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SENSOR AND SIMULATION NOTES

NOTE 68

January 1969

Termination of Two Parallel Semi-Infinite Plates by A Matched Admittance Sheet

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Abstract

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TERMINATION OF TWO PARALLEL SEMI-INFINITE PLATES BY A MATCHED ADMITTANCE SHEET

by

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ABSTRACT

The problem of eliminating reflections from an open, two-parallel-plate transmission line by an admittance sheet is formulated as an exterior electromagnetic boundary-value problem and is solved by the extended Wiener-Hopf technique, a technique specially developed to treat diffraction by a thick semi-infinite plate. The required value of the admittance sheet is obtained numerically and presented graphically. The time variation of the sheet current required for eliminating reflections is also calculated at several points on the sheet when the electric field at the position of the sheet is specified to be a step function in time. Realizability of such an admittance sheet by a grid of passive networks is discussed.

Acknowledgments

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I. Introduction

The present paper is devoted to the solution of the problem suggested in a previous note published in this series.[1] The problem is to calculate the characteristics of an admittance sheet that would be required to terminate a two-plate transmission line for zero reflections (figure 1). For wavelengths much larger than the plate separation, it is well known from transmission-line theory that the required admittance should have the value of the characteristic admittance of the line. For other wavelengths, however, the problem has to be approached from the viewpoint of electromagnetic field theory.

To make the problem amenable to mathematical analysis the real situation, which is 3-dimensional in nature (figure 1), is first idealized to be 2-dimensional. Furthermore, the electrical thickness of the sheet is assumed to be so small that the component of the electric-field vector tangent to the sheet is continuous across the sheet. Then the problem posed above can be stated simply as follows. Given that the component of the electric-field vector tangent to the sheet is constant at the sheet, the magnetic field is to be sought from the exterior boundary-value problem. A knowledge of this magnetic field, together with the assumed magnetic field in the interior region between the plates, will uniquely determine the required current density on the sheet, and hence the required admittance function. The problem just stated can be solved by the extended Wiener-Hopf technique specially developed to treat diffraction by a thick semi-infinite plate.[2,3] It is found that the admittance function can be expressed in a cosine series in the spatial coordinate over the sheet's surface, the coefficients of this series satisfying an infinite set of algebraic equations.

The numerical results show that, for wavelengths less than about three times the spacing between the two plates, the admittance function will have a negative real part at some positions on the sheet. This means that, if such wavelengths are of interest, active elements may be required in constructing the sheet.

In synthesizing the required admittance sheet by passive, lumped elements it is useful to have available the time variation of the current that flows on the sheet when the electric field at the position of the sheet is taken to be a step function in time. This time variation can then be collated with that of the current flowing in a resistor-inductor network connected in series to a step-function voltage source. From this collation one can determine the approximate values of the lumped elements required to synthesize the admittance sheet. This approach is particularly appropriate and, perhaps, the best in the present problem.* It is found that for time larger than the transit time across the plates, a resistive-inductive sheet with no spatial variations would suffice to eliminate reflections from the edges of the plates.

*This approach was brought to the authors' attention by Capt. C. E. Baum.

II. Solution by the Wiener-Hopf Technique

As shown in figure 2, the space outside the two parallel semi-infinite plates is divided into two regions. The problem at hand is to find $H_{1z}(0+,y)$ from the wave equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right) H_z(x,y) = 0 \qquad (1)$$

with H_z satisfying the following conditions:

$$H_z \to 0 \qquad \text{as} \quad |x| \;,\; |y| \to \infty \qquad (2a)$$

$$\frac{\partial}{\partial y} H_z = 0 \qquad \text{when} \quad y = \pm b \;,\; x < 0 \qquad (2b)$$

$$\frac{\partial}{\partial x} H_z = ikY_o E_o \;, \quad \text{when} \quad |y| \leq b \;,\; x = 0 \;. \qquad (2c)$$

Here Y_o is the free-space admittance and E_o is a constant electric-field strength.

Define

$$\Phi_+(\alpha,y) = \frac{1}{2\pi} \int_0^\infty H_z(x,y) e^{i\alpha x} \, dx$$

$$\Phi_-(\alpha,y) = \frac{1}{2\pi} \int_{-\infty}^0 H_z(x,y) e^{i\alpha x} \, dx \;, \qquad (3)$$

where α is a complex variable and $\alpha \equiv \sigma + i\tau$. As is customary in the

Wiener-Hopf technique, one first assumes k to have a positive imaginary part, i.e., $\mathrm{Im}\, k > 0$. When the analysis is complete, one then sets $\mathrm{Im}\, k = 0$. Since $H_z \sim \exp(\mp(\mathrm{Im}\, k)x)$ as $x \to \pm\infty$, Φ_+ and Φ_- as defined in (3) are regular respectively in the upper $(\tau > -\mathrm{Im}\, k)$ and lower $(\tau < \mathrm{Im}\, k)$ half-planes.

In region (I), we multiply (1) by $e^{i\alpha x}/2\pi$, integrate the equation by parts, and use (2c). Then

$$\left(\frac{d^2}{dy^2} - \gamma^2\right)\Phi_{1+}(\alpha,y) = -\frac{i\alpha}{2\pi} H_{1z}(0+,y) + \frac{ik}{2\pi} Y_o E_o \tag{4}$$

where $\gamma = \sqrt{\alpha^2 - k^2}$ and $\gamma \equiv -ik$ when $\alpha = 0$.

To eliminate the unknown H_{1z} in (4) we change α to $-\alpha$ in that equation and add the resulting equation to (4). Then

$$\left(\frac{d^2}{dy^2} - \gamma^2\right)\left[\Phi_{1+}(\alpha,y) + \Phi_{1+}(-\alpha,y)\right] = \frac{ik}{\pi} Y_o E_o \tag{5}$$

the solution of which is

$$\Phi_{1+}(\alpha,y) + \Phi_{1+}(-\alpha,y) = -\frac{ik}{\pi} Y_o E_o \gamma^{-2} + B(\alpha)\cosh \gamma y \quad , \tag{6}$$

where the condition of evenness about the $y = 0$ axis has been used.

