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Sensor and Simulation Notes

Note 79

Three- and Four-branch RLC Network Energy Sources for the Buried Transmission Line EMP Simulator

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ABSTRACT

A method of designing RLC networks to drive a buried transmission-line EMP simulator is presented and illustrated with designs that impress a step function of voltage on the simulator. The closing of a single switch actuates the driving network. The technique can be used to find circuit elements of the network that will synthesize a wide variety of pulse shapes at the terminals of the simulator.

CONTENTS

I. INTRODUCTION

In a previous note[1] the buried transmission line EMP simulation technique was discussed, and in other notes[2,3,4] capacitive and inductive sources were proposed. The driving network presented in this note consists of several RLC series circuits in parallel. The capacitors in each branch are initially charged to some voltage (V_o); the circuit is actuated by closing a single switch in series with the network and the simulator. The shape of the resultant pulse depends, of course, on the values of the parameters in the RLC branches.

The values of these circuit elements are found in two steps. First, the driving-network, short-circuit current is found that will give the desired pulse shape at the terminals of the simulator. Second, a least-squares fitting technique is used to find the circuit parameters of the RLC branches to fit the short-circuit current.

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[1] Lt. Carl E. Baum, Sensor and Simulation Note XXII, A Transmission Line EMP Simulation Technique for Buried Structures, June 1966.

[2] Capt. Carl E. Baum, Sensor and Simulation Note XLIV, The Capacitor Driven, Open Circuited, Buried Transmission Line Simulator, June 1967.

[3] Capt. Carl E. Baum, Sensor and Simulation Note XLIX, The Buried Transmission Line Simulator Driven by Multiple Capacitive Sources, August 1967.

[4] R. W. Latham and K. S. H. Lee, Sensor and Simulation Note L, The Buried Transmission Line Simulator with an Inductive Energy Source, April 1968.

II. ANALYSIS

A. General Method

Figure 1 contains a block diagram of the simulator and driving network. Here Z_s is the impedance of the network and Z_L the impedance of the transmission line simulator in the ground. The voltage V_o is the initial voltage across the capacitors, and s is the Laplace transform variable (all quantities are Laplace transformed). The current I(s) and the voltage across the simulator $V_s(s)$ are given by

$$I(s) = \frac{V_o}{s} \frac{1}{(Z_s + Z_L)}, \tag{1}$$

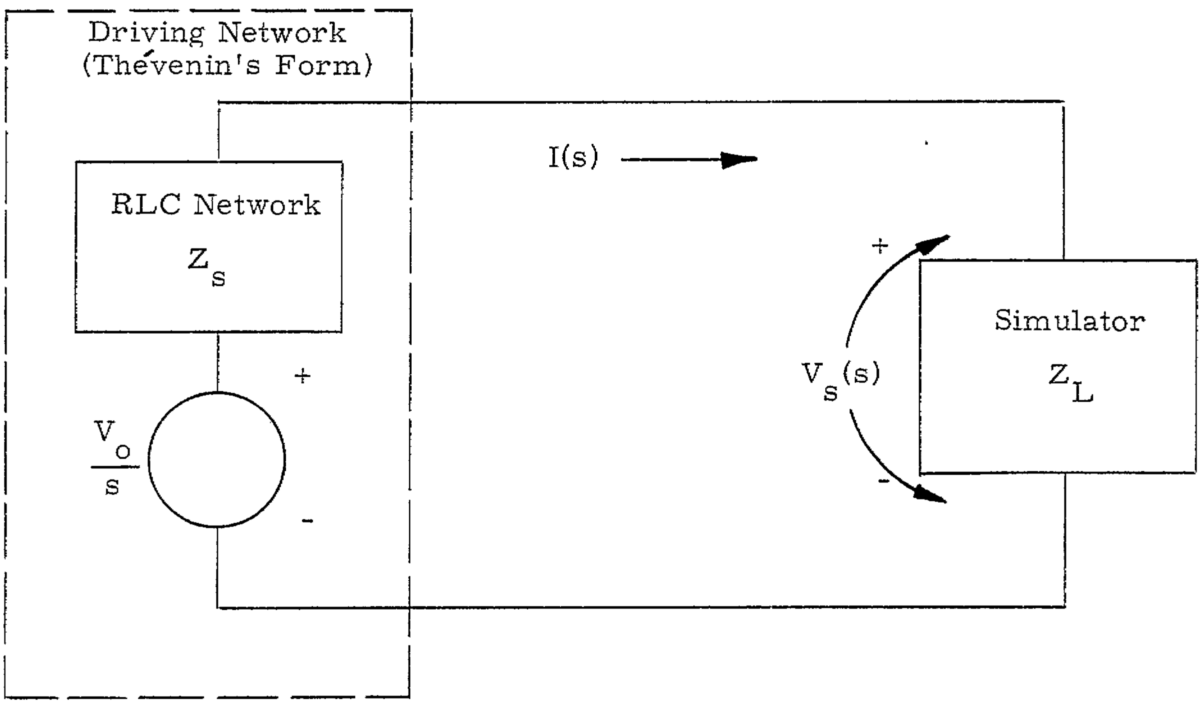


Figure 1

and

$$V_s(s) = \frac{V_o}{s} \frac{Z_L}{(Z_s + Z_L)} . \tag{2}$$

Suppose that one wants the driving network to approximate a desired pulse V_s across the terminals of the simulator. The first step is to find the short-circuit current for a hypothetical driving network that has open-circuit voltage V_o/s, and that produces the desired pulse exactly.

The short-circuit current of the driving network is given by

$$I_{s.c.}(s) = \frac{V_o}{sZ_s} . \tag{3}$$

Solving Eq. (3) for Z_s one obtains

$$Z_s = \frac{V_o}{s\,I_{s.c.}(s)} . \tag{4}$$

Substituting the right-hand side of Eq. (4) into Eq. (2) for Z_s and solving for $I_{s.c.}(s)$, the result is

$$I_{s.c.}(s) = \frac{V_o/s}{Z_L\left(\frac{V_o/s}{V_s(s)} - 1\right)} . \tag{5}$$

To complete the design of the driving network one needs the short-circuit current as a function of time, i.e., the inverse transform of Eq. (5). For many functions V_s this can be found analytically; if simulation of a pulse is desired where analytic inversion of Eq. (5) is impossible, then a numerical inversion can probably be done. For the special case solved in detail in this note (V_s equals a step function of time) the inversion is done analytically.

B. Approximation of Step Function Across the Simulator

From Eqs. (2) and (4) in Sensor and Simulation Note XXII,

$$Z_L = f_g \sqrt{s\,\mu/\sigma} \; ; \tag{6}$$

f_g is the dimensionless geometric factor described in Notes XXI[5] and LII[6], σ is conductivity and μ is permeability. Substituting the right-hand side of Eq. (6) into Eq. (5) for Z_L, and letting $V_s(s) = V_m/s$, V_m a constant,

$$I_{s.c.}(s) = \frac{V_o}{f_g \sqrt{\mu/\sigma}\, s^{3/2} \left(\frac{V_o}{V_m} - 1 \right)} \, . \tag{7}$$

Taking the inverse Laplace transform,[7]

$$\bar{I}_{s.c.}(t) = \frac{2 V_o t^{1/2}}{f_g \sqrt{\mu/\sigma} \left(\frac{V_o}{V_m} - 1 \right) \sqrt{\pi}} \, . \tag{8}$$

[5] Lt. Carl E. Baum, Sensor and Simulation Note XXI, Impedances and Field Distributions for Parallel Plate Transmission Line Simulators, June 1966.

[6] Terry L. Brown and Kenneth D. Granzow, Sensor and Simulation Note LII, A Parameter Study of Two Parallel Plate Transmission Line Simulators of EMP Sensor and Simulation Note XXI, April 1968.

[7] Standard Mathematical Tables, Fifteenth Edition, The Chemical Rubber Company, 18901 Cranwood Parkway, Cleveland, Ohio 44128, June 1967.

A new constant $f_v = V_o/V_m - 1$ is defined and Eq. (8) becomes,

$$\bar{I}_{s.c.}(t) = \frac{2 V_o t^{1/2}}{f_g f_v \sqrt{\pi \mu / \sigma}} \quad . \tag{9}$$

This is an expression for the short-circuit current through a hypothetical driving network, to yield a step function of time at the simulator. The constant f_v must be greater than zero; that is, the initial charging voltage must be greater than the step function voltage that is to appear across the terminals of the simulator.

