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Sensor and Simulation Notes

Note 75

A Model for Transient Reflections from Objects Within an Above-ground Simulator

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ABSTRACT

The model considered in this report uses several sections of lossless transmission line, mismatched at the junctions, as an approximation to a simulator with objects inside. The transient reflections is a suitably constructed transmission line will be roughly similar to the reflections from the objects within the simulator. The source network for the line consists of a charged capacitor and a switch. The voltage at a selected position on the line is first calculated in the frequency domain by an iterative process using conventional equations for a uniform transmission line. The inverse Fourier transform of this result is then calculated to find the voltage in the time domain.

I. THEORY

The formulas for a uniform section of transmission line for the lossless case are[1]:

$$Z = Z_o \frac{\left(Z_R + jZ_o \tan\beta\ell\right)}{Z_o + jZ_R \tan\beta\ell}, \tag{1}$$

$$E(x) = \frac{E_g Z_o\left[\left(Z_o + Z_R\right)e^{j\beta(\ell-x)} - \left(Z_o - Z_R\right)e^{-j\beta(\ell-x)}\right]}{\left(Z_o + Z_g\right)\left(Z_o + Z_R\right)e^{j\beta\ell} - \left(Z_o - Z_g\right)\left(Z_o - Z_R\right)e^{-j\beta\ell}}, \tag{2}$$

where E_g is the open-circuit voltage of the driver, Z_g is the driver impedance, and Z_R is the load impedance; Z_o is the characteristic impedance of the line of length ℓ. The propagation constant γ is:

$$\gamma = \alpha + j\beta .$$

For the lossless case $\alpha = 0$ and $\beta = \omega/c$, where ω is the frequency and c is the speed of light. These formulas then give us Z, the impedance across the input terminals, and E(x), the voltage at a distance x from the input. See Fig. 1.

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[1] Everitt, W. L. and G. E. Anner, Communication Engineering (McGraw-Hill Book Company, Inc., New York, 1950), pp. 337, 361.

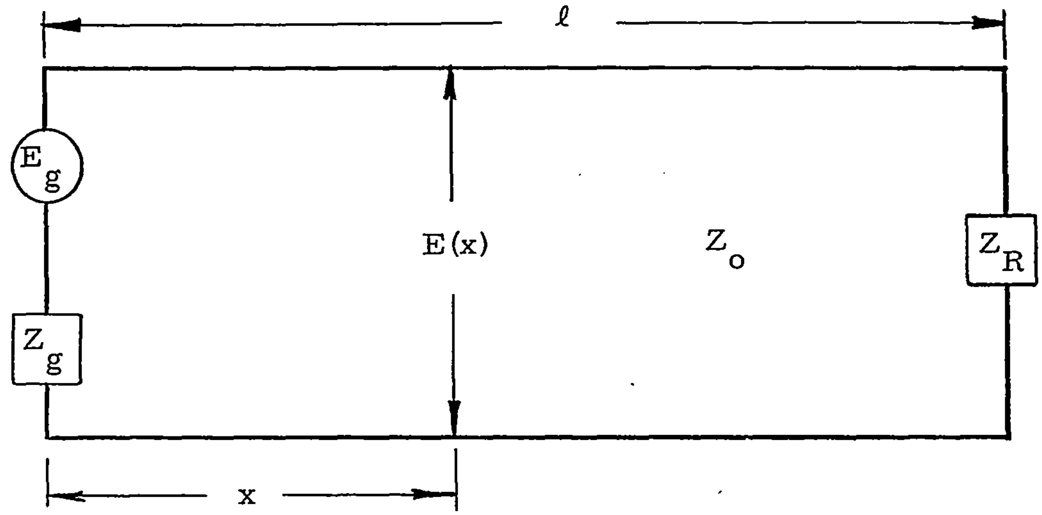

Figure 1

Then for a transmission line composed of n segments (labeled one to n starting at the load end), the load impedance Z_R on Section n is given by:

$$Z_n^R = Z_{n-1}^o \frac{\left(Z_{n-1}^R + j\,Z_{n-1}^o \tan\beta\ell_{n-1}\right)}{\left(Z_{n-1}^o + j\,Z_{n-1}^R \tan\beta\ell_{n-1}\right)}, \tag{3}$$

where Z_1^R is the load impedance for Section 1. Similarily, the input impedance to Section n is

$$Z_n^g = Z_{n+1}^o \frac{\left(Z_{n+1}^g + j\,Z_{n+1}^o \tan\beta\ell_{n+1}\right)}{\left(Z_{n+1}^o + j\,Z_{n+1}^g \tan\beta\ell_{n+1}\right)}, \tag{4}$$

where Z_n^g is the driver impedance seen by the input terminals of the line.