From the definition (3) we have

$$\Phi_{1+}(\alpha,y) + \Phi_{1+}(-\alpha,y) = \frac{1}{\pi}\int_0^\infty H_{1z}(x,y)\cos \alpha x \, dx \quad .$$

Consequently,

$$H_{1z}(x,y) = 2 \int_0^\infty \left[\Phi_{1+}(\alpha,y) + \Phi_{1+}(-\alpha,y)\right]\cos \alpha x \, d\alpha \quad .$$

Substituting (6) into this equation and evaluating one of the integrals by the method of residues we obtain, after setting $x = 0$,

$$H_{1z}(0+,y) = Y_o E_o + 2 \int_0^\infty B(\alpha)\cosh \gamma y \, d\alpha \quad . \tag{7}$$

To find B we differentiate (6) with respect to y and set $y = b$. Thus

$$B(\alpha) = \frac{1}{\gamma \sinh \gamma b}\left[\Phi'_{1+}(\alpha,b) + \Phi'_{1+}(-\alpha,b)\right] \tag{8}$$

where the prime denotes differentiation with respect to y . Thus, finding B is tantamount to finding the quantity in the square bracket in (8), which will now be obtained by the standard procedure in the Wiener-Hopf technique. To this end, we first set $y = b$ in (6) and obtain with the aid of (8)

$$\Phi_{1+}(\alpha,b) + \Phi_{1+}(-\alpha,b) = -\frac{ik}{\pi} Y_o E_o + \frac{\coth \gamma b}{\gamma}\left[\Phi'_{1+}(\alpha,b) + \Phi'_{1+}(-\alpha,b)\right] \quad . \tag{9}$$

We now consider region (II) in figure 2. Multiplying (1) by $e^{i\alpha x}/2\pi$ and integrating the equation with respect to x from $-\infty$ to ∞ we have

$$\left(\frac{d^2}{dy^2} - \gamma^2\right)\left[\Phi_{2+}(\alpha,y) + \Phi_{2-}(\alpha,y)\right] = 0$$

the solution of which, with proper behavior at infinity, is

$$\Phi_{2+}(\alpha,y) + \Phi_{2-}(\alpha,y) = A(\alpha)e^{-\gamma y} \quad , \quad y \geq b \quad . \tag{10}$$

Differentiation of (10) with respect to y gives, after setting $y = b$ and using (2b),

$$A(\alpha) = -\gamma^{-1}\,\Phi'_{2+}(\alpha,b)e^{\gamma b} \quad .$$

Eliminating A from (10) and setting $y = b$ we obtain

$$\Phi_{2+}(\alpha,b) + \Phi_{2-}(\alpha,b) = -\gamma^{-1}\Phi'_{2+}(\alpha,b) \quad . \tag{11}$$

We now subtract (9) from (11), noting that $\Phi_{1+}(\alpha,b) = \Phi_{2+}(\alpha,b)$ and $\Phi'_{1+}(\alpha,b) = \Phi'_{2+}(\alpha,b)$. Thus

$$\frac{S_+(\alpha)}{\gamma e^{-\gamma b}\sinh\gamma b} + \frac{\coth\gamma b}{\gamma}S_+(-\alpha) + \psi_-(\alpha) - \frac{ik}{\pi}\gamma^{-2} = 0 \quad , \tag{12}$$

where $S_+(\alpha) \equiv (Y_oE_o)^{-1}\,\Phi'_{1+}(\alpha,b)$, and $\psi_-(\alpha) \equiv (Y_oE_o)^{-1}\left[\Phi_{2-}(\alpha,b) - \Phi_{1+}(-\alpha,b)\right]$ which is, of course, regular for $\tau < \text{Im}\,k$.

Let the admittance function Y be defined by

$$Y = Y_o - H_{1z}(0+,y)/E_o \quad .$$

Then, in terms of $S_+(\alpha)$ and $S_+(-\alpha)$ we obtain from (7) and (8)

$$\frac{Y}{Y_o} = - \int_{-\infty}^{\infty} \{S_+(\alpha) + S_+(-\alpha)\} \frac{\cosh \gamma y}{\gamma \sinh \gamma b} d\alpha \quad . \tag{13}$$

The remaining problem is first to find $S_+(\alpha)$ from (12) and then to evaluate the integral (13).

Equation (12) is valid within the strip $- \text{Im } k < \tau < \text{Im } k$ in the complex α-plane. Following the usual Wiener-Hopf technique[2] we write

$$(\gamma b)^{-1} e^{-\gamma b} \sinh \gamma b = L_+(\alpha) L_-(\alpha) \quad , \tag{14a}$$

$$\frac{\coth \gamma b}{\gamma} = A_-(\alpha) - \sum_{s=o}^{\infty} \frac{\varepsilon_s}{2} \frac{1}{\alpha_s b(\alpha+\alpha_s)} \quad , \tag{14b}$$

where

$$\varepsilon_s = \left[\begin{array}{ll} 1 & \text{if} \quad s = 0 \\ 2 & \text{if} \quad s > 0 \end{array} \right.$$

$$\alpha_s = k\sqrt{1 - (s\pi/kb)^2} = ik\sqrt{(s\pi/kb)^2 - 1} \quad .$$

It can be easily shown that $A_-(\alpha)$, as defined in (14b), is regular for $\tau < \text{Im } k$. Substitution of (14) into (12) gives

$$\frac{S_+(\alpha)}{(\alpha+k)L_+(\alpha)} = \frac{ikb}{\pi} \frac{L_-(\alpha)}{a + k} - b(\alpha-k)L_-(\alpha)[\psi_-(\alpha) + A_-(\alpha)S_+(-\alpha)]$$

$$+ (a-k)L_-(\alpha)S_+(-\alpha) \sum_{s=o}^{\infty} \frac{\varepsilon_s}{2} \frac{1}{\alpha_s(\alpha+\alpha_s)} \quad .$$

