} - \rho\cos\phi\tan\theta\right) & \text{for } \rho \le (1+\cos^2\phi\tan^2\theta)^{-1/2} \\ & \text{(region } A_3\text{)} \\ 2r_o\sqrt{1-\rho^2} & \text{for } \begin{cases} 1 \ge \rho \ge (1+\cos^2\phi\tan^2\theta)^{-1/2} \\ \pi/2 \le \phi \le 3\pi/2 \end{cases} \\ & \text{(region } A_1\text{)} \end{cases} \quad (8)$$

Since electrode 1 is unshadowed then the Compton current density incident on this electrode, J_{c_1} (ρ, ϕ), is

$$J_{c_1}(\rho, \phi) = J_c(\theta) \quad (9)$$

The current in electrode 1, I_1 (θ), can then be expressed as

$$I_1(\theta) = J_c(\theta)\, r_o^2 \int_0^{2\pi}\int_0^{(1+\cos^2\phi\tan^2\theta)^{-1/2}} \left(1 - e^{-\frac{r_o}{r_\gamma}\left(\sqrt{1-\rho^2} - \rho\cos\phi\tan\theta\right)}\right)\rho\, d\rho\, d\phi \quad \text{(region } A_3\text{)}$$

$$+ J_c(\theta)\, r_o^2 \int_{\pi/2}^{3\pi/2}\int_{(1+\cos^2\phi\tan^2\theta)^{-1/2}}^{1} \left(1 - e^{-\frac{2r_o}{r_\gamma}\sqrt{1-\rho^2}}\right)\rho\, d\rho\, d\phi \quad \text{(region } A_1\text{)} \quad (10)$$

For electrode 2 the thickness, $(\Delta z)_2$, is similar to that for electrode 1.

$$(\Delta z)_2 = \begin{cases} r_0\left(\sqrt{1-\rho^2} + \rho\cos\phi\tan\theta\right) & \text{for } \rho \le (1+\cos^2\phi\tan^2\theta)^{-1/2} \\ & \text{(region } A_3\text{)} \\ 2r_0\sqrt{1-\rho^2} & \text{for } \begin{cases} 1 \ge \rho \ge (1+\cos^2\phi\tan^2\theta)^{-1/2} \\ -\pi/2 \le \phi \le \pi/2 \end{cases} \\ & \text{(region } A_2\text{)} \end{cases} \tag{11}$$

Since electrode 2 is partially shadowed by electrode 1, the Compton current density, $J_{c2}(\rho,\phi)$, incident on this electrode will be reduced by an amount equal to the gamma ray attenuation in electrode 1 (already calculated for eqn. (10)) giving

$$J_{c_2}(\rho,\phi) = \begin{cases} J_c(\theta)\, e^{-\frac{r_0}{r_\gamma}\left(\sqrt{1-\rho^2} - \rho\cos\phi\tan\theta\right)} & \text{for } \rho \le (1+\cos^2\phi\tan^2\theta)^{-1/2} \\ & \text{(region } A_3\text{)} \\ J_c(\theta) & \text{for } \begin{cases} 1 \ge \rho \ge (1+\cos^2\phi\tan^2\theta)^{-1/2} \\ -\pi/2 \le \phi \le \pi/2 \end{cases} \\ & \text{(region } A_2\text{)} \end{cases} \tag{12}$$

Again using eqn. (7) the current in electrode 2, $I_2(\theta)$, is

$$I_2(\theta) = J_c(\theta)\, r_0^2 \int_0^{2\pi}\int_0^{(1+\cos^2\phi\tan^2\theta)^{-1/2}} \left(e^{-\frac{r_0}{r_\gamma}\left(\sqrt{1-\rho^2}-\rho\cos\phi\tan\theta\right)} - e^{-\frac{2r_0}{r_\gamma}\sqrt{1-\rho^2}}\right)\rho\, d\rho\, d\phi \quad \text{(region } A_3\text{)}$$

$$+ J_c(\theta)\, r_0^2 \int_{-\pi/2}^{\pi/2}\int_{(1+\cos^2\phi\tan^2\theta)^{-1/2}}^{1} \left(1 - e^{-\frac{2r_0}{r_\gamma}\sqrt{1-\rho^2}}\right)\rho\, d\rho\, d\phi \quad \text{(region } A_2\text{)}$$

