Sensor and Simulation Notes

Note 77

Waveforms on a Lossy Non-uniform Surface Transmission Line

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Abstract

A non-uniform surface transmission line is investigated as a means of transporting a simulated electromagnetic pulse (EMP) over a lossy surface such as the earth. Current and voltage waveforms on non-uniform lines with various geometries are displayed and compared with those on uniform lines with comparable dimensions. The waveforms demonstrate that a marked improvement in the behavior of an EMP waveform at early time can be obtained with a non-uniform line.

Acknowledgment

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I. Introduction

The effect on physical structures caused by an electromagnetic pulse (EMP) emitted by a nuclear explosion is presently a topic of considerable interest. Since the geometry of the structures is typically very complex, an investigation based solely on computational methods is impossible. In addition to computation, experimental measurements are needed, and these have motivated the development of techniques for simulating EMP artificially. Recently techniques for EMP simulation have become increasingly sophisticated and effective in simulating EMP over large areas.

As discussed in previous notes[1,2] a surface transmission line can be used for transporting simulated EMP over a lossy surface. The early portion of an EMP waveform varies rapidly, and for this reason a surface transmission line must perform well at high frequency to be effective.

In this note we seek to improve the high-frequency performance of a lossy surface transmission line by making it non-uniform. In particular we investigate the possibility of shaping the top plate of a surface line to improve its high-frequency performance. Closely related to this note is a previous note[3] that deals with a technique utilizing Brewster's angle as a means of improving high-frequency performance.

We want to shape the top plate so that as nearly as possible a current waveform excited at one end of a transmission line propagates to the other end with no distortion. Using conventional transmission line theory as it applies to a non-uniform line, we determine a shape that allows the components of a current that are centered about a frequency ω_c to propagate down the line with relatively no distortion. Current and voltage waveforms in the time domain at selected positions along the line are obtained for geometries corresponding to various values of ω_c. These waveforms are displayed along with those for the case of a uniform line with comparable dimensions, placing in evidence the improvement offered by a non-uniform line.

An obvious assumption made in this note is the structure being analogous to a transmission line. Regarding this assumption, the same comments as made in Section IV of Sensor and Simulation Note 60 are pertinent. Additional scrutiny is required in this note because the structure is non-uniform.

Generally speaking, the analogy is valid at frequencies for which shape of the top plate changes gradually enough.

II. Geometry of Non-uniform Line

It turns out that a structure whose top plate slants downward with a constant slope provides a significant improvement over a uniform line with comparable dimensions. Perhaps a transmission line with a slope that is not constant but varies with position along the line should be considered in a future note. A schematic representation of a line with constant slope is shown in figure 1. An excellent three-dimensional representation of a uniform structure, similar to the non-uniform structure being considered in this note, is depicted in figure 1 of Sensor and Simulation Note 60. Imagining that the top plate of the structure in Note 60 were slanted down toward the termination with a constant slope provides a good mental picture of the non-uniform structure in this note.

A step of current with magnitude I_o is excited by an ideal current source at $x = 0$ and propagates over the lossy section $(0 \leq x \leq d)$ to a termination at $x = d$. The bottom plate of the line in the region $(0 \leq x \leq d)$ is the lossy surface itself which has a surface impedance Z_g; the top plate is a perfect conductor. The termination at $x = d$ is the impedance Z_L which is purely resistive throughout the note and is given by $Z_L = y_d Z_o$ where y_d is the height of the line at $x = d$ and Z_o is the wave impedance of free space. A subsequent note is planned in which a complex Z_L at $x = d$ will be considered.

Adhering to the usual convention of transmission line theory, an $e^{j\omega t}$ time dependence is used in this note. From conventional transmission line theory we know that the current on a non-uniform line behaves according to the equation

$$I'' - \frac{Y'}{Y} I' - YZI = 0 \tag{1}$$

where the primes designate derivatives with respect to x. Assuming the effect of the lossy medium (earth) in the region $(0 \leq x \leq d)$ can be taken into account by a surface impedance defined by

$$Z_g = \sqrt{\frac{j\omega\mu_o}{\sigma + j\omega\varepsilon}} \quad , \tag{2a}$$

we have for a width W the impedance Z and the admittance Y, both per unit length, given by

$$Z = \frac{j\omega\mu_o y}{W} + \frac{Z_g}{W} \tag{2b}$$

$$Y = \frac{j\omega\varepsilon_o W}{y} \tag{2c}$$

where y, the height of the top plate, varies with x.

We want to determine a shape for y such that the current waveform propagates along the line without distortion. In the frequency domain no distortion means that, at all frequencies, the amplitude is constant with respect to x and the phase varies linearly with respect to both x and ω. Accordingly we write

$$I = I_o \frac{e^{j\theta}}{j\omega} \tag{3}$$

where I_o is a constant with respect to x, and θ, although presently unspecified, hopefully varies linearly with respect to both x and ω. Substitution of (3) into (1) yields

$$\theta'' - j(\theta')^2 - \frac{Y'}{Y}\theta' - jYZ = 0 \tag{4}$$

which, after separating into real and imaginary parts and eliminating θ, gives

$$y' + \frac{\sqrt{1 - 2a/y}}{1 - a/y} b = 0 \tag{5}$$

where

$$a = -\frac{\operatorname{Im}\left[Z_g/Z_o\right]}{2k_o} \tag{6a}$$

$$b = \operatorname{Re}\left[Z_g/Z_o\right] \quad . \tag{6b}$$

Solving (5) we obtain

$$\frac{(y - a)^2}{a^2} - \frac{(x - c_o/b)^2}{(a/b)^2} = 1 \tag{7}$$

or

$$y = a + \sqrt{a^2 + \gamma^2} \qquad (0 \le x \le c_o/b) \tag{8}$$

where

$$\gamma = c_o - bx \tag{9a}$$

and

$$c_o = \left[y\sqrt{1 - 2a/y}\right]_{x = 0} \quad . \tag{9b}$$

Substitution of (8) into (4) and solution of the resulting differential equation gives

$$\theta = -\frac{k_o}{b}\sqrt{a^2+\gamma^2} + a\ \ln\left[\frac{a^2+\sqrt{a^2+\gamma^2}}{\gamma^2}\right] + c_1 \tag{10}$$

where c_1 is an arbitrary constant.

Equation (7) defines a hyperbola centered at $(x = c_o/b\ ,\ y = a)$ with semitransverse axis a and semiconjugate axis a/b . Clearly y depends on frequency; figure 2 shows this dependence for a representative transmission line. Since we are primarily concerned with the high-frequency performance of the line corresponding to the early time of a pulse, we are interested in those curves for y that result for large values of ω . At large values of ω the term $a \to 0$ and y is closely approximated by the straight-line asymptote of the hyperbola, a convenience from the standpoint of fabricating a non-uniform line. That is,

$$y \approx y_o - bx \tag{11}$$

where

$$y_o = \Big[y(x)\Big]_{x\,=\,0} \quad . \tag{12}$$

Furthermore, as $a \to 0$

$$\theta \approx -k_o x + c_1 \tag{13}$$

and hence for high frequency θ varies linearly with x and ω.

The preceding development shows that a transmission line can be "tuned" to a particular frequency ω_c by shaping the top plate in a prescribed manner. The shape corresponding to large values of ω_c is a monotonic slope defined by (11) with

$$b = \mathrm{Re}\left[Z_g/Z_o\right]_{\omega = \omega_c} \quad . \tag{14}$$

The approximation imposed by (11) can be improved by truncating the sloping top plate at $x = d$ where (7) and (11) do not yet differ significantly from each other and by terminating the line with the impedance $Z_L = y_d Z_o$. Placing an appropriate, frequency-dependent impedance at $x = d$ might prove to be a means of improving the performance of the structure at lower frequencies and, as mentioned previously, will be considered in a future note.

Equation (11) becomes exact as $\omega_c \to \infty$. In this limit $b = 1/\sqrt{\varepsilon_r}$ where ε_r is the relative dielectric constant of the lossy medium. This result corresponds to the high-frequency Brewster's angle discussed in Sensor and Simulation Note 37.