Now it is desired to find RLC component values in Z_s that will approximate the desired short-circuit current expressed in Eq. (9). Figure 2 shows a diagram for an n-branch network. The individual branches serve as pulse shaping devices. An expression for the short-circuit current is,

$$I_{s.c.}(s) = \frac{V_o/s}{Z_1} + \frac{V_o/s}{Z_2} + \ldots + \frac{V_o/s}{Z_n} \quad . \tag{10}$$

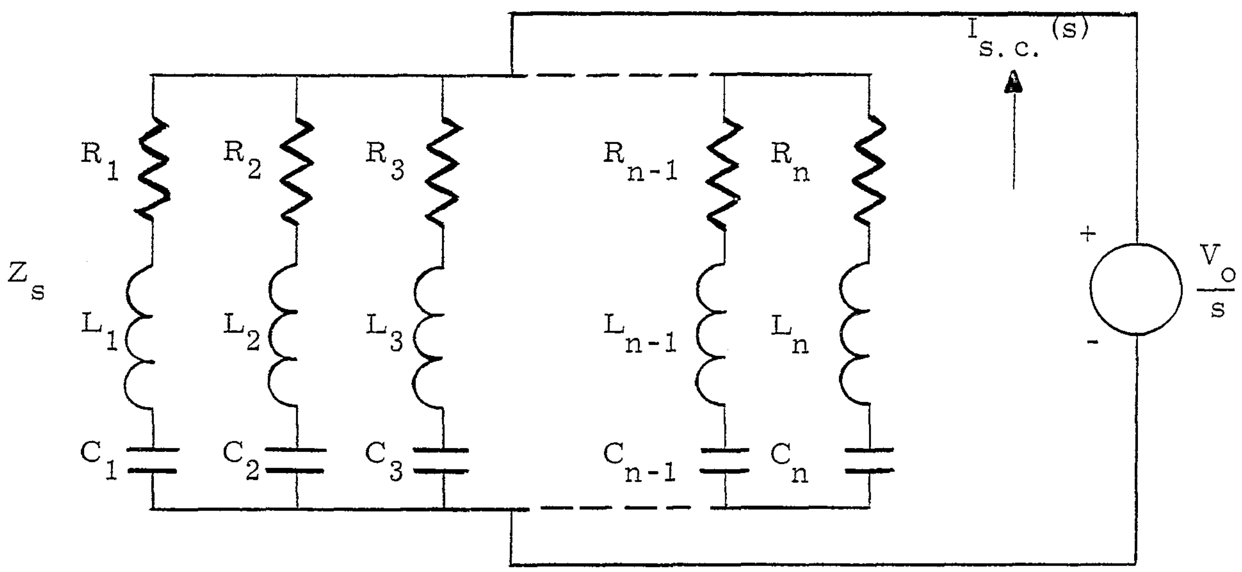


Figure 2

The transformed impedance of the ith branch is,

$$Z_i = s L_i + R_i + \frac{1}{s C_i} . \tag{11}$$

Eq. (10) becomes

$$I_{s.c.}(s) = \sum_{i=1}^{n} \frac{V_o}{s^2 L_i + s R_i + \frac{1}{C_i}} . \tag{12}$$

Another way of expressing Eq. (12) is,

$$I_{s.c.}(s) = \sum_{i=1}^{n} \frac{V_o}{L_i (s - r_{1i})(s - r_{2i})} , \tag{13}$$

where,

$$r_{1i} = \frac{-R_i}{2 L_i} + \sqrt{\frac{R_i^2}{4 L_i^2} - \frac{1}{L_i C_i}} , \tag{14}$$

and,

$$r_{2i} = \frac{-R_i}{2 L_i} - \sqrt{\frac{R_i^2}{4 L_i^2} - \frac{1}{L_i C_i}} . \tag{15}$$

Taking the inverse Laplace transform of Eq. (13), we get

$$\bar{I}_{s.c.}(t) = \sum_{i=1}^{n} \frac{V_o}{L_i} \frac{e^{r_{1i}t} - e^{r_{2i}t}}{r_{1i} - r_{2i}} . \tag{16}$$

Now we have an expression for the short-circuit current which is a function of the parameters as well as time. If we can fit Eq. (16) to Eq. (9) by solving for the parameter values, we can calculate the RLC components which should give us the desired current and voltage wave shapes. In other words, set

$$\sum_{i=1}^{n} \frac{V_o}{L_i} \frac{e^{r_{1i}t} - e^{r_{2i}t}}{r_{1i} - r_{2i}} \cong \frac{2 V_o t^{1/2}}{\sqrt{\pi}\, f_v f_g \sqrt{\mu/\sigma}} , \tag{17}$$

or,

$$g(t, \alpha_k) \cong f(t) , \tag{18}$$

where

$$\alpha_k = L_i, r_{1i}, r_{2i}, i = 1, n . \tag{19}$$

The V_o appears on both sides of Eq. (17) and can be eliminated from the calculation. A least squares fitting scheme may be used to find the α_k. A computer program was written to solve for these parameters and to calculate the component values from the parameters.

The L_i are found directly from the fitting scheme. The R_i and C_i can be calculated by solving Eqs. (14) and (15) simultaneously:

$$r_{1i} + r_{2i} = \frac{-R_i}{L_i} , \tag{20}$$

so,

$$R_i = -L_i(r_{1i} + r_{2i}), \tag{21}$$

also,

$$r_{1i} - r_{2i} = 2\sqrt{\frac{R_i^2}{4L_i^2} - \frac{1}{L_i C_i}}, \tag{22}$$

$$(r_{1i} - r_{2i})^2 = \frac{R_i^2}{L_i^2} - \frac{4}{L_i C_i}, \tag{23}$$

solving for C_i, one gets,

$$C_i = \frac{4}{L_i\left[(r_{1i} + r_{2i})^2 - (r_{1i} - r_{2i})^2\right]}. \tag{24}$$

In writing a computer program to solve for the α_k, three branches were used. The $|r_{2i}|$ were made larger than $|r_{1i}|$, and the radicals in Eq. (14) and (15) were real; hence, the overdamped case was tried initially. In running the program, r_{13} had a tendency to approach r_{23} causing overflow problems in the computer. Since this limit ($r_{13} \longrightarrow r_{23}$) represents the critically damped case, branch three was forced to be critically damped.

The short-circuit current of the third branch became

$$I_{s.c.}(s) = \frac{V_o}{L_3} \frac{1}{(s - r_{13})^2}, \tag{25}$$

taking the inverse Laplace transform,

$$\overline{I}_{s.c.}(t) = \frac{V_o}{L_3} t e^{r_{13} t} . \tag{26}$$

Hence, in computing the α_k Eq. (16) became

$$g(t, \alpha_k) = \sum_{i=1}^{2} \frac{V_o}{L_i} \frac{e^{r_{1i} t} - e^{r_{2i} t}}{r_{1i} - r_{2i}} + \frac{V_o}{L_3} t e^{r_{13} t} . \tag{27}$$

Another case considered was the four-branch network with all the branches critically damped. The transformed short-circuit current is given by

$$I_{s.c.}(s) = \sum_{i=1}^{4} \frac{V_o}{L_i} \frac{1}{(s - r_{1i})^2} , \tag{28}$$

taking the inverse Laplace transform one has,

$$\overline{I}_{s.c.}(t) = \sum_{i=1}^{4} \frac{V_o}{L_i} t e^{r_{1i} t} . \tag{29}$$

Once values for the RLC components are determined for either the three- or four-branch networks, the current and voltage waveforms impressed on the simulator by the design may be obtained by transforming Eqs. (1) and (2) to the time domain. Hence,

$$\overline{I}(t) = \mathfrak{F}^{-1}\left\{I(j\omega)\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t}\, I(j\omega)d\omega, \tag{30}$$

and

$$\overline{V}_s(t) = \mathfrak{F}^{-1}\left\{V_s(s)\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t}\, V(j\omega)d\omega\,. \tag{31}$$

A computer program was written to evaluate Eqs. (30) and (31). The subroutines of Program FORPLEX[8] were used in the inverse fourier transform calculation.

The efficiency of such a driving network is important since the cost is approximately proportional to the initial stored energy and the amount of energy delivered to the simulator is at least a rough measure of the effectiveness of the network. The initial stored energy is given by

$$E_{in} = \frac{V_o^2}{2}\,(C_1 + C_2 + \ldots + C_n). \tag{32}$$

The energy fed into the simulator is

$$E_{sim} = \int_0^t P(t')dt', \tag{33}$$

[8] Frank J. Sulkowski, EMP Mathematics Note II, FORPLEX, A Program to Calculate Inverse Fourier Transforms, November 1966.

where

$$P(t') = \overline{V}_s(t')\,\overline{I}(t'). \tag{34}$$

Percent efficiency would then be,

$$E_{ff} = \frac{E_{sim}}{E_{in}} \times 100. \tag{35}$$

The program which evaluates Eqs. (30) and (31) also does efficiency calculations to two time points. The first is to the ending time of a particular fit and the second is to the point before V_s turns negative as the circuit decays, ($V_s \longrightarrow 0$).

III. RESULTS

Least squares fits were done to three different ending times; namely, 500 μsec., 100 μsec., and 50 μsec. The calculations were made for two different V_o/V_m ratios, and were carried out to two different degrees of accuracy.

The longer time fits produced capacitors of larger values. In some cases, these values were unwieldy. The different V_o/V_m ratios affect the smoothness of the resultant step voltage. A smaller ratio produces a faster rise time, a slower decay, and an overall smoother curve than the larger ratio. The current is also of greater magnitude and tends to decay slower. A major drawback is that the values of the capacitors are much larger.

Fits to different degrees of accuracy were done because, in running the least squares fitting program it appeared that a more accurate fit produced capacitors of larger values.

In the efficiency analysis, it was found that the more efficient simulator is the one whose components were determined from the less accurate fit, and the smaller V_o/V_m ratio. The four-branch network was slightly more efficient than the three-branch network.

A summary of the values calculated can be found in tables in Section IV. Current and Voltage graphs are in Section V. In doing the computer calculations a V_m step voltage of one volt was used. In practice, this would be multiplied many times. The simulator configuration assumed was that where $b/a = 1.0$. This is the ratio of the distance between the plates to the width of the plates in the parallel plate transmission line simulator. The value of the dimensionless geometric factor, f_g, was taken from the tables in Note LII. A ground conductivity of .01 mhos/meter and a permeability of $4\pi \times 10^{-7}$ henrys/meter were used in the calculations.

IV. NUMERICAL RESULTS

The tables in the following pages summarize the calculations performed. RLC component values are listed as well as the various energies and efficiencies involved.