(13)

Strictly speaking these expressions only apply for $0 \leq \theta \leq \pi/2$ because of the assumption that electrode 1 is shadowing electrode 2. However, from symmetry, for $\pi/2 \leq \theta \leq \pi$ the roles of the two currents, $I_1(\theta)$ and $I_2(\theta)$, can be interchanged to give the desired results.

Because of the characteristics of this detector as discussed in Section I it is desirable to express the common mode and differential currents instead of $I_1(\theta)$ and $I_2(\theta)$. This procedure also simplifies the calculations to some extent. Defining these two quantities by convention one has the common mode current, $I_{com}(\theta)$,

$$I_{com}(\theta) = I_1(\theta) + I_2(\theta) \tag{14}$$

and the differential current, I_{dif},

$$I_{dif}(\theta) = \frac{I_1(\theta) - I_2(\theta)}{2} \tag{15}$$

$I_{com}(\theta)$ can now be expressed using eqns. (10) and (13) by noting that the integrals corresponding to region A_3 can be combined making all the integrands identical and thus the three remaining integrals can be combined into one integral over the full circle in the (ρ, ϕ) plane. Thus $I_{com}(\theta)$ is

$$I_{com}(\theta) = J_{c_0} r_0^2 \int_0^{2\pi}\int_0^{1} \left(1 - e^{-\frac{2r_0}{r_\gamma}\sqrt{1-\rho^2}}\right)\rho\, d\rho\, d\phi \tag{16}$$

This same result could have been obtained from a consideration of the current coming from a sphere immersed in the gamma flux. In fact the negative of the exponent in the exponential function in eqn. (16) is just the thickness of such a sphere divided by r_γ.

$I_{dif}(\theta)$ can also be expressed using eqns. (10) and (13) by noting that if the variable, ϕ, in the integral corresponding to region A_1 in eqn. (10) is replaced by $\phi - \pi$, this integral becomes identical in form to the integral corresponding to region A_2 in eqn. (13). Thus, when the electrode currents are differenced these terms drop out and one is left with a single integral over region A_3 so that

$$I_{dif}(\theta) = \frac{J_c(\theta)\, r_o^2}{2} \int_0^{2\pi} \int_0^{(1+\cos^2\phi \tan^2\theta)^{-1/2}} \left(1 + e^{-\frac{2 r_o}{r_\gamma}\sqrt{1-\rho^2}} - 2e^{-\frac{r_o}{r_\gamma}\left(\sqrt{1-\rho^2} - \rho \cos\phi \tan\theta\right)}\right) \rho \, d\rho \, d\phi \tag{17}$$

The problem is now somewhat simplified: (1) $I_{com}(\theta)/J_c(\theta)$ is independent of θ and (2) by symmetry $I_{dif}(\theta)/J_c(\theta)$ can be calculated for values of θ greater than $\pi/2$ by the relation

$$\frac{I_{dif}(\theta)}{J_c(\theta)} = -\frac{I_{dif}(\pi-\theta)}{J_c(\pi-\theta)} \tag{18}$$

The characteristics of $I_{com}(\theta)$ and $I_{dif}(\theta)$ can now be investigated.

