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The Buried Transmission-line Simulator Driven by Multiple Capacitive Sources and an Inductive Source

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Abstract

The time history of voltage and current on a buried transmission-line simulator is obtained for the case of multiple capacitor sources combined with an inductive energy source. Both infinite and finite-length lines are considered. Capacitor parameters are chosen to give less than a ten-percent ripple in the waveforms. The results of calculations are graphically presented to show how the inductance affects the current waveform.

Acknowledgment

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I. Introduction

Buried transmission-line simulators driven by capacitors have been considered in two previous notes. The first note[1] considered the use of a series resistor, which may be chosen large enough to damp out oscillations in the driving current. The second note[2] explains how several capacitors may be switched into the circuit in sequence in order to shape the waveform in the simulator. The multiple capacitor technique achieves considerable flexibility in shaping the waveforms without inserting series resistors, which absorb energy. A special technique, shorting out the capacitors at the time the voltage across them becomes zero, prevents oscillation in the driving current.

Inductive energy sources were recently studied[3] in connection with the problem of shaping the current and voltage waveforms for buried transmission-line simulators. The authors of this investigation were quick to point out that the over-all shape of the current pulse for long times after the initial surge are preferable to those of a single capacitor source. However, the initial surge is not desirable but is unavoidable when the inductive source is used alone.

The present note reveals that added flexibility can be achieved by using capacitive energy sources to initiate the pulse and an inductive source to sustain the pulse. Wave shapes are given for terminal current (or magnetic field) and terminal voltage (or electric field) impressed on the buried transmission line for the cases of: 1) the infinite line and 2) the finite line. The fields as a function of position in the simulator may be determined from the normalized terminal voltage and current by using techniques which were detailed in two previous notes.[4,5] We have assumed the electrical parameters of the ground to be constant over the space in and around the transmission line. Furthermore, in this note, we assume for all frequencies of interest that $\sigma \gg \omega\varepsilon$.

The equivalent circuit impedance developed in a previous note is used to relate the voltage and current at the simulator terminals. Figure 1 illustrates the connection between the N-2 capacitors, the shorting switch and the inductor. Each capacitor C_k is charged initially to a voltage V_k . The inductor has an initial current I_L . The k-th switch is closed at

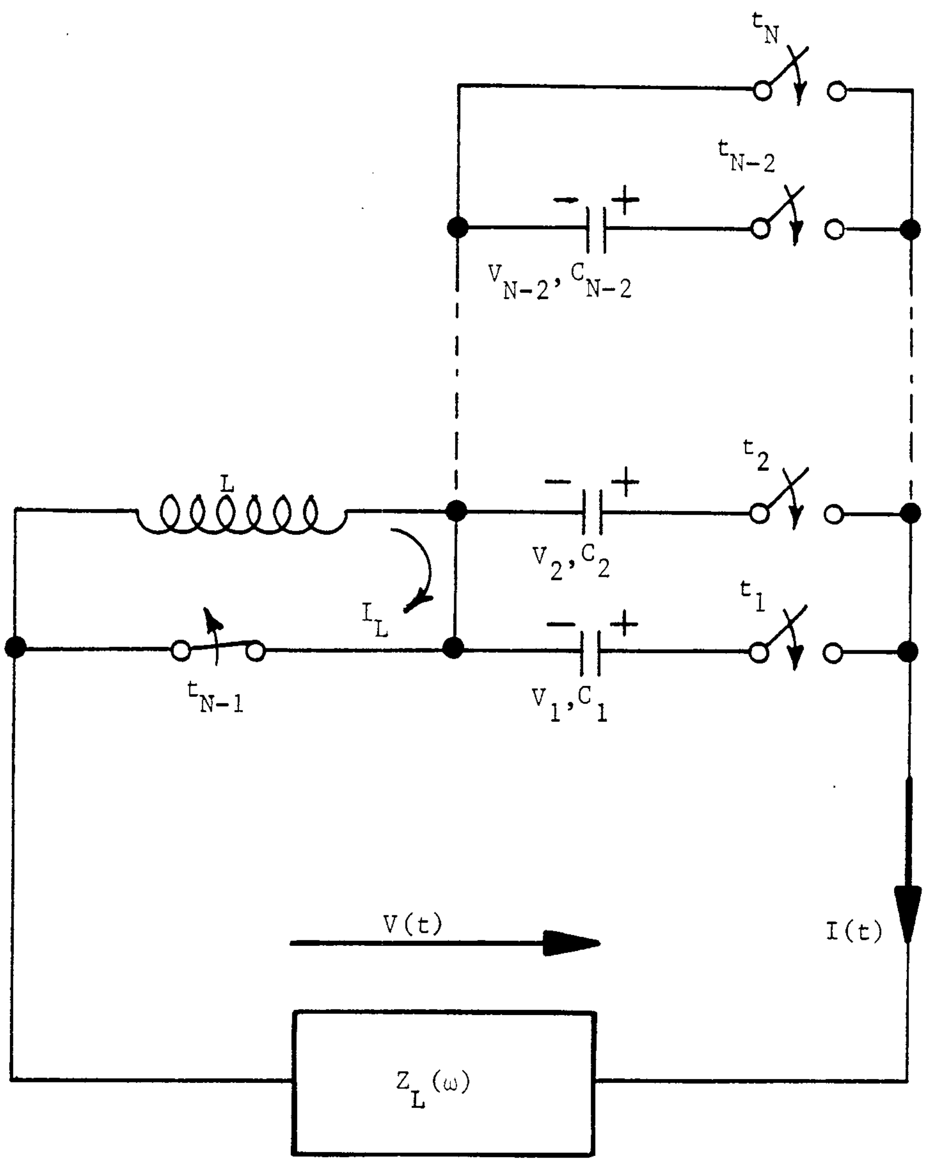

FIGURE 1 EQUIVALENT CIRCUIT FOR THE BURIED-TRANSMISSION-LINE SIMULATOR DRIVEN BY MULTIPLE CAPACITIVE AND SINGLE INDUCTOR SOURCES

time t_k , where $t_k < t_{k+1}$. The voltage, V_{n-1} , is defined as that voltage which happens to reside on the parallel capacitors at the time the inductor switch is opened.

In theory, the switches could be actuated at any time and in any sequence. However, there are definite advantages in choosing carefully the times and sequence of switching. The advantage in choosing the capacitor source first lies in its ability to get a large current flowing in the simulator in a short time. The closing of the first capacitor-source switch under high voltage has been successfully demonstrated by various spark-gap or ionization enhanced switches. The closing of successive capacitor switches occur with no voltage across them and therefore may be mechanical in nature. The closing of each switch corresponding to $k = 1,2,\ldots,N-2$ is assumed to be instantaneous and the switches are assumed to have no resistance when closed.

During the period when energy is being released from the multiple capacitors, the inductor is out of the circuit and the simulator current reaches maximum shortly after the time a capacitor is connected to the simulator. To open the (N-1)th switch without having the current in the simulator line equal to that in the inductor would give an instantaneous voltage spike across the switch similar to that when the inductive source alone energizes the line. The calculations in this note apply for the particular kind of switch where one desires the current to be zero while the switch is opened. The initial inductor current is chosen to be equal to the simulator current at the time when the (N-1)th switch is opened. Furthermore, in order to simplify the problem of timing the switch actuation, the time derivatives of both simulator and inductor current should be equal. For these combined reasons, the inductor switching time is chosen to be the precise time when the simulator current reaches the last peak due to the capacitor sources.

The parameters of the first capacitor, V_1 and C_1 , may be chosen to give a certain rise-time and peak current into the load. The values of t_k , V_k , and C_k for $1 < k < N - 1$ are normalized for convenience. The normalized values are chosen to give the proper degree of ripple in the waveform. The N-th switch is closed at the precise time when the original energy in the capacitors has been completely released. This procedure prevents energy from transferring back into the capacitors from the inductor and simulator. The value of L may be chosen to give the proper decay time and width to the pulse.

The energy stored in the inductor, given by $\frac{1}{2} LI^2$, may account for a large portion of the total energy. When high energy content is desired in each pulse, it is more economical to supply energy to the simulator sources over a relatively long period of time. So long as conventional energy sources are used, the energy build-up will take more time than the energy release into the simulator. The rate at which the initial capacitor voltage and initial inductor current are built up may be determined by the requirements set down on pulse-length and repetition-rate.

II. Problem Solution

The general solution of the circuit equations for transient/pulse generators are normally carried out using Laplace transform techniques. The solution to these equations becomes difficult when the load impedance is a function containing the square-root of frequency. Furthermore, the problem becomes increasingly complicated when a number of different generator sources are switched sequentially onto the load.

This problem is formulated such that a numerical evaluation of integrals will avoid the difficulties experienced by one attempting to obtain a closed form solution. The following solution advances along the lines developed in reference 2.

The voltage and current are complicated functions of time when a pulse generator, containing elements such as those shown in Figure 1, is used for a source. In this case, the load impedance is a function of frequency; the generator internal impedances are functions of frequency; and in addition, the generator impedance changes each time a switch is actuated. The relation between the terminal voltage and current in terms of the line load is

$$\tilde{V}(s) = \tilde{Z}_L(s)\tilde{I}(s) \quad , \qquad (1)$$

where the tilde signifies the Laplace transform of the quantity. The same voltage and current may be expressed in terms of the generator driving voltage and generator internal impedance as,

$$\tilde{V}(s) = \tilde{V}_g(s) - \tilde{I}(s)\tilde{Z}_g(s) \quad , \qquad (2)$$

from which one obtains the solution for the current in the frequency domain as

$$\tilde{I}(s) = \frac{\tilde{V}_g(s)}{\tilde{Z}_L(s) + \tilde{Z}_g(s)} \quad . \tag{3}$$

For simple circuit impedance functions, the inverse transform of equation (3) may be readily obtained by applying the usual inverse Laplace integral transform.

Great difficulty is experienced in obtaining the inverse transform when the denominator of equation (3) contains odd higher powers of the variable $s^{1/2}$, such as $s^{3/2}$, $s^{5/2}$,... etc. To avoid this difficulty, the solution to the simultaneous equations (1) and (2) may be written in the implicit form

$$\begin{aligned} \tilde{I}(s) &= \left[\tilde{V}_g(s) - \tilde{I}(s)\tilde{Z}_g(s)\right]\tilde{G}(s) \\ &= \tilde{V}(s)\tilde{G}(s) \end{aligned} \tag{4}$$

where $G(s) = \left[Z_L(s)\right]^{-1}$ is the Laplace transform of the impulse response of the load admittance.

The convolution theorem[6] may be used to obtain the inverse transform of equation (4). Thus,

$$I(t) = \int_0^t V(t')G(t - t')dt' \quad . \tag{5}$$

In order to perform the integration one must know both the impulse-response admittance, $G(t)$, of the load, and the previous voltage history, $V(t)$, at the common generator-load terminals. For most cases, the admittance function may be found in closed form by taking the inverse Laplace transform of the load admittance. For complicated functions, the time response may be tabulated without creating a problem in the numerical evaluation of the integral in equation (5).

The previous voltage history may be obtained by taking the inverse transform of equation (2). Of course the previous voltage history may be remembered or stored in an array and recalled when needed. Remembering this voltage over an interval, facilitates a solution for different generator circuits. When the voltage is known over an interval, for example the interval between 0 and t_1, equation (5) may be expressed as

$$I(t) = \int_{o}^{t_1} V(t')G(t - t')dt' + \int_{t_1}^{t} V(t')G(t - t')dt' \quad . \qquad (6)$$

This technique may be extended over as many new intervals as one may have the desire to do so, or the capacity to remember data. It should be pointed out that each new current involves the recalculation of each integrand since the admittance function is convoluted.

The complete solution involves the simultaneous solution of equation (6) and the generator circuit equation.

The generator circuit is complicated by the sequential switching of the various energy sources into the line. The straightforward circuit equation for a given period may be written as

$$\left.\begin{aligned} V(t) &= V_k - \frac{S_k}{C_1}\int_{t_k}^{t} I(t')dt' \quad , && \text{for} \quad k = 1 \\ & && \qquad\quad \vdots \\ & && \qquad\quad k = N-2 \\ &= V_{N-1} - \frac{S_{N-2}}{C_1}\int_{t_{N-2}}^{t} I(t')dt' - L\frac{dI(t)}{dt} \quad , && \qquad\quad k = N-1 \\ &= -L\frac{dI(t)}{dt} \quad , && \qquad\quad k = N \quad , \end{aligned}\right| \qquad (7)$$

where $S_k \equiv \left[\frac{1}{C_1} \sum_{n=1}^{k} C_n\right]^{-1}$.

Numerical techniques may be used to closely approximate the combined expressions (6) and (7) such that the current at time t may be solved for in terms of the values at time $t - \Delta t$ and earlier times.

In some cases a numerical instability may exist between the discrete time representation of the circuit equations and the discrete time approximation to the convolution integral. In such cases, the solution may be enacted by first integrating equation (6) by parts to get

$$I(t) = V(t')F(t - t')\Big|_{t'=o}^{t'=t} - \int_o^t V'(t')F(t - t')dt' \tag{8}$$

where $F(t - t')$ and $V'(t')$ are defined by

$$F(t - t') = \int_o^{t'} G(t - t'')dt'' \tag{9}$$

and

$$V'(t') = \frac{dV(t')}{dt'} \quad . \tag{10}$$

Equation (8) may be combined with the generator circuit equations (7) to complete a numerical evaluation of the current. For most problems this later formulation involves more numerical work than the straightforward convolution integral.

III. Infinite-length Transmission Line

Consider the buried transmission line whose length is great enough for the diffusion time to be much longer than the times of interest. If one ignores the end effects at the connection to the transmission line, the impedance of the infinite line is

$$Z_L = f_g \sqrt{\frac{s\mu}{\sigma}} \quad , \tag{11}$$

where f_g is a dimensionless factor which correlates the impedance of the line with the wave impedance, and μ and σ are the permeability and conductivity respectively of the soil.

A list of normalizing equations have been worked out which give the circuit equations and the convolution equation in simplified form. They are:

$$\left.\begin{aligned}
t_c &\equiv \left(C_1^2 f_g^2 \frac{\mu}{\sigma}\right)^{1/3} , &\quad \tau_c &\equiv \frac{t}{t_c} \\
\tau_{c_k} &\equiv \frac{t_k}{t_c} , &\quad s_c &= s t_c \\
v_k &\equiv \frac{V_k}{V_1} , &\quad e_c(\tau_c) &= \frac{V(\tau_c)}{V_1} \\
I_c &\equiv \frac{V_1 C_1}{t_c} , &\quad h_c(\tau_c) &= \frac{I(\tau_c)}{I_c} \\
t_L &\equiv \sqrt{LC_1} , &\quad \frac{t_L}{t_c} &= \left[\frac{L\sigma^{2/3}}{C_1^{1/3} f_g^{4/3} \mu^{2/3}}\right]^{1/2}
\end{aligned}\right| \tag{12}$$

The normalized Laplace transforms of the current, voltage and line admittance are related by

$$\tilde{h}_c(s_c) = \tilde{e}_c(s_c)g_c(s_c) \tag{13}$$

where $g_c(s_c) = \dfrac{1}{\sqrt{s_c}}$.

In the normalized time domain, the admittance function is

$$g_c(\tau_c) = \frac{1}{\sqrt{\pi\tau_c}} \tag{14}$$

where the region of interest is $0 \leq \tau_c$. The convolution integral becomes

$$h_c(\tau_c) = \int_0^{\tau_c} e_c(\tau_c')g_c(\tau_c - \tau_c')d\tau_c' \tag{15a}$$

or alternatively,

$$h_c(\tau_c) = - e_c(o)F(\tau_c) - \int_0^{\tau_c} \frac{de_c(\tau')}{d\tau_c} F(\tau_c - \tau_c')d\tau_c' \tag{15b}$$

where

$$F(\tau_c) = - \frac{2}{\sqrt{\pi}} \tau_c^{1/2} \quad . \tag{15c}$$

The generator voltage equations after normalization are,

$$e_c(\tau_c) = v_k - S_k \int_{\tau_{c_k}}^{\tau_c} h_c(\tau_c')d\tau_c' \qquad , \qquad \text{for} \quad k = 1 \tag{16}$$

$$\vdots$$

$$k = N-2$$

$$e_c(\tau_c) = v_{N-1} - S_{N-2} \int_{\tau_{c_{N-1}}}^{\tau_c} h_c(\tau_c')d\tau_c' - \frac{t_L^2}{t_c^2} \frac{dh_c(\tau_c)}{d\tau_c} \qquad , \qquad k = N-1 \tag{17}$$

$$e_c(\tau_c) = - \frac{t_L^2}{t_c^2} \frac{dh_c(\tau_c)}{d\tau_c} \qquad , \qquad k = N \quad . \tag{18}$$

The voltage expression in the frequency domain is complicated and the use of this convolution technique facilitates the solution without using it.

To obtain a numerical evaluation of (15) through (18), define a set of variables, E_I , H_I , and G_I which correspond to the voltage current and admittance for discrete times. Discrete time being defined by

$$T_I \equiv (I - 1)\Delta T \qquad , \tag{19}$$

where I is a positive integer and ΔT is a positive time increment. The time corresponding to $I = 1$ is $t = 0$. The values given by

$$E_1 = 1 \qquad , \qquad H_1 = 0 \tag{20}$$

are the initial conditions.