To find the equivalent open-circuit driver voltage across Section n, Eq. (2) is used for the open-circuit case where $Z_R = \infty$ and $x = \ell$. Then one obtains

$$E_n^g = \frac{E_{n+1}^g Z_{n+1}^o}{Z_{n+1}^o \cos\beta\ell_{n+1} + j Z_{n+1}^g \sin\beta\ell_{n+1}}, \tag{5}$$

where Z_n^g is given by Eq. (4). See Fig. 2.

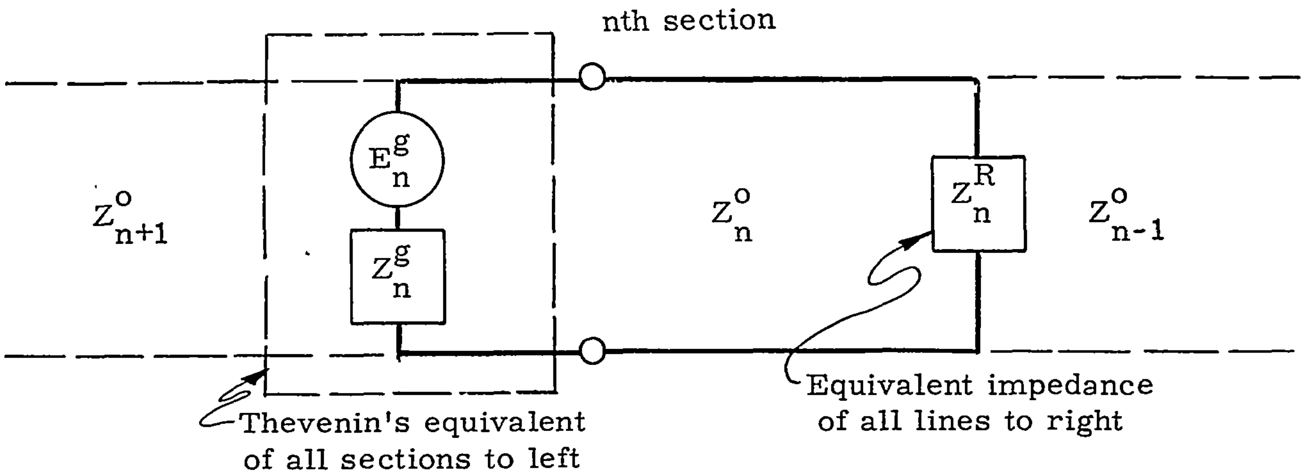

Figure 2

The voltage at a position ξ from the driver end of the line may then be calculated first by finding in which section ξ lies. Referring to this section as n, one may then calculate $E(\xi)$ from

$$E(\xi) = \frac{E_n^g Z_n^o\left(Z_n^R \cos\phi + j Z_n^o \sin\phi\right)}{\left(Z_n^o Z_n^g + Z_n^o Z_n^R\right)\cos\beta\ell_n + j\left[\left(Z_n^o\right)^2 + Z_n^R Z_n^g\right]\sin\beta\ell_n}, \tag{6}$$

where $\phi = \beta\left(\ell_n - x\right) = \beta\left[\left(\sum_{i=n}^{n} \ell_i\right) - \xi\right]$.

To find the time-domain response at a position ξ, one must take the inverse transform of $E(\xi)$. This transform can be numerically performed for a wide variety of driver networks; for this study an initially charged capacitor and a switch was chosen which yields the following transform functions that define the driver circuit;

$$E_g = -\frac{j}{\omega},$$

$$Z_g = -\frac{j}{\omega C},$$

where C is the value of the capacitor. Then the Fourier inverse of $E(\xi, \omega)$ gives the voltage at ξ as a function of time.

II. COMPUTER PROGRAM TL

To obtain the most efficient use of central processor time on the computer, the best way to use Eqs. (3), (4), and (5) is to first convert them into real-part plus imaginary-part form. That is to represent the complex equations in the form

$$X = A + jB .$$

Assuming Z_n^o is real, this form of Eqs. (3), (4), and (5) is given by

$$Z_n^R = Z_{n-1}^o \frac{\left[x_R + j\left(y_R\left(\cos^2\theta_{n-1} - \sin^2\theta_{n-1}\right) + \sin\theta_{n-1}\cos\theta_{n-1}\left(1 - x_R^2 - y_R^2\right)\right)\right]}{\cos^2\theta_{n-1} + \left(x_R^2 + y_R^2\right)\sin^2\theta_{n-1} - 2y_R\sin\theta_{n-1}\cos\theta_{n-1}} , \quad (7)$$

where $\theta_n \equiv \omega \ell_n / c$, $\quad x_R + j y_R \equiv Z_{n-1}^R / Z_{n-1}^o$;