VALUES OF R, L, AND C FOR 3 BRANCH NETWORKS

THE FOLLOWING ARE FOR FITS DONE TO AN ACCURACY OF ≤ 1.0 PERCENT ERROR

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 500 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT = 2.00				
	1	2.73675E 00	1.30307E-05	1.12937E-05
	2	1.50882E-01	5.64573E-05	1.93366E-02
	3	1.17539E 00	4.05147E-05	1.17303E-04
V-NAUGHT = 1.20				
	1	5.47349E-01	2.60614E-06	5.64684E-05
	2	3.01763E-02	1.12915E-05	9.66831E-02
	3	2.35078E-01	8.10293E-06	5.86514E-04

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 100 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT = 2.00				
	1	6.11955E 00	5.82751E-06	1.01014E-06
	2	3.37382E-01	2.52485E-05	1.72952E-03
	3	2.62825E 00	1.81187E-05	1.04919E-05
V-NAUGHT = 1.20				
	1	1.22391E 00	1.16550E-06	5.05069E-06
	2	6.74763E-02	5.04970E-06	8.64760E-03
	3	5.25650E-01	3.62374E-06	5.24594E-05

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 50 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT = 2.00				
	1	8.65436E 00	4.12067E-06	3.57138E-07
	2	4.77130E-01	1.78534E-05	6.11478E-04
	3	3.71691E 00	1.28119E-05	3.70944E-06
V-NAUGHT = 1.20				
	1	1.73087E 00	8.24134E-07	1.78569E-06
	2	9.54258E-02	3.57067E-06	3.05739E-03
	3	7.43382E-01	2.56237E-06	1.85472E-05

VALUES OF R, L, AND C FOR 3 BRANCH NETWORKS

THE FOLLOWING ARE FOR FITS DONE TO AN ACCURACY OF ≤ 4.0 PERCENT ERROR

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 500 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT =	2.00			
	1	3.83806E 00	1.49399E-05	1.62497E-05
	2	1.62227E-01	4.87958E-05	9.61265E-03
	3	1.55045E 00	3.63059E-05	6.04118E-05
V-NAUGHT =	1.20			
	1	7.67612E-01	2.98797E-06	8.12484E-05
	2	3.24453E-02	9.75915E-06	4.80632E-02
	3	3.10090E-01	7.26118E-06	3.02059E-04

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 100 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT =	2.00			
	1	8.58217E 00	6.68131E-06	1.45342E-06
	2	3.62750E-01	2.18221E-05	8.59781E-04
	3	3.46691E 00	1.62365E-05	5.40339E-06
V-NAUGHT =	1.20			
	1	1.71643E 00	1.33626E-06	7.26708E-06
	2	7.25499E-02	4.36443E-06	4.29891E-03
	3	6.93382E-01	3.24730E-06	2.70170E-05

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 50 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT =	2.00			
	1	1.21370E 01	4.72440E-06	5.13860E-07
	2	5.13006E-01	1.54306E-05	3.03979E-04
	3	4.90295E 00	1.14809E-05	1.91039E-06
V-NAUGHT =	1.20			
	1	2.42740E 00	9.44880E-07	2.56930E-06
	2	1.02601E-01	3.08612E-06	1.51989E-03
	3	9.80591E-01	2.29619E-06	9.55194E-06

VALUES OF R, L, AND C FOR 4 BRANCH NETWORKS

THE FOLLOWING ARE FOR FITS DONE TO AN ACCURACY OF ≤ 1.0 PERCENT ERROR

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 500 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT =	2.00			
	1	4.64199E 00	2.00449E-05	3.72097E-06
	2	1.30999E-01	6.33481E-05	1.47658E-02
	3	1.08112E 00	4.80906E-05	1.64579E-04
	4	3.29179E 00	3.81249E-05	1.40736E-05
V-NAUGHT =	1.20			
	1	9.28398E-01	4.00898E-06	1.86048E-05
	2	2.61998E-02	1.26696E-05	7.38290E-02
	3	2.16224E-01	9.61813E-06	8.22895E-04
	4	6.58358E-01	7.62499E-06	7.03679E-05

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 100 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT =	2.00			
	1	1.23028E 01	9.59260E-06	2.53505E-07
	2	2.93450E-01	2.82485E-05	1.31216E-03
	3	2.40037E 00	2.10637E-05	1.46230E-05
	4	6.82762E 00	1.44793E-05	1.24242E-06
V-NAUGHT =	1.20			
	1	2.46057E 00	1.91852E-06	1.26752E-06
	2	5.86901E-02	5.64970E-06	6.56078E-03
	3	4.80075E-01	4.21274E-06	7.31150E-05
	4	1.36552E 00	2.89585E-06	6.21210E-06

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 50 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT =	2.00			
	1	1.60203E 01	6.66154E-06	1.03823E-07
	2	4.14533E-01	2.00067E-05	4.65711E-04
	3	3.40423E 00	1.50355E-05	5.18966E-06
	4	1.00430E 01	1.11106E-05	4.40631E-07
V-NAUGHT =	1.20			
	1	3.20406E 00	1.33231E-06	5.19114E-07
	2	8.29067E-02	4.00134E-06	2.32855E-03
	3	6.80846E-01	3.00709E-06	2.59483E-05
	4	2.00859E 00	2.22213E-06	2.20316E-06

VALUES OF R, L, AND C FOR 4 BRANCH NETWORKS

THE FOLLOWING ARE FOR FITS DONE TO AN ACCURACY OF $\leq$ 4.0 PERCENT ERROR

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 500 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT = 2.00				
	1	7.12203E 00	2.26268E-05	1.78433E-06
	2	1.56529E-01	4.96605E-05	8.10735E-03
	3	1.19987E 00	2.65083E-05	7.36503E-05
	4	7.12203E 00	2.26268E-05	1.78433E-06
V-NAUGHT = 1.20				
	1	1.42441E 00	4.52535E-06	8.92165E-06
	2	3.13059E-02	9.93211E-06	4.05368E-02
	3	2.39974E-01	5.30166E-06	3.68252E-04
	4	1.42441E 00	4.52535E-06	8.92165E-06

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 100 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT = 2.00				
	1	1.59253E 01	1.01190E-05	1.59595E-07
	2	3.50011E-01	2.22089E-05	7.25144E-04
	3	2.68299E 00	1.18549E-05	6.58748E-06
	4	1.59253E 01	1.01190E-05	1.59595E-07
V-NAUGHT = 1.20				
	1	3.18507E 00	2.02380E-06	7.97977E-07
	2	7.00021E-02	4.44177E-06	3.62572E-03
	3	5.36597E-01	2.37097E-06	3.29374E-05
	4	3.18507E 00	2.02380E-06	7.97977E-07

THE FOLLOWING VALUES WERE CALCULATED FROM FITS DONE TO 50 MICROSECONDS

	BRANCH	R(OHMS)	L(HENRYS)	C(FARADS)
V-NAUGHT = 2.00				
	1	2.25218E 01	7.15521E-06	5.64255E-08
	2	4.94990E-01	1.57040E-05	2.56377E-04
	3	3.79432E 00	8.38265E-06	2.32903E-06
	4	2.25218E 01	7.15521E-06	5.64255E-08
V-NAUGHT = 1.20				
	1	4.50437E 00	1.43104E-06	2.82127E-07
	2	9.89979E-02	3.14081E-06	1.28189E-03
	3	7.58863E-01	1.67653E-06	1.16451E-05
	4	4.50437E 00	1.43104E-06	2.82127E-07

ENERGY AND EFFICIENCY VALUES

3 BRANCH NETWORKS

THE FOLLOWING ARE FOR FITS DONE TO AN ACCURACY OF ≤ 1.0 PERCENT ERROR

FIT (USEC)	V-NAUGHT (VOLTS)	ENERGY-IN (JOULES)	ENERGY-OUT TO END OF FIT (JOULES)	ENERGY-OUT $V \rightarrow 0$ (JOULES)	EFFICIENCY TO END OF FIT (PERCENT)	EFFICIENCY $V \rightarrow 0$ (PERCENT)
500	2.0	3.89304E-02	1.58614E-03	1.17654E-02	4.07431E 00	3.02217E 01
500	1.2	7.00748E-02	1.58704E-03	4.19293E-02	2.26477E 00	5.98351E 01
100	2.0	3.48204E-03	1.41876E-04	1.05151E-03	4.07451E 00	3.01980E 01
100	1.2	6.26768E-03	1.41957E-04	3.80167E-03	2.26490E 00	6.06551E 01
50	2.0	1.23109E-03	5.01642E-05	3.71147E-04	4.07478E 00	3.01479E 01
50	1.2	2.21596E-03	5.01920E-05	1.32512E-03	2.26502E 00	5.97989E 01

THE FOLLOWING ARE FOR FITS DONE TO AN ACCURACY OF ≤ 4.0 PERCENT ERROR

FIT (USEC)	V-NAUGHT (VOLTS)	ENERGY-IN (JOULES)	ENERGY-OUT TO END OF FIT (JOULES)	ENERGY-OUT $V \rightarrow 0$ (JOULES)	EFFICIENCY TO END OF FIT (PERCENT)	EFFICIENCY $V \rightarrow 0$ (PERCENT)
500	2.0	1.93786E-02	1.58184E-03	6.64146E-03	8.16278E 00	3.42721E 01
500	1.2	3.48815E-02	1.58545E-03	2.25594E-02	4.54525E 00	6.46743E 01
100	2.0	1.73328E-03	1.41493E-04	5.94219E-04	8.16330E 00	3.42830E 01
100	1.2	3.11990E-03	1.41818E-04	2.01649E-03	4.54558E 00	6.46333E 01
50	2.0	6.12306E-04	5.00291E-05	2.09304E-04	8.16394E 00	3.41549E 01
50	1.2	1.10305E-03	5.01423E-05	7.12676E-04	4.54579E 00	6.46097E 01