By using the simple trapezoidal approximation, the convolution integral, equation (15a), may be approximated by

$$H_I \simeq \frac{\Delta T}{2} \sum_{J=1}^{I-2} \left\{E_J G_{I-J+1} + E_{J+1} G_{I-J}\right\} + \frac{1}{2} \left\{E_{I-1} + E_I\right\} \int_{T_{I-1}}^{T_I} \frac{d\tau_c'}{\sqrt{\pi (T_I - \tau_c')}} \tag{21}$$

where the last term is an approximation for the interval near the singularity of the admittance function used in convolution. Equation (21) reduces to

$$H_I \simeq B_{I-1} + \sqrt{\frac{\Delta T}{\pi}}\, E_I \tag{22}$$

where

$$B_{I-1} \equiv \frac{\Delta T}{2} \left\{E_1 G_I + E_{I-1} G_2\right\} + \Delta T \sum_{J=2}^{I-2} E_J G_{I-J+1} + \sqrt{\frac{\Delta T}{\pi}}\, E_{I-1} \tag{23}$$

contains the contribution to H_I due to the previous voltage on the simulator terminals. For $I = 2$ and $I = 3$, the appropriate terms must be removed from B_{I-1}.

The alternative discrete time expression for the convolution integral after integration by parts is

$$H_I \simeq D_{I-1} - F_2 E_I \tag{24}$$

where

$$D_{I-1} \equiv F_2 E_{I-1} - \frac{1}{2} F_I (E_1 + E_2) - \sum_{J=3}^{I-1} (E_J - E_{J-1}) F_{I-J+2} \tag{25}$$

contains the contribution to H_I due to the previous history of voltage on the simulator. The integral of the line admittance, equation (9), is given in discrete time by

$$F_I \equiv F(T_I) \tag{26}$$

where values are taken from equation (15c).

In the following paragraphs we will develop an expression for E_I in terms of H_I and the generator parameters used in equations (16), (17) and (18), so that substitution into either equation (22) or (24) will yield an explicit equation for H_I. Knowing H_I, the value of E_I may immediately be found from the generator circuit relations. Each value of E_I and H_I may be stored or recorded for use in subsequent calculations of B_{I-1} and D_{I-1}.

Capacitor Drivers $\quad t < t_{N-1}$

At the beginning, the energy sources in the circuit are only capacitors which are individually switched into the line. The inductance is zero during this period $(k \leq N-2)$; so equation (16) is pertinent.

Using the trapezoidal approximation for the integral, the discrete time approximation of the generator circuit equation yields a recurrence relation

$$E_I \simeq E_{I-1} - S_k \frac{H_{I-1} + H_I}{2} \Delta T \quad . \tag{27}$$

Grouping all the terms containing the previous values of E_I and H_I into one term,

$$A_{I-1} \equiv E_{I-1} - \frac{1}{2} S_k \Delta T \; H_{I-1} \quad , \tag{28}$$

the recurrence relation becomes

$$E_I \simeq A_{I-1} - \frac{1}{2} S_k \Delta T\ H_I \quad . \tag{29}$$

Substitution of equation (29) into equation (22) provides an expression for H_I .

$$H_I \simeq \frac{\sqrt{\frac{\Delta T}{\pi}}\ A_{I-1} + B_{I-1}}{1 + \frac{S_k \Delta T}{2} \sqrt{\frac{\Delta T}{\pi}}} \quad . \tag{30}$$

By making ΔT small, this equation may be used to solve for successive values of the current up to the time the inductor is switched into the circuit. When improved accuracy is desired, ΔT may be decreased.

After each calculation of current, the voltage must be found for use in calculating the next value of current. The voltage is easily found from the recurrence relation

$$E_I \simeq E_{I-1} - \frac{1}{2} S_k \Delta T \left(H_I + H_{I-1} \right) \quad . \tag{31}$$

<u>Capacitor-Inductor Drivers</u> $\quad t_{N-1} \leq t < t_N$

In the introduction we pointed out certain switching times and relative current amplitudes which give the least difficulty in performing the appropriate switching of idealized circuit components. These times and amplitudes also give good results in our attempt to generate a current waveform with low ripple. For this reason, we will proceed in this analysis under the assumption that the inductor is switched into the circuit at a time when all the capacitors are already in the circuit. During the time interval when $k = N-1$, the generator voltage equation, (17) may be approximated in discrete time as

$$E_I \simeq E_{I'} - S_{N-2} \sum_{J=I'+1}^{I} \frac{H_J + H_{J-1}}{2} \Delta T - \frac{t_L^2}{t_c^2} \frac{H_I - H_{I-1}}{\Delta T} \tag{32}$$

where $E_{I'}$ is the capacitor voltage at time $t = t_{N-1}$. The recurrence relation may be obtained by writing the (I-1)th term and subtracting it from equation (32) in order to eliminate $E_{I'}$.

The recurrence relation is

$$E_I \simeq AA_{I-1} - \left(\frac{t_L^2}{t_c^2 \Delta T} + S_{N-2} \frac{\Delta T}{2} \right) H_I \quad , \tag{33}$$

where

$$AA_{I-1} \equiv E_{I-1} - \left(S_{N-2} \frac{\Delta T}{2} - \frac{2t_L^2}{\Delta T t_c^2} \right) H_{I-1} - \frac{t_L^2}{t_c^2 \Delta T} H_{I-2}$$

The simultaneous solution of equation (33) and (22) provide an algebraic expression for H_I .

$$H_I \simeq \frac{B_{I-1} + \sqrt{\frac{\Delta T}{\pi}}\, AA_{I-1}}{1 + \sqrt{\frac{\Delta T}{\pi}} \left(S_{N-2} \frac{\Delta T}{2} + \frac{t_L^2}{t_c^2 \Delta T} \right)} \tag{34}$$

It is important to note that in the development of this equation, the current in the inductor at two previous discrete times contributes to the voltage. During the two intervals of time before the inductor switch opened, the inductor current in equation (32) was assumed to be equal to the simulator current, i.e. the inductor current and its first derivative are equal to the simulator current and its first derivative at the time the switch is opened.

If the conditions are not matched in this way, for the first two time intervals prior to the switching time, the inductor current is different from the simulator current. Equation (34) becomes invalid for evaluating the simulator current for the first two intervals of time after the switch is opened. A separate expression must be developed for each interval. By designating the switching time as $T_{I'}$ the generator voltage at time $T_{I'+1}$ may be expressed as

$$E_{I'+1} = AA'_{I'} - \left(S_{N-2} \frac{\Delta T}{2} + \frac{t_L^2}{t_c^2 \Delta T} \right) H_{I'+1} \quad , \tag{35}$$

where

$$AA'_{I'} \equiv E_{I'} - S_{N-2} \frac{\Delta T}{2} H_{I'} + \frac{t_L^2}{t_c^2 \Delta T} H'_{I'}$$

and H'_I is the normalized value of inductor current before the switch is opened. In the limit as ΔT approaches zero, the right side of equation (35) becomes infinite. If the inductor current is greater than the simulator current, there is a positive voltage spike at the time the inductor is placed in the circuit. If the inductor current is less than the simulator current, there is a negative voltage spike at the time the inductor is placed in the circuit. However, if finite time differences are used, the voltage remains finite so that the voltage may be substituted into equation (22) to yield the relation.

$$H_{I'+1} \simeq \frac{B_{I'} + \sqrt{\frac{\Delta T}{\pi}}\, AA'_{I'}}{1 + \sqrt{\frac{\Delta T}{\pi}} \left(S_{N-2} \frac{\Delta T}{2} + \frac{t_L^2}{t_c^2 \Delta T} \right)} \quad . \tag{36}$$

At the conclusion of the second discrete time interval after t_{N-1} , the current is expressed by

$$H_{I'+2} \approx \frac{B_{I'+1} + \sqrt{\frac{\Delta T}{\pi}}\, AA'_{I+1}}{1 + \sqrt{\frac{\Delta T}{\pi}} \left\{ S_{N-2} \frac{\Delta T}{2} + \frac{t_L^2}{t_c^2 \Delta T} \right\}} \tag{37}$$

where

$$AA'_{I'+1} \equiv E_{I'} - \frac{S_{N-2}\Delta T}{2} H_{I'} + \left\{ \frac{t_L^2}{t_c^2 \Delta T} - S_{N-2}\, \Delta T \right\} H_{I'+1}$$

The first two intervals adequately express the transient condition, so equation (32) may be used for $I \geq I' + 3$. Of course the effect of the transient voltage is always present to some degree in the waveform of current since the B_{I-1} term of equation (32) contains a summation that includes $E_{I'+1}$.

After each calculation of current, the corresponding voltage must be found and used in calculating all succeeding values of current. The voltage could be found directly from the recurrence relation shown in equation (33) except for the fact that this numerical expression is sensitive to the size of ΔT . In the limit as ΔT approaches zero, E_I approaches minus infinity.

By solving equation (24) for the voltage, E_I , this difficulty is overcome and the voltage may be calculated using the equation

$$E_I = \frac{D_{I-1} - H_I}{F_2} \quad . \tag{38}$$

In order to determine when to close the N-th switch, the time at which no energy remains in the capacitors must be determined. This may be done by finding the time the voltage on the capacitors goes to zero. The sum of the first two terms of equation (32) gives the capacitor voltage E_c ,

$$E_{c_I} = E_{I'} - S_{N-2} \frac{\Delta T}{2} \sum_{J=I'+1}^{I} (H_J + H_{J-1}) \tag{39}$$

When this voltage becomes zero, the N-th switch should be closed in order to prevent oscillations. From that time on, the inductor remains in the circuit alone.

<u>Inductor Driver</u> $t_N < t$

After the N-th switch is closed, the capacitor sources are removed from the circuit and equation (18) is valid. Since the current flowing at t_k is not zero, and furthermore, since the voltage drop across the inductor is not zero, it is preferable to use the following alternative relationship between the generator voltage and current:

$$I(t) = -\frac{1}{L} \int_{t_N}^{t} V(t)dt + I_L(t_N) \quad . \tag{40}$$

In the normalized discrete time approximation, the current may be expressed in terms of the voltage and current $H_{I''}$ that flows in the simulator at time t_N . Using the trapazoidal approximation for the integral, the current after time t_N may be expressed as

$$H_I \simeq H_{I''} - \frac{t_c^2}{t_L^2} \sum_{J=I''+1}^{I} \frac{E_J + E_{J-1}}{2} \Delta T \quad . \tag{41}$$

Thus, the following relation may be found for E_I :

$$E_I \simeq AAA_{I-1} - \left(\frac{2}{\Delta T}\right) \frac{t_L^2}{t_c^2} H_I \quad , \tag{42}$$

where

$$AAA_{I-1} \equiv \left(\frac{2}{\Delta T}\right) \frac{t_L^2}{t_c^2} H_{I-1} - E_{I-1} \quad .$$

Substituting equation (42) into (22) yields an expression for the normalized current:

$$H_I \simeq \frac{B_{I-1} + \sqrt{\frac{\Delta T}{\pi}}\, AAA_{I-1}}{1 + \sqrt{\frac{\Delta T}{\pi}} \left(\frac{2}{\Delta T}\right) \frac{t_L^2}{t_c^2}} \quad . \tag{43}$$

As in the previous case, the voltage may be found by substituting the current into equation (38). If the switch shorting out the capacitors is actuated at a time other than that when the energy in the capacitors is zero, appropriate changes must be made in equation (41) to account for the line voltage E_{I-1}, not being the inductor voltage at the time the switch is thrown.

Numerical Results

Three case examples were computed in order to show what effect that a change in inductance has on the current pulse decay time. The cases presented are one capacitor, two capacitor, and three capacitor cases; each is followed by an inductive source. The inductive source is initially charged with a continuous current equal to that flowing in the transmission line at the time the inductor switch is opened. In all cases, a shorting switch removes the capacitor/capacitors from the circuit once their energy is spent.

Figures 2, 3 and 4 present the voltage current in normalized time, τ_c, with seven different normalized inductance values t_L/t_c. They are 0, 1, 2, 4, 9, 16 and 36. The data for $t_L/t_c = 0$ agrees quite closely to that presented in Note 49 with no inductor.[2] The capacitor values and voltages were chosen to give approximately 10% ripple in the current waveform up to the time the inductor is switched into the circuit. Table 1 is a summary of the important characteristics of the curves. The initial peak of h_c in all cases is 0.722. The normalized time increment $\Delta T = 0.025$ was selected for use in all calculations on the basis that a 100% increase in ΔT causes less than 1% change in h_{max}.

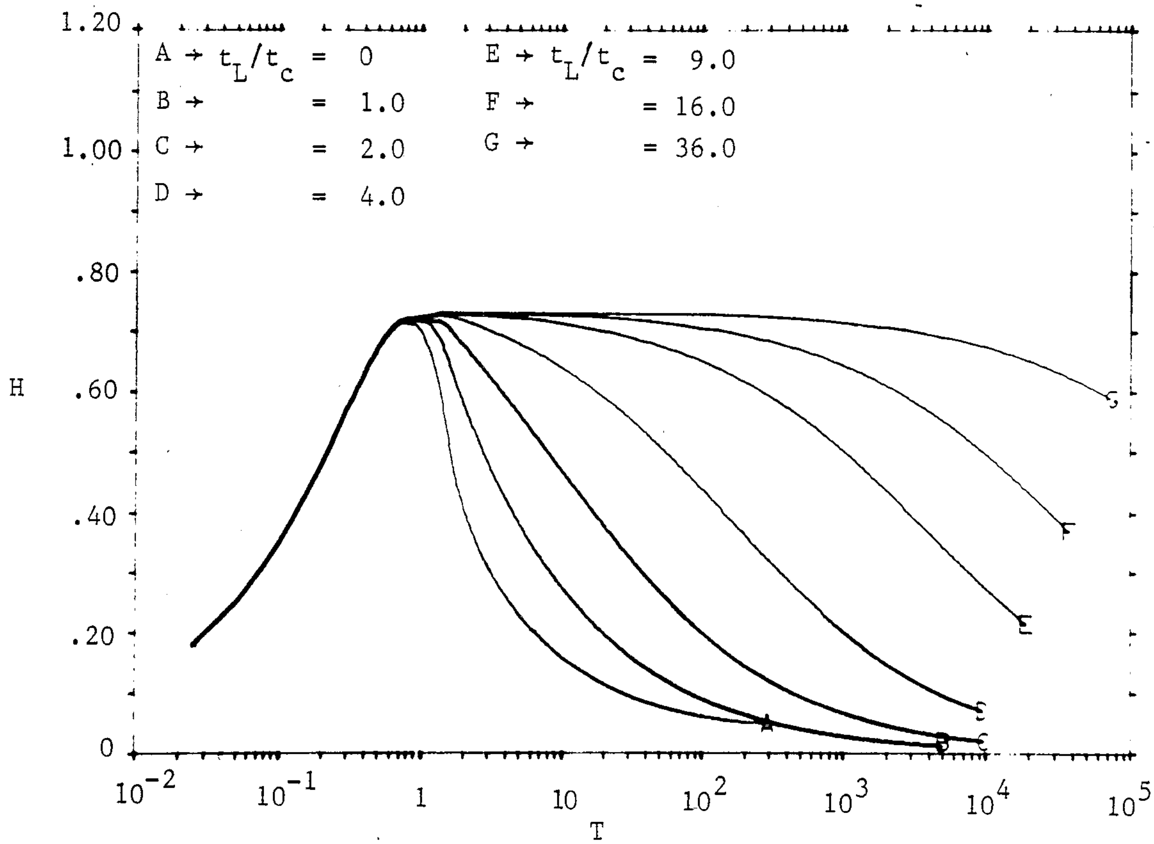

Figure 2a. Current versus Time; Infinite Line with One Capacitor.

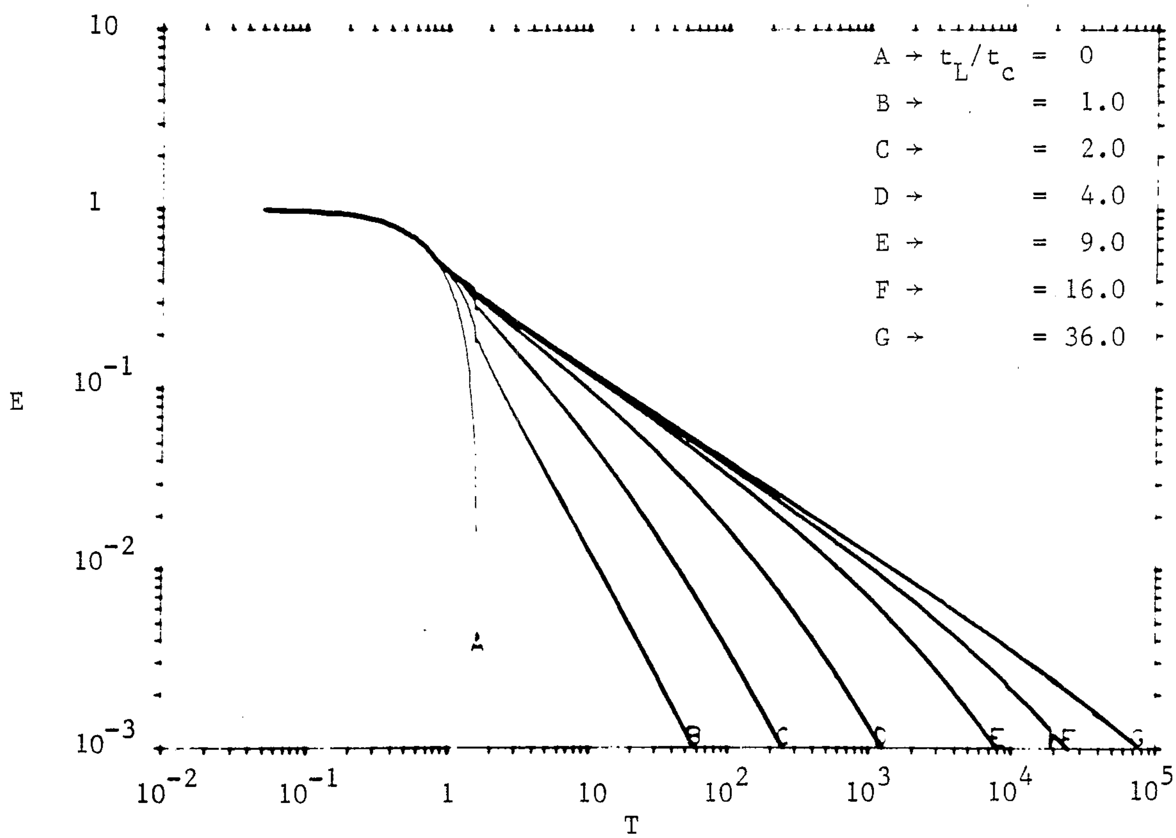

Figure 2b. Voltage versus Time; Infinite Line with One Capacitor.