$$Z_n^g = Z_{n+1}^o \frac{\left[x_g + j\left(y_g\left(\cos^2\theta_{n+1} - \sin^2\theta_{n+1}\right) + \sin\theta_{n+1}\cos\theta_{n+1}\left(1 - x_g^2 - y_g^2\right)\right)\right]}{\cos^2\theta_{n+1} + \left(x_g^2 + y_g^2\right)\sin^2\theta_{n+1} - 2y_g\sin\theta_{n+1}\cos\theta_{n+1}} , \quad (8)$$

where $\quad x_g + j y_g \equiv Z_{n+1}^g / Z_{n+1}^o$;

and

$$E_n^g = \Big\{\Big[\left(E_1 x_g - E_2 y_g\right) x_g \cos\theta_{n+1} + \left(x_g E_2 + y_g E_1\right)\left(y_g \cos\theta_{n+1} + \sin\theta_{n+1}\right)$$

$$+ j\Big[x_g \cos\theta_{n+1}\left(x_g E_2 + y_g E_1\right) - \left(y_g \cos\theta_{n+1} + \sin\theta_{n+1}\right)\left(x_g E_1 - y_g E_2\right)\Big]\Big\}$$

$$/\Big[\left(x_g^2 + y_g^2\right)\cos^2\theta_{n+1} + \sin^2\theta_{n+1} + 2y_g \sin\theta_{n+1}\cos\theta_{n+1}\Big], \tag{9}$$

where

$$E_1 + jE_2 \equiv E_{n+1}^g,$$

$$x_g + jy_g \equiv Z_{n+1}^o / Z_{n+1}^g .$$

Because Eq. (6) is the final equation and not subject to an iterative process as are Eqs. (3), (4), and (5), it is not used separated into real and imaginary parts.

We must also consider the cases when the denominators in these equations reduce to zero and their subsequent effect on Eq. (6). Equations (7) and (8) have the same form for the denominator. The condition of a zero denominator is:

$$\cos^2\theta_n + \left(x^2 + y^2\right)\sin^2\theta_n - 2y\sin\theta_n\cos\theta_n = 0,$$

which reduces to

$$\left(x^2 + y^2\right)\tan^2\theta_n - 2y\tan\theta_n + 1 = 0,$$

or

$$\tan\theta_n = \frac{y \pm j|x|}{x^2 + y^2}$$

Since $\tan\theta_n$ is real, $x = 0$ is the only meaningful solution; then $\tan\theta_n = 1/y$, or equivalently for Eq. (7)

$$Z^R_{n-1} = j\, Z^o_{n-1} \cot\theta_{n-1} \,. \tag{10}$$

It may then be verified that $Z^R_n = \infty$ and

$$Z^R_{n+1} = -j\, Z^o_n \cot\theta_n \,. \tag{11}$$

Similarily for Eq. (8) the zero denominator condition is

$$Z^g_{n+1} = j\, Z^o_{n+1} \cot\theta_{n+1} \tag{12}$$

which results in $Z^g_n = \infty$, and

$$Z^g_{n-1} = -j\, Z^o_n \cot\theta_n \,. \tag{13}$$

The condition for a vanishing denominator of Eq. (9) is again Eq. (12) with the effect that $E^g_n \rightarrow \infty$ for non-vanishing E^g_{n+1} . Because of the rarity in the occurrence of Eq. (12) and the complexity in the expression for $E(\xi)$ for this special case, when Eq. (12) occurs the program produces an informative diagnostic giving the section n and the frequency ω, causing E^g_n to become infinite and stops calculation of $E(\xi)$ at that frequency.

III. RESULTS

Typical output from the Calcomp plotter is shown in Figs 4 through 7. For each graph produced, a table giving all the pertinent data of that particular run is also produced. These tables are shown at the left of each figure. On the right of each figure is a plot of the voltage in retarded time.

To verify the graphed results consider the case represented in Fig. 2. The transmission line for which this graph was made is shown below.