III. Current Waveform

We want to solve (1) for the case of a non-uniform line with a top plate whose slope is described by (11); accordingly we substitute (11) into (1). For convenience we introduce the change of variable

$$I = I_o u \frac{e^{-jk_o x}}{j\omega}$$

where u varies with both x and ω ; hence equation (1) becomes

$$u'' - \left[2jk_o + b/y(x)\right]u' + \frac{jk_o}{y(x)}\left[b - Z_g/Z_o\right]u = 0 \quad . \tag{15}$$

The purpose of this change of variable is to remove oscillations from the solution of (15) at high frequency which is helpful from the standpoint of numerical computation. Two boundary conditions for (15) are

$$(u'/u)_{x = d} = 0 \tag{16}$$

$$(u)_{x = 0} = 1 \quad . \tag{17}$$

Equation (15) is solved numerically for values of $u(x)$ by starting at $x = d$ where we assume the boundary conditions $u' = 0$ and $u = u_o$, an arbitrary constant, and by using the Runge-Kutta[4] method to compute backward along x to $x = 0$. The value of u at $x = 0$ obtained in this way is compared with the correct value given by (17). From this comparison we can determine the factor by which $u(x)$ should be multiplied to scale its magnitude correctly so that the arbitrariness of u_o is removed.

The Fourier inversion of the solution of (1) gives the current waveform in the time domain. In order to perform this inversion numerically, a singularity at $\omega = 0$ of the form ω^{-1} must be removed. Doing this, we obtain

$$h(x,\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B(x,\omega) e^{j\omega\frac{d}{c}\tau} \, d\omega + U(\tau) \tag{18}$$

with

$$B(x,\omega) = \frac{u(x,\omega) - 1}{j\omega} \tag{19a}$$

$$\tau = \frac{t - x/c}{d/c} \tag{19b}$$

where $h(x,\tau) = i(x,\tau)/I_o$ is a normalized current.

The initial-value theorem applied to expressions developed in a previous note[2] gives the current on a uniform line at $\tau = 0$, i.e.,

$$h(x,\tau = 0) = e^{-\frac{x}{2\sqrt{\varepsilon_r}\,y}} \tag{20}$$

where, as mentioned previously, ε_r is the relative dielectric constant of the lossy medium and y is the height of the top plate. For the case of a non-uniform line, the solution of (1) is obtained in the limit as $\omega \to \infty$ by using the WBK method before applying the initial-value theorem; the result is

$$h(x,\tau = 0) = \left[y(x)/y_o\right]^{\beta} \tag{21}$$

where

$$\beta = \frac{1}{2}\left(\frac{1}{b\sqrt{\varepsilon_r}} - 1\right) \quad . \tag{22}$$

From these expressions we see that by properly adjusting b the initial value of the current on a non-uniform line can be made larger than on a comparable uniform line. In fact, for $b = 1/\sqrt{\varepsilon_r}$ we see that $\beta = 0$ and hence the current at $\tau = 0$ on a non-uniform line is exactly unity for all values of x, an attractive feature if the waveform for $\tau > 0$ behaves properly. Unfortunately this is not the case. As b approaches the value $1/\sqrt{\varepsilon_r}$ the initial value of the current approaches unity but the waveform for $\tau > 0$ becomes distorted.

IV. Voltage Waveform

In the frequency domain the voltage is given by

$$V = Z_i I \tag{23}$$

where

$$Z_i = yZ_o\left[1 + j\,\frac{u'/u}{k_o}\right] \quad . \tag{24}$$

The voltage has two singularities at $\omega = 0$ that must be removed before a numerical Fourier inversion can be performed. A singularity of the form $\omega^{-\frac{1}{2}}$ is present due to Z_g, the surface impedance of the lossy section; a singularity of the form ω^{-1} is present due to Z_L, the purely resistive impedance at $x = d$. Removing these singularities, we have a normalized voltage $e(x,\tau) = v(x,\tau)/(I_o y_o Z_o)$ given by

$$e(x,\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(x,\omega)e^{j\omega\frac{d}{c}\tau}\,d\omega + \frac{K_1}{\sqrt{\frac{d}{c}\,\pi\tau}} + K_2U(\tau) \tag{25}$$

where

$$A(x,\omega) = \frac{y(x)}{y_o}\left[u + j\,\frac{u'}{k_o}\right]\frac{1}{j\omega} - \frac{K_1}{\sqrt{j\omega}} - \frac{K_2}{j\omega} \tag{26a}$$

with

$$K_1 = \frac{d - x}{y_o}\sqrt{\frac{\varepsilon_o}{\sigma}} \tag{26b}$$

$$K_2 = y_d/y_o \quad . \tag{26c}$$

At $x = 0$ a plot of the normalized voltage $e(0,\tau)$ versus τ is actually a plot of the normalized input impedance $z_{in}(\tau) = Z_{in}(\tau)/(y_o Z_o)$ because the normalized current $h(o,\tau)$ is unity for $\tau > 0$ at the ideal current source. The input impedance $Z_{in}(\tau)$, although displayed in the time domain, is conventional in the sense that it is obtained by dividing the total voltage $v(x,\tau)$ by the total current $i(x,\tau)$ at $x = 0$. Curves of $e(0,\tau)$ versus τ are presented in this note for various non-uniform transmission lines ; ideally we would like it to be unity.

V. Waveforms for Various Geometries

The curves contained in this note are parameterized in terms of

$$\eta = y_o/d \tag{27a}$$

$$b = \frac{y_d - y_o}{d} \tag{27b}$$

$$\xi = x/d \quad . \tag{27c}$$

We have assigned the transmission line a length of $d = 50$ meters and assumed for the lossy medium a conductivity $\sigma = 10^{-2}$ mhos/meter and relative permittivity $\varepsilon_r = 10$.

Waveforms of normalized current and voltage versus τ are presented for several normalized heights η . For each value of η waveforms for several representative values of b are displayed, including $b = 0$.

For the purpose of showing how the slope b is related to ω_c we include Table 1.

Table 1. Values of $b(\omega_c)$

ω_c	b
∞	.3162
10^9	.3149
10^8	.2347
5×10^7	.1687
10^7	.0693
5×10^6	.0481
10^6	.0212

It is interesting to note from this table that even the gradual slopes correspond to relatively high frequency.

As mentioned previously a slope of $b = 1/\sqrt{\varepsilon_r}$, which corresponds to $\omega_c = \infty$, gives a normalized current of unity at $\tau = 0$ for all values of x . The waveforms displayed in this note illustrate, however, that the waveform for $\tau > 0$ deteriorates as b approaches the value $1/\sqrt{\varepsilon_r}$. Better waveforms result for values of b somewhat less than $1/\sqrt{\varepsilon_r}$. In fact the best waveforms seem to result for values of b corresponding to frequencies in the vicinity of $\omega_c = 10^7$.

Figure 3 demonstrates the current waveform on a lossy surface transmission line at early time can be improved considerably by slanting the top plate with a constant slope. Waveform A results on a uniform line with normalized height $\eta = .06$; waveform B on a non-uniform line with $\eta = .12$ and slope $b = .06$; waveform C on a line with $\eta = .20$ and $b = .08$. The initial rise of the current waveform on B and C is significantly faster than on A . At $\xi = .5$ the current on A reaches 90% of its final value of unity at $\tau = 1.2$ whereas B reaches this value at $\tau = .04$ and C at $\tau = .01$.

This improved behavior at early time is not due entirely to the effect of sloping the line, for increasing the height also contributes. To illustrate how both the height and slope influence the waveform at early time, figures 4 through 11 are included. Clearly increasing the height improves the waveform, but for a given height sloping the line provides further improvement.

At $\xi = .5$ the waveforms, especially the current, have a pronounced "hump" beginning at $\tau = 1.0$ that is due to reflections from the discontinuity at $\xi = 1.0$. This hump causes the waveform to "overshoot" the desired value of unity. Placing an appropriate impedance Z_L at $\xi = 1.0$ might reduce this overshoot considerably. The proper impedance might also allow a steeper slope to be used without introducing as much distortion for $\tau > 0$ as is the case for the impedance $Z_L = y_d Z_o$ being presently considered in this note.