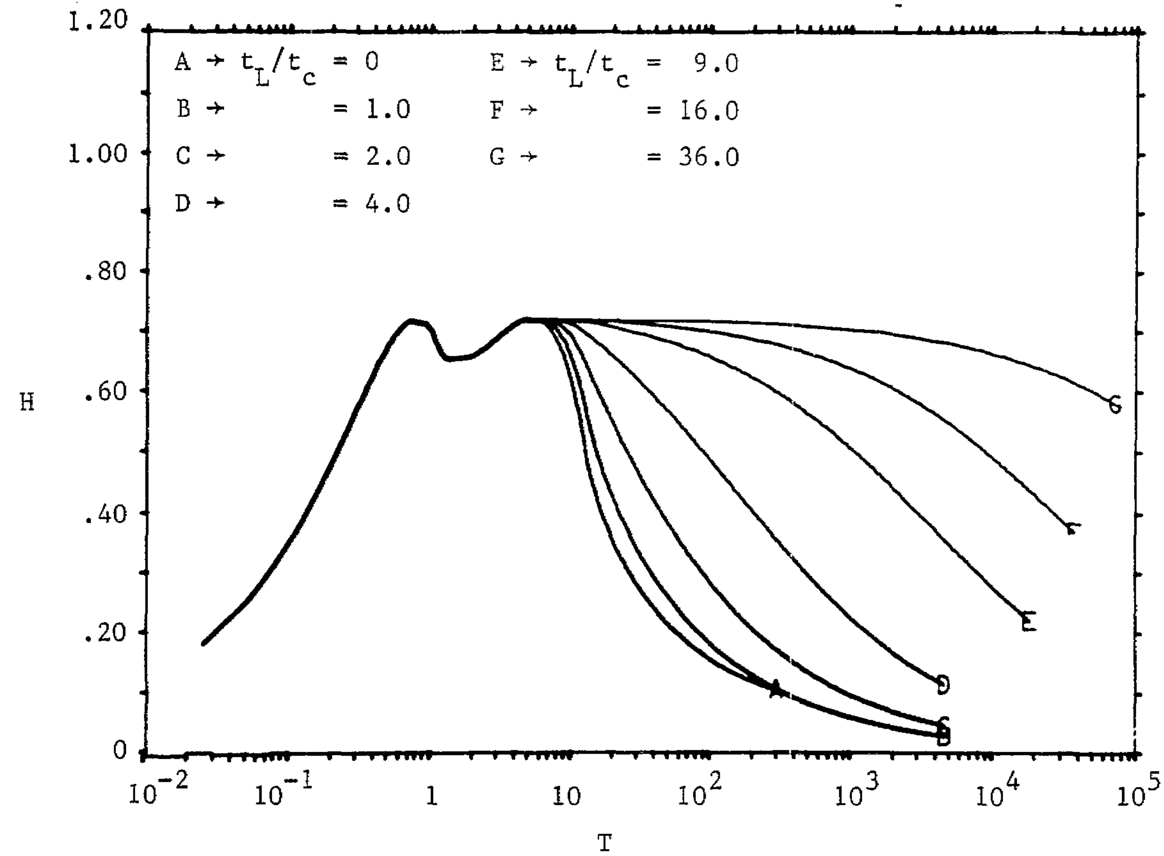

Figure 3a. Current versus Time; Infinite Line with Two Capacitors.

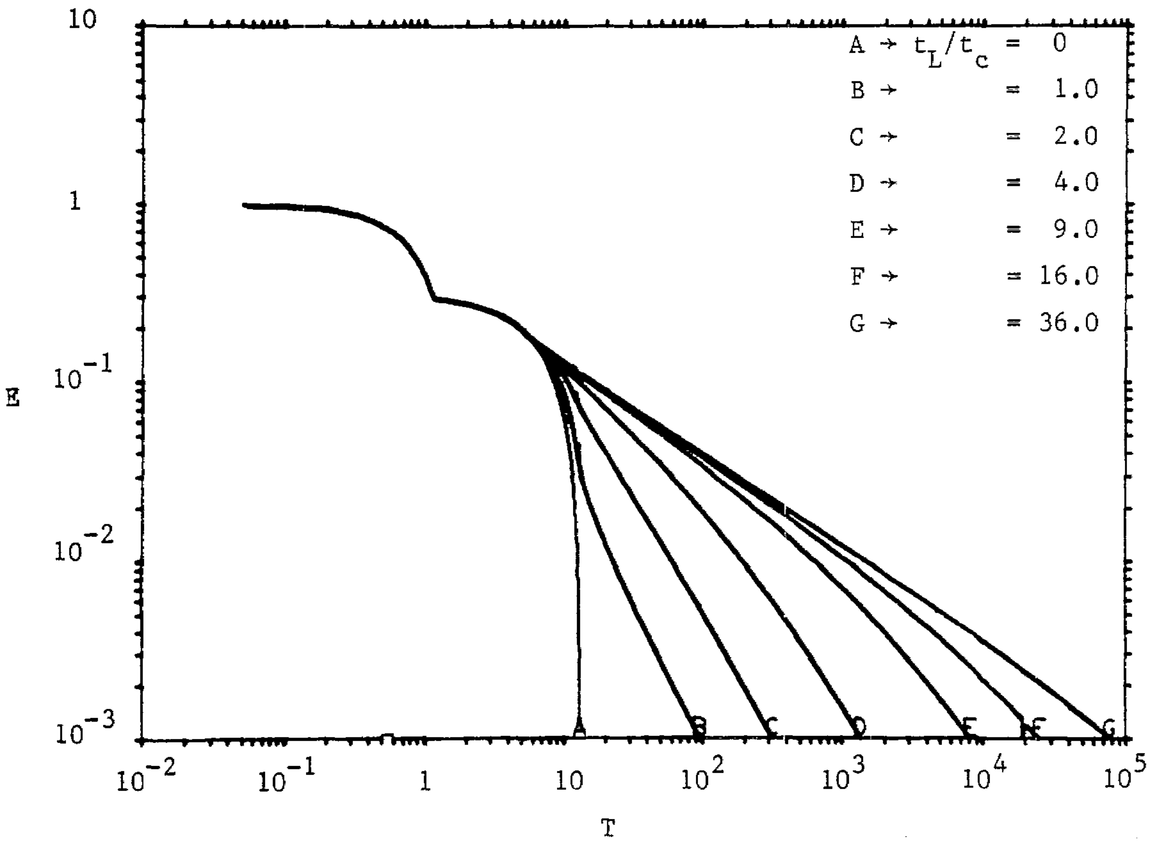

Figure 3b. Voltage versus Time; Infinite Line with Two Capacitors.

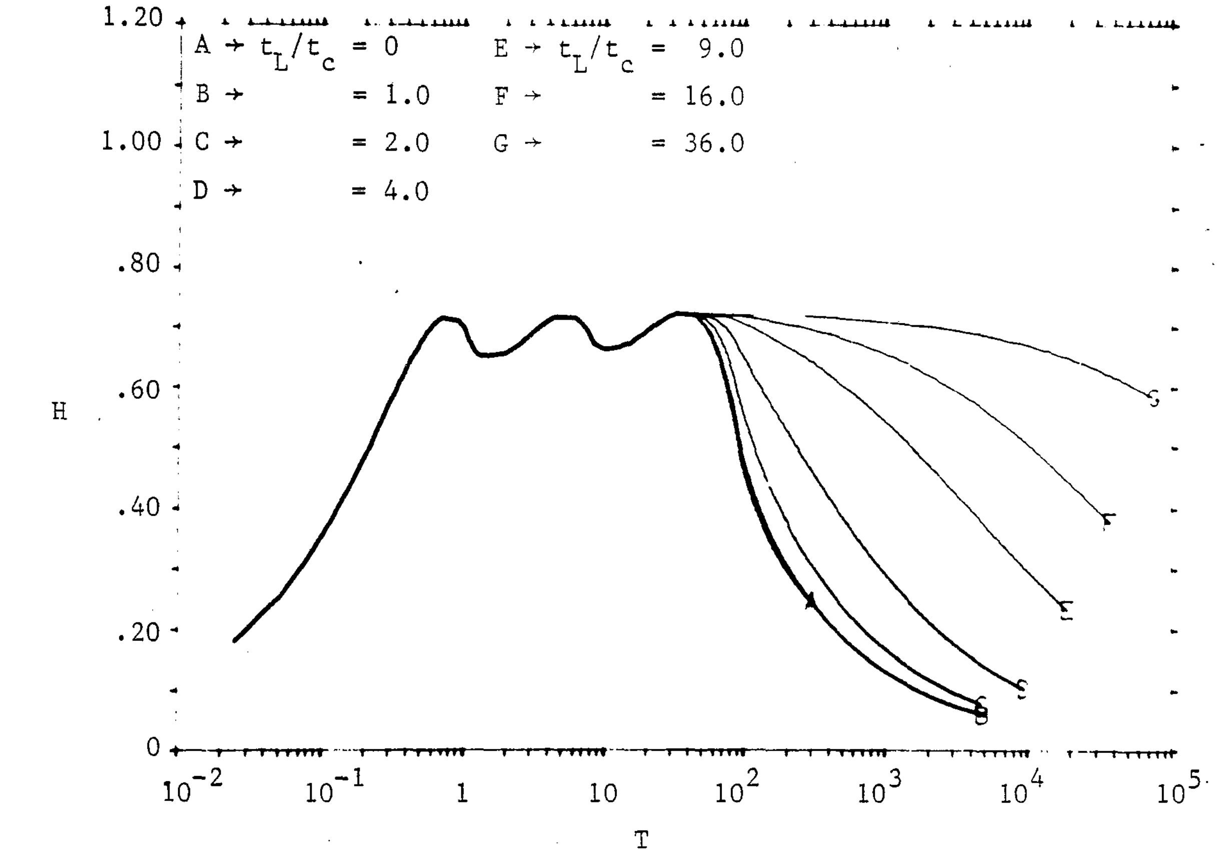

Figure 4a. Current versus Time; Infinite Line with Three Capacitors.

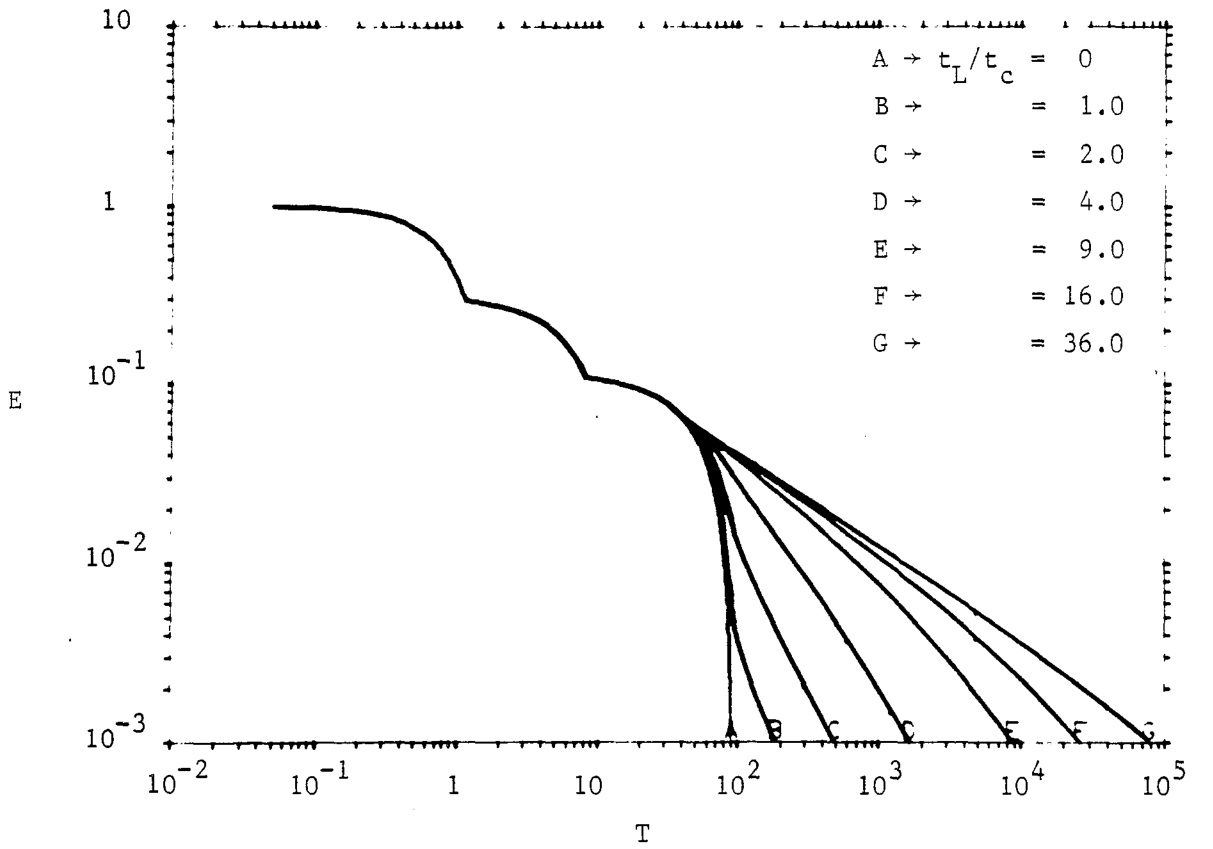

Figure 4b. Voltage versus Time; Infinite Line with Three Capacitors.

Table 1: Infinite Line Data Summary

	Number of Capacitors	1 (S_1 = 1)			2 (S_2 = .038)			3 (S_3 = .002)		
	t_L/t_c	V_3	τ_{c_3}	Fixed Values	V_4	τ_{c_4}	Additional Fixed Values	V_5	τ_{c_5}	Additional Fixed Values
↑	36	.34	1.55	V_2 = .55 τ_{c_2} = .80	.13	12.18	V_2 = .30 τ_{c_2} = 1.15 V_3 = .19 τ_{c_3} = 5.35	.010	84.83	V_3 = .11 τ_{c_3} = 8.23 V_4 = .07 τ_{c_4} = 36.40
	16	.34	1.55		.13	12.18		.010	84.88	
Larger	9	.34	1.55		.13	12.20		.010	85.03	
Inductor	4	.32	1.55		.12	12.23		.009	85.78	
	2	.28	1.58		.09	12.38		.008	87.55	
	1	.19	1.58		.05	12.68		.006	89.65	
		V_2	τ_{c_2}	Fixed Values	V_3	τ_{c_3}	Additional Fixed Values	V_4	τ_{c_4}	Additional Fixed Values
No Inductor	0	0	1.68	--	0	13.10	V_2 = .30 τ_{c_2} = 1.15	0	91.15	V_3 = .11 τ_{c_3} = 8.23

IV. Finite-length Open-end Transmission Line

An analysis of interest in practical transmission-line simulator work is the one containing no approximation in the input impedance based on the diffusion time being long with respect to the times of interest. This section will consider the terminal current and voltage in a simulator of finite length, ℓ, which has a diffusion time

$$t_\ell = \frac{\mu\sigma\ell^2}{4} \quad . \tag{44}$$

The reflections from the bottom end of the transmission line are significant for the times and depths of interest. The input impedance of the open-end transmission line is[5]

$$Z_{LO} = 2R_o\sqrt{st_\ell}\,\frac{1 + e^{-4\sqrt{st_\ell}}}{1 - e^{-4\sqrt{st_\ell}}} \tag{45}$$

where $R_o \equiv f_g/(\ell\sigma)$ is the resistance of the open-circuited line at zero frequency.