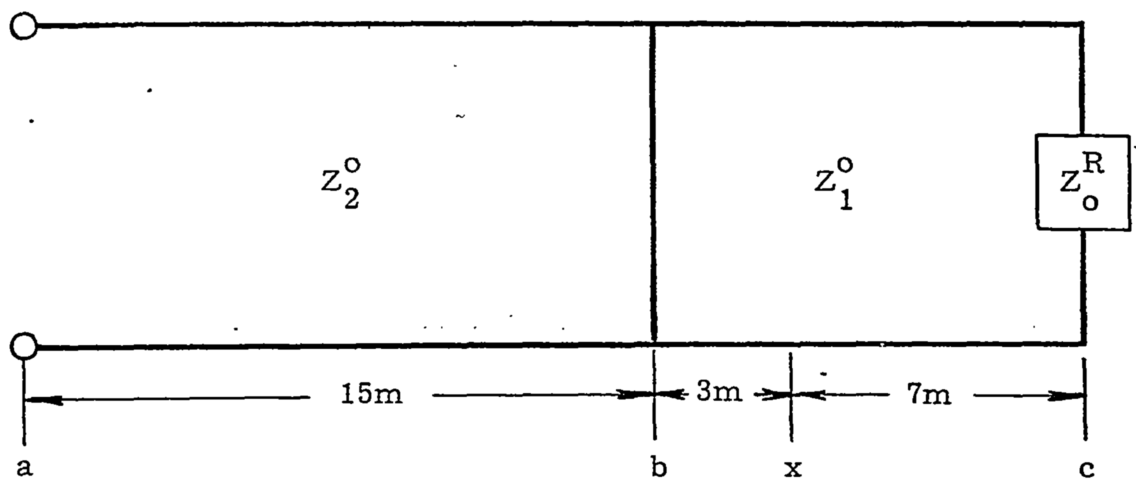

Figure 3

Retarded time equals zero is the time at which the initial pulse arrives at the observation point x. The first reflection observable at x is the pulse that reflects from boundary c back to x, a total distance of 14 meters. A subsequent reflection will occur at b for this reflected pulse and will arrive at x after traveling an additional six meters, or 20 meters total. It is evident that the pulse will continue to reflect at b and c decreasing in amplitude due to energy absorbed at c and transmitted at b. Thus one will continue to see reflections at position x every 14 and six meters or at 14m, 20m, 34m, 40m, etc. However, in the first pass down the line, a portion of the pulse is also reflected at b before it reaches x. The total distance from b back to a and forward to x is 33 meters. Since the initial reflection occurs three meters in front of x, the pulse will arrive at x at a retarded time corresponding to only 30 meters. These distances may be converted to times simply by dividing by the speed of light. This conversion is shown in the table below.

Distance	Retarded Time
14m	4.67×10^{-8} sec
20m	6.67×10^{-8} sec
30m	1.00×10^{-7} sec
34m	1.13×10^{-7} sec
40m	1.33×10^{-7} sec

As one can see from the graph, these reflections all occur at the specified times within the accuracy of the calculations. For this particular graph, the time increment is 2.5×10^{-9} sec and all reflected pulses, occurring at times t, are seen at times $t \pm 2.5 \times 10^{-9}$ seconds. This accuracy may be increased somwhat by using a smaller time increment. It should be noted that the reflected pulses should be very nearly step functions. The variations appearing on the graph are quite simply "computational noise." This may be reduced by using a finer frequency point spacing in the Fourier inversion scheme. However, any increase in the number of time or frequency points will cause a roughly proportional increase in computer run time.