Waveforms for structures with heights of 3, 4, 6, 8, 10, 12 and 16 meters are represented in figures 3 through 11. Hopefully this range of heights -- beginning with $y_o = 3$ meters, the height of a simulator actually being used, and ranging up to 16 meters, a rather tall structure -- includes most of the structures that might be considered for actual simulators. At least this range will serve to demonstrate the effect of a non-uniform line with constant slope.

For each height waveforms for three slopes are depicted. In general, these include a slope of zero, an intermediate slope, and a relatively extreme slope for which the waveform is clearly distorted.

The waveforms are depicted at selected positions along the line. In particular, the current is displayed at $\xi = .5$ and $\xi = 1$. The current at $\xi = 0$ is identically unity for $\tau > 0$; so it is not included. The voltage is displayed at $\xi = 0$, where it is the same as the normalized input impedance, and at $\xi = .5$.

An especially promising structure is the one that produces waveform B in figure 3. It is 6 meters tall at $\xi = 0$ and slopes downward to 3 meters at $\xi = 1$. A uniform line with a height of 3 meters -- a structure actually being used in EMP simulation -- produces waveform A in figure 3. This uniform structure could be altered to give waveform B by simply increasing the height at $\xi = 0$ to 6 meters. As pointed out, waveform B rises much faster than waveform A . They both have approximately the same amount of overshoot. Much of this overshoot, however, might be removed by placing the proper termination at $\xi = 1.0$ for the case of the non-uniform line.

VI. Power Considerations

It should be pointed out that, although the initial rise of the current is improved by increasing the height y_o , the amount of power required to establish a given current at positions on the line near the source increases as the height is increased. The reason is that increasing the height increases the impedance which, in turn, increases the voltage required to establish a current; hence more power is required. How much more power is not obvious and requires futher computation to determine.

For the case of a losseless uniform line with a matched termination the amount of additional power varies linearly with an increase in height, e.g., doubling the height requires doubling the power to maintain the same current. For the same line except with a lower plate having a surface impedance Z_g the linear relationship between height and power no longer holds because the lossy effect of Z_g diminishes as the height is increased. This point is illustrated by Table 2 which is based on computations made using (20). In this table h_1 and p_1 are the current and the instantaneous power in a wave on a lossy uniform surface line with a height $y_1 = 3$ meters ; they are computed at a time slightly after $\tau = 0$ and at a position ξ along the line (d = 50 meters). Similarly h_2 and p_2 are on the same kind of line except with a height $y_2 = 6$ meters .

Table 2. Current and Power at $\tau = 0$ on Two Lossy Uniform Lines. $P_1 = y_1 Z_o I_o^2$

ξ	h_1	h_2	p_1/P_1	p_2/P_1
.0	1.000	.707	1.000	1.000
.2	.590	.543	.348	.590
.4	.349	.417	.122	.349
.6	.206	.321	.042	.206
.8	.121	.246	.015	.121
1.0	.072	.189	.005	.072

We have set up Table 2 so that the sources at $\xi = 0$ impart the same amount of power to waves on both lines. At $\xi = 0$ the taller line has a smaller current because, since it is taller, more of its power is in the voltage. The table shows that, as the waves propagate along the lines, the current and the instantaneous power diminish more slowly on the taller line. In fact, at a distance not too far from the source both of these quantities are larger on the taller line even though at $\xi = 0$ the current was smaller on the taller line and the power was the same. Although the current and the power have been considered only at $\tau = 0$, we see the relationship between height and power is not linear. Of course the waveforms in this note can be used to compare the current and the instantaneous power for lines of different heights at times other than $\tau = 0$.

In Table 3 the current and the instantaneous power on a lossy uniform surface line with height $y_1 = 3$ meters are compared with those on a lossy non-uniform surface line with height $y_o = y_2 = 6$ meters and slope $b = .06$ ($d = 50$ meters for both lines). The ideal current source, which excites waveforms A and B on these lines as shown in figure 3, supplies an instantaneous power that varies with time. To emphasize how current and instantaneous power compare for these two lines, we have adjusted Table 3 so that the sources at $\xi = 0$ are not ideal current sources but instead put out the same amount of power at both $\tau = 0$ and $\tau = .5$ for both lines.

Table 3. Current and Power at $\tau = 0$ and $\tau = .5$ on Lossy Uniform and Non-uniform Lines. $P_1 = y_1 Z_o I_o^2$

ξ	h_1	h_2	p_1/P_1	p_2/P_1
		$\tau = 0$		
.0	1.000	.707	1.000	1.000
.5	.268	.382	.072	.220
1.0	.072	.161	.005	.026
		$\tau = .5$		
.0	.846	.675	1.000	1.000
.5	.683	.670	.640	.758
1.0	.594	.695	.350	.482

Table 3 shows that increasing the height of a uniform line at $\xi = 0$ to form a non-uniform line requires additional power but the increase is not linear, i.e., doubling y_o from 3 to 6 meters while keeping $y_d = 3$ meters does not require doubling the power. In Table 3 multiplying h_2 and p_2 by factors of $\sqrt{2}$ and 2, respectively, corresponds to doubling the power, an increase larger than is necessary to make the wave on the non-uniform line significantly better than on the uniform line.

VII. Optimum Non-uniform Line

It is not clear how to specify an optimum non-uniform line with constant slope. Two factors to consider are how fast an initial rise is desired and how much distortion for $\tau > 0$ can be tolerated. Another factor is that a steep slope requires a relatively tall structure at $\xi = 0$ which imposes difficulties from the standpoint of both power and construction practicalities.

Examining the waveforms at $\xi = .5$, we conclude that a gradual slope around $b = .08$ corresponding to approximately $\omega_c = 10^7$ produces the best current waveform at early time, independent of the termination impedance Z_L at $\xi = 1$. Slopes greater than $b = .08$ cause the current to overshoot the ideal value of unity during the time $(0 \leq \tau \leq 1)$. Since reflections from the discontinuity at $\xi = 1$ do not arrive before $\tau = 1$, adjusting Z_L will not reduce overshoot occurring before $\tau = 1$. Incremental slopes greater than $b = .08$ may prove effective, however, for the case of a non-uniform line with a top plate whose slope is not constant but varies with ξ .

Overshoot occurring after $\tau = 1$ is certainly affected by the nature of Z_L . The low-frequency components of the current "see" a discontinuity that depends on the difference between the impedances y_oZ_o and Z_L . Since we have taken $Z_L = y_dZ_o$ in this note, the discontinuity at low frequency depends on the difference $y_o - y_d$. Accordingly making Z_L frequency dependent with limits y_oZ_o at low frequency and y_dZ_o at high frequency would probably reduce the overshoot that occurs at later time.

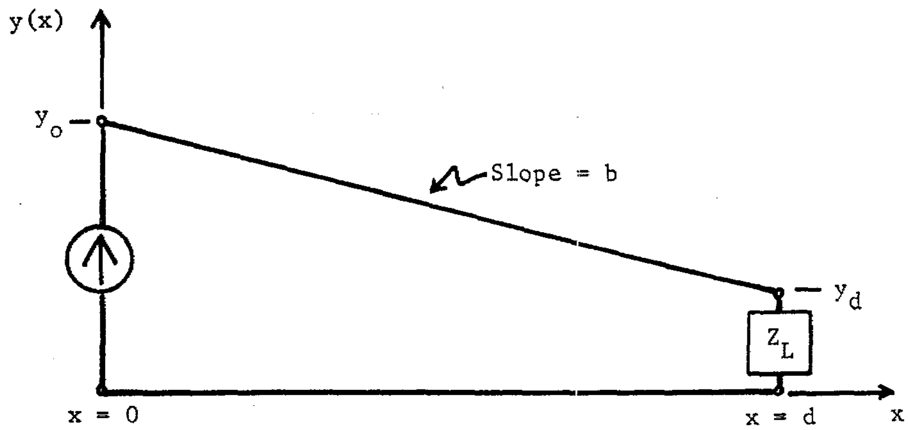

Figure 1. Schematic of lossy non-uniform line with constant slope.