The generator circuit is identical with that used to drive the infinite-length line. Equation (2) may be written for the finite line and then normalized to become

$$\tilde{h}_\ell(s_\ell) = \tilde{e}_\ell(s_\ell)\tilde{g}(s_\ell) \tag{46}$$

where the following normalizing parameters apply:

$$t_o \equiv R_oC_1 \quad , \quad \tau_\ell = \frac{t}{t_\ell}$$

$$t'_L \equiv L/R_o \quad , \quad t'_L/t_\ell = \frac{4L}{f_g\mu\ell}$$

$$S'_k \equiv \frac{t_\ell}{t_o} S_k \quad , \quad t_o/t_\ell = \frac{4f_gC_1}{\mu\sigma^2\ell^3} \tag{47}$$

$$I_o \equiv V_1/R_o \quad , \quad s_\ell \equiv t_\ell s$$

$$h_\ell(\tau_\ell) = \frac{I(\tau_\ell)}{I_o} \quad , \quad e_\ell(\tau_\ell) = \frac{V(\tau_\ell)}{V_1}$$

The normalized line admittance in the frequency domain is

$$\tilde{g}(s_\ell) \equiv \frac{1}{2s_\ell^{\frac{1}{2}}} \left(\frac{1 - e^{-4\sqrt{s_\ell}}}{1 + e^{-4\sqrt{s_\ell}}} \right) \quad . \tag{48}$$

The inverse transform of the input admittance of the open-circuited line has been used previously in two equivalent forms. The first form,[5] more convenient for small times $\tau_\ell \leq 1$, is

$$g(\tau_\ell) = \frac{1}{2\sqrt{\pi\tau_\ell}} \left(1 + 2 \sum_{n=1}^{\infty} (-1)^n e^{-4n^2/\tau_\ell} \right) \quad , \tag{49}$$

and an equivalent expression,[3] useful for $\tau_\ell > 1$, is

$$g(\tau_\ell) = \frac{e^{-\pi^2\tau_\ell/16}}{2}\left[1 + \sum_{n=1}^{\infty} e^{-n(n+1)\pi^2\tau_\ell/4}\right] \quad . \tag{50}$$

The convolution integral becomes,

$$h_\ell(\tau_\ell) = \int_0^{\tau_\ell} e_\ell(\tau_\ell')g(\tau_\ell - \tau_\ell')d\tau_\ell' \tag{51}$$

or alternatively

$$h_\ell(\tau_\ell) = f(\tau_\ell) - \int_0^{\tau_\ell} \frac{de_\ell(\tau_\ell')}{d\tau_\ell} f(\tau_\ell - \tau_\ell')d\tau_\ell' \tag{52}$$

where

$$f(\tau_\ell) \equiv -1 + \frac{8}{\pi^2} e^{\frac{-\pi^2\tau_\ell}{16}} \sum_{n=o}^{\infty} \frac{e^{\frac{-n(n+1)\pi^2\tau_\ell}{4}}}{(2n+1)^2} \quad . \tag{53}$$

Normalization when applied to the generator circuit equations gives,

$$e_\ell(\tau_\ell) = v_k - S_k' \int_{\tau_{\ell_k}}^{\tau_\ell} h(\tau_\ell')d\tau_\ell' \quad , \qquad \text{for} \quad k = 1, \ldots, k = N-2 \tag{54}$$

$$e_\ell(\tau_\ell) = v_{N-1} - S'_{N-2} \int_{\tau_{\ell_{N-1}}}^{\tau_\ell^-} h(\tau_\ell')d\tau_\ell' - \frac{t_L'}{t_\ell}\frac{dh(\tau_\ell)}{d\tau_\ell} \quad , \quad k = N-1 \tag{55}$$

$$e_\ell(\tau_\ell) = -\frac{t_L'}{t_\ell}\frac{dh(\tau_\ell)}{d\tau_\ell} \quad , \qquad k = N \quad . \tag{56}$$

The discrete time form for the admittance is

$$GL_I \equiv g(T_I) \quad . \tag{57}$$

The corresponding normalized voltage and current for the finite line are EL_I and HL_I respectively. The convolution integral may be written in the discrete approximation as

$$HL_I \simeq BL_{I-1} + \frac{1}{2}\sqrt{\frac{\Delta T}{\pi}}\, EL_I \quad , \tag{58}$$

where we define

$$BL_{I-1} \equiv \frac{\Delta T}{2}\left\{EL_1 GL_I + EL_{I-1} GL_2\right\} + \Delta T \sum_{J=2}^{I-2} EL_J GL_{I-J+1} + \frac{1}{2}\sqrt{\frac{\Delta T}{\pi}}\, EL_{I-1} \quad . \tag{59}$$

Equation (58) may be solved simultaneously with one of the generator voltage-current expressions (54) through (56) to obtain the value of HL_I. The inverse relationship analogous to equation (22) may be solved to give EL_I in terms of the known current and past voltage.

$$EL_I = 2\sqrt{\frac{\pi}{\Delta T}}\left\{HL_I - BL_{I-1}\right\} \tag{60}$$

These relations in discrete time differ very little from those of the finite line (see equations 22 and 23).

By defining the integral of the line admittance in discrete time as

$$F_I \equiv f(T_I) \quad , \tag{61}$$

the alternative form of the convolution integral may be approximated by equations (24) and (25), which were developed to approximate the infinite line convolution integral. These equations may be used for the finite line by replacing E_I by EL_I and H_I by HL_I.

The discrete time approximation of the circuit equations are exactly like those of the finite line (equations 27, 33 and 42) except that S_k and $(t_L/t_c)^2$ are replaced by S_k' and t_L'/t_ℓ respectively. Solving these equations over their respective time intervals yields the current and voltage expressions that follow.

<u>Capacitor Sources</u> $0 \leq t < t_{N-1}$

From the straight-forward convolution integral

$$HL_I \simeq \frac{BL_{I-1} + \frac{1}{2}\sqrt{\frac{\Delta T}{\pi}}\, AL_{I-1}}{1 + \frac{1}{2}\sqrt{\frac{\Delta T}{\pi}}\, S_k'\, \Delta T} \tag{62}$$

where

$$AL_{I-1} \equiv EL_{I-1} - S_k' \frac{\Delta T}{2} HL_{I-1} \quad ,$$

and

$$EL_I = 2\sqrt{\frac{\pi}{\Delta T}}\left(HL_I - BL_{I-1}\right) \quad . \tag{63}$$

The alternate form of the convolution integral, after integration by parts, becomes

$$HL_I \simeq \frac{D_{I-1} - AL_{I-1}F_2}{1 - \frac{\Delta T}{2} S_k'\, F_2} \tag{64}$$

and

$$EL_I = \frac{D_{I-1} - HL_I}{F_2} \tag{65}$$

<u>Inductor and Capacitor Sources</u> $t_{N-1} \le t < t_N$

The straightforward integral becomes,

$$HL_I \simeq \frac{BL_{I-1} + \frac{1}{2}\sqrt{\frac{\Delta T}{\pi}}\, AAL_{I-1}}{1 + \frac{1}{2}\sqrt{\frac{\Delta T}{\pi}}\left(\frac{t_L'}{t_\ell \Delta T} + S_k' \frac{\Delta T}{2}\right)} \quad , \tag{66}$$

where

$$AAL_{I-1} \equiv EL_{I-1} - \left(S_k' \frac{\Delta T}{2} - \frac{2t_L'}{t_\ell \Delta T}\right) HL_{I-1} - \frac{t_L'}{t_\ell \Delta T}\, HL_{I-2} \tag{67}$$

and

$$EL_I = 2\sqrt{\frac{\pi}{\Delta T}}\left(HL_I - BL_{I-1}\right) \quad . \tag{68}$$

Again, the alternative form for the convolution integral may be approximated using the discrete-time formulation as,

$$HL_I \simeq \frac{D_{I-1} - AAL_{I-1}\, F_2}{1 - F_2\left(\frac{t_L'}{t_\ell \Delta T} + S_k' \frac{\Delta T}{2}\right)} \tag{69}$$

with

$$EL_I = \frac{D_{I-1} - HL_I}{F_2} \quad . \tag{70}$$

<u>Inductor Alone</u> $\quad t_N \leq t$

The approximation of the straight-forward integral is

$$HL_I \simeq \frac{BL_{I-1} + \frac{1}{2}\sqrt{\frac{\Delta T}{\pi}}\, AAAL_{I-1}}{1 + \frac{t_L'}{t_\ell \Delta T}\sqrt{\frac{\Delta T}{\pi}}} \tag{71}$$

where

$$AAAL_{I-1} \equiv \left(\frac{2}{\Delta T}\right) \frac{t_L'}{t_\ell} HL_{I-1} - EL_{I-1} \tag{72}$$

and

$$EL_I = 2\sqrt{\frac{\pi}{\Delta T}} \left(HL_I - BL_{I-1}\right) \quad . \tag{73}$$

The alternative form for which is,

$$HL_I = \frac{D_{I-1} - AAAL_{I-1}\, F_2}{1 - F_2 \frac{2t_L'}{\Delta T t_\ell}} \tag{74}$$

and

$$EL_I = \frac{D_{I-1} - HL_I}{F_2} \quad . \tag{75}$$

By taking ΔT small enough, EL_I and HL_I closely approximate $e(\tau_\ell)$ and $h(\tau_\ell)$. As in the case of the infinite-length line, the initial inductor current used in these expressions is matched to the line current at t_{N-1}. Expressions which take into account the possibility of the inductor current not being matched to the line current would be similar to those in Section III.

Numerical Results

Results have been worked out for four separate ratios of initial capacitor discharge time to line diffusion time, i.e. t_o/t_ℓ. In each of these cases, the amount of inductance was varied from zero upward through six values to include t_L'/t_ℓ equal to 900. Graphical data is presented for both voltage and current versus time in figures 5 through 15. These graphs indicate that a high ratio of inductance will broaden the pulse.

The capacitor discharge time is equal to the diffusion time in figures 5 and 6 where $t_o/t_\ell = 1.0$. As t_o/t_ℓ ratio gets smaller, the relative pulse width gets broader (compare figures 5a, 7a, 10a and 13a). These curves also indicate that the current pulse broadens at a rate approximately linearly with the increase in inductance ratio t_L'/t_ℓ so long as this ratio is larger than 16. For smaller ratios, the variation in pulse width is less than linear and reaches a limit of zero when t_L'/t_ℓ equal zero.

When two or three capacitors are used, the time that the inductor is brought into the circuit may be large compared to the diffusion time, as in figures 6, 8, 9 and 12. The transmission line at this point in time appears more like a resistive load. In these cases, the pulse width is primarily controlled by the amount of energy stored in the second or third capacitor. If one tries to achieve a relatively flat-topped current pulse using the capacitors, one finds that when these sources are switched into the circuit at a sufficiently long relative time, the source will no longer effect a current increase. (Unless, of course the capacitive source is originally at a voltage exceeding the voltage on the line.) To extend the width of the pulse, one must use either an extremely large capacitive source or a very large inductive source.

In the data presented, the capacitor switching times were chosen to be those which would approximately give a 10% ripple in the current waveform. When the capacitive sources failed to give sufficient current rise, the ripple was decreased in order to achieve successive current peaks at or near the initial current peak. All current peaks were maintained to within a few percent of their initial peaks. These peak magnitudes are listed along with their respective times in Table 2. Table 2 also indicates the initial normalized time increment which was used in the calculations. After all switching had taken place, $\Delta\tau$ was increased so that data could be obtained for large normalized time. On the basis of data obtained from the finite line, the $\Delta\tau$ listed below gives the initial h_{max} to an accuracy of a few percent.

Table 2: Values of normalized current and normalized time for first peak in current waveform on the finite line

t_o/t_ℓ	h_{peak}	$\tau_{\ell_{peak}}$	$\Delta\tau$
1	.451	1.18	.025
.1	.211	0.28	.010
.01	.0981	0.060	.005
.001	.0457	0.013	.005

Table 3 summarizes the important voltages and switching times for the curves with no inductor. They agree quite well with the data obtained by C. Baum.[2] The data having various inductor ratio values are summarized in Table 4. This table lists the line voltage and actuation time of the last switch for the various inductor ratios. The last switch short-circuits the capacitors when they have no voltage. The fixed values give the line voltage and corresponding time of all previous switch actuations.

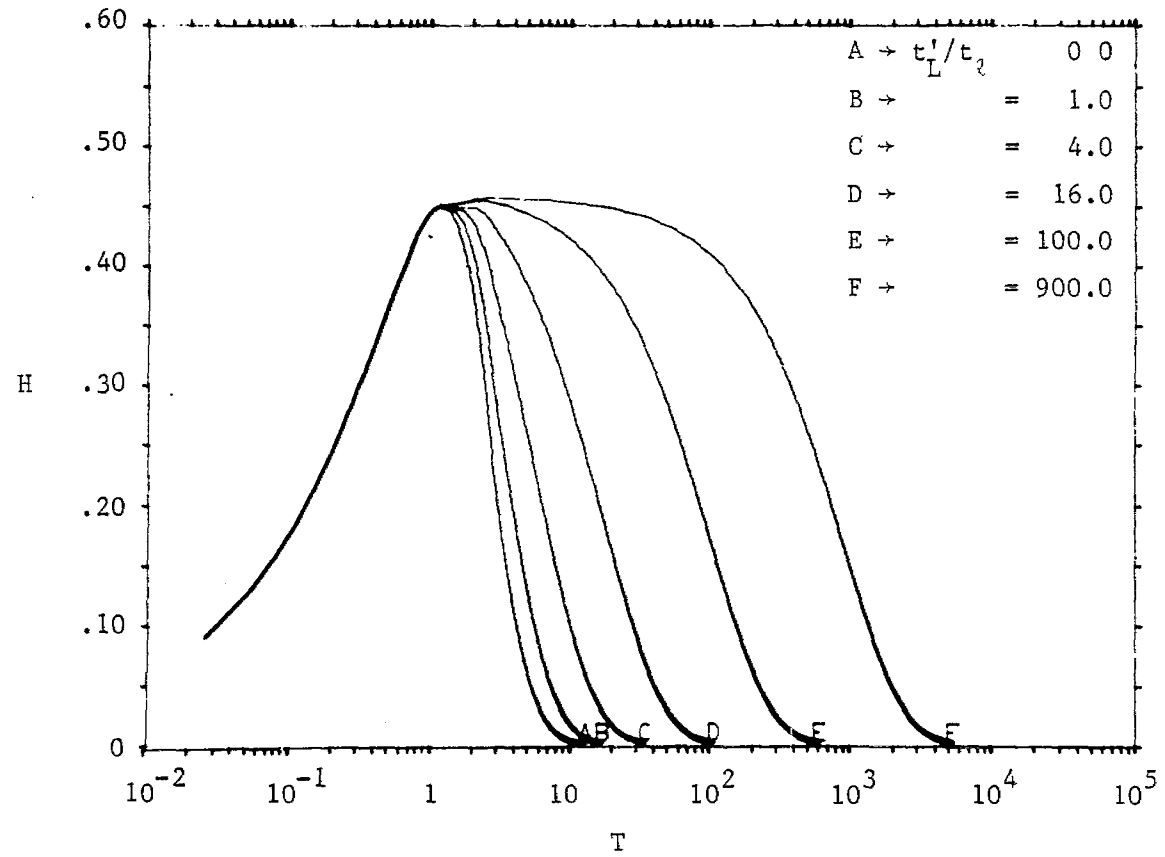

Figure 5a. Current versus Time; Finite Line, t_o/t_ℓ = 1.0, One Capacitor.

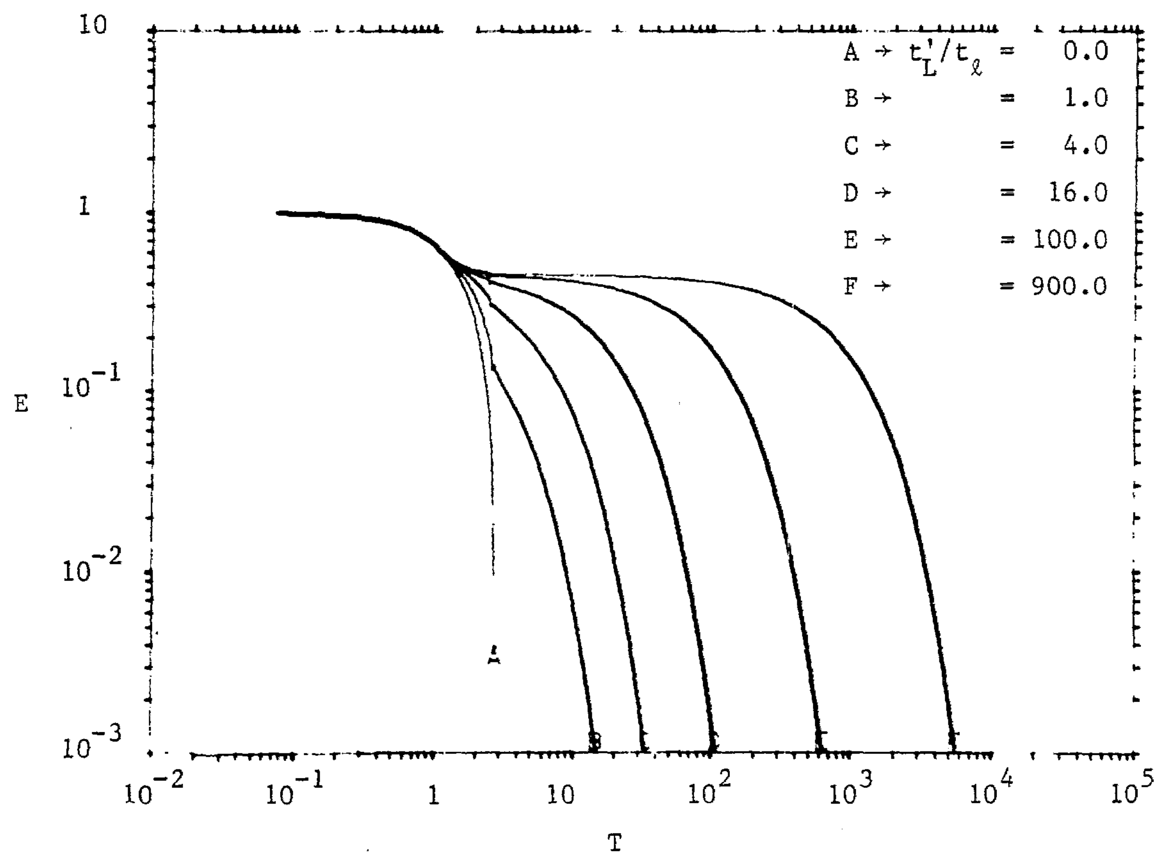

Figure 5b. Voltage versus Time; Finite Line, t_o/t_ℓ = 1.0, One Capacitor.