4 BRANCH NETWORKS

THE FOLLOWING ARE FOR FITS DONE TO AN ACCURACY OF ≤ 1.0 PERCENT ERROR

FIT (USEC)	V-NAUGHT (VOLTS)	ENERGY-IN (JOULES)	ENERGY-OUT TO END OF FIT (JOULES)	ENERGY-OUT $V \rightarrow 0$ (JOULES)	EFFICIENCY TO END OF FIT (PERCENT)	EFFICIENCY $V \rightarrow 0$ (PERCENT)
500	2.0	2.98963E-02	1.58703E-03	1.08061E-02	5.30845E 00	3.61451E 01
500	1.2	5.38134E-02	1.58733E-03	3.57942E-02	2.94969E 00	6.65154E 01
100	2.0	2.65656E-03	1.41956E-04	9.60320E-04	5.34360E 00	3.61490E 01
100	1.2	4.78179E-03	1.41984E-04	3.18678E-03	2.96926E 00	6.66441E 01
50	2.0	9.42890E-04	5.01930E-05	3.40002E-04	5.32331E 00	3.60596E 01
50	1.2	1.69720E-03	5.02015E-05	1.12813E-03	2.95790E 00	6.64702E 01

THE FOLLOWING ARE FOR FITS DONE TO AN ACCURACY OF ≤ 4.0 PERCENT ERROR

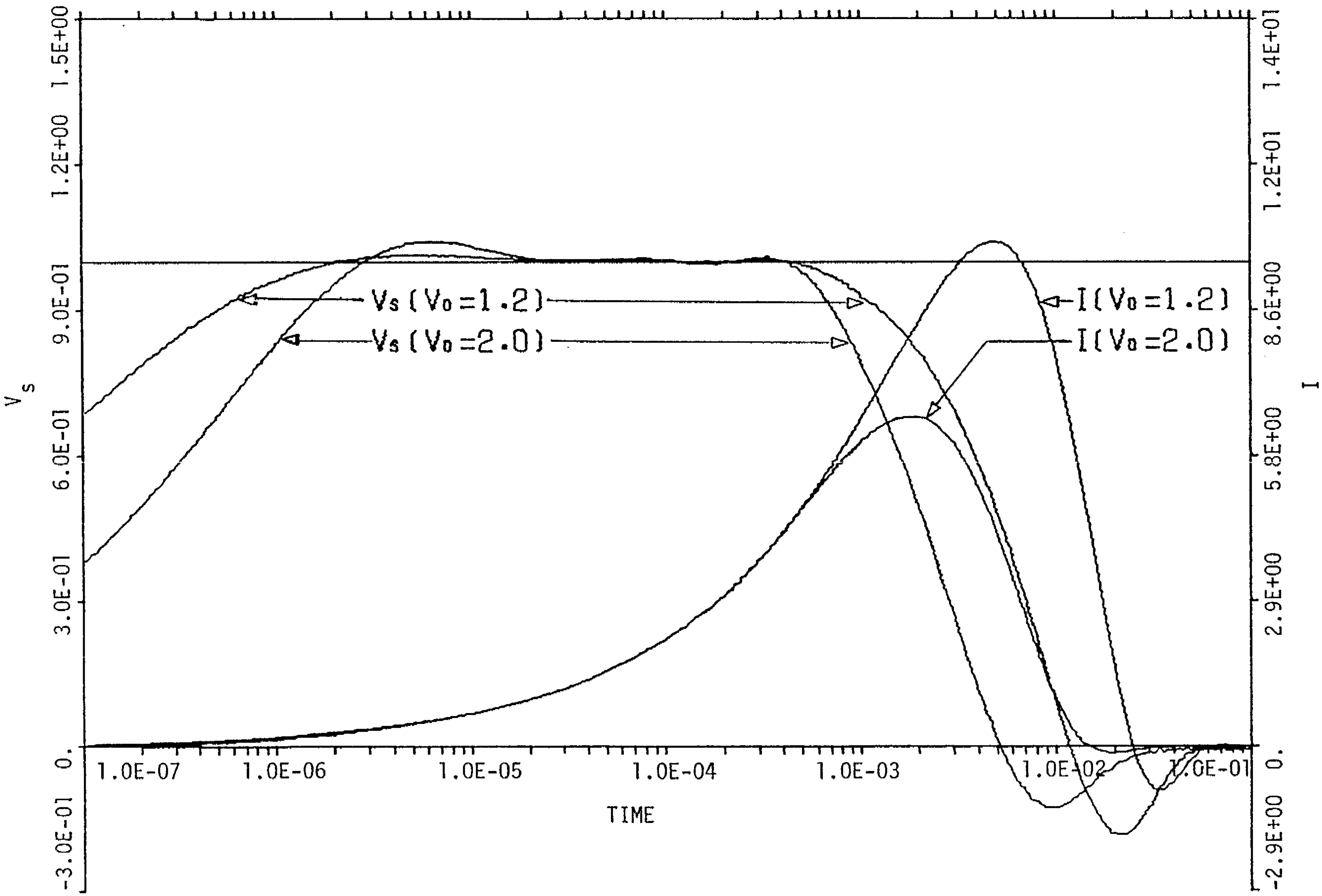
FIT (USEC)	V-NAUGHT (VOLTS)	ENERGY-IN (JOULES)	ENERGY-OUT TO END OF FIT (JOULES)	ENERGY-OUT $V \rightarrow 0$ (JOULES)	EFFICIENCY TO END OF FIT (PERCENT)	EFFICIENCY $V \rightarrow 0$ (PERCENT)
500	2.0	1.63691E-02	1.57973E-03	6.00159E-03	9.65067E 00	3.66641E 01
500	1.2	2.94645E-02	1.58474E-03	1.97238E-02	5.37848E 00	6.69409E 01
100	2.0	1.46410E-03	1.41297E-04	5.37044E-04	9.65080E 00	3.66808E 01
100	1.2	2.63538E-03	1.41753E-04	1.78493E-03	5.37884E 00	6.77295E 01
50	2.0	5.17638E-04	4.99606E-05	1.88929E-04	9.65165E 00	3.64983E 01
50	1.2	9.31752E-04	5.01192E-05	6.22923E-04	5.37903E 00	6.68551E 01

V. GRAPHS OF VOLTAGES AND CURRENTS AT THE SIMULATOR

The following graphs were copied directly from CALCOMP plots.

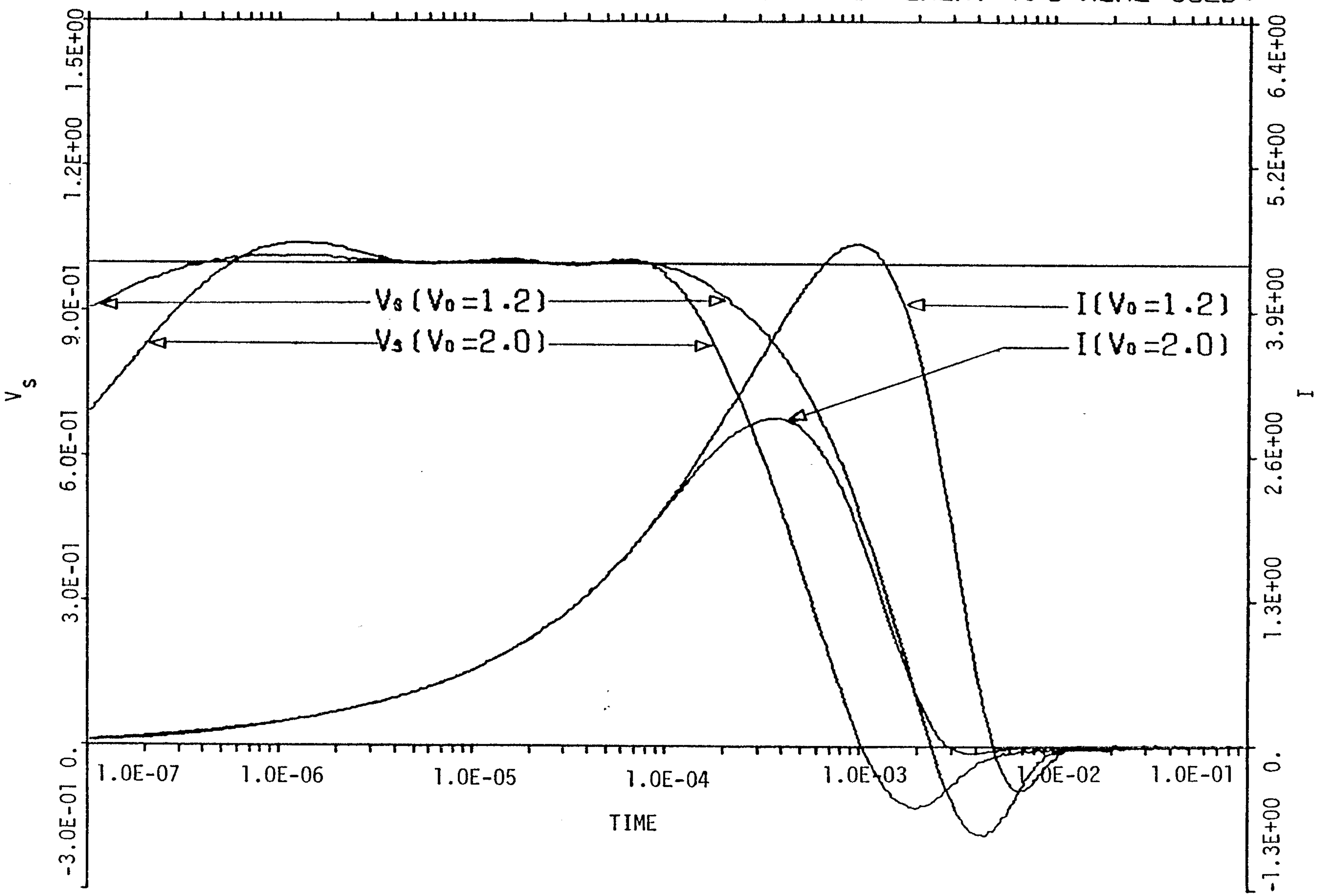
INVERSE FOURIER TRANSFORM CURVES

THESE ARE VOLTAGE AND CURRENT CURVES FOR A 3-BRANCH NETWORK.
THE LEAST SQUARES FIT TO PRODUCE PARAMETERS FOR THE NETWORK WAS DONE TO 500 MICROSECONDS AND WITH AN ACCURACY OF ≤ 1.0% ERROR.
2 NETWORKS ARE REPRESENTED ON THIS GRAPH: DIFFERENT V_0'S WERE USED.