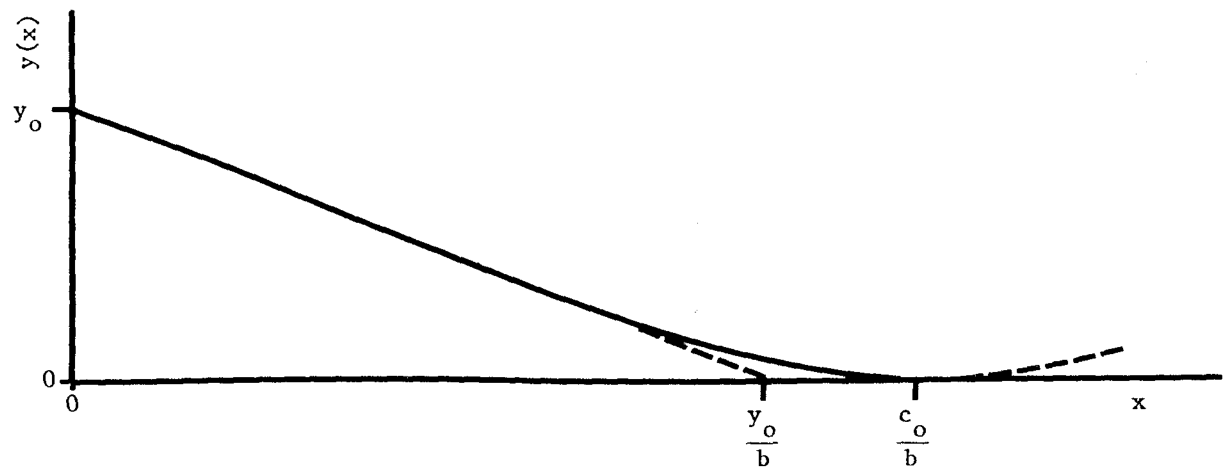

Figure 2a. General hyperbolic curve for $y(x)$.

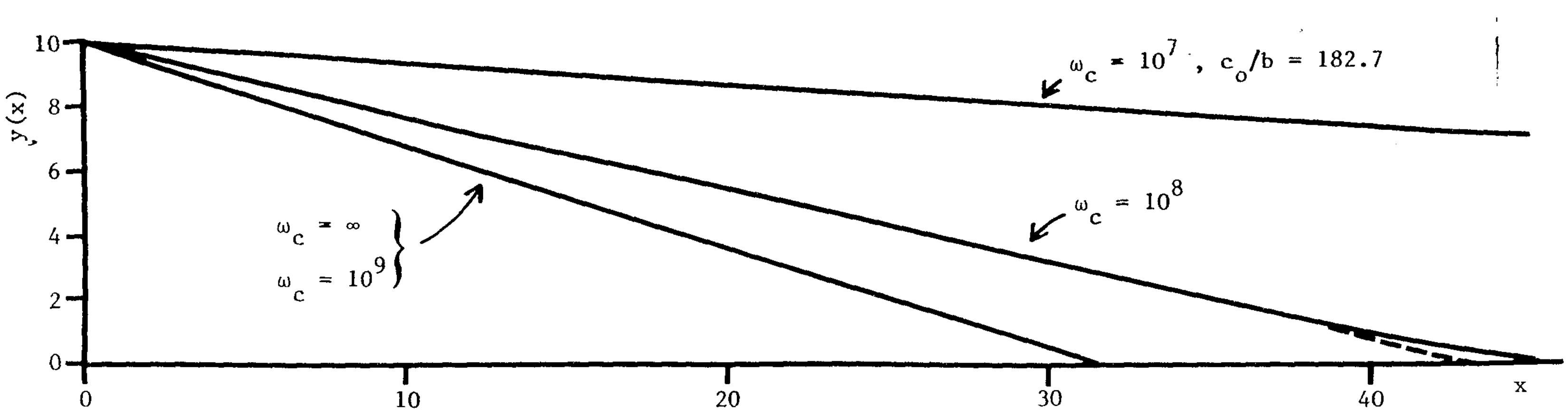

Figure 2b. Representative curves of $y(x)$ versus x with ω_c as a parameter ($y_o = 10$ meters, $\sigma = 10^{-2}$ mhos/meter, $\varepsilon_r = 10$) .

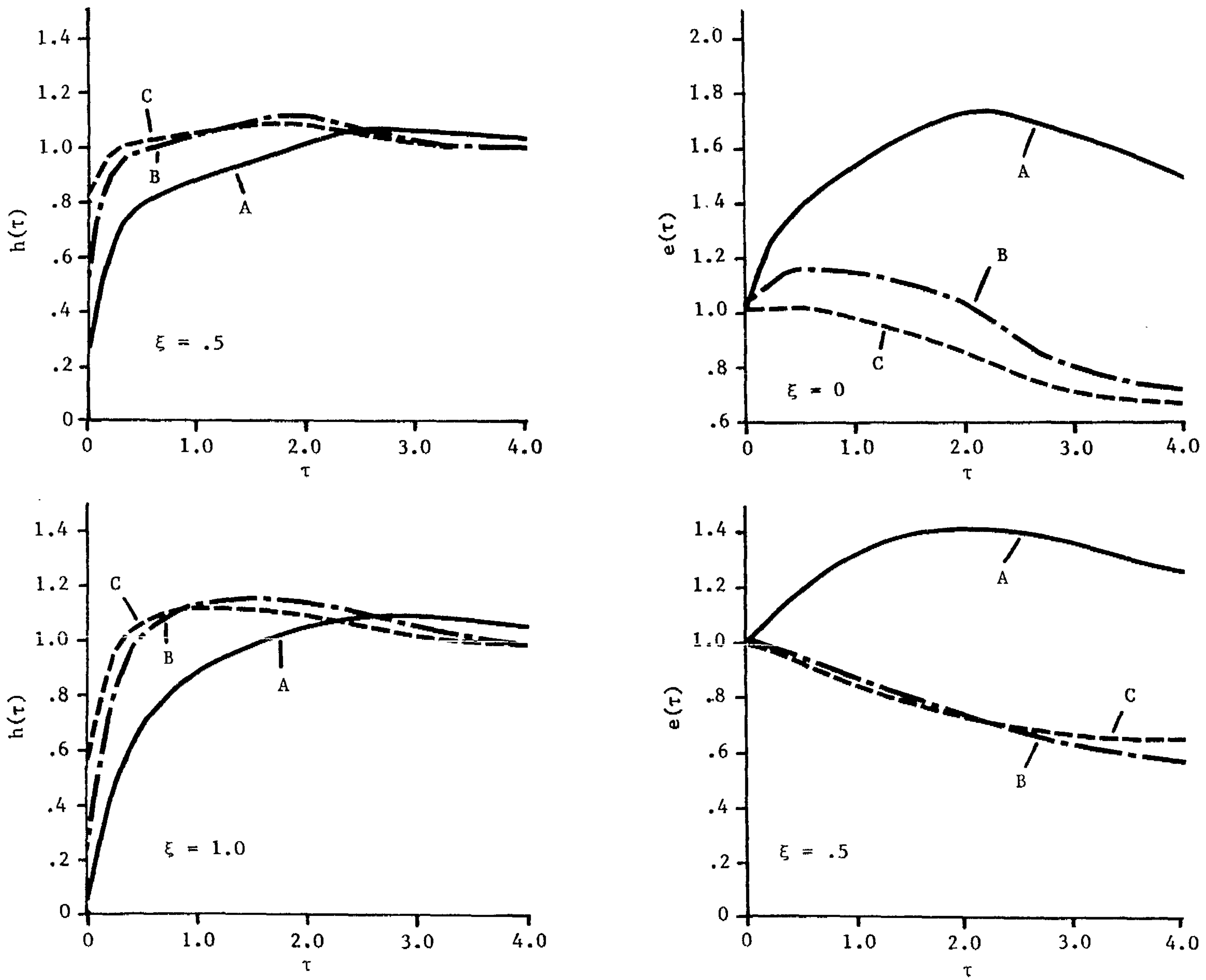

Figure 3. Comparison of current and voltage waveforms for three representative configurations.
A: $\eta = .06$, $b = 0$; B: $\eta = .12$, $b = .06$; C: $\eta = .20$, $b = .08$.