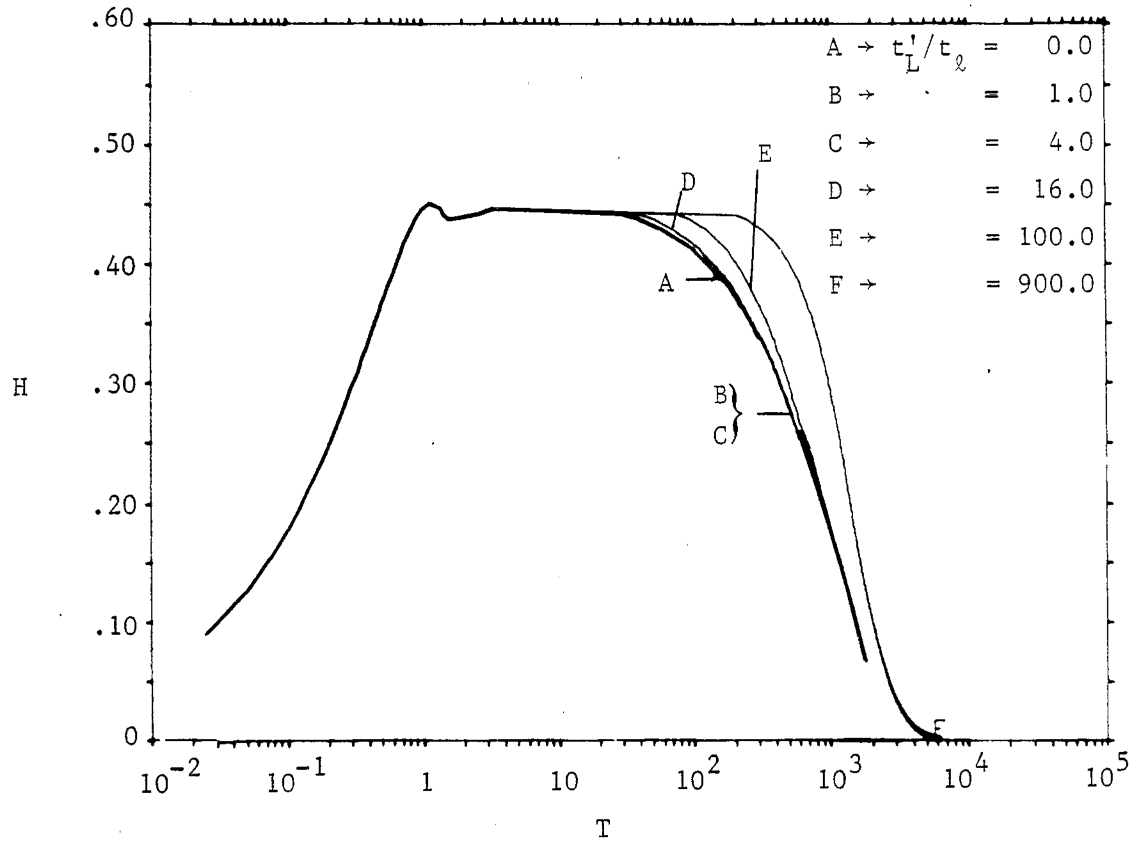

Figure 6a. Current versus Time; Finite Line, t_o/t_ℓ = 1.0, Two Capacitors.

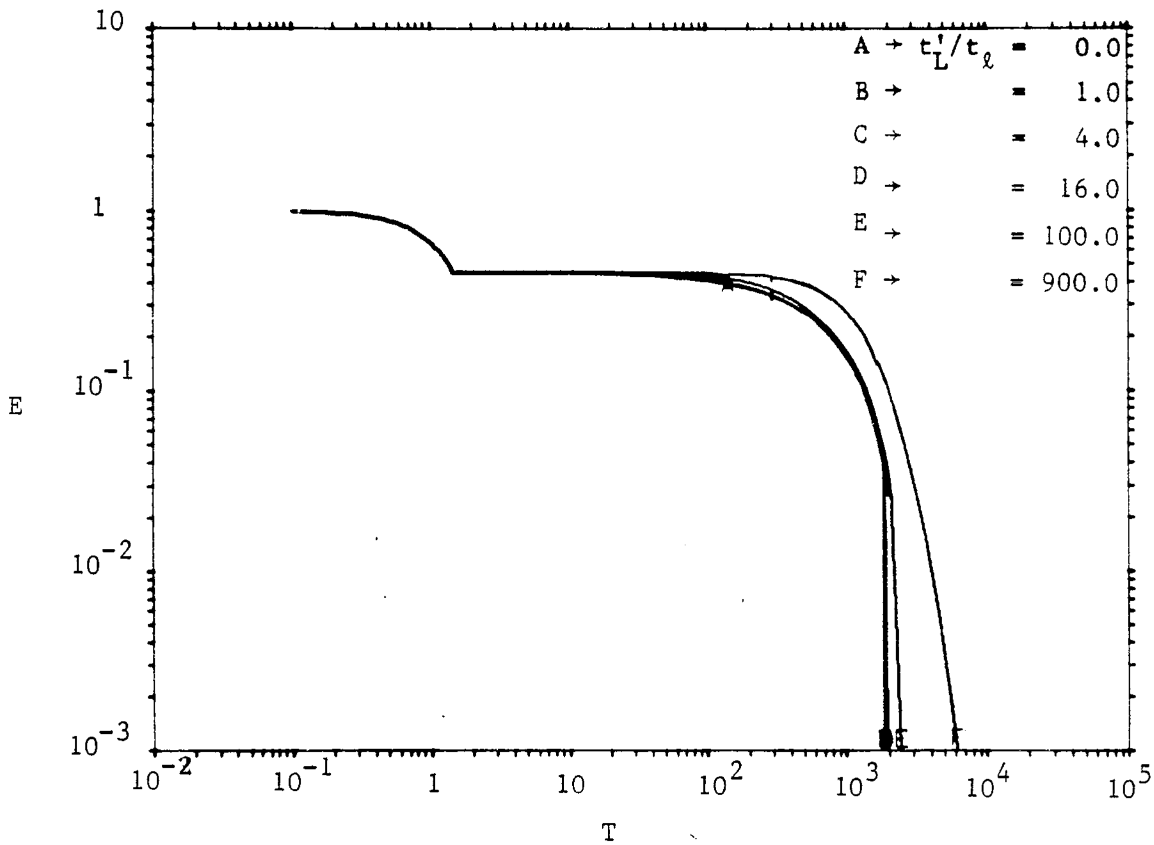

Figure 6b. Voltage versus Time; Finite Line, t_o/t_ℓ = 1.0, Two Capacitors.

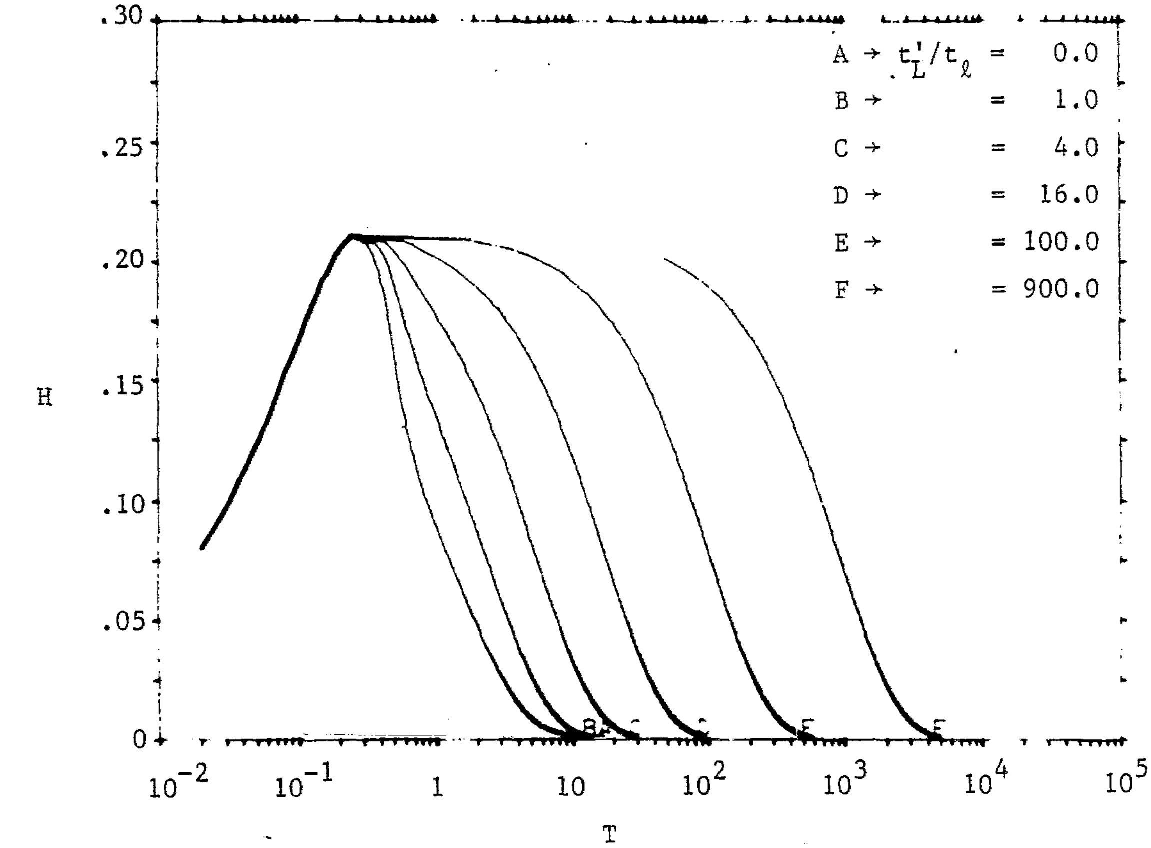

Figure 7a. Current versus Time; Finite Line, t_o/t_ℓ = 0.1, One Capacitor.

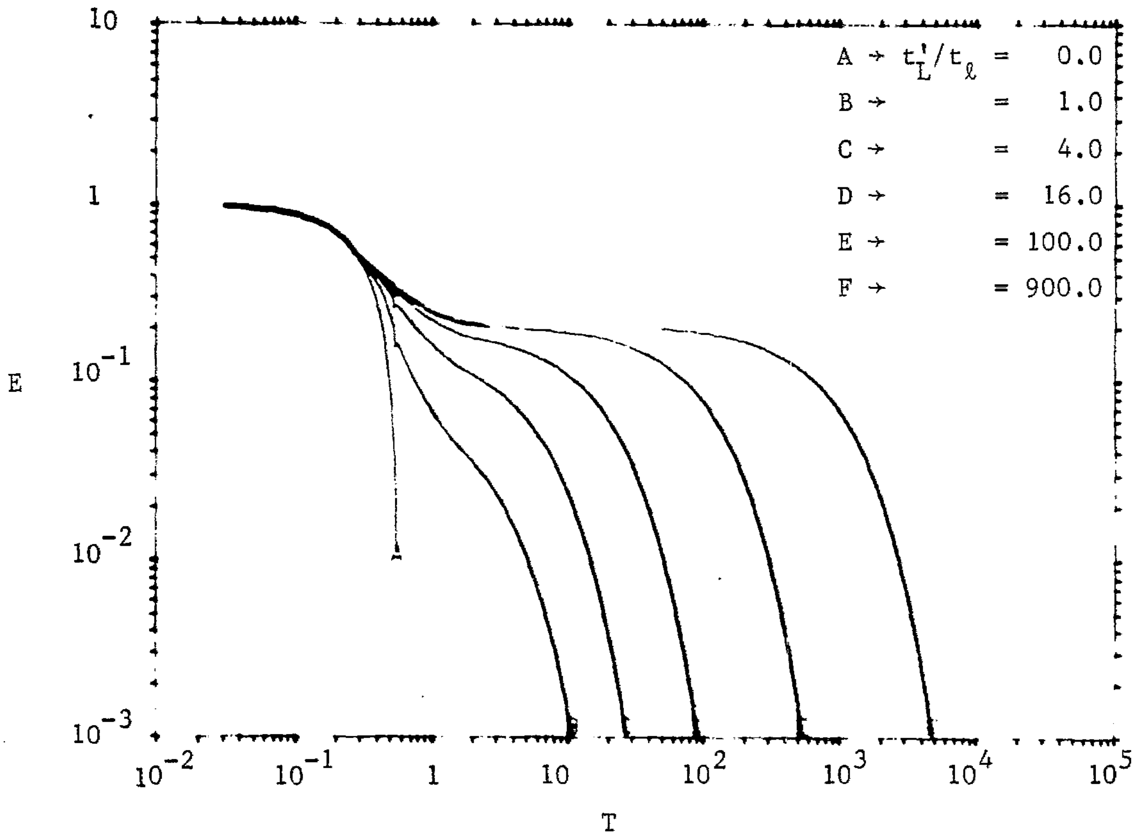

Figure 7b. Voltage versus Time; Finite Line, t_o/t_ℓ = 0.1, One Capacitor.

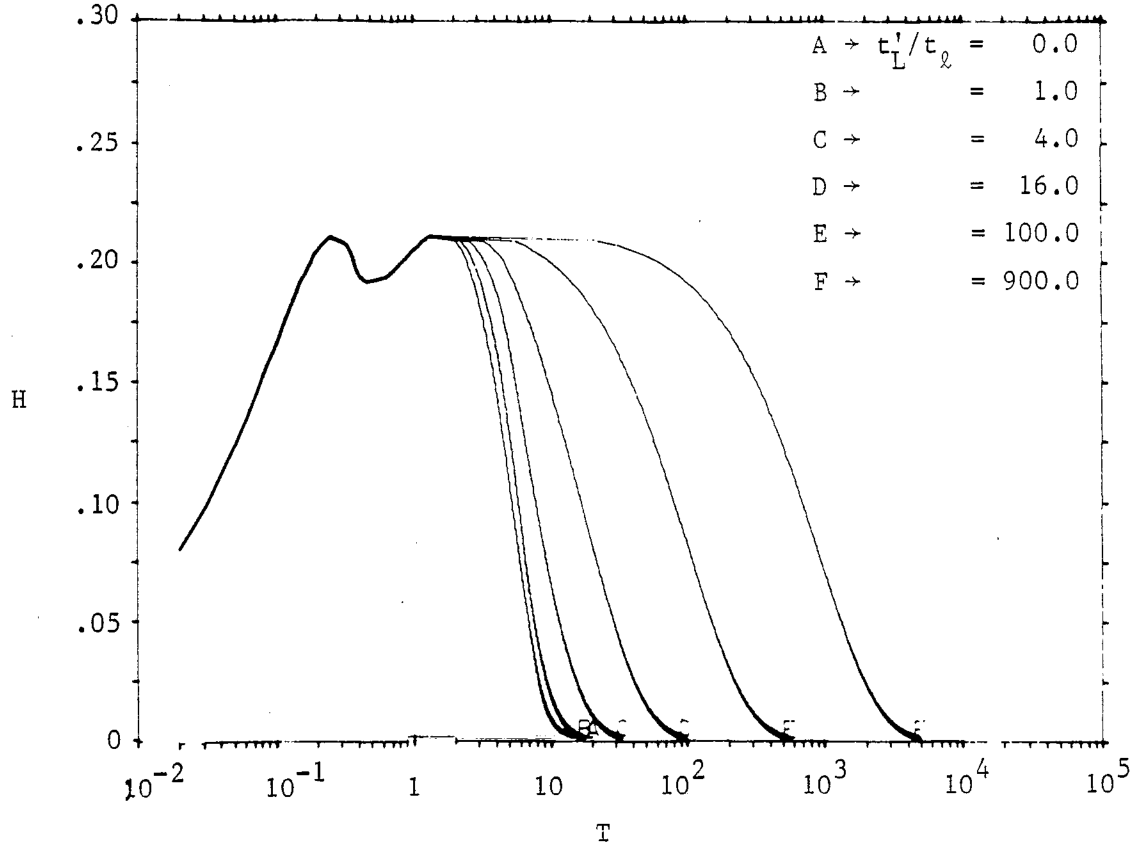

Figure 8a. Current versus Time; Finite Line, t_o/t_ℓ = 0.1, Two Capacitors.

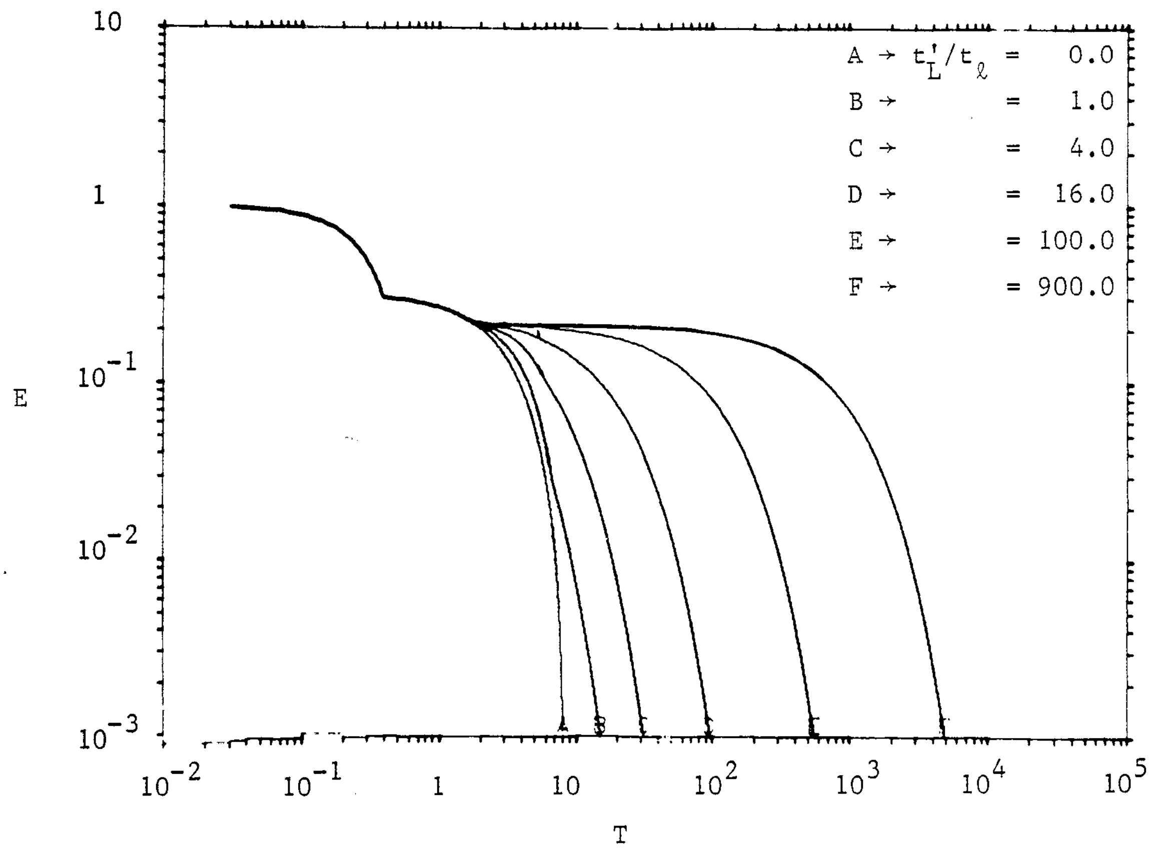

Figure 8b. Voltage versus Time; Finite Line, t_o/t_ℓ = 0.1, Two Capacitors.