C. Characteristics of the Common Mode Current

To evaluate $I_{com}(\theta)$ first note that in eqn. (16) the integrand and limits are independent of ϕ so that the equation can be reduced to

$$I_{com}(\theta) = J_c(\theta)\, r_o^2\, (2\pi) \int_0^1 \left(1 - e^{-\frac{2r_o}{r_\gamma}\sqrt{1-\rho^2}}\right) \rho\, d\rho \tag{19}$$

Next making the substitution

$$\psi = \sqrt{1-\rho^2} \tag{20}$$

or

$$\psi^2 = 1-\rho^2 \tag{21}$$

and which makes the differential become

$$\psi\, d\psi = -\rho\, d\rho \tag{22}$$

eqn. (19) becomes

$$I_{com}(\theta) = J_c(\theta)\, r_o^2\, (2\pi)(-) \int_1^0 \left(1 - e^{-\frac{2r_o}{r_\gamma}\psi}\right) \psi\, d\psi \tag{23}$$

The limits on this integral are then inverted and the integrand multiplied out.

$$I_{com}(\theta) = J_c(\theta)\, r_o^2\, (2\pi) \int_0^1 \left(\psi - \psi e^{-\frac{2r_o}{r_\gamma}\psi}\right) d\psi \tag{24}$$

This is integrated to give

$$I_{com}(\theta) = J_c(\theta)\, r_o^2\, (2\pi) \left\{ \frac{\psi^2}{2}\Big|_0^1 - \frac{e^{-\frac{2r_o}{r_\gamma}\psi}}{\left(2\frac{r_o}{r_\gamma}\right)^2} \left(-2\frac{r_o}{r_\gamma} - 1\right)\Big|_0^1 \right\} \tag{25}$$

or

$$I_{com}(\theta) = J_c(\theta)\, \pi r_o^2 \left[1 - \frac{1}{2}\left(\frac{r_\gamma}{r_o}\right)^2 + \left(\frac{r_\gamma}{r_o} + \frac{1}{2}\left(\frac{r_\gamma}{r_o}\right)^2\right) e^{-\frac{2r_o}{r_\gamma}}\right] \tag{26}$$

Thus, with the approximations used one is able to explicitely solve for the common mode current in terms of the Compton current density associated with $\gamma(\theta)$, the radius of the electrodes, and the gamma mean free path in the electrodes. The independence of $I_{com}(\theta)/J_c(\theta)$ on θ shows the isotropic nature of this detector in the common mode so that I_{com} will be proportional to the integral of $\gamma(\theta)$ over the entire solid angle, γ.

Ideally for an infinitely thick, detector $I_{com}(\theta)$ is just $J_c(\theta)\pi r_0^2$, the total Compton current incident on the detector. This makes it convenient to define I_{com} as

$$I_{com}(\theta) = J_c(\theta)\pi r_0^2 f_{com} \tag{27}$$

where f_{com} is the sum of terms in the brackets in eqn. (26). The value of f_{com} lies between 0.0 and 1.0 and is the fraction of the Compton current incident on the detector which is collected and is thus a measure of how close the detector approaches the ideal case ($f_{com} = 1.0$).

For certain values of the parameter, r_0/r_γ, f_{com} can be approximated by simpler expressions. For $r_0/r_\gamma >> 1.0$, f_{com} becomes simply

$$f_{com} \simeq 1 - \frac{1}{2}\left(\frac{r_\gamma}{r_0}\right)^2 \tag{28}$$

The opposite case, $r_0/r_\gamma << 1.0$, is more complex because the exponential must be expanded as far as the cubic term to remove the terms in r_γ/r_0. Since

$$e^{-2\frac{r_0}{r_\gamma}} = 1 - 2\frac{r_0}{r_\gamma} + 2\left(\frac{r_0}{r_\gamma}\right)^2 - \frac{4}{3}\left(\frac{r_0}{r_\gamma}\right)^3 \cdots \tag{29}$$

then for $r_0/r_\gamma << 1.0$

$$\begin{aligned} f_{com} = 1 - \frac{1}{2}\left(\frac{r_\gamma}{r_0}\right)^2 + \frac{r_\gamma}{r_0} - 2 + 2\frac{r_0}{r_\gamma} - \frac{2}{3}\left(\frac{r_0}{r_\gamma}\right)^2 \cdots \\ + \frac{1}{2}\left(\frac{r_\gamma}{r_0}\right)^2 - \frac{r_\gamma}{r_0} + 1 - \frac{2}{3}\frac{r_0}{r_\gamma} \cdots \end{aligned} \tag{30}$$