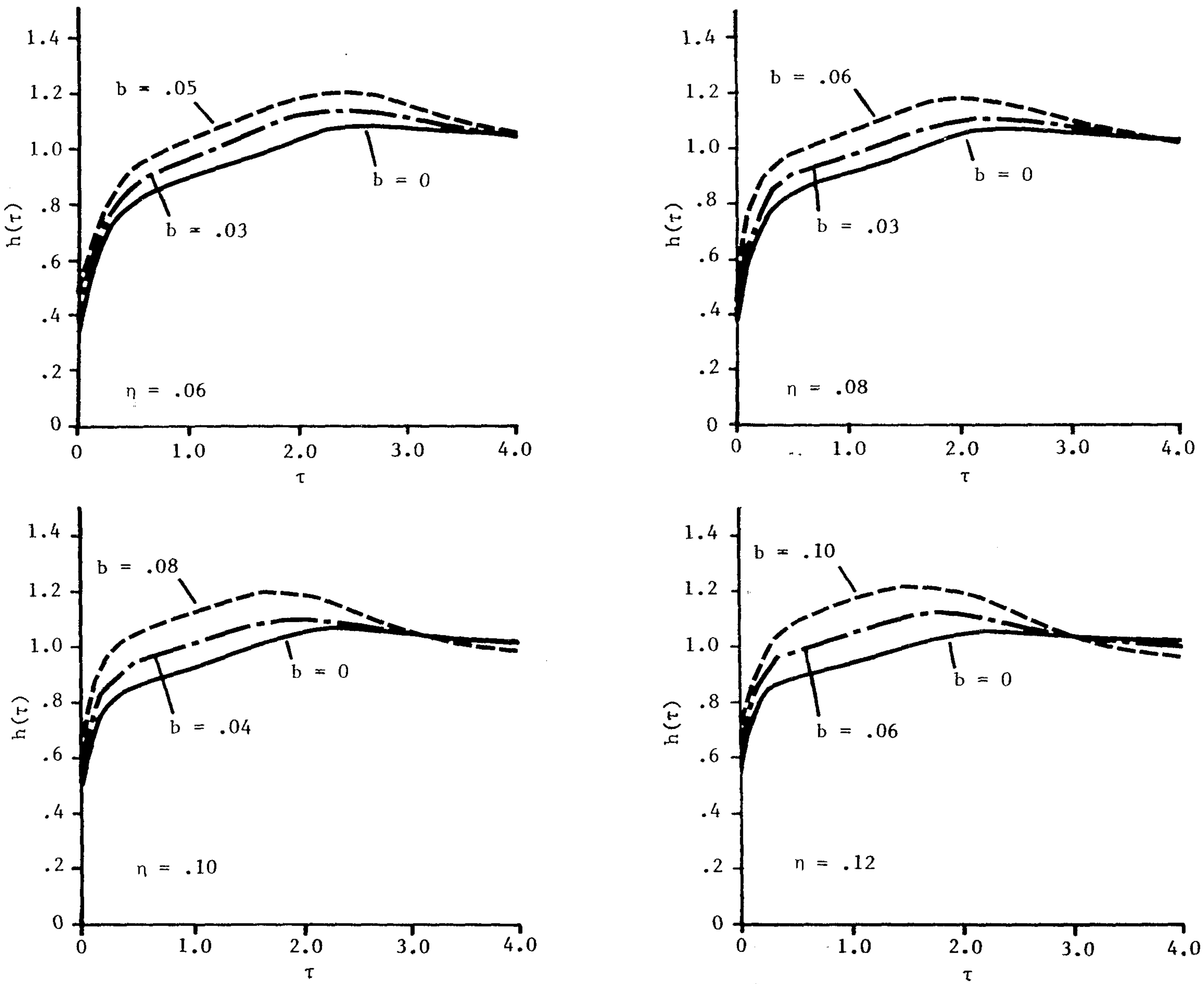

Figure 4. Current waveforms with b and η as parameters (ξ = .5) .

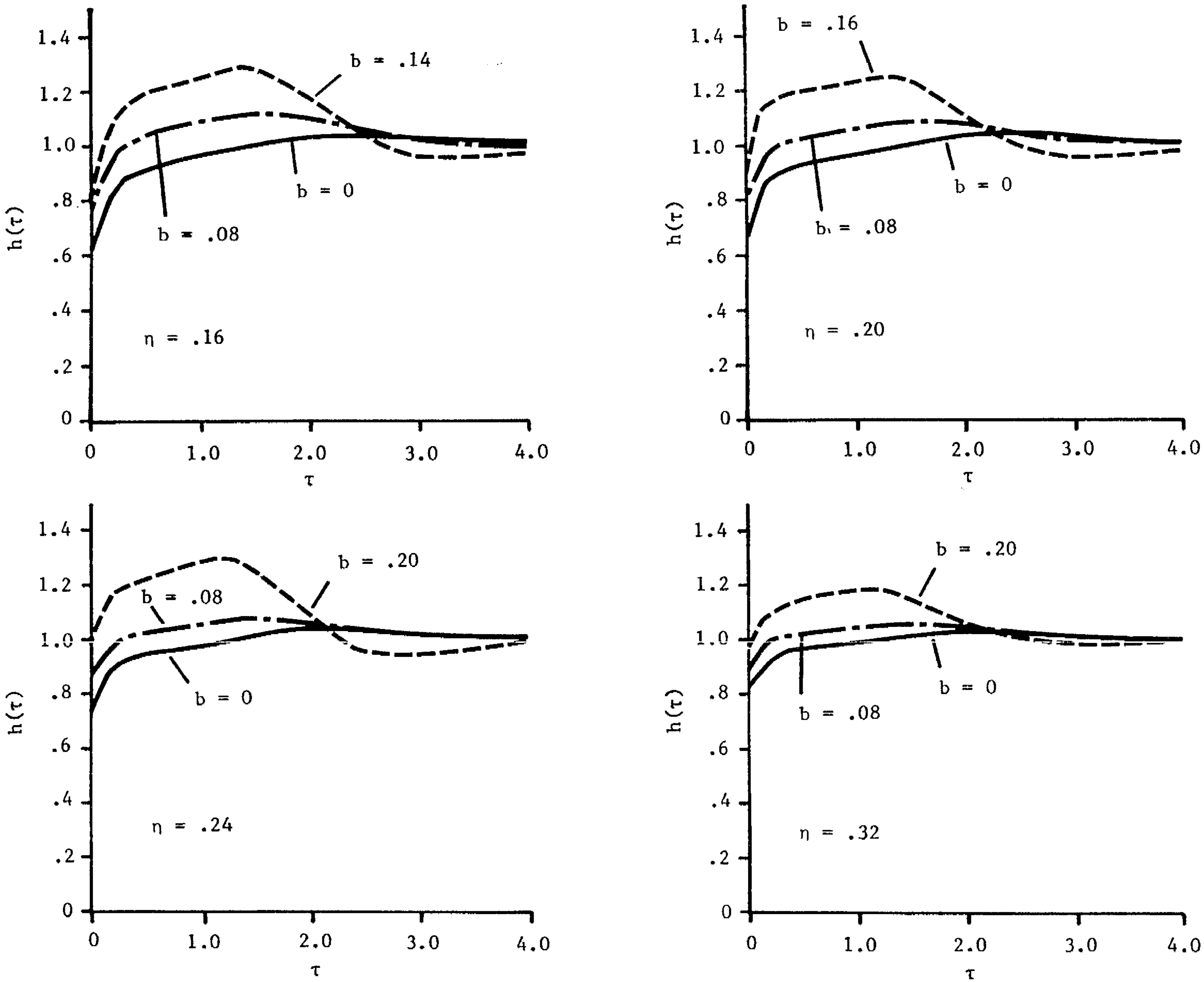

Figure 5. Current waveforms with b and η as parameters (ξ = .5) .

