NOTE 8

MAXIMIZING FREQUENCY RESPONSE OF A $\dot{B}$ LOOP

by

C.E. Baum
Air Force Weapons Laboratory

9 December 1964

Sensor and Simulation Notes VIII

Maximizing Frequency Response of a $\dot{B}$ Loop

I. Introduction

A typical method of constructing a $\dot{B}$ (time rate of change of the magnetic field) loop uses a pair of coaxial cables driving a twinax cable. The coaxial cables are tied together at the end opposite the twinax to form a loop of either the split shield type or the moibus type described in a previous note. However, considering that the loops must drive typically a 100-ohm characteristic impedance cable, the limitation of the frequency response comes not from the loop transit time but from the time constant of the loop inductance and the impedance driven by the loop. Any desired frequency response can be obtained by lowering the size of the loop to decrease the inductance but this also lowers its sensitivity.

For EMP work sensitivity may not be a problem, but signal to noise ratio is. The radiation noise signal from Compton currents in the cable center conductors is determined by the length of both signal cable and cable in the loop. In addition the EMP itself will induce noise signals in the signal cables and, since the loop sensitivity and thus the signal strength goes as the square of the loop dimensions, there may be some advantage in optimizing the sensitivity for a given frequency response. If less sensitivity is desired, then an optimization procedure should raise the frequency response of the loop as an added bonus. The purpose of this note is to discuss the maximization of the frequency response of a $\dot{B}$ loop.

The problem of the air conductivity and its effect on the $\dot{B}$ loop response will not be discussed here and all the calculations assume a negligible conductivity.

II. Transit Time Limitation

The first and most basic limitation to the frequency response of a $\dot{B}$ loop is the transit time characteristic of the loop dimensions. For the loop in figure 1 driving a resistive load Z_1 (given by the cable impedances and manner of hookup, e.g., split shield or moibus loop) L.Libby has shown (in the I.R.E., Sept. 1946) that, for purposes of calculation, this loop structure can be considered as shown in figure 2, in which the loop structure is extended to form a shorted transmission line with the loop area maintained the same as in figure 1. From this model one can calculate a transit time limitation.

For the optimum situation in which Z_1 equals the characteristic impedance of this equivalent transmission line, any signals will be terminated. Since the only place which these signals can be introduced is at the opening (through Z_1) at the top, the time, $\mathcal{T}_t$, required to get the signal back and terminated into the coaxial lines (represented by Z_1) is just the round trip transit time or

$$\mathcal{T}_t \approx \frac{2 \pi a}{c} \tag{1}$$

where c is the velocity of light.

This time, $\mathcal{T}_t$, represents a time dispersion introduced into the signal by the loop and thus corresponds to the fastest possible rise time of the loop as a sensor of $\dot{B}$. From this, one can calculate a maximum frequency response, f_{max}, of the $\dot{B}$ loop as

$$f_{max} \approx \frac{1}{\pi \mathcal{T}_t} \approx \frac{c}{2 \pi^2 a} \tag{2}$$

This last equation assumes that the time constant appropriate for calculating the rolloff frequency is approximately twice the rise time, $\mathcal{T}_t$. An upper bound can also be put on the loop radius, a_{max}, for given desired frequency response, f_c.

$$a_{max} \approx \frac{c}{2\pi^2 f_c} \tag{3}$$

This is illustrated in the following table for a few values of frequency response.

f_c (Megacycles)	a_{max} (Centimeters)
100	15.2
200	7.6
300	5.1

Table 1: Maximum loop radius for a desired frequency response

Thus a first bound is placed on the problem, i.e., one of the loop dimensions has been limited, automatically limiting the allowable loop area.

III. Loop Inductance

Now that the transit time constraints are calculated for the loop the problem becomes one of how to realize this maximum frequency response. To do this one can look at the equivalent loop structure given in figure 2 and constrain the characteristic impedance of this structure to be equal to Z_1 so that all signals on the loop are terminated. Another way to consider this problem is to calculate the low frequency inductance, L, of the loop and calculate the rise time, $\mathcal{T}_L$, for the response to a step $\dot{B}$ signal as

$$\mathcal{T}_L = 2\ L/Z_1 \tag{4}$$

To optimize the frequency response, one can equate this rise time to the round trip transit time, $\mathcal{T}_t$, giving