The constants and terms in r_γ/r_0 cancel leaving only terms in r_0/r_γ. If terms of order $(r_0/r_\gamma)^2$ and higher are ignored then

$$f_{com} \simeq \frac{4}{3}\frac{r_0}{r_\gamma} \tag{31}$$

This approximation illustrates the possible undesireability of having a thin detector because its sensitivity (to the Compton current) will be energy dependent as reflected in r_γ while for the thick detector eqn. (28) indicates that this effect will appear only as a small correction term.

D. Characteristics of the Differential Mode Current

The evaluation of $I_{dif}(\theta)$ (from eqn. (17)) is a much more complex matter than the evaluation of $I_{com}(\theta)$ because of the appearance of ϕ both in the arguements of the exponentials and in one of the limits of the integral over ρ. However, much can be learned by looking at certain approximations and certain cases.

1. Very Thick Detector

If it is assumed that $r_1/r_\gamma \gg 1$, then over the region of integration the exponentials in the integrand of eqn. (17) contribute negligibly to the integral since their exponents are always negative or zero which makes these terms arbitrarily small over all but a vanishingly small part of the region of integration. Then

$$I_{dif}(\theta) \simeq \frac{J_c(\theta)\, r_0^2}{2}\int_0^{2\pi}\int_0^{(1+\cos^2\phi\tan^2\theta)^{-1/2}} \rho\, d\rho\, d\phi \tag{32}$$

However, this integral just represents the area of region A_3 in the (ρ, ϕ) plane. Since this is an ellipse with major axis 1.0 and minor axis $\cos\theta$ the value of this integral is simply $\pi\cos\theta$ so that

$$I_{dif}(\theta) \simeq \frac{J_c(\theta)\, \pi r_0^2}{2}\cos\theta \tag{33}$$

showing that in the approximation of a thick detector the response is proportional to $\cos\theta$ as desired.

This equation applies for all θ since the symmetry requirement of eqn. (10) is satisfied by $\cos\theta$.

This result shows that the magnitude of $I_{dif}(\theta)$ has a maximum value of $J_c(\theta)\pi r_o^2/2$ which makes it convenient to define $I_{dif}(\theta)$ as

$$I_{dif}(\theta) = \frac{J_c(\theta)\pi r_o^2}{2} f_{dif}(\theta) \tag{34}$$

where

$$f_{dif}(\theta) = \frac{1}{\pi}\int_0^{2\pi}\int_0^{(1+\cos^2\phi\tan^2\theta)^{-1/2}} \left(1+e^{-2\frac{r_o}{r_\gamma}\sqrt{1-\rho^2}} - 2e^{-\frac{r_o}{r_\gamma}(\sqrt{1-\rho^2}-\rho\cos\phi\tan\theta)}\right)\rho\, d\rho\, d\phi \tag{35}$$

The factor $f_{dif}(\theta)$ now varies between -1.0 and +1.0.

2. Very Thin Detector

Unfortunately, since exponential functions do not expand as a sum of one over their arguements this kind of a scheme cannot be used to see how closely $f_{dif}(\theta)$ approaches the $\cos\theta$ dependence of large r_o/r_γ. However, the exponentials of eqn. (35) can be expanded in powers of r_o/r_γ to evaluate this dependence for small r_o/r_γ. Thus

$$e^{-2\frac{r_o}{r_\gamma}\sqrt{1-\rho^2}} = 1 - 2\frac{r_o}{r_\gamma}(1-\rho^2)^{1/2} + 2\left(\frac{r_o}{r_\gamma}\right)^2(1-\rho^2)\cdots \tag{36}$$