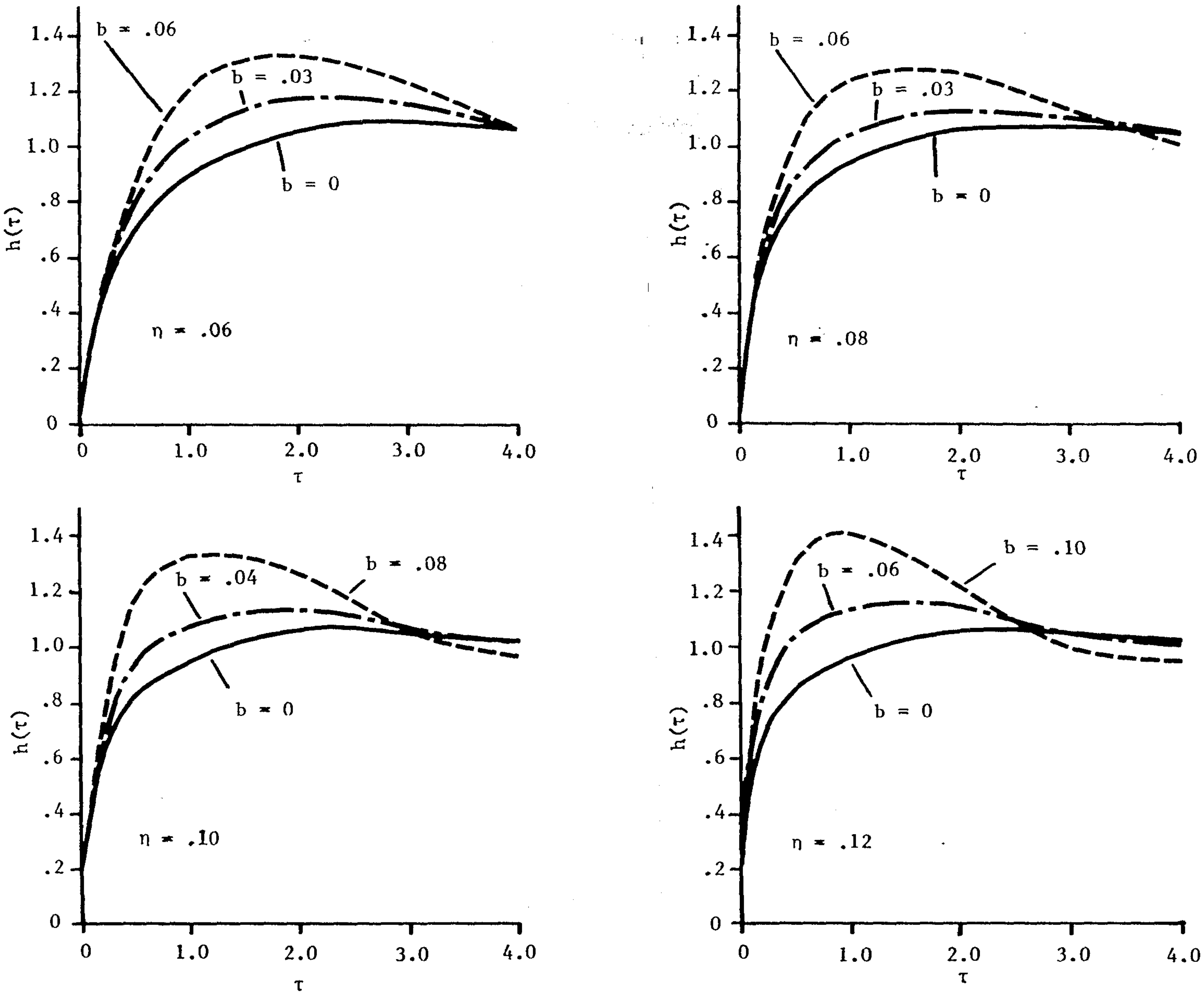

Figure 6. Current waveforms with b and η as parameters (ξ = 1.0) .

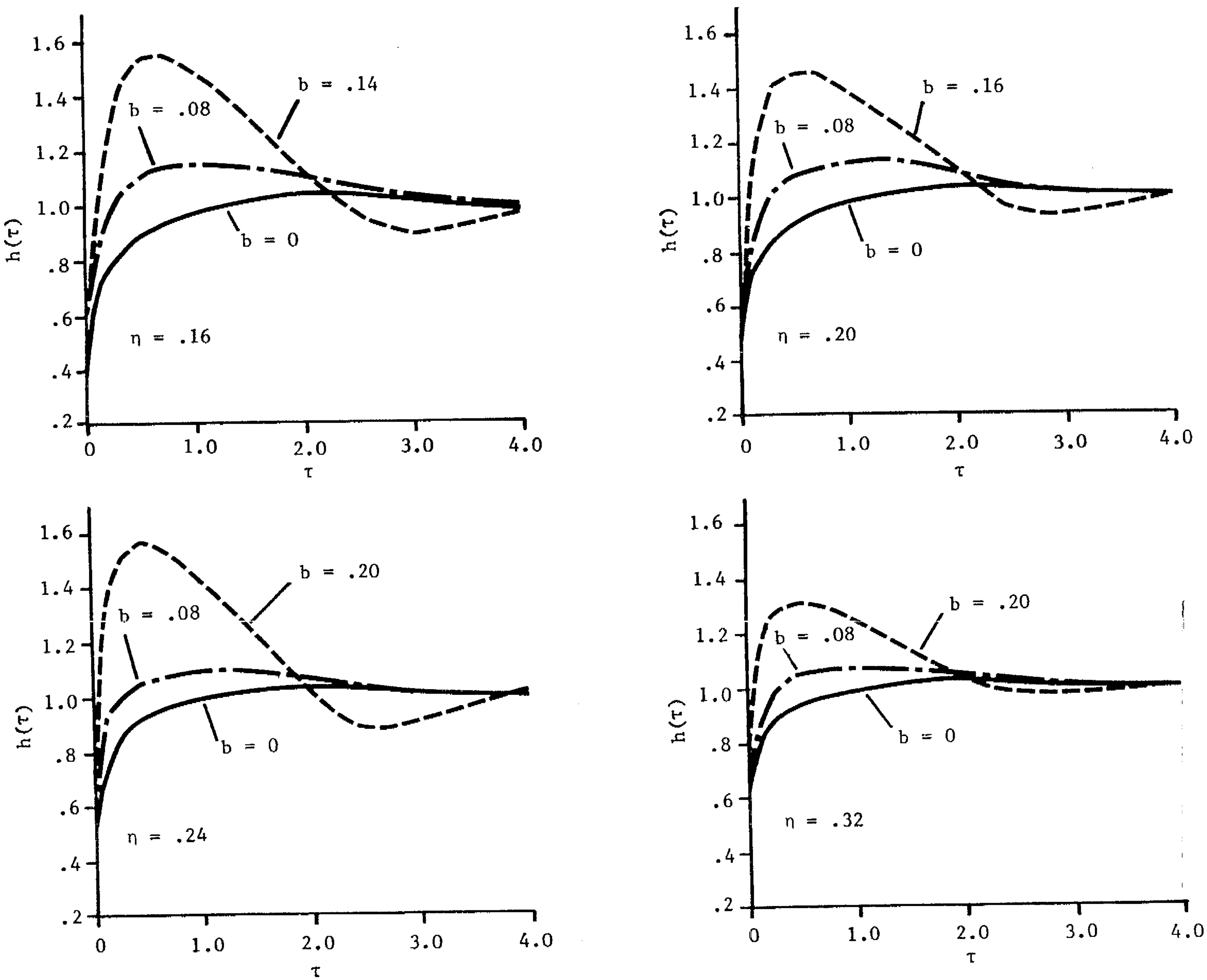

Figure 7. Current waveforms with b and η as parameters ($\xi = 1.0$) .