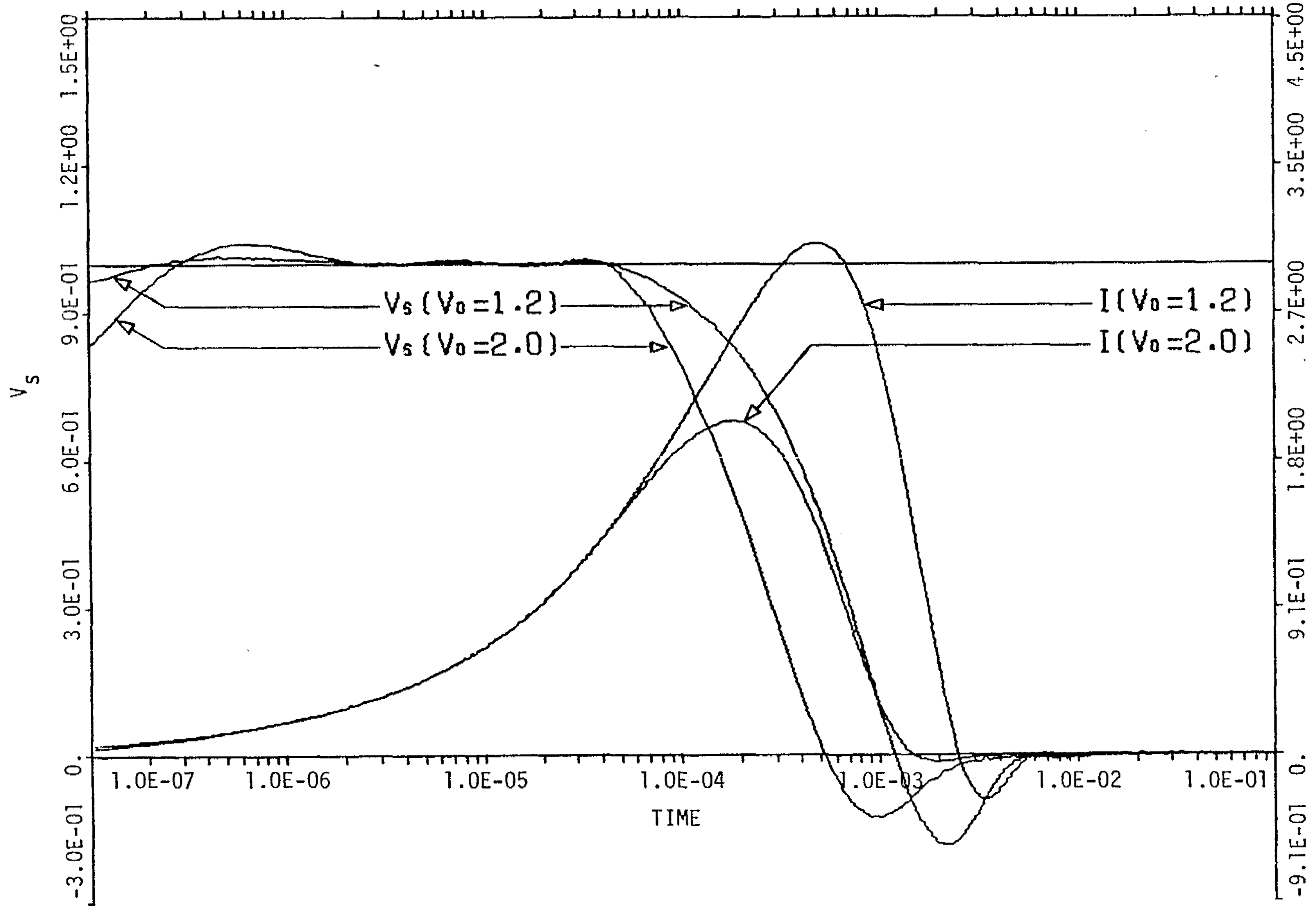
INVERSE FOURIER TRANSFORM CURVES

THESE ARE VOLTAGE AND CURRENT CURVES FOR A 3-BRANCH NETWORK.
THE LEAST SQUARES FIT TO PRODUCE PARAMETERS FOR THE NETWORK WAS DONE TO 100 MICROSECONDS AND WITH AN ACCURACY OF ≤ 1.0% ERROR.
2 NETWORKS ARE REPRESENTED ON THIS GRAPH; DIFFERENT V_0'S WERE USED.



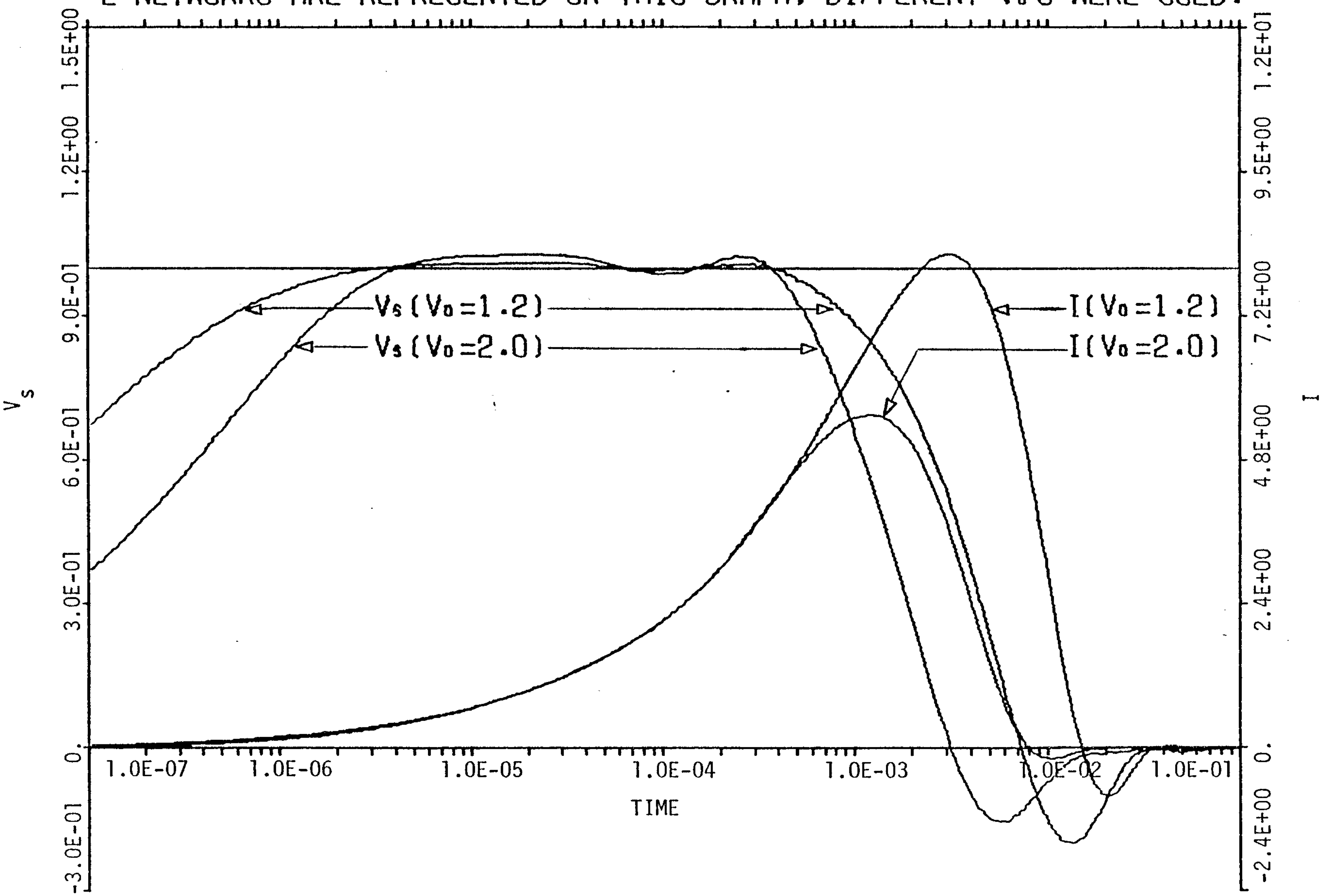
INVERSE FOURIER TRANSFORM CURVES

THESE ARE VOLTAGE AND CURRENT CURVES FOR A 3-BRANCH NETWORK. THE LEAST SQUARES FIT TO PRODUCE PARAMETERS FOR THE NETWORK WAS DONE TO 50 MICROSECONDS AND WITH AN ACCURACY OF ≤ 1.0% ERROR. 2 NETWORKS ARE REPRESENTED ON THIS GRAPH; DIFFERENT V_o'S WERE USED.