and

$$\begin{aligned} e^{-\frac{r_o}{r_\gamma}(\sqrt{1-\rho^2}-\rho\cos\phi\tan\theta)} = 1 &- \frac{r_o}{r_\gamma}(1-\rho^2)^{1/2} + \frac{r_o}{r_\gamma}\rho\cos\phi\tan\theta \\ &+ \frac{1}{2}\left(\frac{r_o}{r_\gamma}\right)^2(1-\rho^2) - \frac{1}{2}\left(\frac{r_o}{r_\gamma}\right)^2(1-\rho^2)^{1/2}\rho\cos\phi\tan\theta \\ &+ \frac{1}{2}\left(\frac{r_o}{r_\gamma}\right)^2\rho^2\cos^2\phi\tan^2\theta\ \cdots \end{aligned} \tag{37}$$

Combining these expressions with eqn. (35) gives

$$f_{dif}(\theta) \simeq \frac{1}{\pi}\int_0^{2\pi}\int_0^{(1+\cos^2\phi\tan^2\theta)^{-1/2}}\Big[\left(\frac{r_o}{r_\delta}\right)^2(1-\rho^2)-2\frac{r_o}{r_\delta}\rho\cos\phi\tan\theta$$
$$+\left(\frac{r_o}{r_\delta}\right)^2(1-\rho^2)^{1/2}\rho\cos\phi\tan\theta-\left(\frac{r_o}{r_\delta}\right)^2\rho^2\cos^2\phi\tan^2\theta\Big]\rho\,d\rho\,d\phi \tag{38}$$

Because of the symmetry of the limits of this integral around $\phi = \pi/2$ and $\phi = -\pi/2$, the terms in the integral involving $\cos\phi$ to an odd power will give no net contribution to the integral. Eqn. (38) then reduces to

$$f_{dif}(\theta) \simeq \frac{1}{\pi}\left(\frac{r_o}{r_\delta}\right)^2\int_0^{2\pi}\int_0^{(1+\cos^2\phi\tan^2\theta)^{-1/2}}\left[1-\rho^2(1+\cos^2\phi\tan^2\theta)\right]\rho\,d\rho\,d\phi \tag{39}$$

This integral can be solved by first shifting back to rectangular coordinates using the relationships

$$\eta = \rho\cos\phi$$
$$\vartheta = \rho\sin\phi$$
$$d\eta\,d\vartheta = \rho\,d\rho\,d\phi \tag{40}$$

which transforms the integral to

$$f_{dif}(\theta) \simeq \frac{1}{\pi}\left(\frac{r_o}{r_\delta}\right)^2\int_{-1}^{1}\int_{-\sqrt{1-\left(\frac{\eta}{\cos\theta}\right)^2}}^{+\sqrt{1-\left(\frac{\eta}{\cos\theta}\right)^2}}\left[1-\eta^2-\vartheta^2-\eta^2\tan^2\theta\right]d\eta\,d\vartheta \tag{41}$$

where

$$\vartheta = \pm\sqrt{1-\left(\frac{\eta}{\cos\theta}\right)^2} \tag{42}$$

is the equation of the elliptical boundary of region A_3 in these normalized rectangular coordinates. Eqn. (41) reduces further to

$$f_{dif}(\theta) \simeq \frac{1}{\pi}\left(\frac{r_0}{r_\gamma}\right)^2 \int_{-1}^{+1} \int_{-\sqrt{1-\left(\frac{\eta}{\cos\theta}\right)^2}}^{+\sqrt{1-\left(\frac{\eta}{\cos\theta}\right)^2}} \left[1 - \left(\frac{\eta}{\cos\theta}\right)^2 - \upsilon^2\right] d\eta\, d\upsilon \tag{43}$$