First, we add extra terms to both sides of this equation to make the left-hand side regular for $\tau > -\,\mathrm{Im}\,k$ and the right-hand side regular for $\tau < \mathrm{Im}\,k$. Then, we apply the Wiener-Hopf technique and assume that both sides are identically zero at infinity. Thus

$$\frac{S_+(\alpha)}{(\alpha+k)L_+(\alpha)} + \sum_{s=o}^{\infty} \frac{\varepsilon_s}{2} \frac{(\alpha_s+k)L_+(\alpha_s)}{\alpha_s(\alpha+\alpha_s)} S_+(\alpha_s) - \frac{ikb}{\pi} \frac{L_+(k)}{\alpha + k} = 0 \quad , \tag{15}$$

where we have made use of the fact $L_+(\alpha) = L_-(-\alpha)$.[2] By setting $\alpha = \alpha_n$ $(n = 0, 1, 2, \ldots)$ in (15) one then obtains an infinite set of simultaneous linear algebraic equations for $S_+(\alpha_n)$.

To reduce (15) to dimensionless form we define

$$a_n = \varepsilon_n \frac{S_+(\alpha_n)}{\alpha_n b} ,$$

$$\lambda_n = \sqrt{n^2 - \theta^2} = -\,i\sqrt{\theta^2 - n^2}$$

$$\theta = \frac{kb}{\pi} \quad .$$

Then (15) becomes

$$\sum_{s=o}^{\infty} \left[2\frac{\lambda_n}{\varepsilon_n}\delta_{ns} + \frac{(\lambda_n+\lambda_o)(\lambda_s+\lambda_o)}{\lambda_n + \lambda_s} L_n L_s \right] a_s = \frac{2i\lambda_o}{\pi} L_o L_n \tag{16}$$

$$n = 0, 1, 2, \ldots .$$

where δ_{ns} is the Kronecker delta function, and L_n denotes $L_+(\lambda_n)$, which will be examined in the next section.

We now return to the integral (13). Since $S_+(\alpha)$ is regular in the upper half-plane, the integral involving $S_+(\alpha)$ can be evaluated by the method of residues by closing the contour at infinity in the upper half-plane. The only poles within the contour are the zeros of $(\gamma b)\sinh(\gamma b)$. The contribution from the integral over the semi-infinite circle can be shown to be zero. The integral involving $S_+(-\alpha)$ can be evaluated in the same manner by noting that $S_+(-\alpha)$ is regular in the lower half-plane. Thus, the evaluation of (13) gives

$$\frac{Y}{Y_o} = 2\pi i \sum_{n=o}^{\infty} (-)^{n+1} a_n \cos\frac{n\pi y}{b} \quad ; \qquad -b \le y \le b \tag{17}$$

where the coefficients a_n satisfy (16).

III. Computation of $L_+(\lambda_n)$

In this section, we shall briefly discuss how to numerically compute $L_+(\lambda_n)$, which is denoted by L_n in (16). In addition, we shall discuss two limiting cases of (17), namely, $\theta \to \infty$ and $\theta \to 0$. The factorization of $(\gamma b)^{-1} e^{-\gamma b} \sinh \gamma b$ into $L_+(\alpha) L_-(\alpha)$ can be found in Reference 2 where L_+ is, in terms of our notation, given by

$$L_+(\lambda_n) = e^{-F(\lambda_n)} \prod_{m=1}^{\infty} \left\{\sqrt{1 - (\theta/m)^2} + \lambda_n/m\right\} e^{-\lambda_n/m} , \tag{18}$$

where

$$F(\lambda_n) = \frac{\pi}{2}|\lambda_n| - i\left[n \cos^{-1} \frac{|\lambda_n|}{\theta} + |\lambda_n|(1 + \ln 2 - C - \ln \theta)\right] , \quad \text{if} \quad \theta > n$$

$$= \lambda_n(1 + \ln 2 - C - \ln \theta) - n \ln \frac{n+\lambda_n}{\theta} - i\,\frac{\pi}{2}(n - \lambda_n) , \quad \text{if} \quad \theta < n . \tag{19}$$

Here $\cos^{-1}\phi = \pi/2$ if $\phi = 0$, and C is Euler's constant.

In computing L_+ the infinite products may cause some difficulty, which may be overcome by introducing the Gamma function Γ :

$$\frac{1}{\Gamma(\lambda_n)} = \lambda_n e^{\lambda_n C} \prod_{m=1}^{\infty} (1 + \lambda_n/m) e^{-\lambda_n/m} . \tag{20}$$

Combining (18) and (20) we have, after making use of the infinite-product representation of $x^{-1}\sin x$,

$$L_+ = \lambda_n^{-1} \Gamma^{-1}(\lambda_n) \exp(-F - \lambda_n C)(\sin \theta\pi/\theta\pi)^{1/2} \prod_{m=1}^{\infty} (1 + \lambda_n/\lambda_m)(1 + \lambda_n/m)^{-1} , \tag{21}$$

where, as before, $\sqrt{-1}$ is understood as $-i$. The infinite products in (21) can be written in the form

$$\prod_{m=1}^{\infty} = \exp\left[\sum_{m=M}^{\infty} \ln(1 + \lambda_n/\lambda_m)(1 + \lambda_n/m)^{-1}\right] \prod_{m=1}^{M} \quad , \tag{22}$$

where, for brevity, the factor $(1 + \lambda_n/\lambda_m)(1 + \lambda_n/m)^{-1}$ has been omitted in the products. From (22) one can readily deduce that, if $M > (\lambda_n\theta^2/2)^{1/3}$,

$$\prod_{m=1}^{M} < \prod_{m=1}^{\infty} < \exp[(M + 5)(\lambda_n\theta^2)/(4M^3)] \prod_{m=1}^{M} \quad . \tag{23}$$

Thus, by choosing M properly one can compute the infinite products to any degree of accuracy.