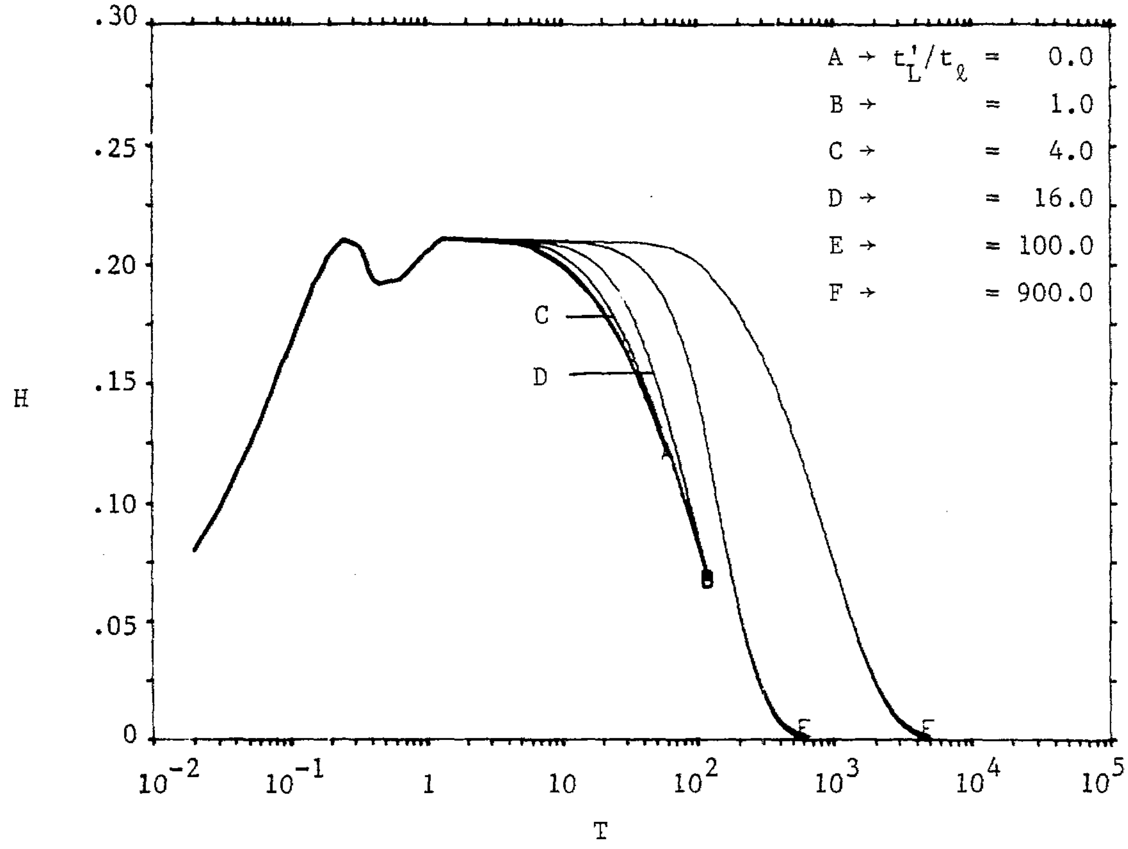

Figure 9a. Current versus Time; Finite Line t_o/t_ℓ = 0.1, Three Capacitors.

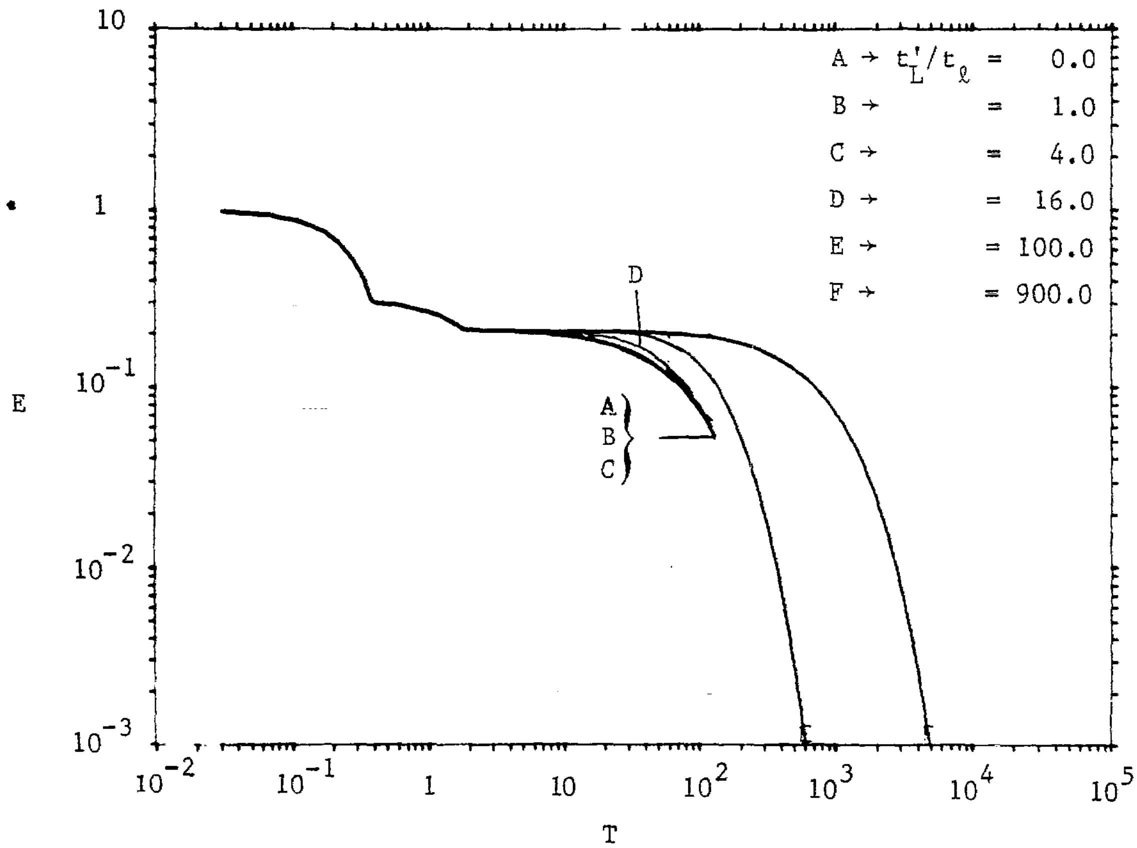

Figure 9b. Voltage versus Time; Finite Line, t_o/t_ℓ = 0.1, Three Capacitors.

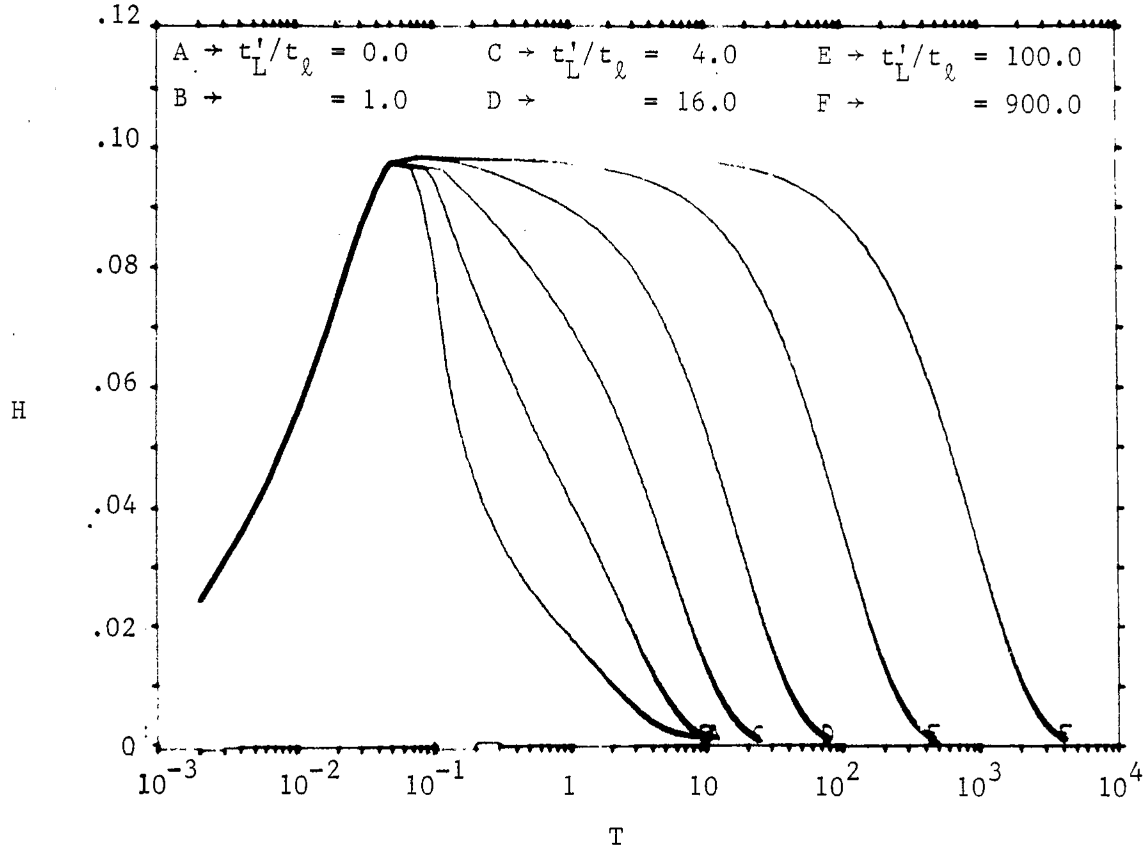

Figure 10a. Current versus Time; Finite Line $t_o/t_\ell = 0.01$, One Capacitor.

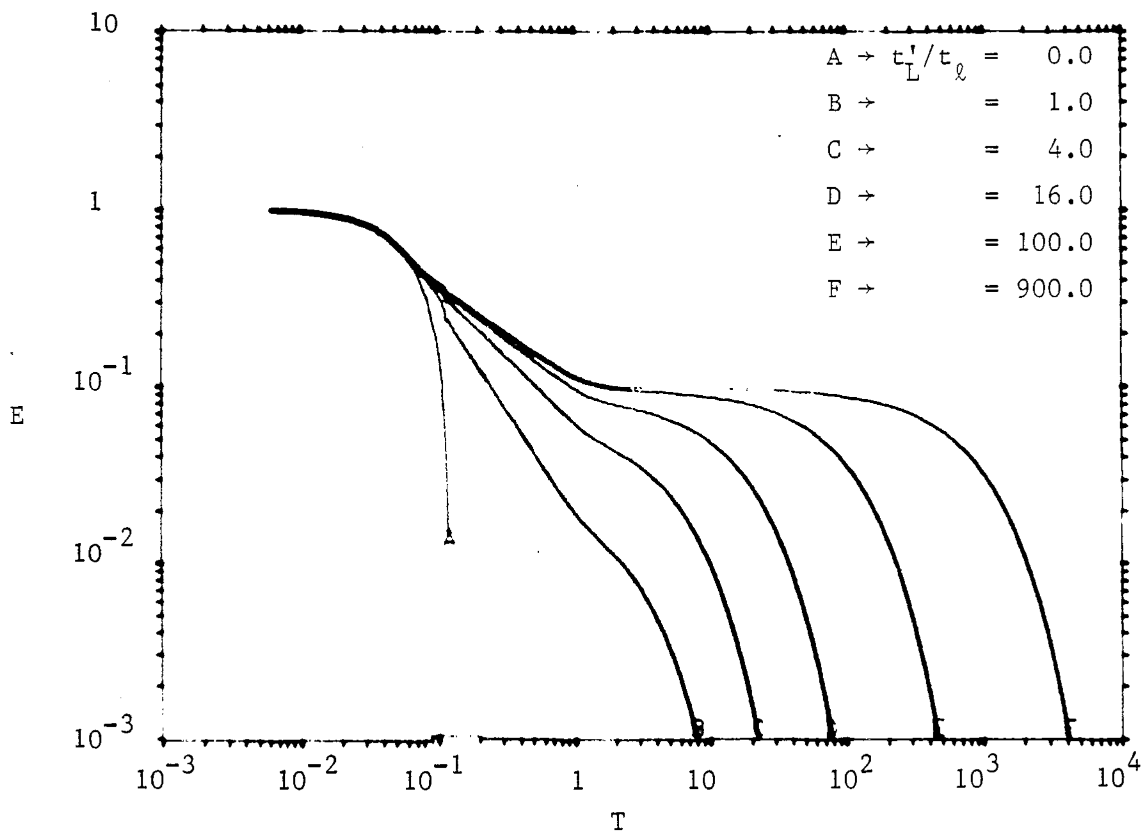

Figure 10b. Voltage versus Time; Finite Line, $t_o/t_\ell = 0.01$, One Capacitor.

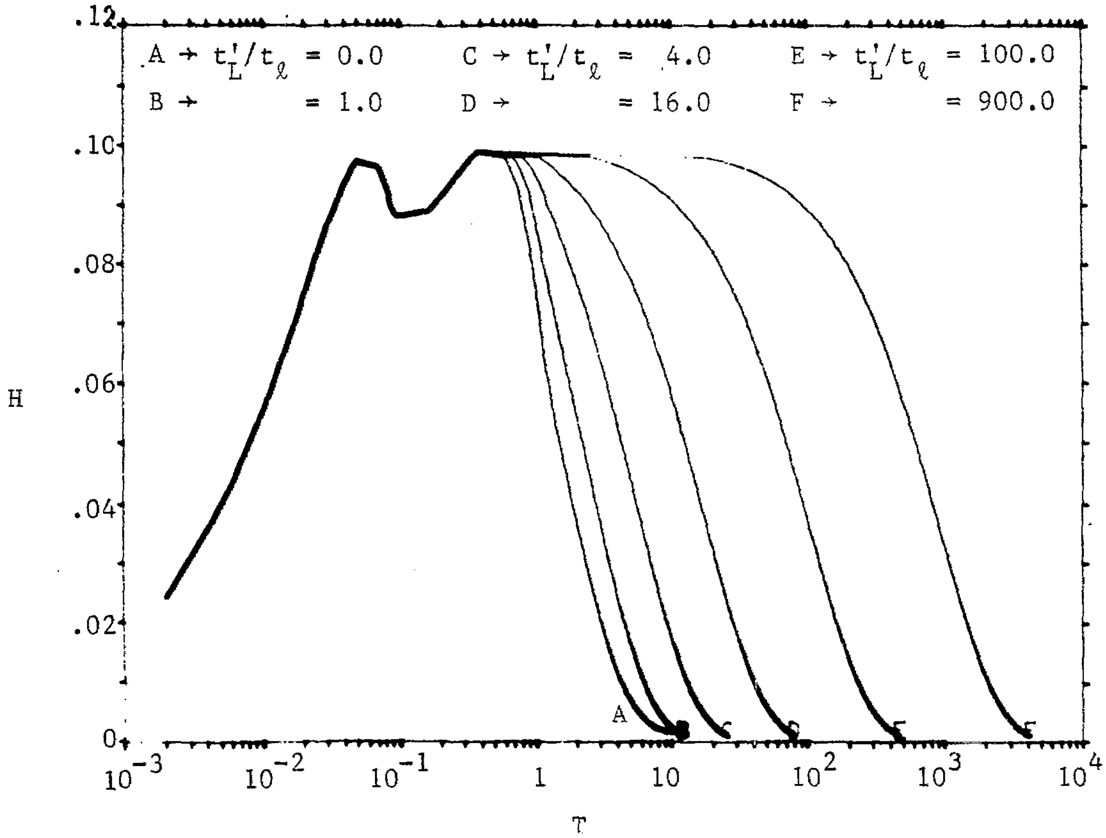

Figure 11a. Current versus Time; Finite Line, t_o/t_ℓ = 0.01, Two Capacitors.