This reduced equation further lends itself to a substitution of the form

$$\xi = \frac{\eta}{\cos\theta}$$

$$d\xi = \frac{1}{\cos\theta}\, d\eta \tag{44}$$

which leads to

$$f_{dif}(\theta) \simeq \frac{1}{\pi}\left(\frac{r_0}{r_\gamma}\right)^2 \cos\theta \int_{-1}^{+1} \int_{-\sqrt{1-\xi^2}}^{+\sqrt{1-\xi^2}} \left[1 - \xi^2 - \upsilon^2\right] d\xi\, d\upsilon \tag{45}$$

which is now in a form where the θ dependence is removed from the integral. Using another set of substitutions

$$\xi = \rho' \cos\phi'$$

$$\upsilon = \rho' \sin\phi'$$

$$d\xi\, d\upsilon = \rho' d\rho' d\phi' \tag{46}$$

then

$$f_{dif}(\theta) \simeq \frac{1}{\pi}\left(\frac{r_0}{r_\gamma}\right)^2 \cos\theta \int_0^{2\pi} \int_0^{1} \left[1 - \rho'^2\right] \rho' d\rho' d\phi' \tag{47}$$

Since the integrand and the limits are independent of ϕ' the integral over ϕ' will only give a factor of 2π leaving

$$f_{dif}(\theta) \simeq 2\left(\frac{r_0}{r_\gamma}\right)^2 \cos\theta \int_0^1 (\rho' - \rho'^3)\, d\rho' \tag{48}$$

The integral over ρ' can be easily evaluated as

$$f_{dif}(\theta) \simeq 2\left(\frac{r_0}{r_\gamma}\right)^2 \cos\theta \left[\frac{1}{2} - \frac{1}{4}\right] = \frac{1}{2}\left(\frac{r_0}{r_\gamma}\right)^2 \cos\theta \tag{49}$$

This gives the very encouraging result that for $r_0/r_\gamma << 1$ the detector sensitivity is still proportional to $\cos\theta$ indicating that the angular dependence may not be too sensitive to the parameter r_0/r_γ. It therefore may not be too difficult then to construct such a detector with a reasonable approximation to the $\cos\theta$ dependence.

3. Exact solution for $\theta = 0.0$

In order to get some estimate of how closely the approximation of a thick detector ($r_0/r_\gamma >> 1$) has been achieved, eqn. (35) for $f_{dif}(\theta)$ can be exactly solved for the case $\theta = 0$, i.e.

$$f_{dif}(0) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 \left(1 + e^{-2\frac{r_0}{r_\gamma}\sqrt{1-\rho^2}} - 2e^{-\frac{r_0}{r_\gamma}\sqrt{1-\rho^2}}\right)\rho\, d\rho\, d\phi \tag{50}$$

Again the integral over ϕ can be removed to give

$$f_{dif}(0) = 2\int_0^1 \left(1 + e^{-2\frac{r_0}{r_\gamma}\sqrt{1-\rho^2}} - 2e^{-\frac{r_0}{r_\gamma}\sqrt{1-\rho^2}}\right)\rho\, d\rho \tag{51}$$

Making the substitutions of eqns. (20) through (22) as were used in the evaluation of the common mode current

$$f_{dif}(0) = 2\,(-)\int_1^0 \left(1 + e^{-2\frac{r_0}{r_\gamma}\gamma} - 2e^{-\frac{r_0}{r_\gamma}\gamma}\right)\gamma\, d\gamma \tag{52}$$

or

$$f_{dif}(0) = 2\int_0^1 \left(\gamma + \gamma e^{-2\frac{r_0}{r_\gamma}\gamma} - 2\gamma e^{-\frac{r_0}{r_\gamma}\gamma}\right) d\gamma \tag{53}$$

Integrating equation (53) gives

$$f_{dif}(0) = 2\left[\frac{\gamma^2}{2} + \frac{e^{-2\frac{r_0}{r_\gamma}\gamma}}{\left(2\frac{r_0}{r_\gamma}\right)^2}\left(-2\frac{r_0}{r_\gamma}\gamma - 1\right) - 2\frac{e^{-\frac{r_0}{r_\gamma}\gamma}}{\left(r_0/r_\gamma\right)^2}\left(-\frac{r_0}{r_\gamma}\gamma - 1\right)\right]\Bigg|_0^1 \tag{54}$$

or

$$f_{dif}(0) = 1 - \frac{7}{2}\left(\frac{r_\gamma}{r_0}\right)^2 + 4\left(\frac{r_\gamma}{r_0} + \left(\frac{r_\gamma}{r_0}\right)^2\right)e^{-\frac{r_0}{r_\gamma}} - \left(\frac{r_\gamma}{r_0} + \frac{1}{2}\left(\frac{r_\gamma}{r_0}\right)^2\right)e^{-2\frac{r_0}{r_\gamma}} \tag{55}$$