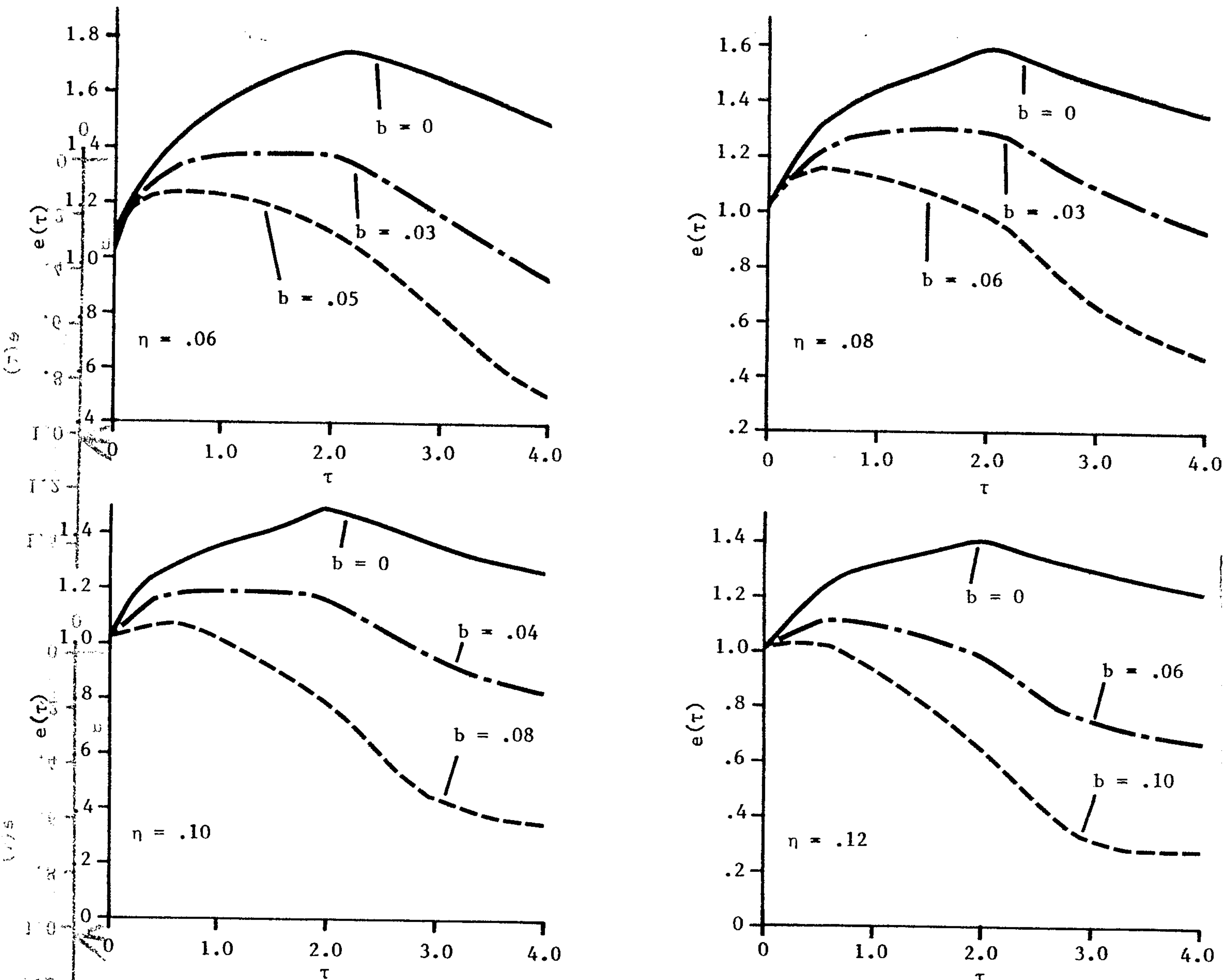

Figure 8. Voltage waveforms with b and η as parameters ($\xi = 0$) .

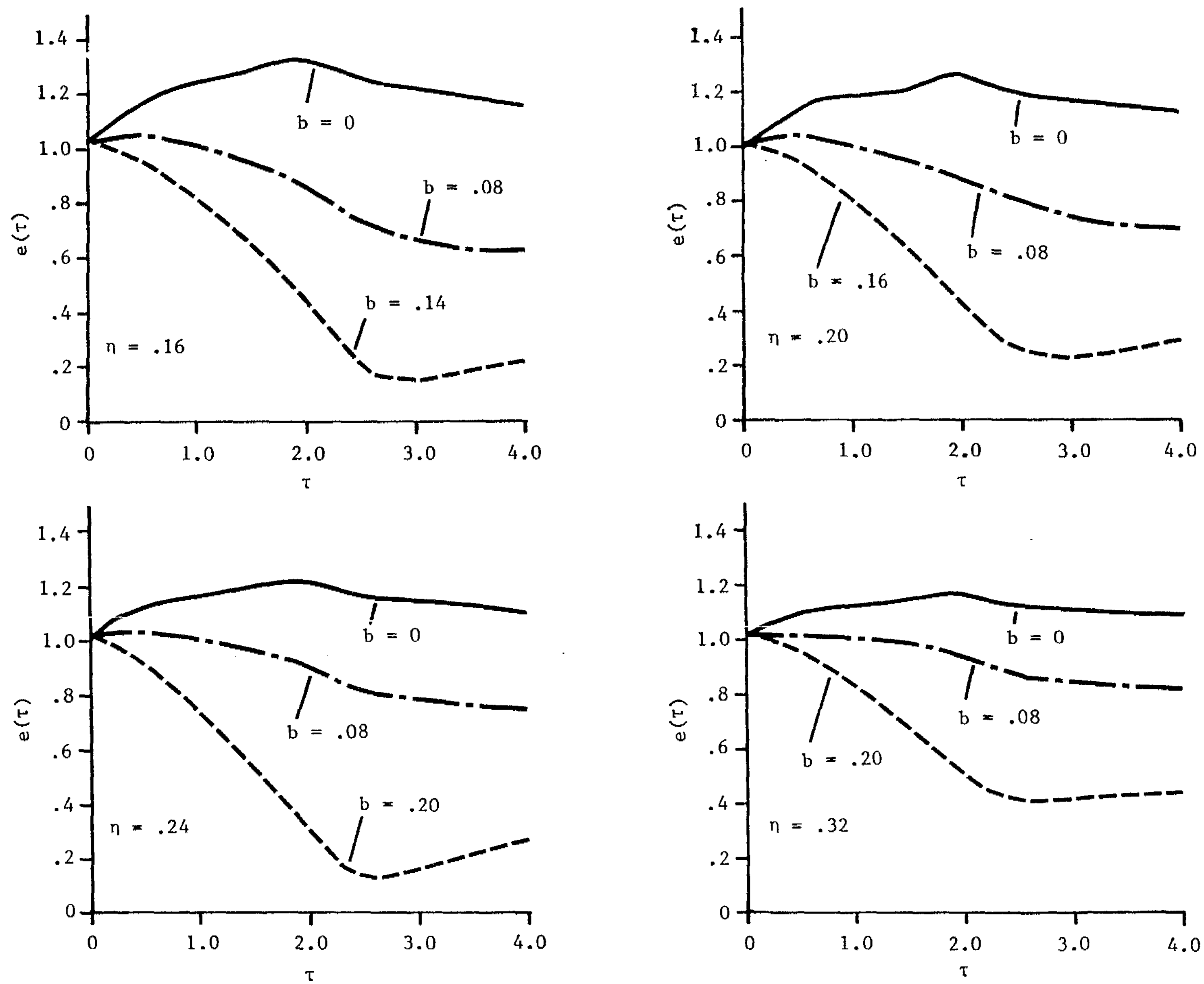

Figure 9. Voltage waveforms with b and η as parameters (ξ = 0) .