$$\frac{L}{Z_1} = \frac{\pi\ a}{c} \tag{5}$$

The inductance of the loop constructed from two coaxial lines, L_1, is given by

$$L_1 = \mu_o\ a\left(\ln\left(\frac{8a}{b}\right) - 2\right) = \frac{120\ \pi}{c}\ a\left(\ln\left(\frac{8a}{b}\right) - 2\right) \tag{6}$$

where a and b are as given in figure 1, c is the velocity of light and 120π is the impedance of free space (ohms). This expression is accurate only for large a/b. Substituting this into eqn. (5) then

$$\ln\left(\frac{8a}{b}\right) = \frac{Z_1}{120} + 2 \tag{7}$$

For a typical value of Z_1 of 100 Ω for the split shield loop

$$a/b \doteq 2.1 \tag{8}$$

At this point the ratio a/b is too small for eqn. (6) to be accurate but this calculation does point out that to minimize the inductance of the loop (maximize the frequency response) the coax size must be an appreciable fraction of the loop size. Thus, the effective loop area, and the loop sensitivity, will be harder to calculate and may become frequency dependent. The problem becomes worse for the moibus strip loop where Z_1 may be typically 25Ω.

One way to overcome this is to use extremely high impedance coaxial cables for the loops and resistively match these coaxes into the twinax, resulting in loss of signal. Another way to solve this problem is to effectively extend the coax dimensions in the direction of the loop axis but not in the direction of the loop radius. In this construction the loop is extended into a cylinder as shown in figure 3. Neglecting the manner of coupling of the signal cables to the cylinder the inductance of this structure, L_2, can be approximated as

$$L_2 = \mu_0 \frac{\pi a^2}{\ell} = \frac{120\pi}{c} \frac{\pi a^2}{\ell} \tag{9}$$

The length, ℓ, of this cylinder can be varied to give any desired L_2. Optimizing L_2 by eqn. (5) then gives

$$120 \pi \frac{a}{\ell} = Z_1 \tag{10}$$

This result is the same as that which applies to the loop equivalent transmission line impedance, Z_2, i.e.,

$$Z_2 = 120\pi \frac{a}{\ell} \tag{11}$$

implying that this equivalent transmission line is terminated.

For maximum frequency response, this places a constraint on the dimensions

$$\frac{\ell}{a} = \frac{120 \pi}{Z_1} \tag{12}$$

In the following table some values of this ratio of dimensions are given for selected values of Z_1.

Z_1 (ohms)	ℓ/a	$\ell/2a$
25	15.1	7.53
50	7.53	3.77
100	3.77	1.88
200	1.88	.94

Table 2: Ratio of dimensions for cylindrical loop

If these ratios of dimensions are attainable and the geometry is truly cylindrical then a loop of this kind can be used to optimize the frequency response by lowering the loop inductance to the point where this limitation is the same as the transit time limitation.

IV. Limitations of the Cylindrical Loop

The cylindrical loop shown in figure 3 still has not completely optimized the frequency response because the signal is introduced into the coaxial cables at only one point. The transit time limitations of this structure can be approximated by considering the distance along the wall of the cylinder from the point of signal introduction to the cable to the farthest point on the cylindrical shell giving

$$\tau_t \approx \frac{2}{c}\sqrt{(\pi a)^2 + (\ell/2)^2} \tag{13}$$

From this consideration the loop radii for various frequency responses will have to be reduced from those given in Table 1. However there are techniques for circumventing this limitation such as distributing the signal input to the coaxial lines by increasing the number of coaxial cable inputs, i.e., by splitting each coaxial cable into several coaxial cables as in figure 4. The coaxial cables on each side of the cylindrical loop are wired in parallel before joining the twinax cable and the transit times of these coaxes are constrained to be identical. Finally if the number of coaxes on each arm of this cylindrical loop is n and if they are driving a twinax of differential impedance Z, each of these coaxes must have an impedance, Z_3, given by

$$Z_3 = n\,(Z/2) \tag{14}$$

since n coaxes in parallel have their total impedance reduced as 1/n and the required impedance for a single coax is Z/2. Using this technique, the revised transit time limitation of eqn. (13) can be made to approach a minimum value given by eqn. (1). However, it would not seem profitable to extend this procedure much beyond the point where the distance between the coaxes on each arm of the cylindrical loop is comparable to the loop circumference.

This technique can also be modified to get around the large length-to-diameter ratios for low impedance structures (necessary for the moibus strip loop) by splitting the cylindrical structure into two or more such structures as in figure 5.