For $r_0/r_\gamma >> 1$ this becomes approximately

$$f_{dif}(0) \simeq 1 - \frac{7}{2}\left(\frac{r_\gamma}{r_0}\right)^2 \tag{56}$$

This last expression shows how rapidly the approximation of a thick detector (from the viewpoint of the differential signal) is approached. The approximation of a thin detector has already been considered in eqn. (49).

E. Summary

In designing such a detector where it is desirable that $r_0/r_\gamma >> 1$ there are three calculations to make: (1) f_{com} from eqns. (27) and (26). This gives the degree to which the common mode signal is independent of r_0/r_γ. (2) $f_{dif}(0)$ from eqn. (55). This gives the same calculation for the differential signal. (3) the ratio of $f_{dif}(0)$ to f_{com}. This is a form of signal to noise ratio.

Unfortunately, the author is unable to evaluate eqn. (35) analytically for f_{dif} for all θ. This could easily be done on a computer but since second order scattering of the photons has not been considered a computer calculation of this integral may not be very profitable.

IV. Response Time of Detector

Another point to be considered in the design of this detector is its capability for time resolution of the gamma pulse. A convenient method of calculating the time resolution is to calculate the capacitance of the detector in both common and differential modes and using the common and differential mode impedance of the twinax to calculate a time constant or risetime for a step radiation input.

The detector capacitance in the common mode, C_{com}, can be calculated by considering the two electrodes as one spherical electrode of radius r_0 inside a conducting case of radius r_1. The dielectric constant is K (or ϵ/ϵ_0). Thus,

$$C_{com} = K \frac{4\pi\epsilon_0 r_0 r_1}{r_1 - r_0} \tag{57}$$

This gives a rise time,

$$t_{r_{com}} \simeq 2 Z' C_{com} = Z' K \frac{8\pi\epsilon_0 r_0 r_1}{r_1 - r_0} \tag{58}$$

The detector capacitance in the differential mode, C_{dif}, has two contributions; (1) the direct capacitance across the boundary between the two electrodes, and (2) the capacitance of one electrode to the outer shell and in turn to the other electrode. The direct capacitance, C_{direct}, is just

$$C_{direct} = K \epsilon_0 \pi \frac{r_0^2}{d} \tag{59}$$

The capacitance through the outer shell, $C_{indirect}$, is calculated by considering that the capacitance of one electrode to the case is just $C_{com}/2$ and that this contribution to the differential capacitance is represented by two such capacitors in series or

$$C_{indirect} = \frac{C_{com}}{4} = K \frac{\pi\epsilon_0 r_0 r_1}{r_1 - r_0} \tag{60}$$

Thus,

$$C_{dif} = C_{direct} + C_{indirect} = K\pi\epsilon_0 r_0\left(\frac{r_0}{d} + \frac{r_1}{r_1 - r_0}\right) \tag{61}$$

and the risetime, $t_{r_{dif}}$, is

$$t_{r_{dif}} \simeq 2Z\,C_{dif} = 2Z\,K\pi\epsilon_0 r_0\left(\frac{r_0}{d} + \frac{r_1}{r_1 - r_0}\right) \tag{62}$$

From these risetime calculations some constraints can then be placed on the detector dimensions.