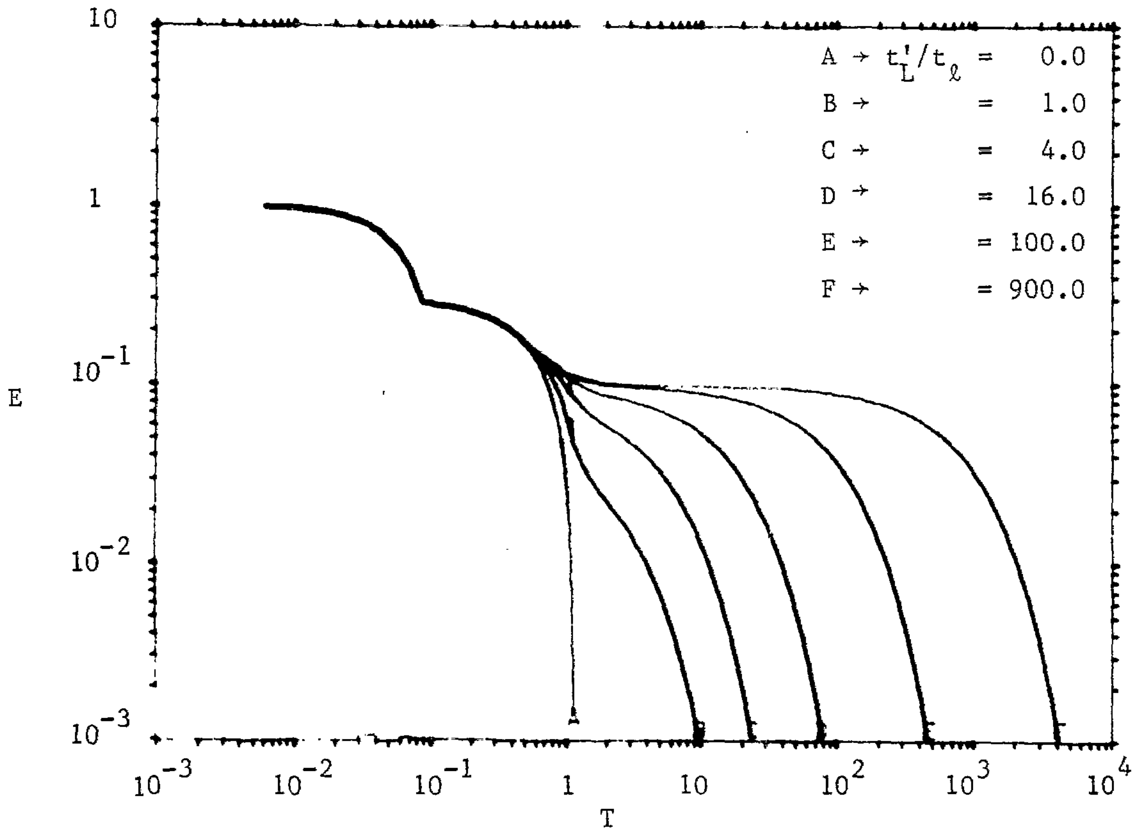

Figure 11b. Voltage versus Time; Finite Line, t_o/t_ℓ = 0.01, Two Capacitors.

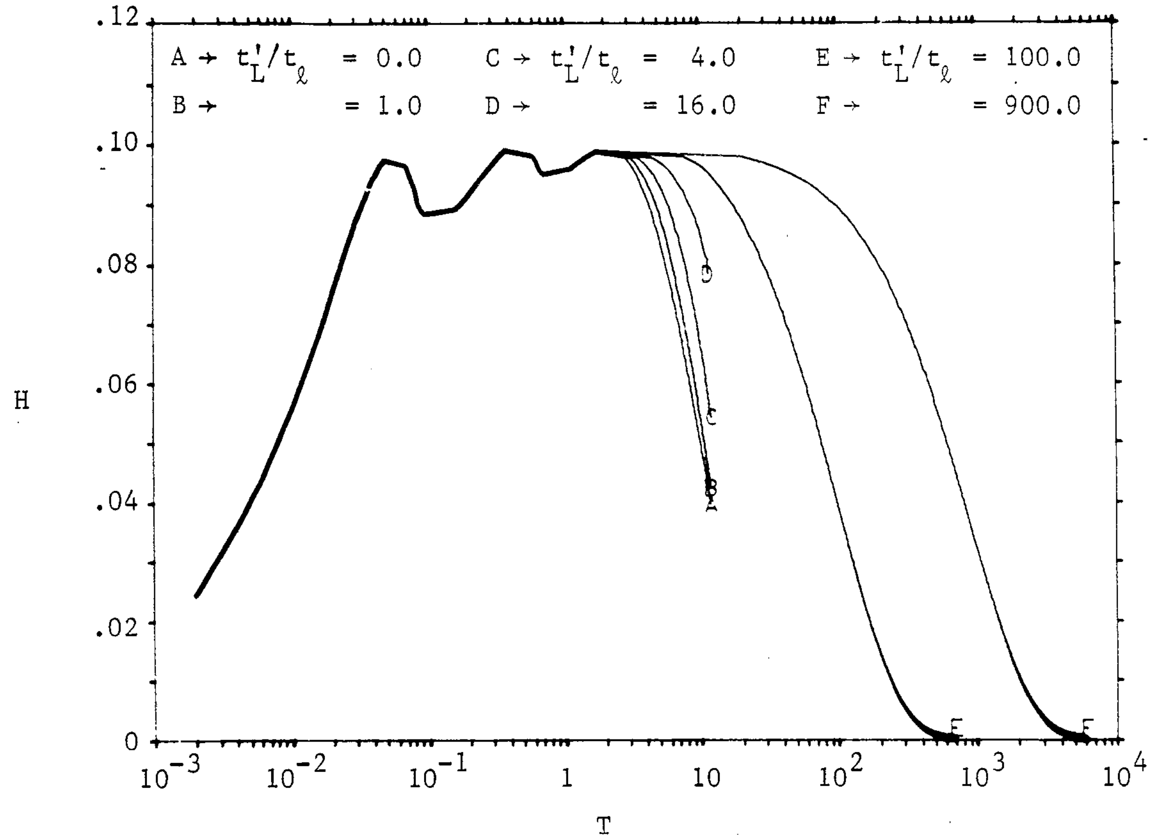

Figure 12a. Current versus Time; Finite Line, t_o/t_ℓ = 0.01, Three Capacitors.

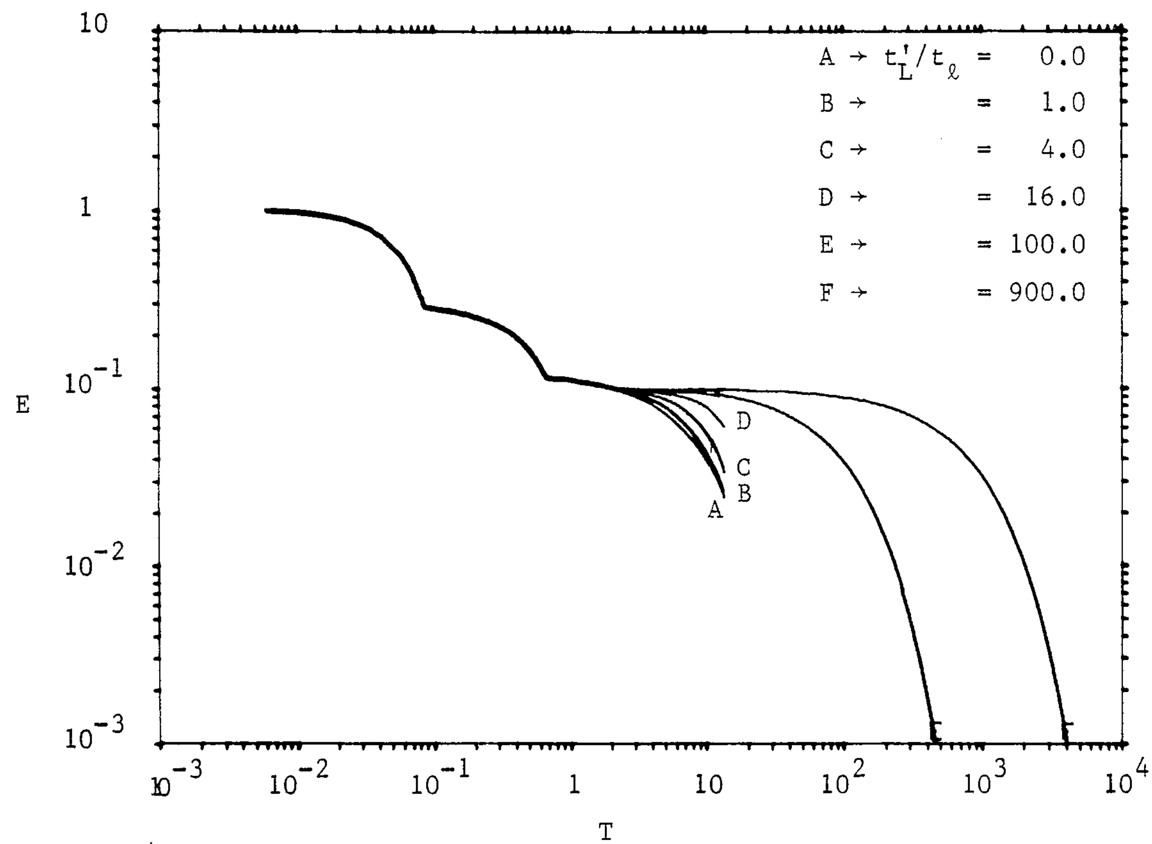

Figure 12b. Voltage versus Time; Finite Line, t_o/t_ℓ = 0.01, Three Capacitors.

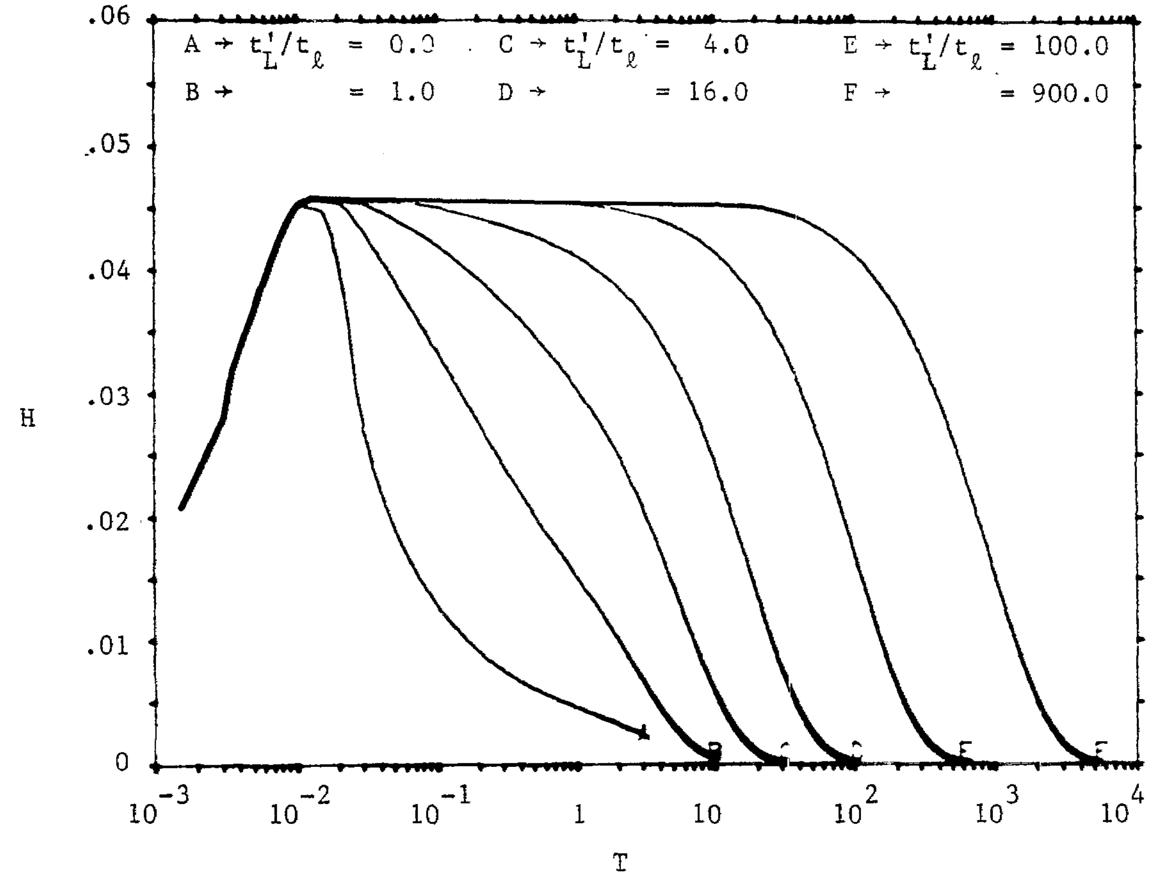

Figure 13a. Current versus Time; Finite Line, t_o/t_ℓ = 0.001, One Capacitor.

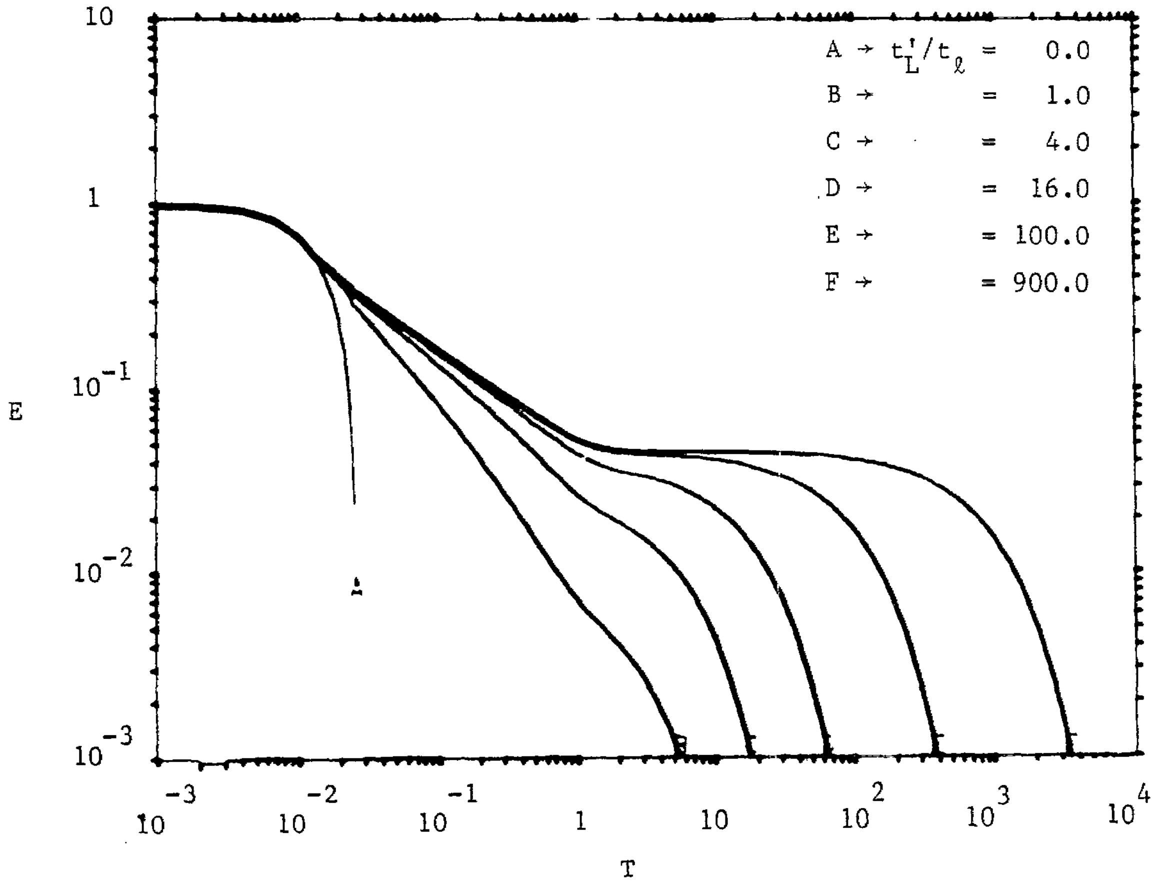

Figure 13b. Voltage versus Time; Finite Line, t_o/t_ℓ = 0.001, One Capacitor.