Here again the coaxial arms of the loop are wired in parallel but account must now be taken of the difference in arrival time of the EM signal at each loop. This means that the coaxial lines on the cylindrical structures closer to the EM source must have their transit times longer than the coaxes on cylindrical structures more distant from the EM source so that the signal from all the cylindrical loops will arrive at the twinax junction at the same time.

These two techniques make the loop either (1) a distributed device or (2) a travelling wave device. By judiciously combining these two techniques an optimum frequency response can be obtained and the $\dot{B}$ sensor can be kept from becoming unwieldy in dimensions.

Finally the junction of the loop structure to the cables must be considered. In effect there must be as smooth a transition as possible between the equivalent transmission line describing the loop structure and the coaxial cables which carry the signal to the twinax, or reflections will occur at these junctions decreasing the frequency response.

V. Summary

Four limitations must be considered in the design of a $\dot{B}$ loop for maximum frequency response:

(1) The loop radius. This determines the signal transit time on the loop structure.

(2) The matching of the impedances of the equivalent transmission line of the loop and the group of coaxial lines leading to the twinax. This can be done with a cylindrical loop.

(3) The number of coaxial lines on each arm of a cylindrical loop. This must be such that the separation between cables on an arm is less than the loop circumference.

(4) The size of the structure. A cylindrical loop can be split into more than one loop to shorten its length and give a more manageable structure.

A remaining problem is the smoothing of all transitions to minimize reflections. Again, it must be emphasized that the effects of air conductivity have not been included in this discussion.