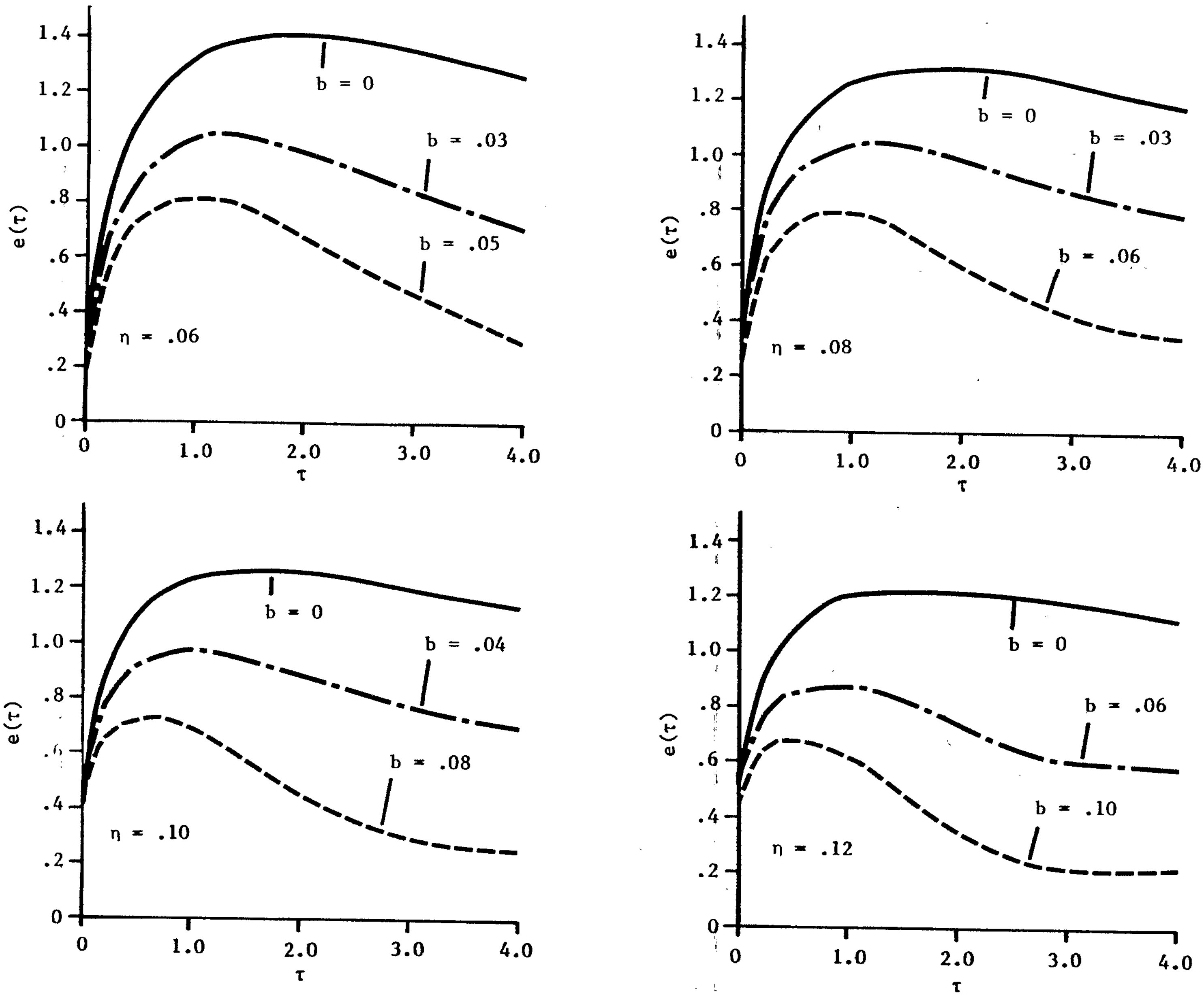

Figure 10. Voltage waveforms with b and η as parameters ($\xi = .5$) .

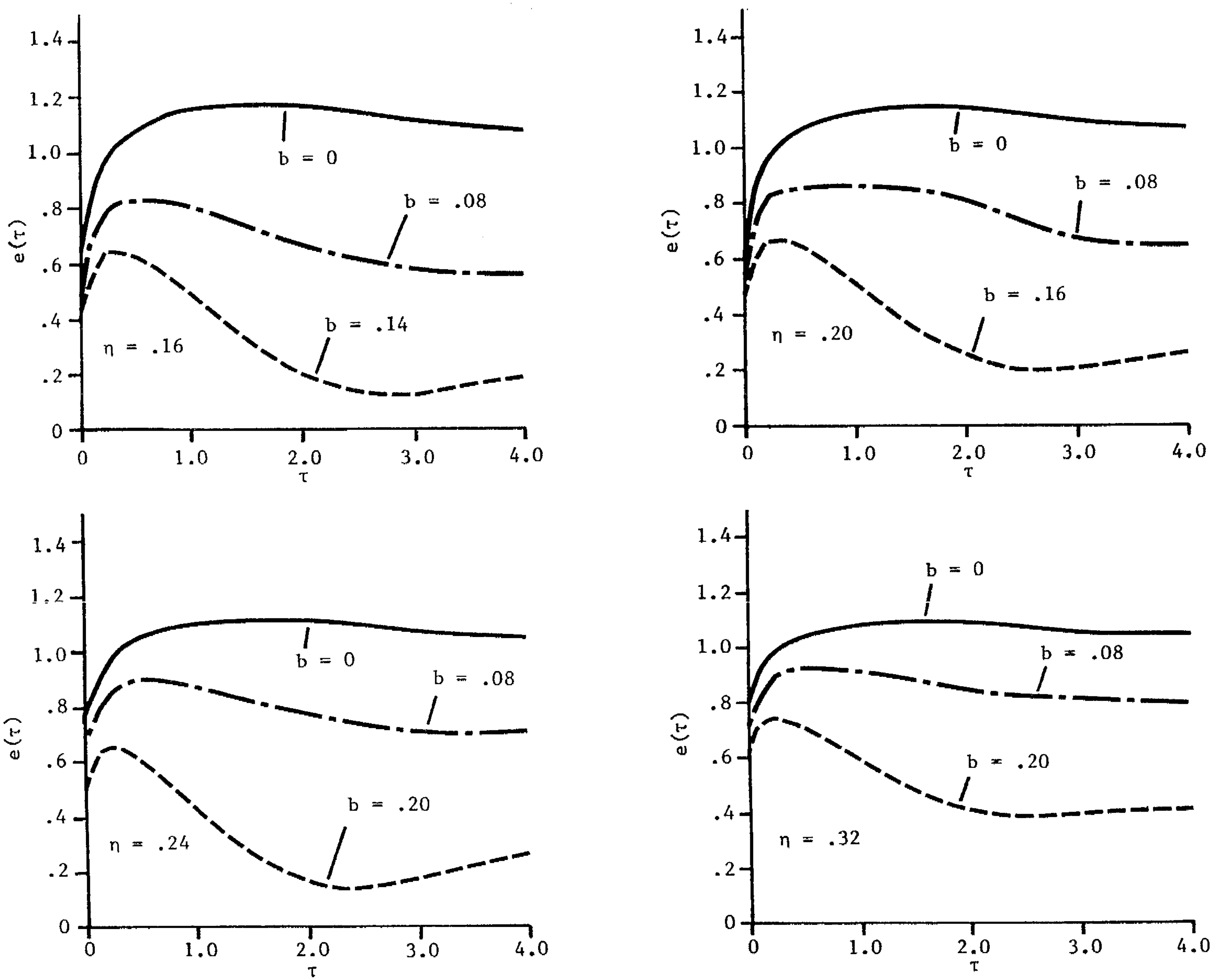

Figure 11. Voltage waveforms with b and η as parameters (ξ = .5) .

References

1. Carl E. Baum, Sensor and Simulation Note 46, "The Single-Conductor, Planar, Uniform Surface Transmission Line, Driven from One End," July 1967.
2. R. W. Latham and K. S. H. Lee, Sensor and Simulation Note 60, "Waveforms on a Surface Transmission Line with an Inductive Load," August 1968.
3. Carl E. Baum, Sensor and Simulation Note 37, "The Brewster Angle Wave Matcher," March 1967.
4. J. B. Scarborough, Numerical Mathematical Analysis, The John Hopkins Press, 1962, p. 358.
5. Carl E. Baum, Electromagnetic Pulse Theoretical Note 25, "The Reflection of Pulsed Waves from the Surface of a Conducting Dielectric," February 1967.