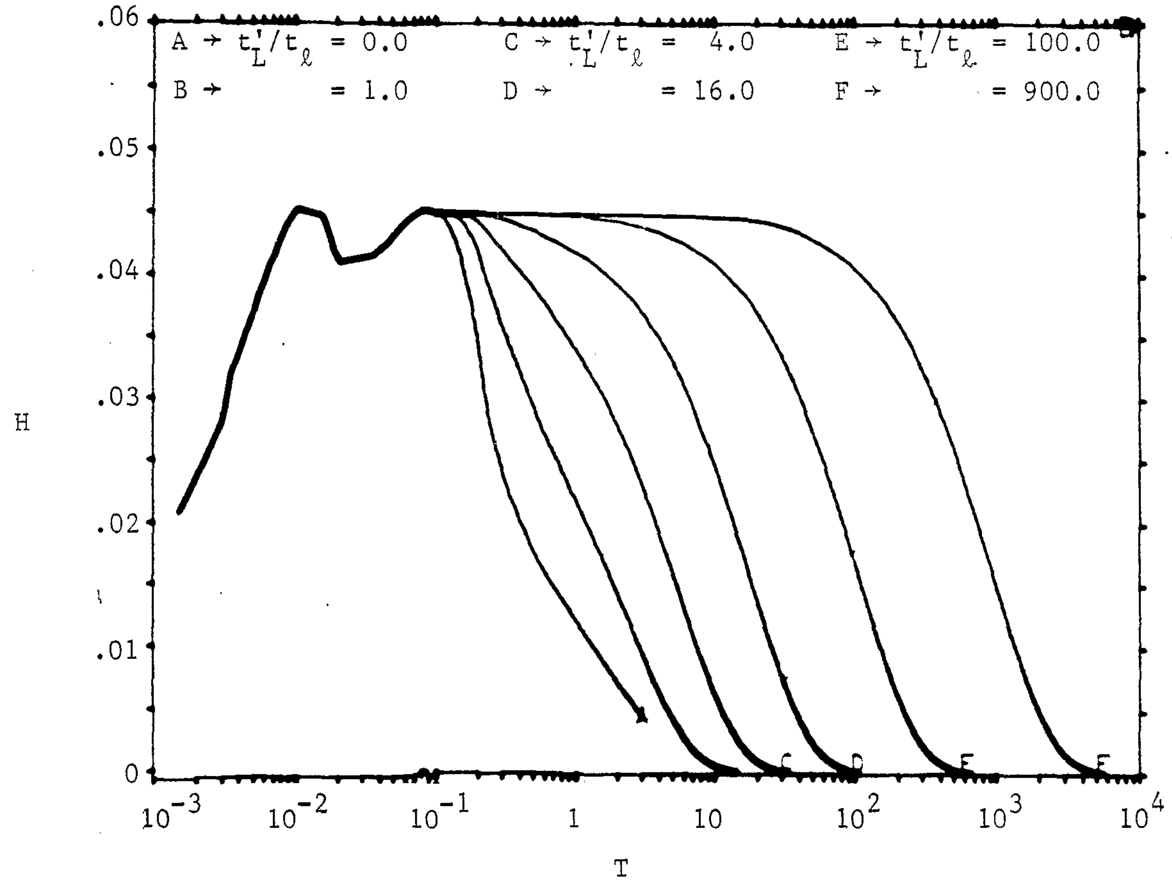

Figure 14a. Current versus Time; Finite Line, t_o/t_ℓ = 0.001, Two Capacitors.

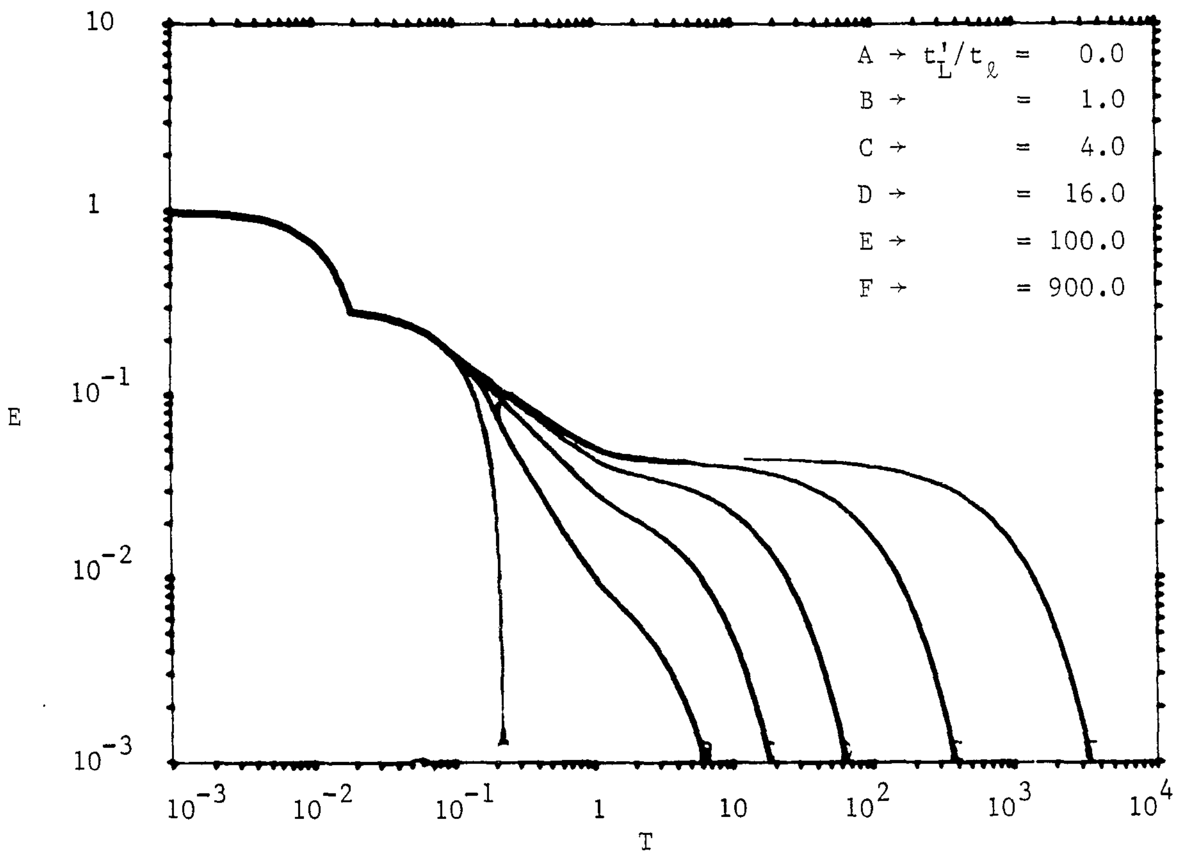

Figure 14b. Voltage versus Time; Finite Line, t_o/t_ℓ = 0.001, Two Capacitors

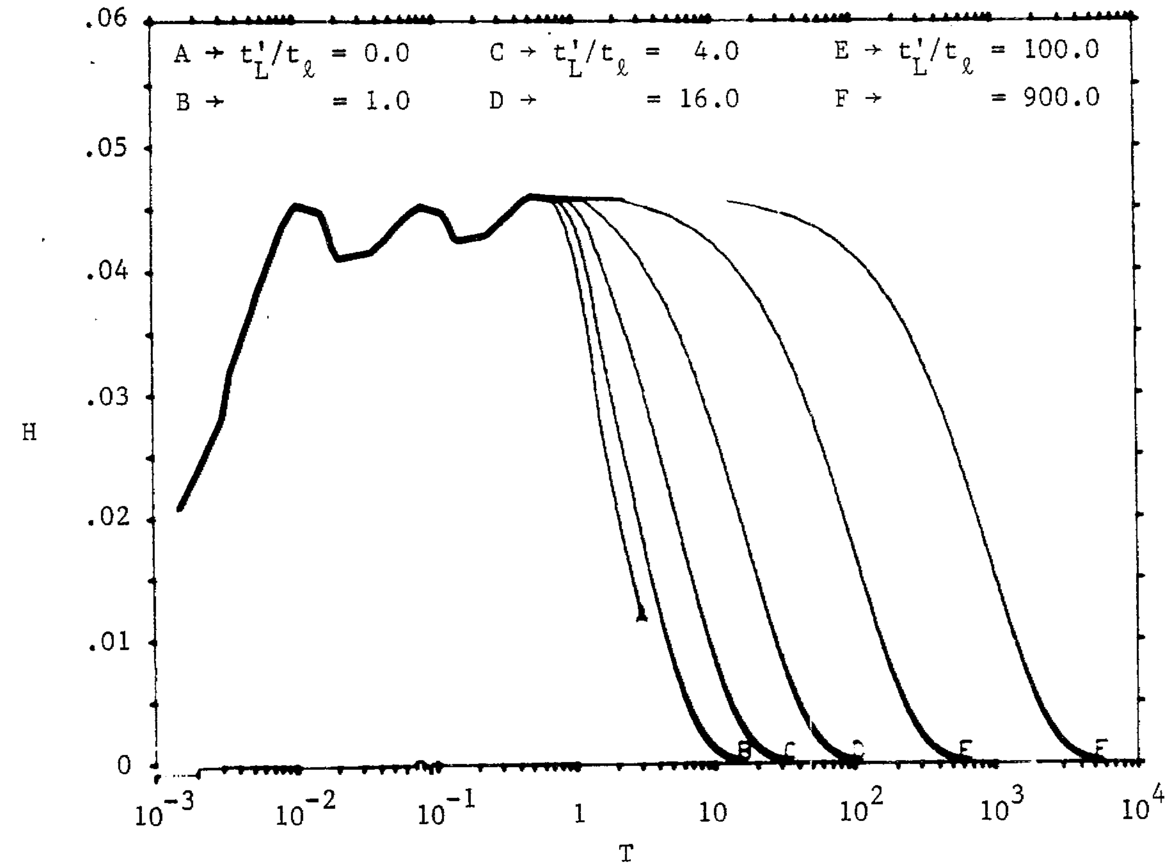

Figure 15a. Current versus Time; Finite Line t_o/t_ℓ = 0.001, Three Capacitors.

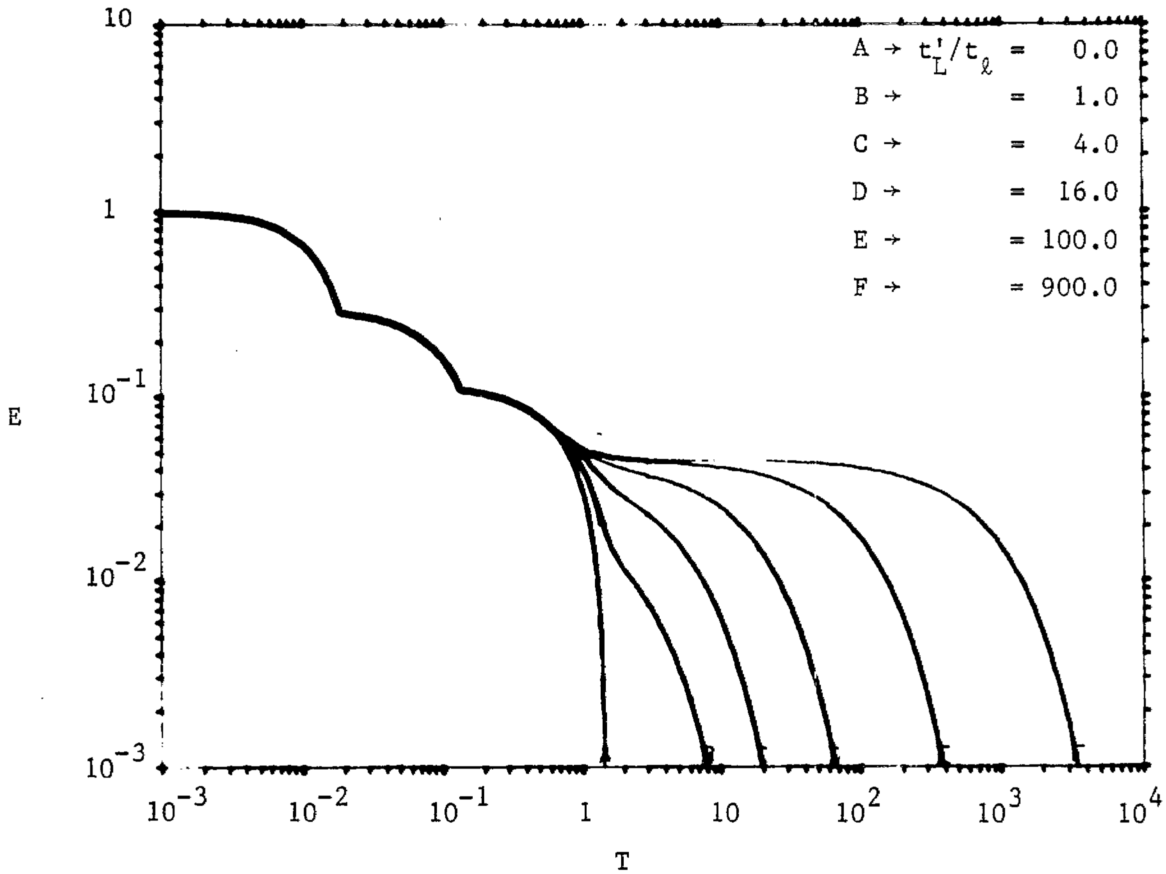

Figure 15b. Voltage versus Time; Finite Line, t_o/t_ℓ = 0.001, Three Capacitors.

Table 3: Data Summary for Finite Line Driven by Capacitors and no Inductor

Number of Capacitors		1			2			3		
	t_o/t_ℓ	V_2	τ_{ℓ_2}	Fixed Values	V_3	τ_{ℓ_3}	Additional Fixed Values	V_4	τ_{ℓ_4}	Additional Fixed Values
$t_L/t_\ell = 0$ [No Inductor]	1	0	2.78	$S_1 = 1$, $V_1 = 1$, $\tau_{\ell_1} = 0$	0	> 300*	$S_2 = 10^{-4}$, $V_2 = .44$, $\tau_{\ell_2} = 1.52$	--	---	---
	.1	0	0.57	$S_1 = 1$, $V_1 = 1$, $\tau_{\ell_1} = 0$	0	8.09	$S_2 = .03$, $V_2 = .31$, $\tau_{\ell_2} = .390$	0	> 60.0*	$S_3 = .001$, $V_3 = .212$, $\tau_{\ell_3} = 1.89$
	.01	0	0.124	$S_1 = 1$, $V_1 = 1$, $\tau_{\ell_1} = 0$	0	1.142	$S_2 = .03$, $V_2 = .30$, $\tau_{\ell_2} = .086$	0	> 12.0*	$S_3 = .001$, $V_3 = .115$, $\tau_{\ell_3} = .678$
	.001	0	0.026	$S_1 = 1$, $V_1 = 1$, $\tau_{\ell_1} = 0$	0	0.025	$S_2 = .034$, $V_2 = .30$, $\tau_{\ell_2} = .019$	0	1.458	$S_3 = .002$, $V_3 = .11$, $\tau_{\ell_3} = .137$

* Computation terminated before data point.

t_o/t_ℓ \ Number of Capacitors		1			2			3		
	t_L/t_ℓ	V_3	τ_{ℓ_3}	Fixed Values	V_4	τ_{ℓ_4}	Fixed Values	V_5	τ_{ℓ_5}	Additional Fixed Values
1	1	.131	2.63	$S_1 = 1$	<0.424	> 150	$S_2 = 10^{-4}$	---	---	---
	4	.332	2.53	$V_2 = .577$	<0.424		$V_2 = .44$	---	---	---
	16	.404	2.50	$\tau_{\ell_2} = 1.20$	<0.424		$\tau_{\ell_2} = 1.52$	---	---	---
	100	.442	2.48		<0.427		$V_3 = .430$	---	---	---
	900	.449	2.48		<0.429	> 150	$\tau_{\ell_3} = 7.52$	---	---	---
.1	1	.199	.540	$S_1 = 1$	.034	6.42	$S_2 = .03$	< .116	> 12.0	$S_3 = .001$
	4	.263	.540	$V_2 = .550$	.112	5.63	$V_2 = .31$			$V_3 = .212$
	16	.305	.540	$\tau_{\ell_2} = .280$	.178	5.32	$\tau_{\ell_2} = .39$			$\tau_{\ell_3} = 1.89$
	100	.319	.540		.225	5.22	$V_3 = .230$			$V_4 = .210$
	900	.322	.540		.230	5.21	$\tau_{\ell_3} = 1.60$	< .116	> 12.0	$\tau_{\ell_4} = 2.73$
.01	1	.233	.116	$S_1 = 1$	.065	1.092	$S_2 = .03$	< .097	> 12.0	$S_3 = .001$
	4	.299		$V_2 = .55$	.103	1.072	$V_2 = .30$			$V_3 = .115$
	16	.321		$\tau_{\ell_2} = .060$	.122	1.064	$\tau_{\ell_2} = .086$			$\tau_{\ell_3} = .678$
	100	.328			.128	1.062	$V_3 = .176$			$V_4 = .101$
	900	.329	.116		.130	1.062	$\tau_{\ell_3} = .472$	< .097	> 12.0	$\tau_{\ell_4} = 2.11$
.001	1	.285	.025	$S_1 = 1$	.075	.211	$S_2 = .034$	.020	1.39	$S_3 = .002$
	4	.323		$V_2 = .55$	.101	.209	$V_2 = .30$	.037	1.36	$V_3 = .11$
	16	.334		$\tau_{\ell_2} = .013$	.104	.208	$\tau_{\ell_2} = .019$	.045	1.35	$\tau_{\ell_3} = .137$
	100	.337			.113	.208	$V_3 = .180$	.048	1.35	$V_4 = .071$
	900	.337	.025		.114	.208	$\tau_{\ell_3} = .091$	.048	1.35	$\tau_{\ell_4} = .578$

Table 4: Data Summary for Finite Line Driven by Capacitors and an Inductor

V. Conclusion

The use of capacitive energy sources followed by an inductive energy source significantly broadens the pulse-width of the input current of a buried transmission line. The addition of a switch used to short-circuit the capacitive sources prevents oscillations without the need of inserting series resistance, which would absorb energy.

The number of capacitive energy sources chosen to initiate the current pulse depends fundamentally upon the opening speed and timing accuracy of the available inductor-shorting switch. When very rapidly acting switches are available, only one capacitive source need be used.

The calculations have been directed toward achieving a roughly flat-topped waveform for the current (magnetic field) with no more than 10% ripple after the initial rise. The choice of the waveform is somewhat arbitrary and used here only for illustration. Other waveforms may be calculated using the same techniques. One such possibility is that of opening the capacitor shorting switch at some time after the inductor has released most of the stored energy. This would truncate the waveform and allow damped oscillations at a low level.

For the infinite line, the energy released by the inductor is approximately $\frac{1}{2}\left(t_L^2/t_c^2\right)$ times the initial energy stored in the first capacitor. For the finite line, the inductive energy required varies with the diffusion time constant, but is proportional to t_L'/t_ℓ times the initial energy stored in the first capacitor so long as the line parameters are held constant.

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