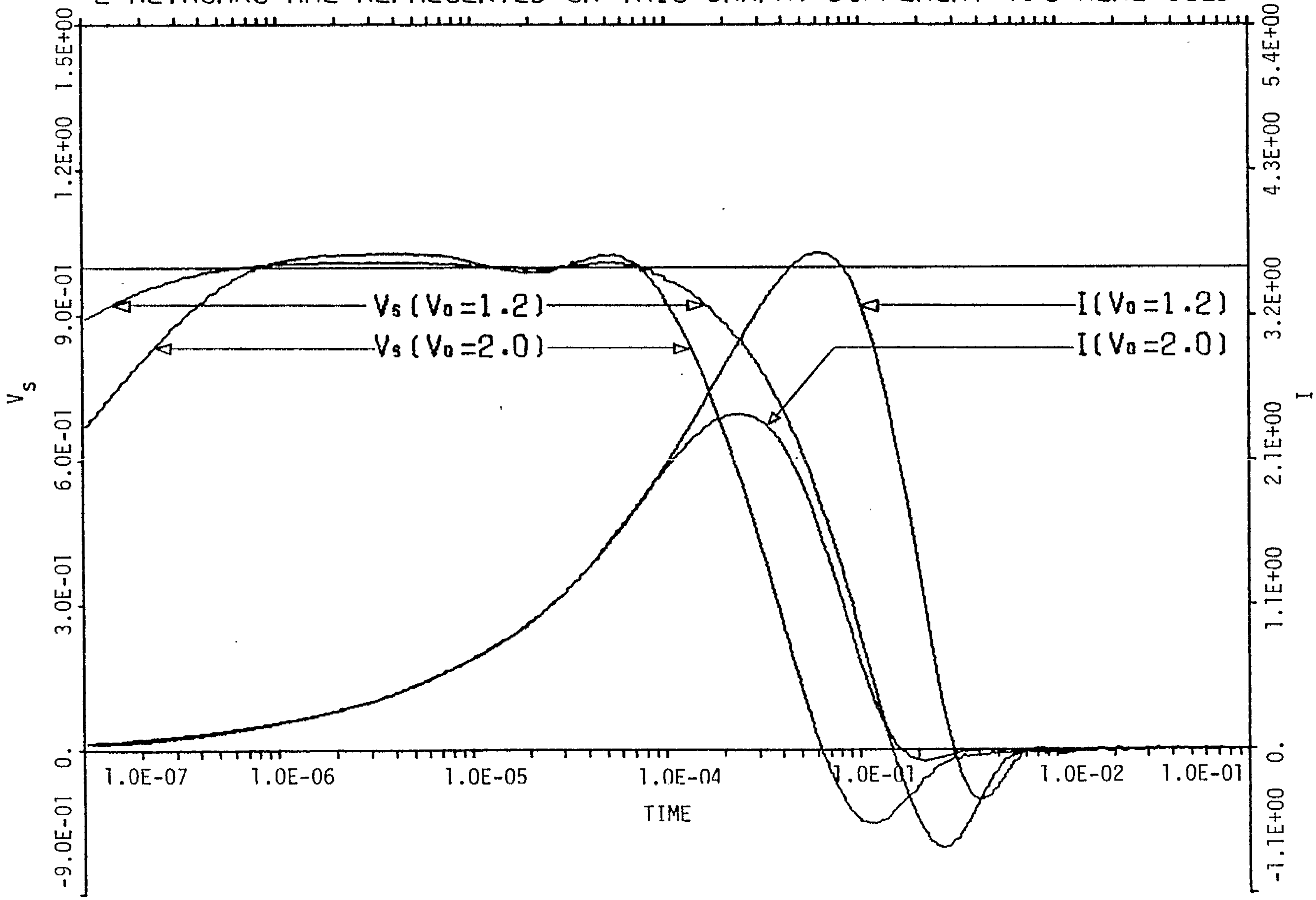
INVERSE FOURIER TRANSFORM CURVES

THESE ARE VOLTAGE AND CURRENT CURVES FOR A 3-BRANCH NETWORK.
THE LEAST SQUARES FIT TO PRODUCE PARAMETERS FOR THE NETWORK WAS DONE TO 500 MICROSECONDS AND WITH AN ACCURACY OF ≤ 4.0% ERROR.
2 NETWORKS ARE REPRESENTED ON THIS GRAPH; DIFFERENT V_0'S WERE USED.



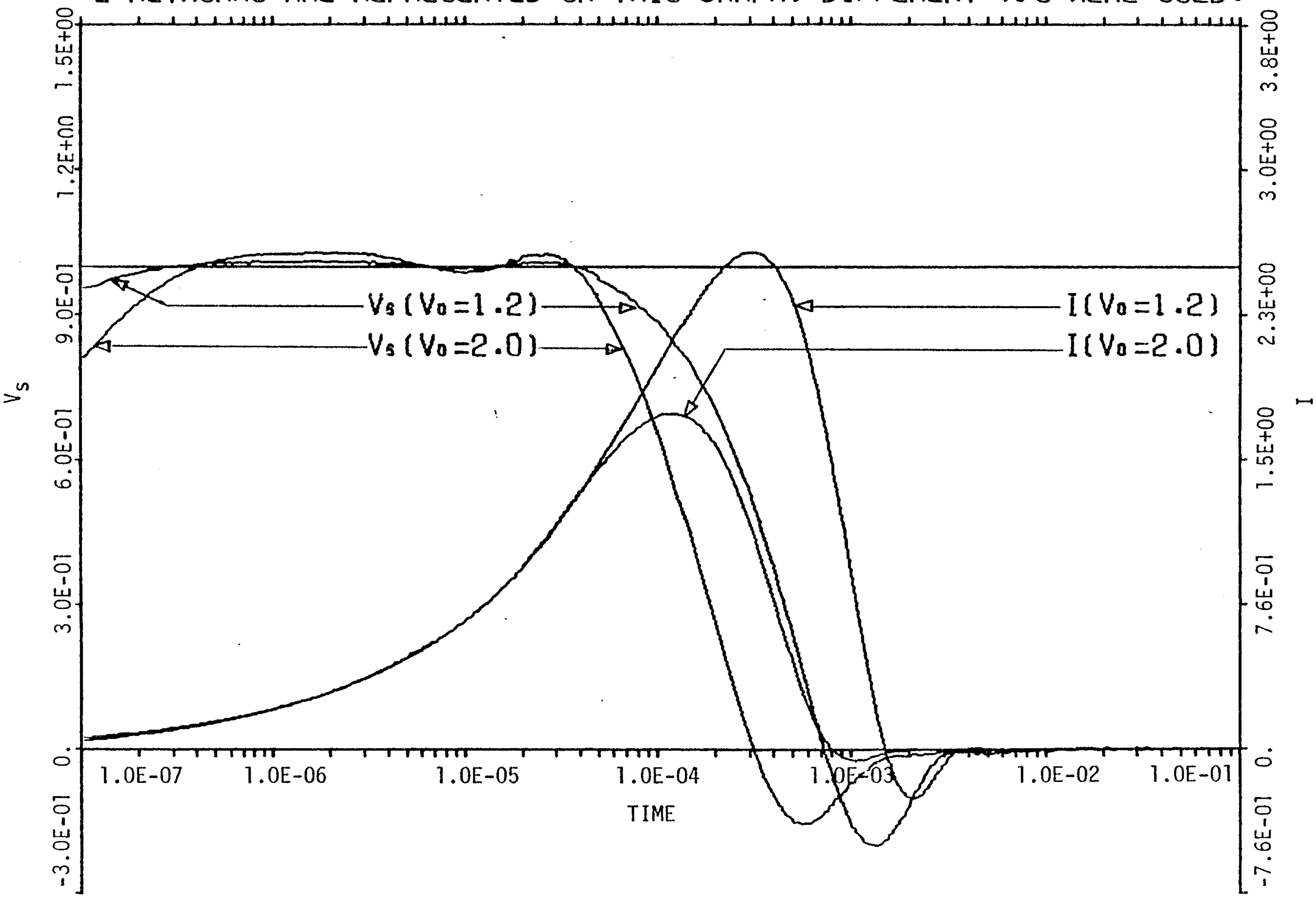
INVERSE FOURIER TRANSFORM CURVES

THESE ARE VOLTAGE AND CURRENT CURVES FOR A 3-BRANCH NETWORK.
THE LEAST SQUARES FIT TO PRODUCE PARAMETERS FOR THE NETWORK WAS DONE TO 100 MICROSECONDS AND WITH AN ACCURACY OF ≤ 4.0% ERROR.
2 NETWORKS ARE REPRESENTED ON THIS GRAPH; DIFFERENT V_0'S WERE USED.