CARL E. BAUM, 1/LT, USAF
9 December 1964

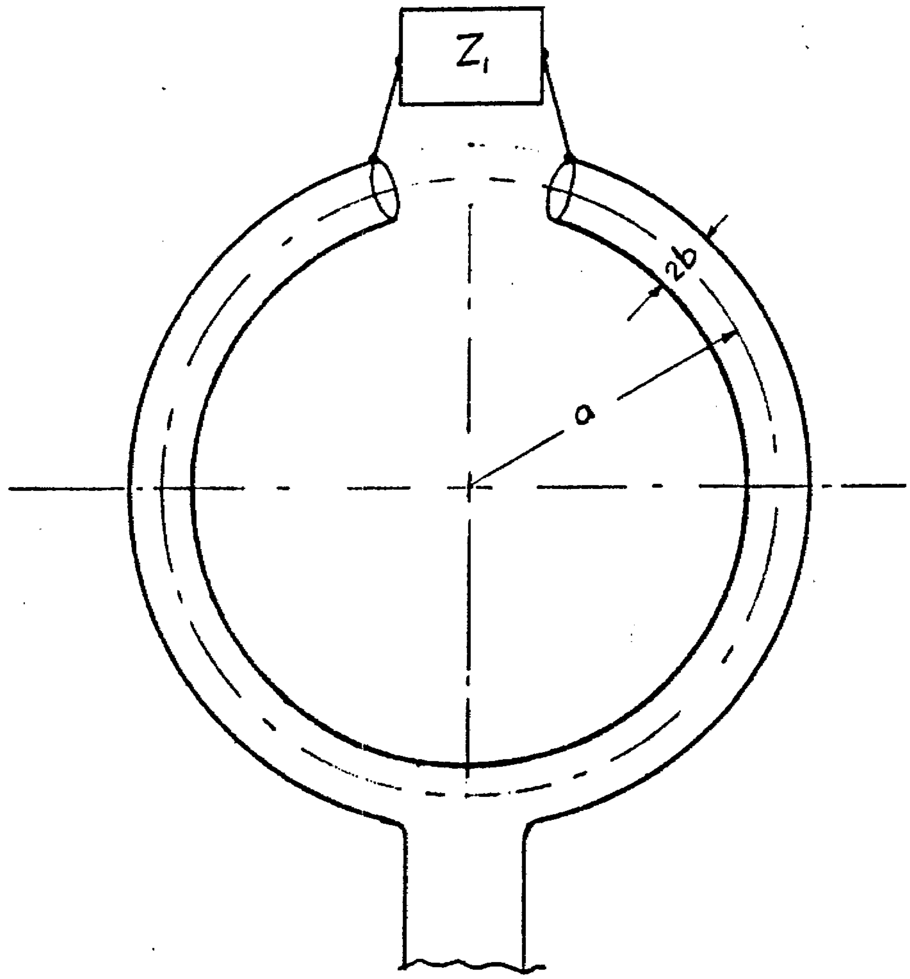

Fig. 1. Basic $\dot{B}$ Loop

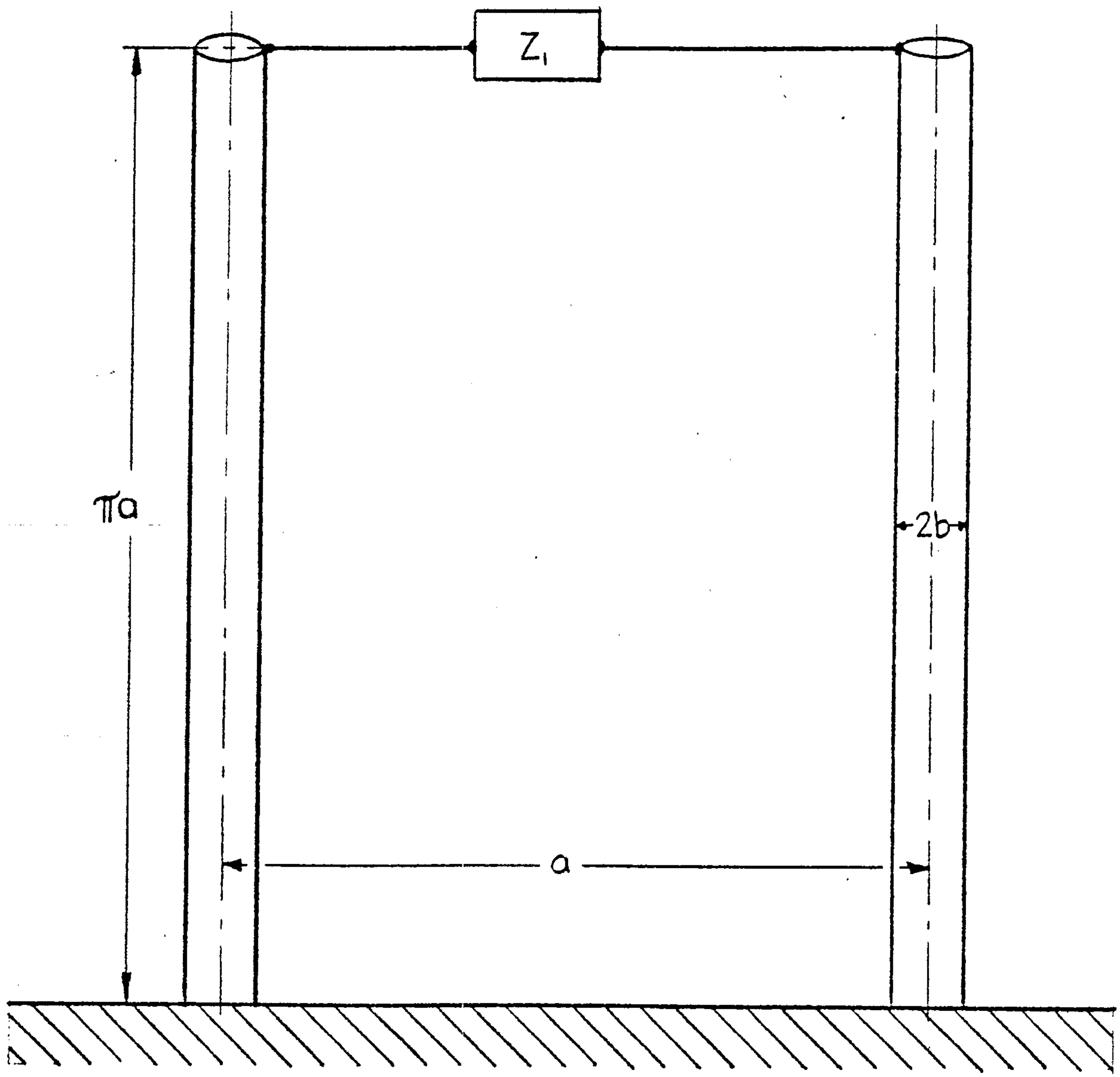