RUN CHARACTERISTICS

TIME 19.10.12.
DATE 14FEB69

INPUT:
CHARGED CAPACITOR
$C = 1.00E-05$
SWITCH
$V = U(T)$

LOAD:
IMPEDANCE
$Z_R = 2.50E+01$

TIME STEPS:
407 POINTS

FREQUENCY STEPS:
4970 POINTS

$X = 18.000$

$F_{MIN} = 9.35E-05$
$F_{MAX} = 9.99E+10$

LINE CHARACTERISTICS

SECTION	Z_0	L
1	6.0	10.0
2	25.0	15.0

Figure 4a

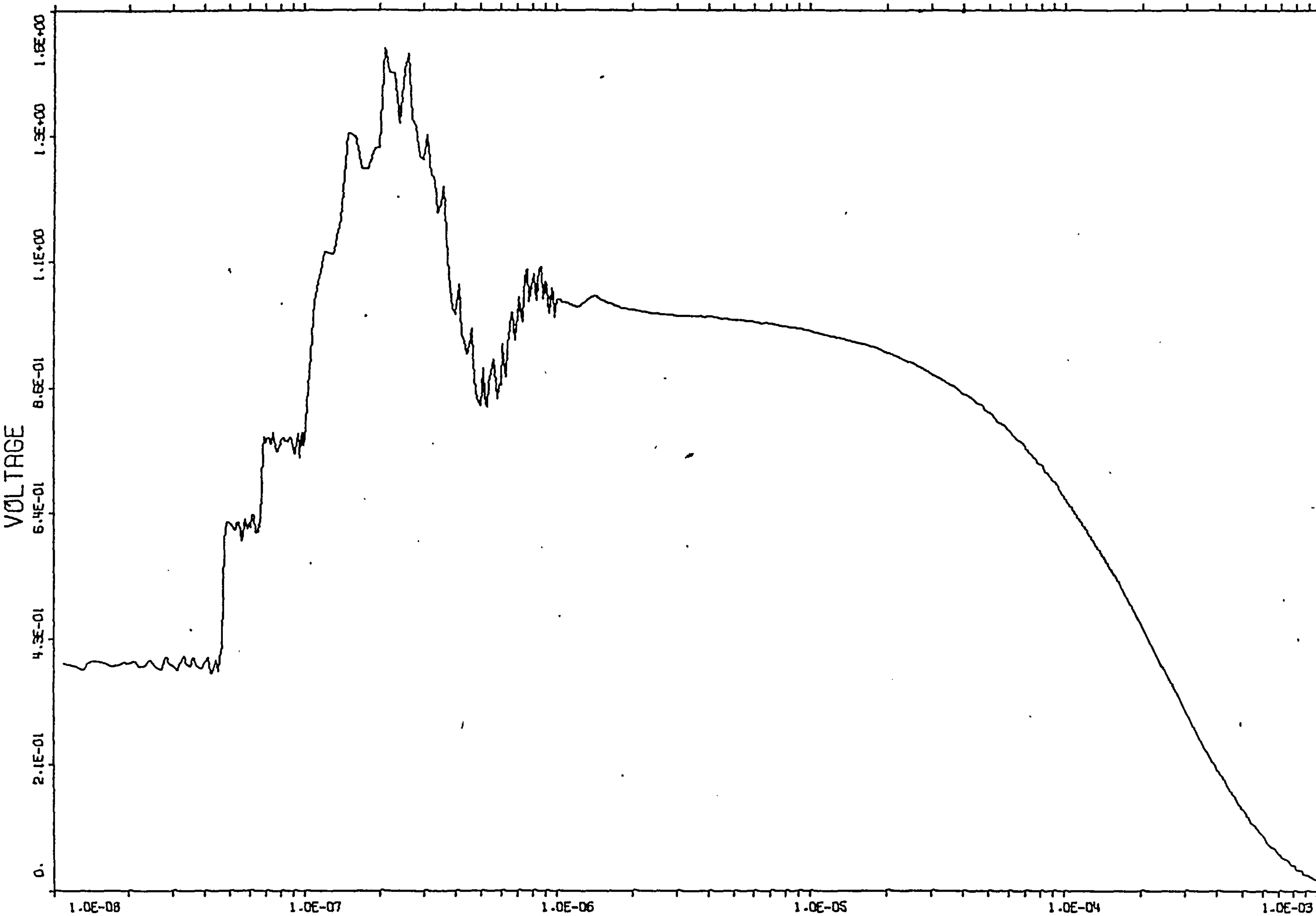

Figure 4b

75-12

AFWL EMP 1-5

RUN CHARACTERISTICS

TIME *19.42.51.
DATE 14FEB69

INPUT:
CHARGED CAPACITOR
C = 1.00E-05
SWITCH
V = U(T)