It is interesting to examine (17) for two limiting cases, namely, $\theta \to \infty$ and $\theta \to 0 (\theta = kb/\pi)$. In the first case $(\theta \to \infty)$, one has

$$\lambda_s \to -i\theta \quad , \quad \text{for all } s < \infty$$

$$L_+(\lambda_s) \to \theta^{-1/2} \quad .$$

Substitution of these into (16) gives $a_n = 0(\theta^{-1})$ for all n . Thus, from (17) we see that Y goes to zero as $\theta \to \infty$, as it should.

In the second limiting case $(\theta \to 0)$, one may make use of the limiting forms of L_+ ,

$$L_+(\lambda_o) \to 1 + 0(\theta \ln \theta)$$

$$L_+(\lambda_n) \to \frac{e^{-n} n^{n-1}}{\Gamma(n)} \qquad (n \neq 0)$$

which can be obtained from (18) and (19), to show from (16) that

$$a_o \to \frac{i}{2\pi} + 0(\theta)$$

(24)

$$a_n \to 0(\theta) \quad , \quad (n \neq 0) \quad .$$

Thus, as expected, Y approaches Y_o as $\theta \to 0$.

IV. Discussion of Numerical Results

Figure 4 through 9 illustrate the variation of the admittance function (17) with frequency ($\theta = kb/\pi$) for some typical points (y/b) on the sheet, while, in figures 10 through 19, the admittance function is plotted against position for several representative frequencies. In these figures, G and B are related to Y by

$$\frac{Y}{Y_o} = G + iB \quad . \tag{25}$$

Due to our choice of the time-harmonic factor $e^{-i\omega t}$, the case $B > 0$ means an inductor in parallel with G, whereas the case $B < 0$ implies a capacitor in parallel with G.

Examination of these curves shows that $G > 0$ if the wavelength $\geq$ three times the plate separation; otherwise, G can be negative at some positions on the sheet. That the required values of G may be negative is also true for the case where the two parallel semi-infinite plates have a perfectly conducting, right-angled flange. In this case, the required admittance function for zero reflections can be easily found to be

$$\frac{Y}{Y_o} = 1 - \frac{1}{2}\left\{\int_o^{k(b+y)} + \int_o^{k(b-y)}\right\} H_o^{(1)}(u)du \quad , \tag{26}$$

where $H_o^{(1)}$ is the Hankel function of the first kind of order zero. These integrals are tabulated in Reference 4. The variation of (26) with frequency at a given position on the sheet is very similar to that of (17) in the unflanged case.

In practice, the required admittance sheet for a given frequency is probably synthesized by a grid of passive-element networks. In figure 3, the sheet is divided into meshes, the maximum linear dimension of each mesh being smaller than the wavelength of interest. Then, within each mesh the variation of H_z with y is negligible (E_y being constant by assumption); that is to say, one may regard the admittance function given by (17) within each mesh as the wave impedance for the wave with propagation vector normal to the sheet. For the mn-th mesh (figure 3) the required lumped admittance $(Y_L)_{mn}$ is equal to the wave admittance (17) multiplied by a geometrical factor, viz.

$$(Y_L)_{mn} = \frac{W}{M}\frac{N}{b} Y = 2\pi i \frac{N}{M}\frac{W}{b} Y_o \sum_{p=o}^{\infty} (-)^{p+1} a_p \cos \frac{p\pi}{b} y_{mn} \quad , \tag{27}$$

where W denotes the width of the sheet and $M \times N$ equals the number of meshes.

For very low frequency we may talk about the total admittance of the sheet and obtain from (24) and (27)

$$(Y_L)_{total} = \frac{1}{\sum\limits_n \frac{1}{\sum\limits_m (Y_L)_{mn}}} \to \frac{W}{b} Y_o \quad , \tag{28}$$

which simply insures the correctness of the definition (27) for the mesh input admittance.

The simplest, and perhaps the best, way to synthesize the admittance function (27) is the following. At the sheet we assume the electric field to be a unit step in time, i.e., $E_y = E_o U(t)$, and calculate the time variations of the required sheet current at various positions on the sheet. Then, we try to approximate the time variations by those of some passive-element networks, thereby determining the required values of the lumped elements. Defining the sheet current K by

$$K(\omega,y/b) = H_{1z}(0-,y/b) - H_{1z}(0+,y/b)$$

we obtain, from the definition of Y/Y_o and for $E_y = E_o U(t)$ at $x = 0$,

$$\frac{K(\tau, y/b)}{K_o} = U(\tau) - \frac{1}{2\pi i} \int_{-\infty}^{\infty} \left(\frac{Y}{Y_o} - 1\right) \frac{e^{-i(\pi\tau)\theta}}{\theta} d\theta \quad , \tag{29}$$

where $K_o = Y_o E_o$, $\tau = ct/b$, and Y/Y_o is given by (17). Equation (29) was computed by a CDC 6600 and the results are shown in figure 20, which illustrates the time variations of K/K_o at several positions on the sheet. In the flanged case, the situation is largely the same as that shown in Fig. 20 except for smaller rise time. Substitution of (26) into (29) gives, after carrying the Fourier inversion,

$$\frac{K_f(\tau, y/b)}{K_o} = U(\tau) - \frac{1}{2} U(1-y/b-\tau) - \frac{1}{2} U(1+y/b-\tau)$$

$$- \frac{1}{\pi} \sin^{-1}\left(\frac{1-y/b}{\tau}\right) U(\tau-1+y/b) - \frac{1}{\pi} \sin^{-1}\left(\frac{1+y/b}{\tau}\right) U(\tau-1-y/b) \quad .$$

When a voltage source, $V(t) = RI_o U(t)$, is connected to a series combination of an inductor L and a resistor R , the current is given by

$$\frac{I(t)}{I_o} = \left(1 - e^{-\frac{R}{L} t}\right) U(t) \quad .$$

In terms of the variable $\tau (= ct/b)$ we have

$$\frac{I(\tau)}{I_o} = (1 - e^{-\beta\tau}) U(\tau) \quad , \tag{30}$$

where $\beta = Rb/(Lc)$. If one takes

$$R = 1/Y_o = \sqrt{\mu_o/\epsilon_o} \quad , \tag{31a}$$

then

$$L = \mu_o b/\beta \quad . \tag{31b}$$

Equation (30) is plotted in Fig. 20 in broken lines for $\beta = 1$ and 0.5. It is seen that the equation (30) with $\beta \approx 0.8$ fits all the solid curves very well for $\tau \geq 2$.