Fig. 2 Equivalent Loop Structure

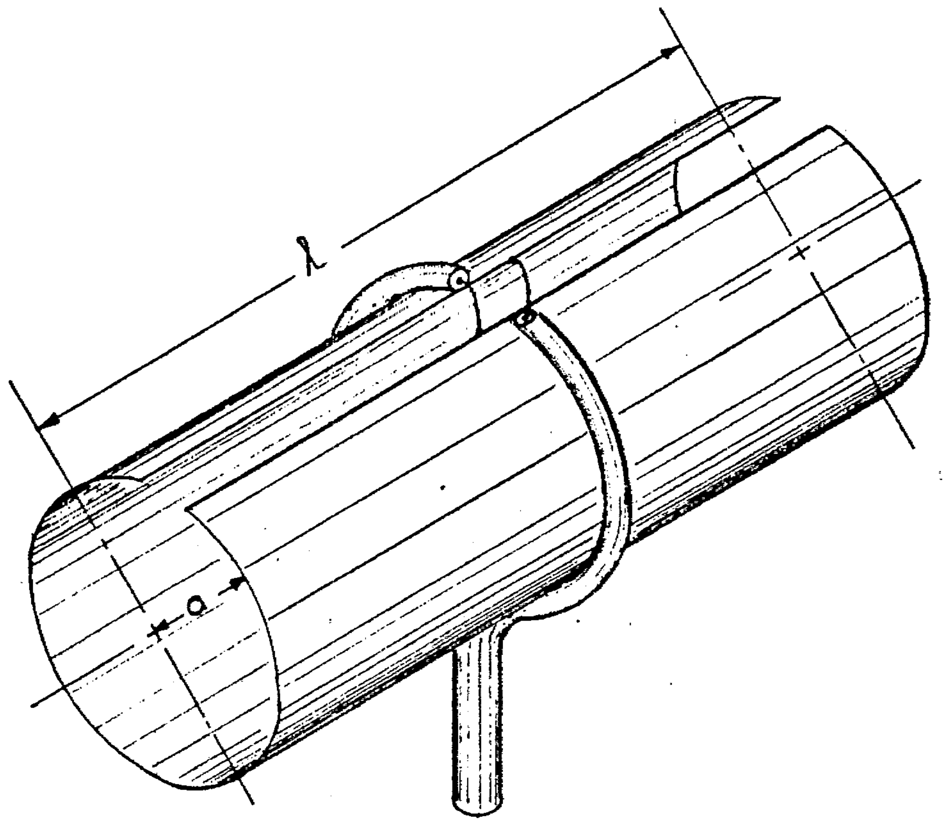

Fig. 3 Cylindrical Loop