LOAD:
IMPEDANCE
Z_R = 2.50E+01

TIME STEPS:
401 POINTS

FREQUENCY STEPS:
4944 POINTS

X = 18.000

F_{MIN} = 1.01E-03
F_{MAX} = 9.99E+12

LINE CHARACTERISTICS

SECTION	Z_0	L
1	6.0	10.0
2	25.0	15.0

Figure 5a

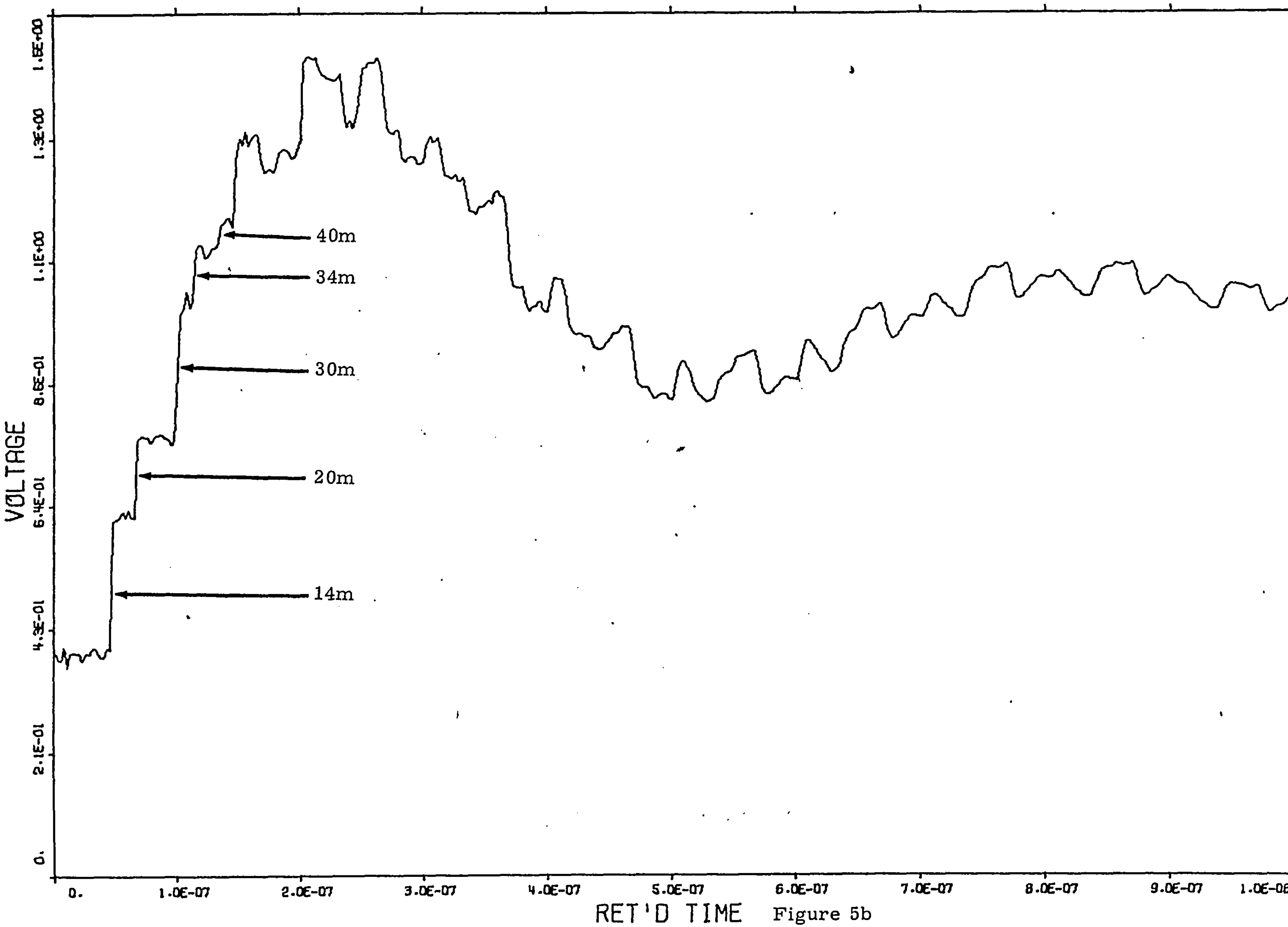
VOLTAGE
RET'D TIME
40m
34m
30m
20m
14m
0.
2.1E-01
4.3E-01
6.4E-01
8.6E-01
1.1E+00
1.3E+00
1.5E+00
1.0E-07
2.0E-07
3.0E-07
4.0E-07
5.0E-07
6.0E-07
7.0E-07
8.0E-07
9.0E-07
1.0E-06

Figure 5b

RUN CHARACTERISTICS

TIME #19.16.07.
DATE 14FEB69

INPUT:
CHARGED CAPACITOR
C = 1.00E-05
SWITCH
V = U(T)