Figure 21 through 25 plot $Re(Y_o/Y)$ and $Im(Y_o/Y)$ against θ at several points on the sheet. We wish to approximate these curves by the input impedance of a R-L network. In accord with the preceding paragraph we set $R = Z_o$ and choose L appropriately. In terms of θ and β defined in (31b) we have

$$\frac{\omega L}{Z_o} = \frac{\pi L}{\mu_o b} \theta = \pi\beta^{-1}\theta = \alpha\theta \quad . \tag{32}$$

Equation (32) is plotted in broken lines for some appropriate values of α in figures 21-25. These curves may be used to supplement figure 20, although they do not yield additional new information in our method of synthesizing the admittance sheet.

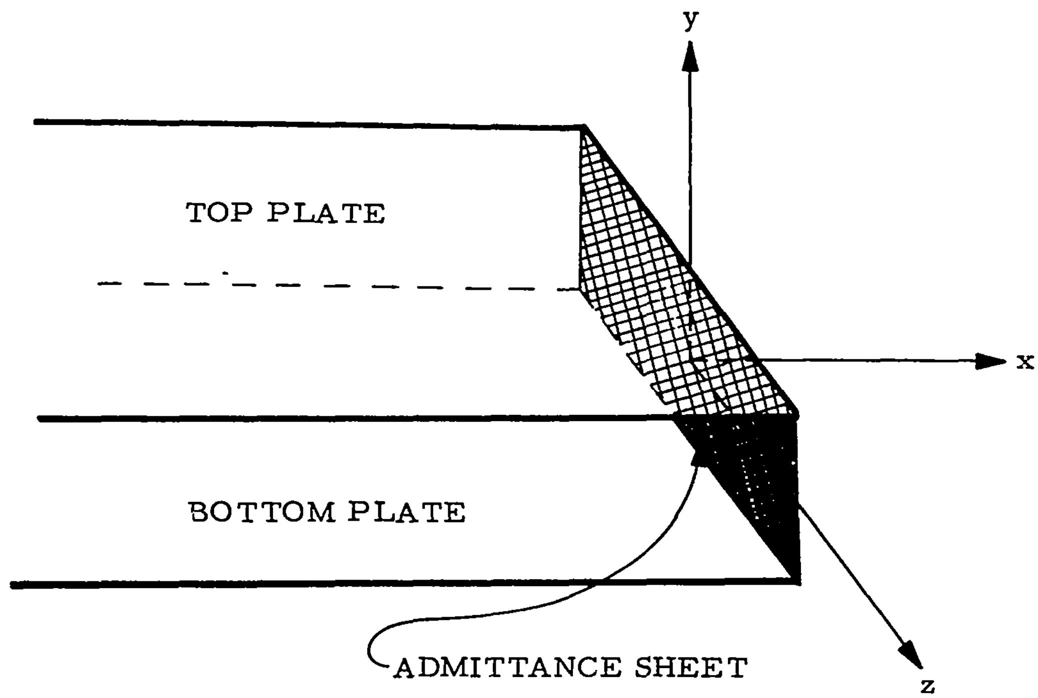