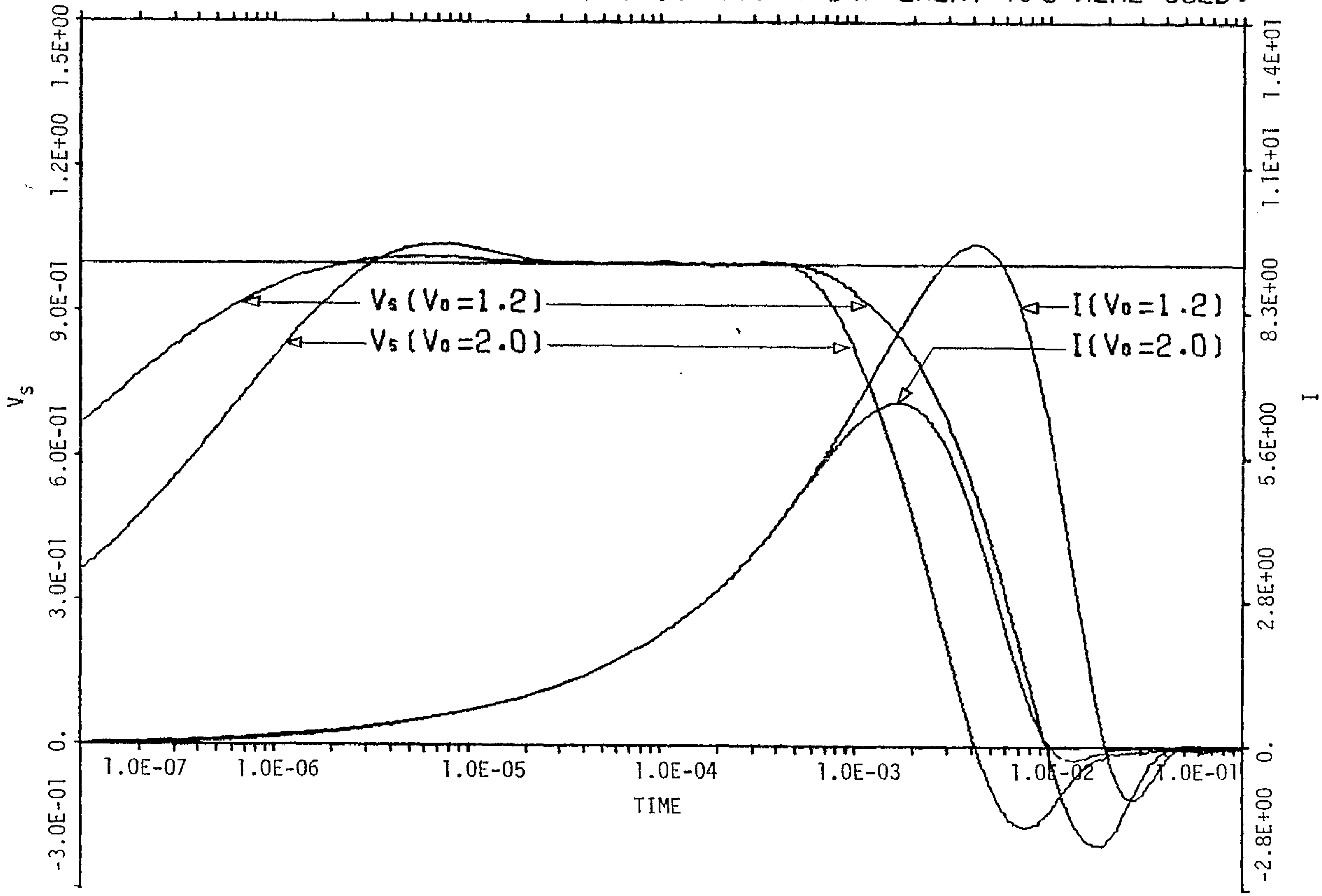
INVERSE FOURIER TRANSFORM CURVES

THESE ARE VOLTAGE AND CURRENT CURVES FOR A 3-BRANCH NETWORK. THE LEAST SQUARES FIT TO PRODUCE PARAMETERS FOR THE NETWORK WAS DONE TO 50 MICROSECONDS AND WITH AN ACCURACY OF ≤ 4.0% ERROR. 2 NETWORKS ARE REPRESENTED ON THIS GRAPH; DIFFERENT V_0'S WERE USED.



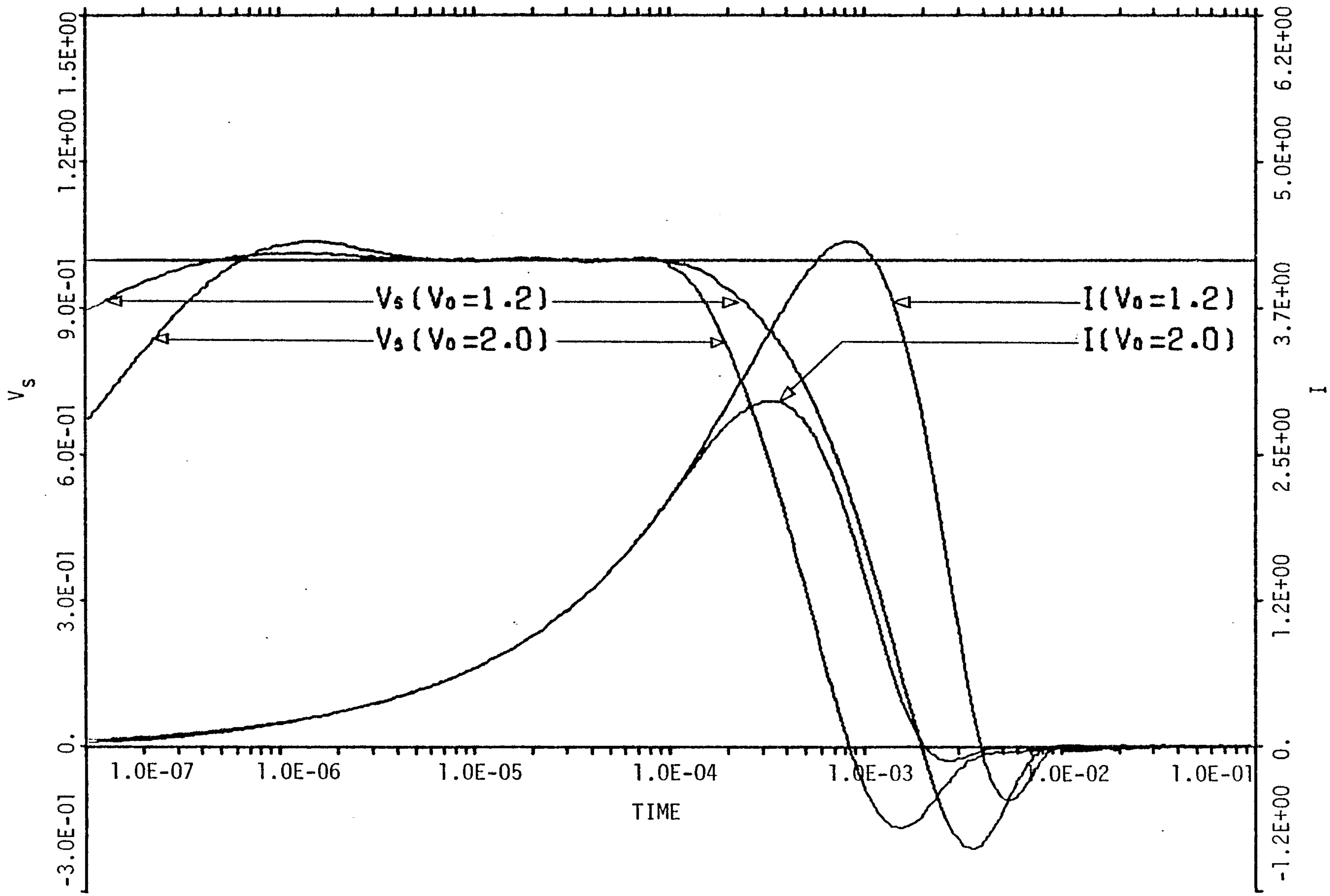
INVERSE FOURIER TRANSFORM CURVES

THESE ARE VOLTAGE AND CURRENT CURVES FOR A 4-BRANCH NETWORK.
THE LEAST SQUARES FIT TO PRODUCE PARAMETERS FOR THE NETWORK WAS
DONE TO 500 MICROSECONDS AND WITH AN ACCURACY OF ≤ 1.0% ERROR.
2 NETWORKS ARE REPRESENTED ON THIS GRAPH: DIFFERENT V_0'S WERE USED.



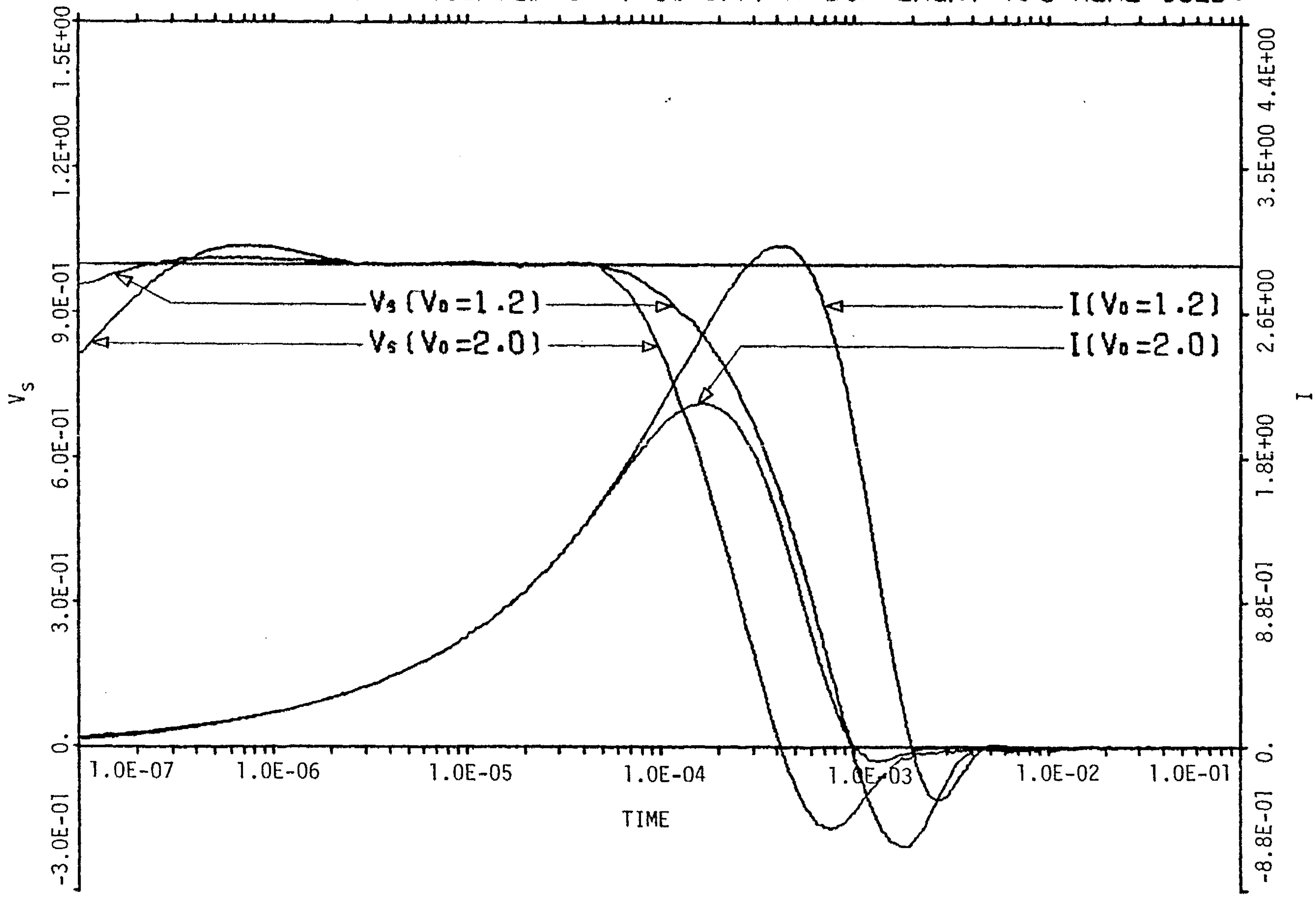
INVERSE FOURIER TRANSFORM CURVES

THESE ARE VOLTAGE AND CURRENT CURVES FOR A 4-BRANCH NETWORK.
THE LEAST SQUARES FIT TO PRODUCE PARAMETERS FOR THE NETWORK WAS DONE TO 100 MICROSECONDS AND WITH AN ACCURACY OF ≤ 1.0% ERROR.
2 NETWORKS ARE REPRESENTED ON THIS GRAPH: DIFFERENT V_0'S WERE USED.