LOAD:
IMPEDANCE
Z_R = 2.50E+01

TIME STEPS:
407 POINTS

FREQUENCY STEPS:
4970 POINTS

X = 13.000

F_{MIN} = 9.35E-05
F_{MAX} = 9.99E+10

LINE CHARACTERISTICS

SECTION	Z_0	L
1	6.0	10.0
2	25.0	15.0

Figure 6a

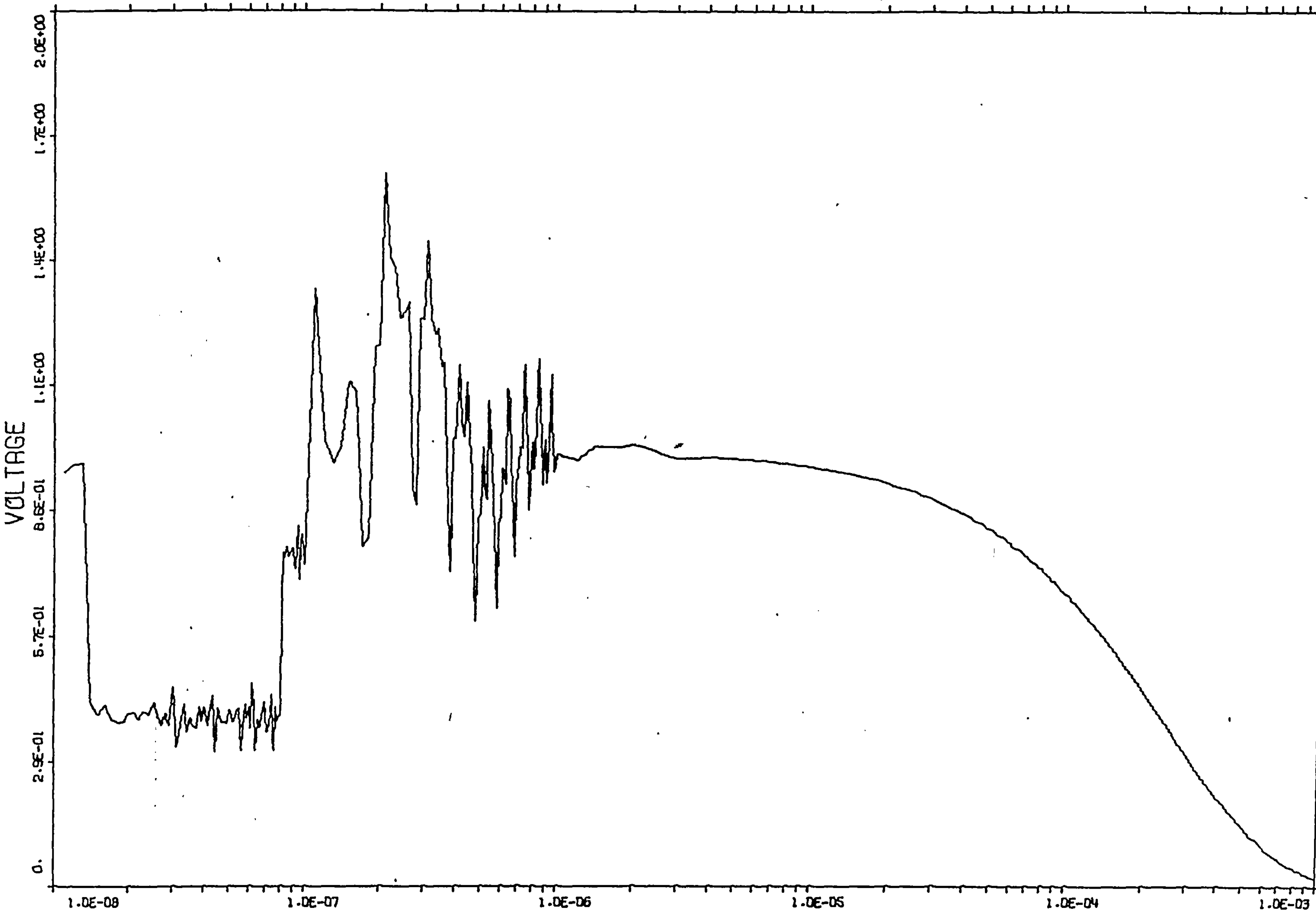

Figure 6b

AFWL EMP 1-5

75-16

RUN CHARACTERISTICS

TIME *19.19.09.
DATE 14FEB69

INPUT:
CHARGED CAPACITOR
$C = 1.00E-05$
SWITCH
$V = U(T)$

LOAD:
IMPEDANCE
$Z_R = 2.50E+01$

TIME STEPS:
401 POINTS

FREQUENCY STEPS:
4944 POINTS

X = 13.000

$F_{MIN} = 1.01E-03$
$F_{MAX} = 9.99E+12$

LINE CHARACTERISTICS

SECTION	Z_0	L
1	6.0	10.0
2	25.0	15.0

Figure 7a

75-18

AFWL EMP 1-5

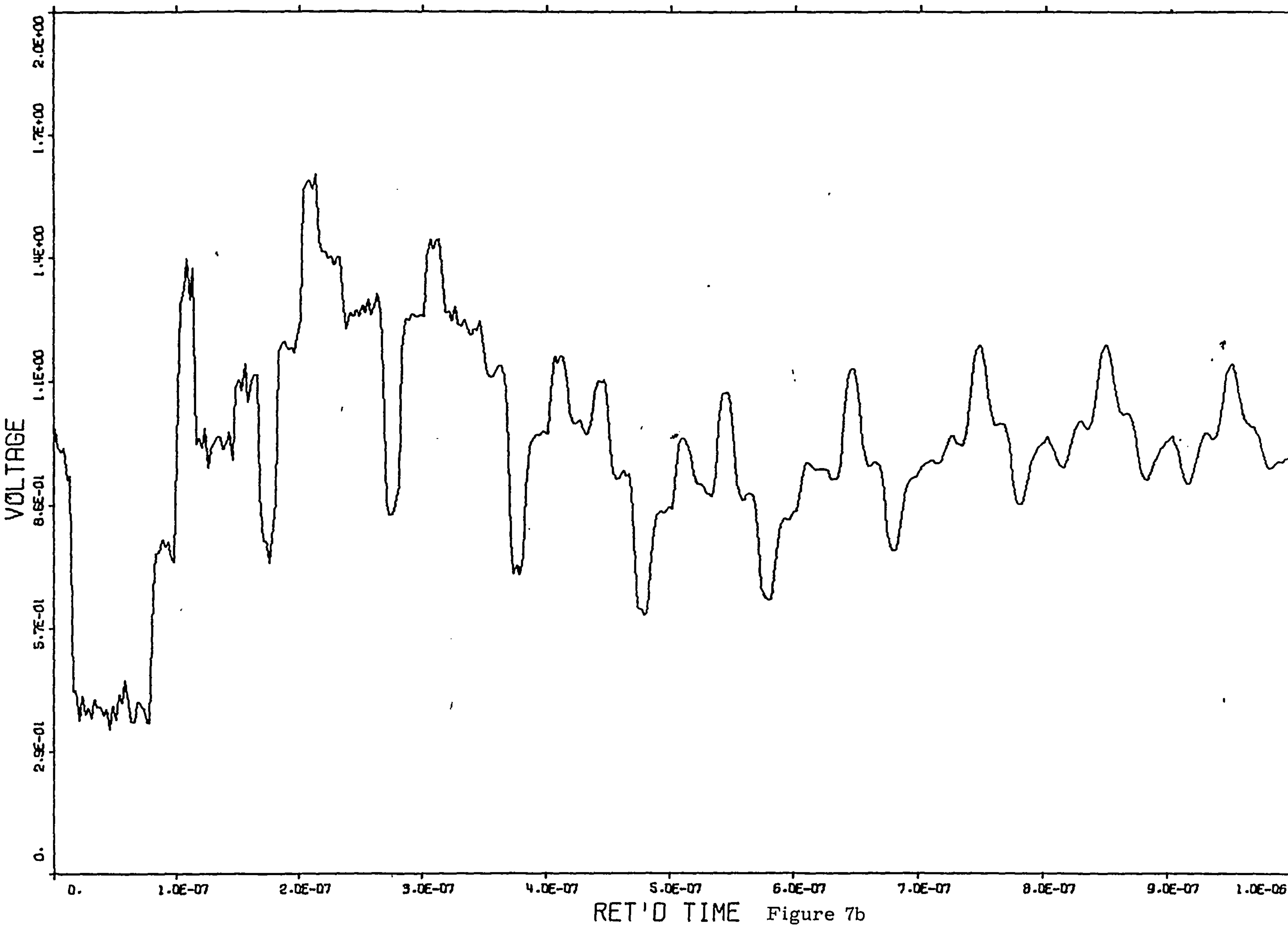

Figure 7b

IV. OPERATION

The Fourier inverse transform routines were modified from Program FORPLEX[2]. These are input on cards. The basic modifications are: the transform is calculated only once, the time to frequency conversion directly extends the frequency range by σ, an integer; and the size of $\Delta\omega$ is calculated to give the number of frequency points determined by the input variable DIMEN. Subroutine FIPI does the frequency range calculations and Subroutines WRSET and WSET convert this range into intervals and associated increments and into a frequency point array. Subroutine CALCLT does the actual calculations for inversion.

Subroutine VOLTAGE calculates the transform using equivalent input voltage and impedance supplied by Subroutine ZECALC and an equivalent load supplied by ZCALC. Both Subroutines ZCALC and ZECALC use the iterative process previously mentioned.