Figure 1. Two parallel semi-infinite plates with an admittance sheet

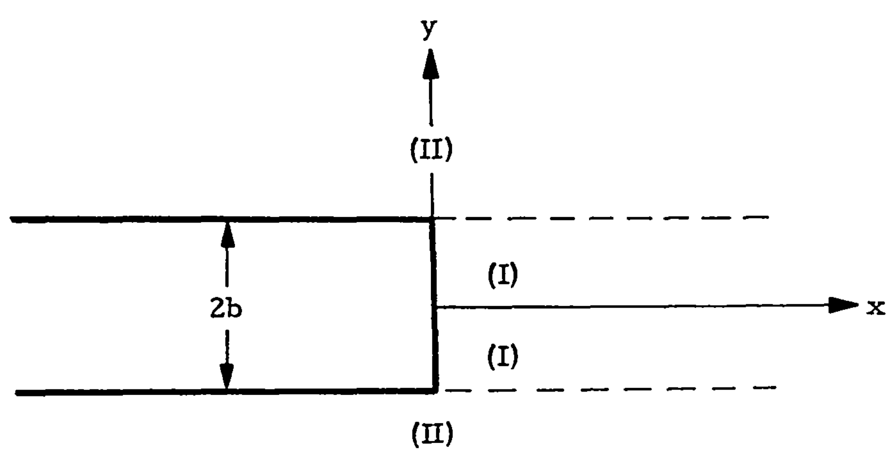


Figure 2. The exterior boundary-value problem

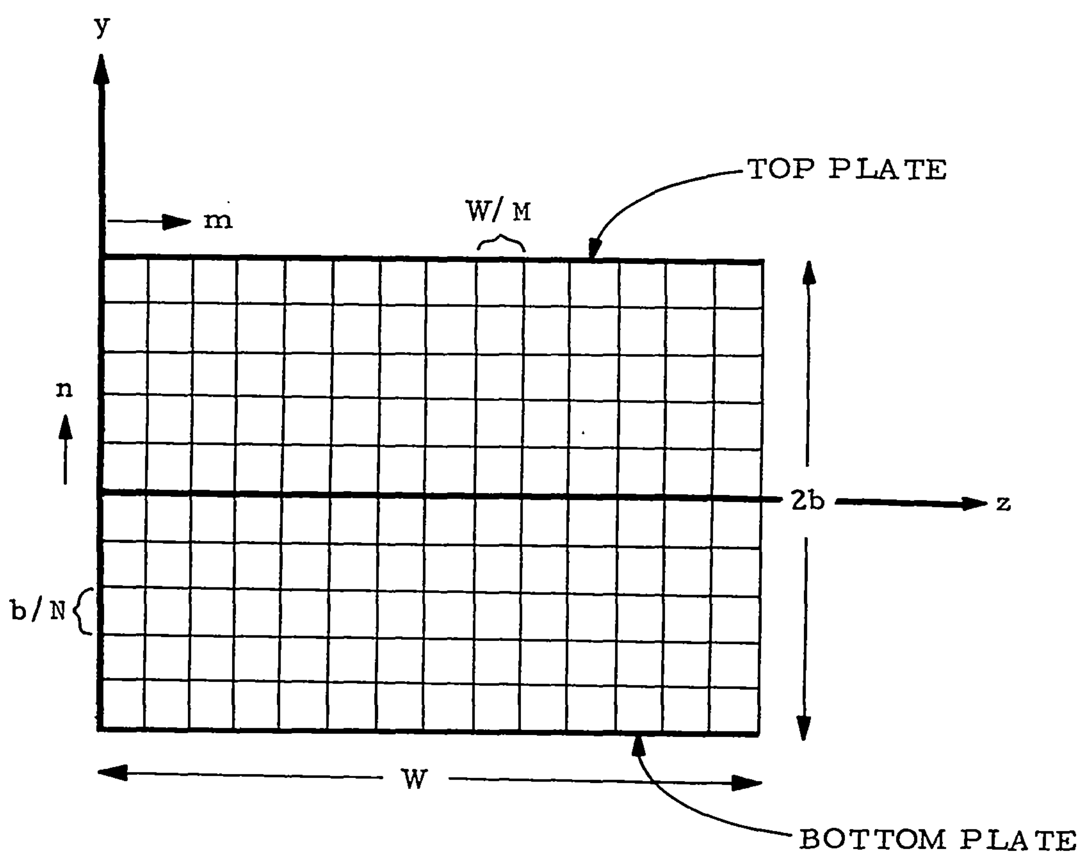


Figure 3. Admittance sheet divided into meshes

Fig. 4 $y/y_o = G + iB$

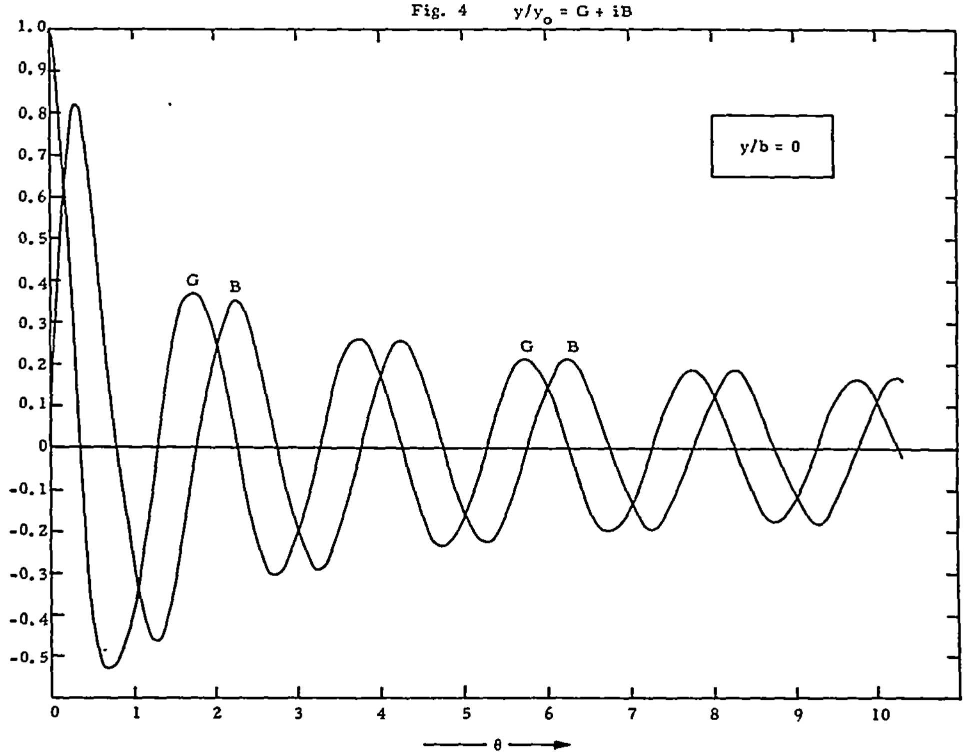


Fig. 5 $y/y_o = G + iB$

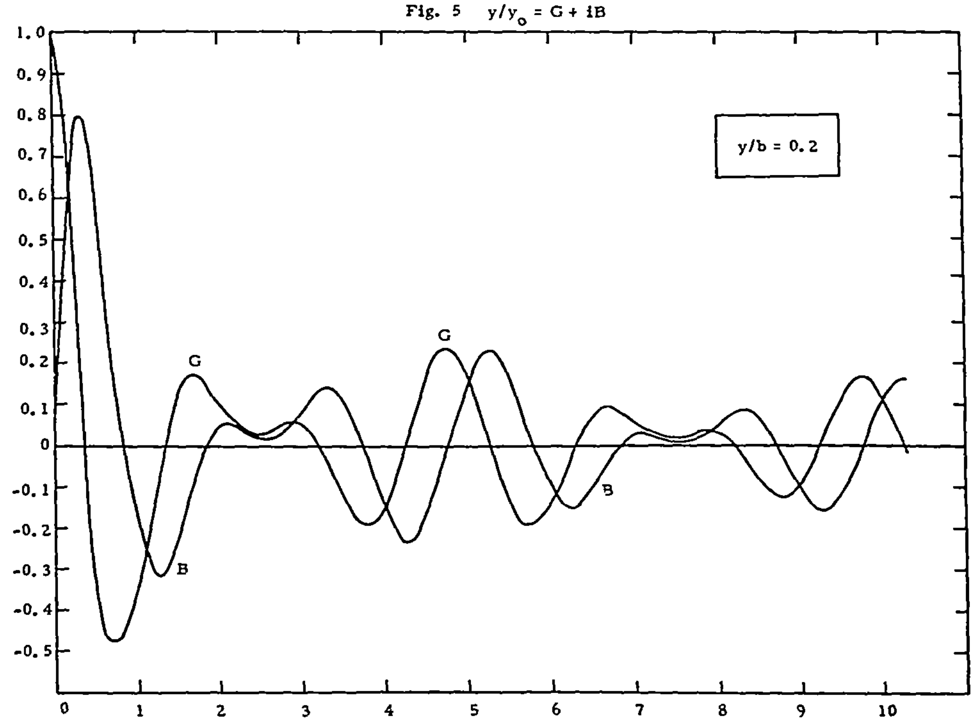