V. Sample Calculation

To illustrate how the previous calculations might be used and to get a feel for the numbers involved it is useful to make a sample calculation. Assume that the electrode material is nickel (which has been used by LASL for Compton diodes) and that it is desired to have r_0 give a decade attenuation, i.e.,

$$\frac{r_0}{r_\gamma} \simeq 2.3 \tag{63}$$

For nickel r_γ is about 3.5 cm., near the minimum of the gamma-ray total cross section curve. This would make r_0 about 8.05 cm. (3.17 in.). From eqns. (26) and (27) then

$$f_{com} \simeq .91 \tag{64}$$

and from eqn. (55)

$$f_{dif}(0) \simeq .58 \tag{65}$$

These numbers indicate the closeness of approximation to a thick detector.

Assume further that the desired risetime in both modes is 3 nanoseconds, that the dielectric is some non-hydrogenous material with a dielectric constant similar to teflon ($K \simeq 2.1$), and that all spacings (d and $r_1 - r_0$) are 2 cm. Then the capacitances are from eqns. (57) and (61)

$$\begin{aligned} C_{com} &\simeq 98 \text{ pf.} \\ C_{dif} &\simeq 48 \text{ pf.} \end{aligned} \tag{66}$$

and the impedances from eqns. (58) and (62) are

$$Z' \simeq 15\,\Omega$$
$$Z \simeq 31\,\Omega \qquad (67)$$

These values are lower than standard twinax cable impedances but could be achieved by resistive matching at the input.

Finally, a sensitivity can be calculated for this detector by considering the detector cross section and a relation between the Compton current and gamma current as

$$J_c(\theta) \approx -2\times10^{-8}\,\gamma(\theta) \qquad (68)$$

where J_c is in amps per square meter and γ is in roentgens per second (air equivalent dose). The common mode sensitivity, S_{com}, is then about

$$S_{com} = \frac{J_c(\theta)}{\gamma(\theta)}\,\pi r_o^2 f_{com} \approx -3.7\times10^{-10}\ \frac{\text{coulombs}}{\text{roentgen}} \qquad (69)$$

and the differential mode sensitivity, S_{dif}, is about

$$S_{dif} = \frac{J_c(\theta)}{\gamma(\theta)}\,\frac{\pi r_o^2}{2} f_{dif}(0) \approx -1.2\times10^{-10}\ \frac{\text{coulombs}}{\text{roentgen}} \qquad (70)$$

However, if resistive matching were used these sensitivities would be reduced accordingly.

The design of such a detector is clearly a matter of trading off various desirable characteristics to achieve an optimum set of parameters.

VI. Generalization of Detector

This scheme for measuring a vector component of $\vec{\gamma}$ can be generalized to measure two or even all three components. As illustrated in figure 1 the electrode material has been split once to form two hemispheres and this arrangement measures the vector component of $\vec{\gamma}$ normal to this plane of separation. If this sphere of electrode material is split again to form two different hemispheres and signal cable is attached to all four of the electrodes then by appropriately summing and differencing the electrode signals two vector components (plus the isotropic flux) can be obtained. Finally, by splitting the sphere into eight parts along three mutually orthogonal planes with common point of intersection in the center of the sphere, the three orthogonal vector components of $\vec{\gamma}$ and the isotropic γ can all be obtained.

Another method to obtain the three vector components together with the isotropic γ is to use three single-vector-component detectors. This last alternative may be simpler and provide more flexibility in use.

VII Summary

In its simplest form, as shown in figure 1 (on page 2), this detector can give two bits of information on one twinax cable; (1) the isotropic γ in the common mode, and (2) one vector component of $\vec{\gamma}$ in the differential mode. This detector concept can also be expanded to measure any desired set of vector components of $\vec{\gamma}$ either by making the detector more complex or by using multiple detectors. Ideally the electrode radius is much greater than the γ-ray mean free path but practically this is difficult to realize. Care should be exercised in the selection of an electrode material and perhaps the electrode should be a composite of more than one material. The requirement for a thick electrode makes the capacitance large, perhaps requiring a slight lowering of the detector sensitivity to achieve a fast rise time.

However, the greatest advantage of such a detector is its reliability, ruggedness, and simplicity of operation.

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