The graphing package used is PLOT000003, a collection of subroutines to store plot data for the Calcomp plotter. It is on the Air Force Weapons Laboratory Library Tape at Kirtland Air Force Base and is a basic part of the software for their CDC 6600 computer.

The load impedance of the entire line Z_o^R, and the characteristic impedances of each section Z_n^o are assumed to be purely resistive. The number of mismatched sections in the lines is limited only by practical considerations; current storage allocation allows for up to 100 sections.

Output consists of a listing of the voltage and the real-time for which the voltage was calculated. The Calcomp plot data is for a voltage, retarded-time graph. One may select the type of graph scaling for time through the

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[2] Sulkowski, Frank J., "FORPLEX, A Program to Calculate Inverse Fourier Transform," Mathematics Note II, Air Force Weapons Laboratory, Kirtland Air Force Base, November 10, 1966.

variable TYPE; a value of three gives a linear time scale and four gives a log scale. The voltage scale is linear. One tape is required in Program TL and it is to store the plot data. It must be a half-inch tape referred to as TAPE9 and the information is to be recorded at 200 bits per inch.

Input to Program TL is shown below.

Card	Columns	Format	Description
1	1-10	E10.3	σ, ω range extension term
2	1-10	E10.3	ξ
	11-20	E10.3	TYPE, determines time axis scaling
3	1-5	I5	NT, number of time intervals
4 . . . . . . NT + 3	1-10	E10.3	t_i in retarded time
	11-20	E10.3	Δt_i in retarded time
NT + 4	1-10	E10.3	DIMEN
NT + 5	1-10	E10.3	Z_o^R
NT + 6	1-10	E10.3	C, input capacitance
NT + 7	11-20	E10.3	N, number of sections of transmission line
NT + 8 . . . . . NT + N+ 7	1-10	E10.3	Z_n^o
	11-20	E10.3	ℓ_n