INVERSE FOURIER TRANSFORM CURVES

THESE ARE VOLTAGE AND CURRENT CURVES FOR A 4-BRANCH NETWORK.
THE LEAST SQUARES FIT TO PRODUCE PARAMETERS FOR THE NETWORK WAS DONE TO 50 MICROSECONDS AND WITH AN ACCURACY OF ≤ 1.0% ERROR.
2 NETWORKS ARE REPRESENTED ON THIS GRAPH: DIFFERENT V_0'S WERE USED.



INVERSE FOURIER TRANSFORM CURVES

THESE ARE VOLTAGE AND CURRENT CURVES FOR A 4-BRANCH NETWORK.
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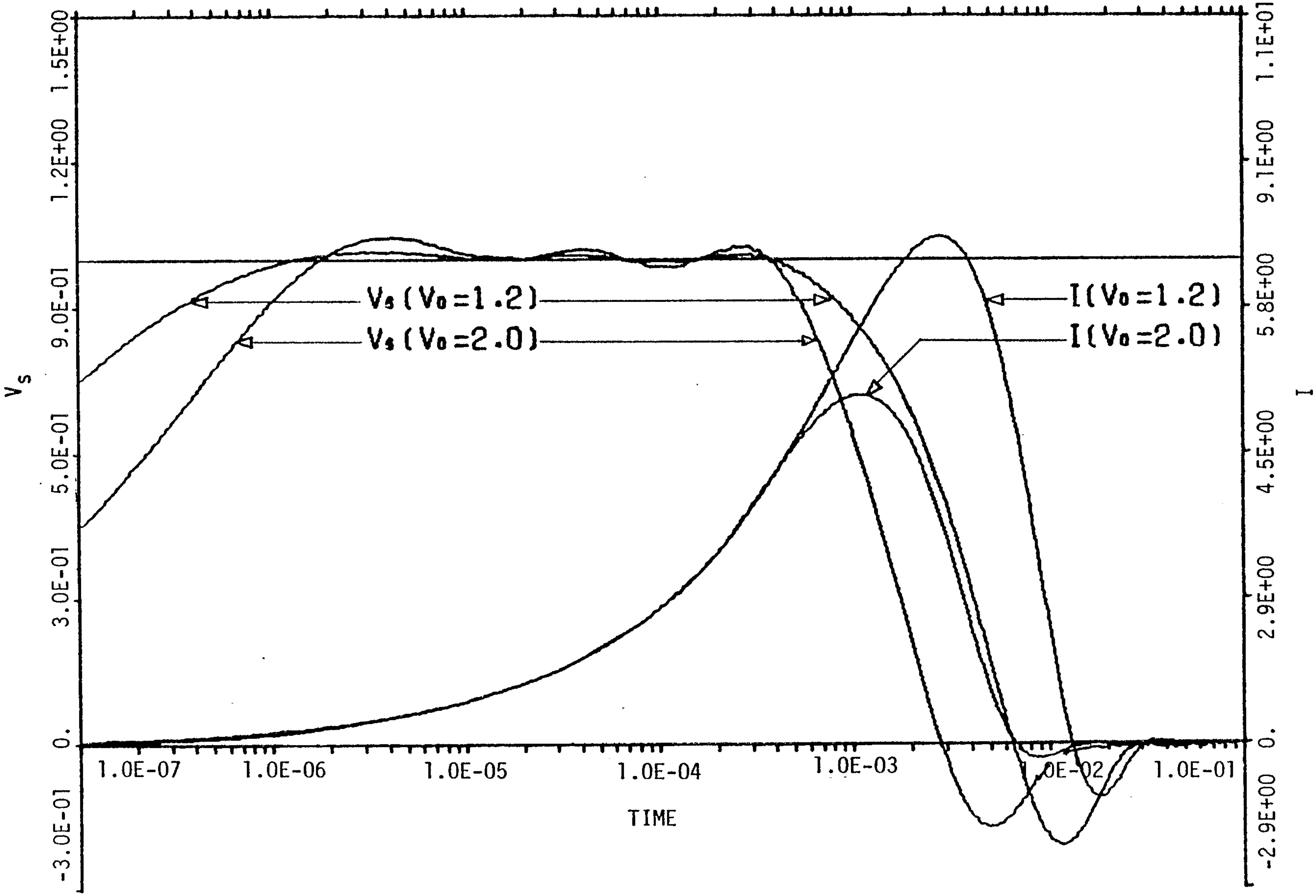
2 NETWORKS ARE REPRESENTED ON THIS GRAPH: DIFFERENT V_o'S WERE USED.



INVERSE FOURIER TRANSFORM CURVES

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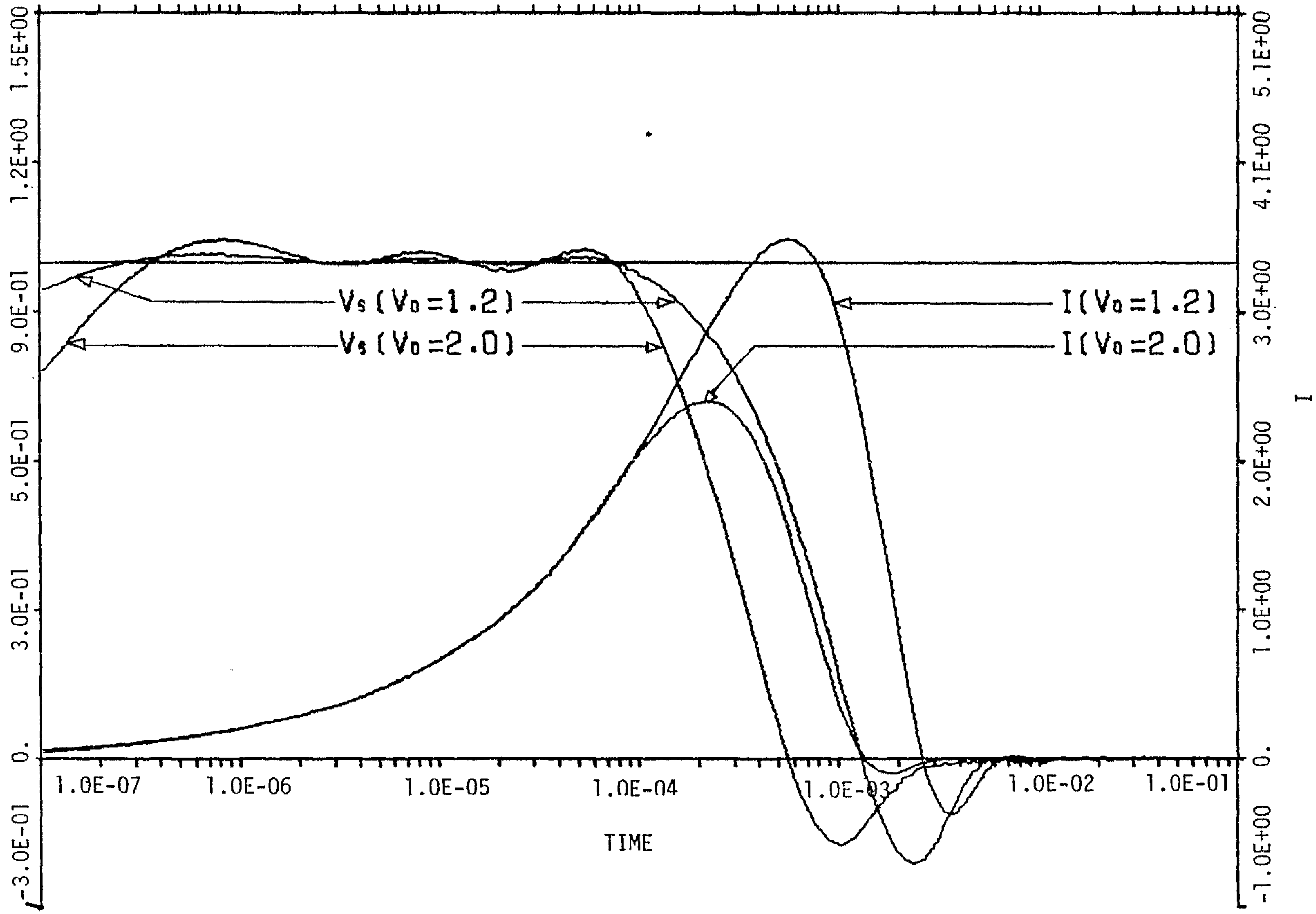

INVERSE FOURIER TRANSFORM CURVES

THESE ARE VOLTAGE AND CURRENT CURVES FOR A 4-BRANCH NETWORK. THE LEAST SQUARES FIT TO PRODUCE PARAMETERS FOR THE NETWORK WAS DONE TO 50 MICROSECONDS AND WITH AN ACCURACY OF ≤ 4.0% ERROR. 2 NETWORKS ARE REPRESENTED ON THIS GRAPH; DIFFERENT V_0'S WERE USED.