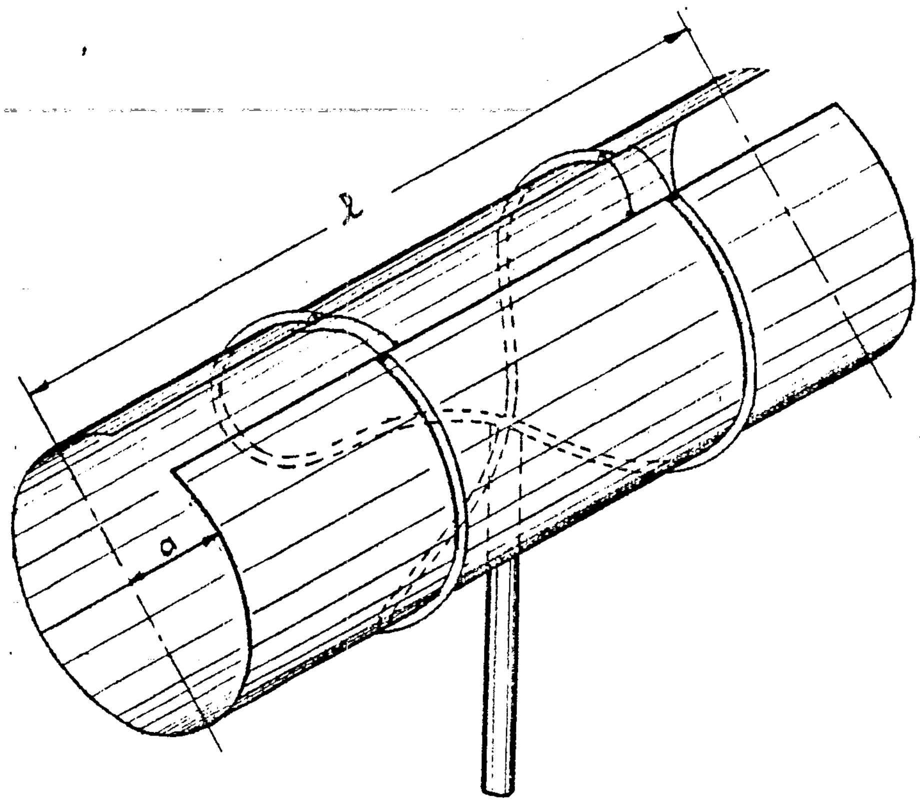

Fig.4 Approximation to Distributed Loop

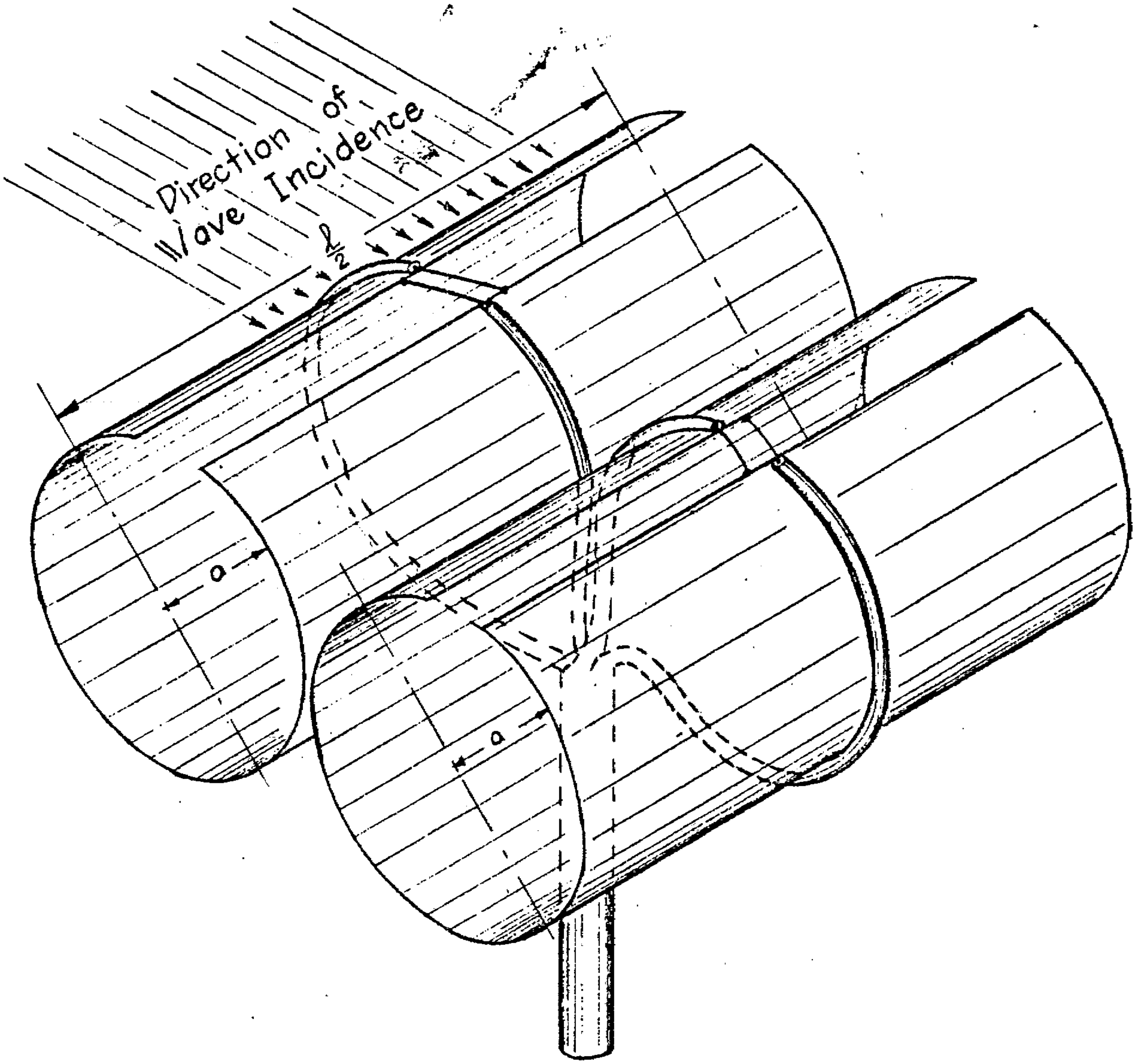

Fig. 5 Multiple Phase Added Loops