Fig. 6 $y/y_0 = G + iB$

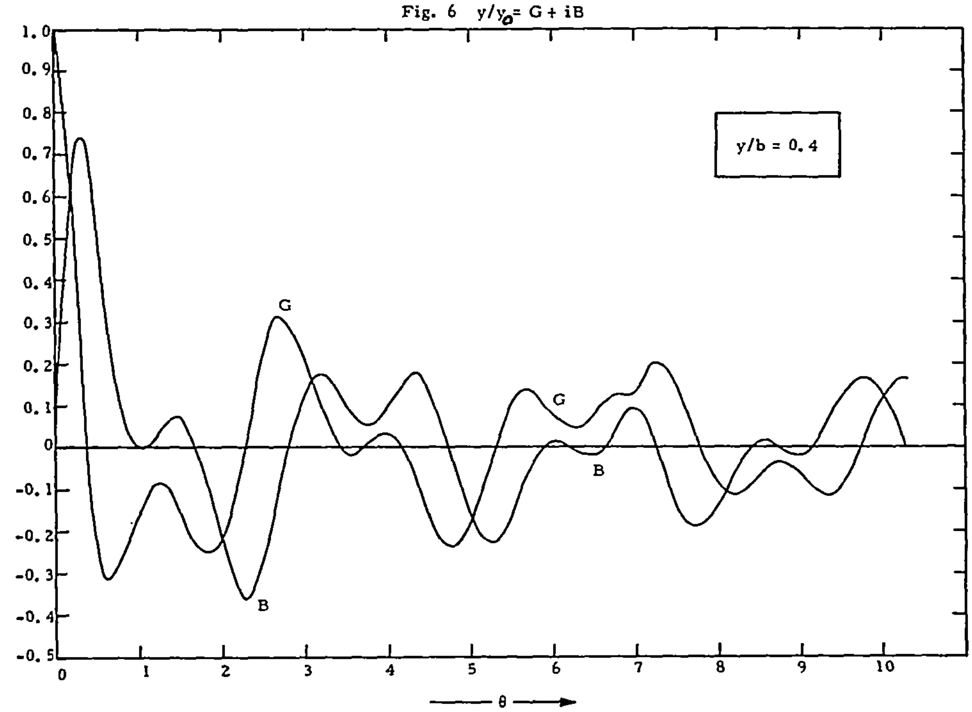


Fig. 7 $y/y_o = G + iB$

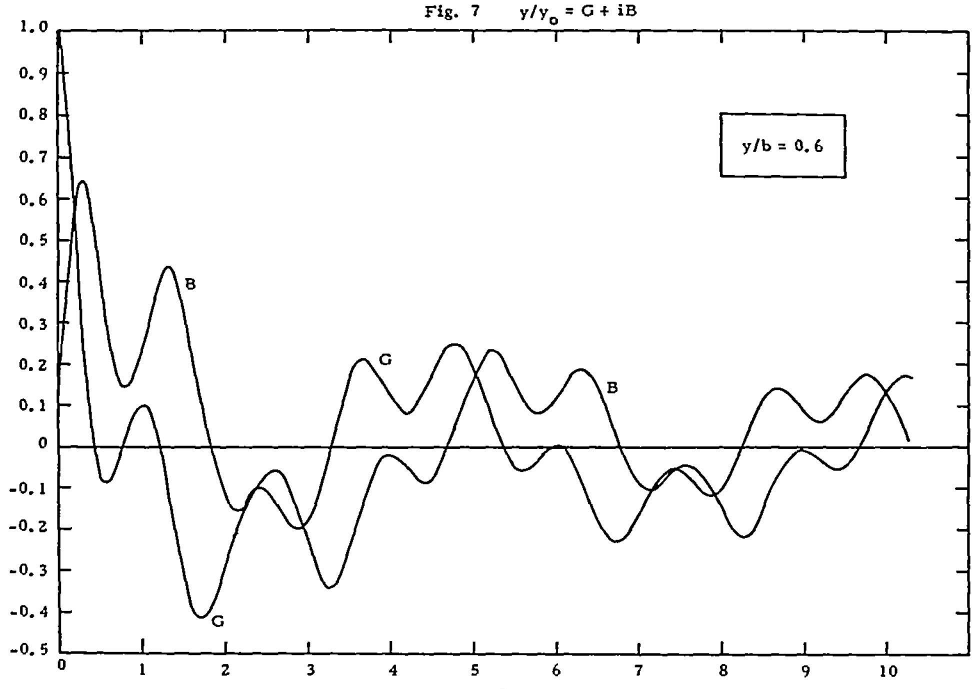


Fig. 8 $Y/Y_o = G + iB$

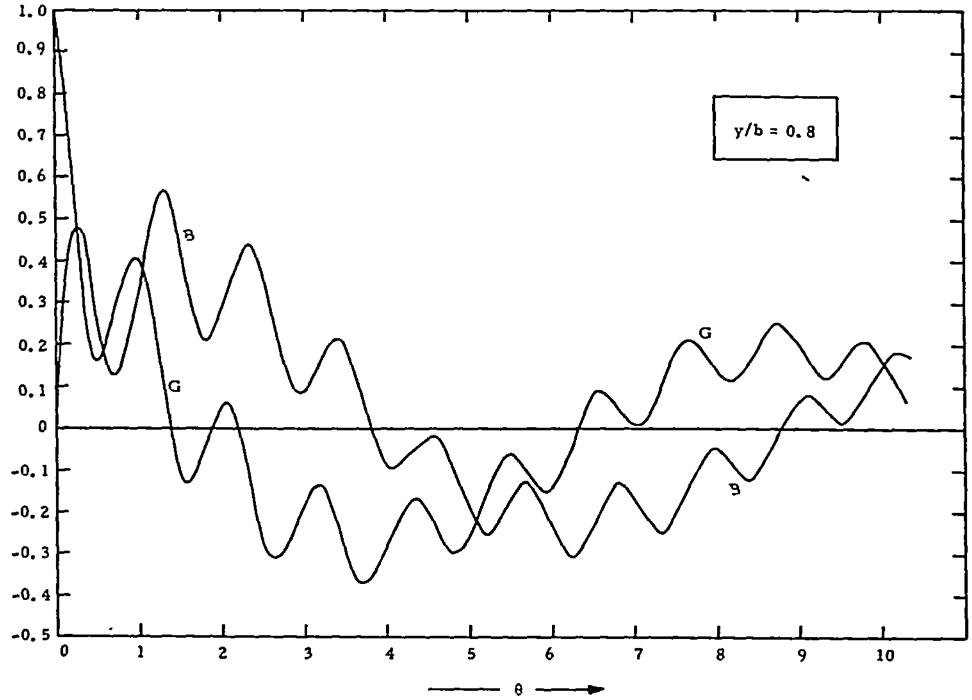


Fig. 9 $y/y_o = G + iB$

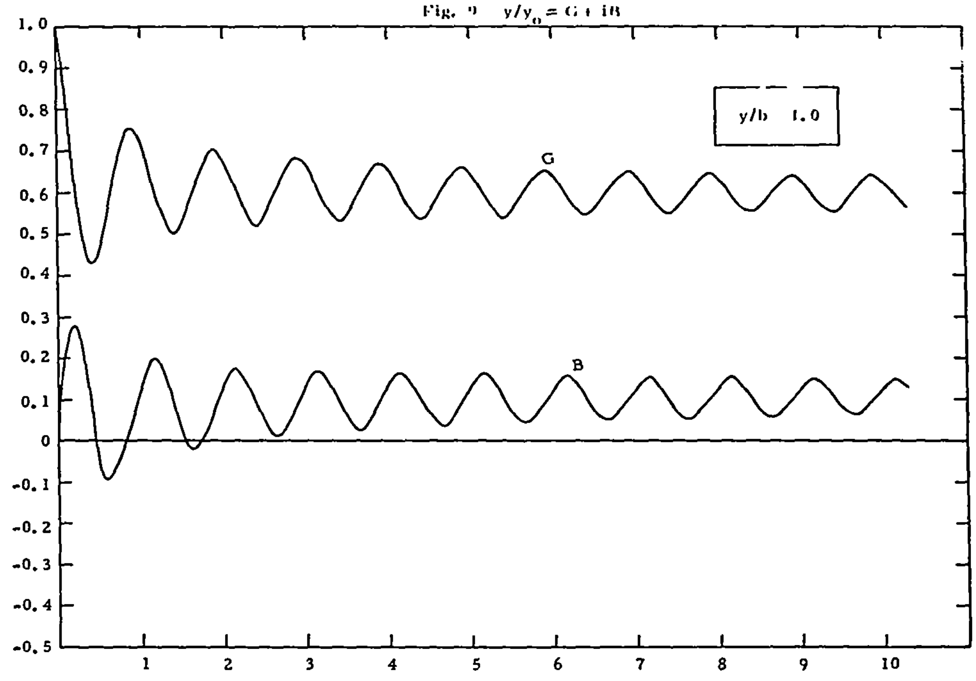


Fig. 10 G vs y/b

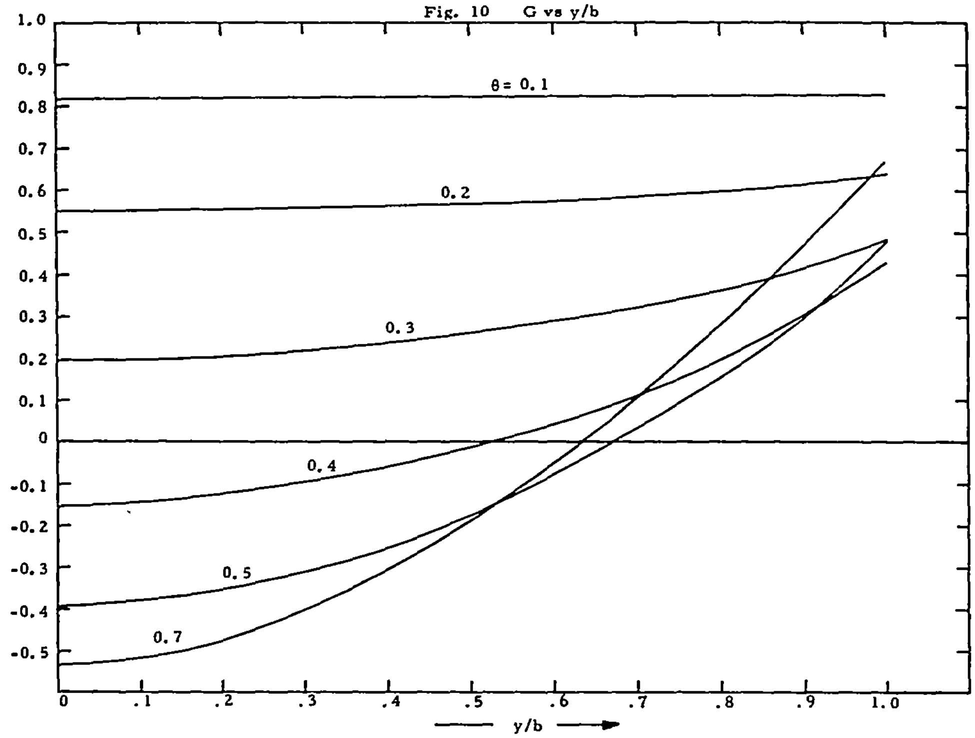


Fig. 11 B vs y/b

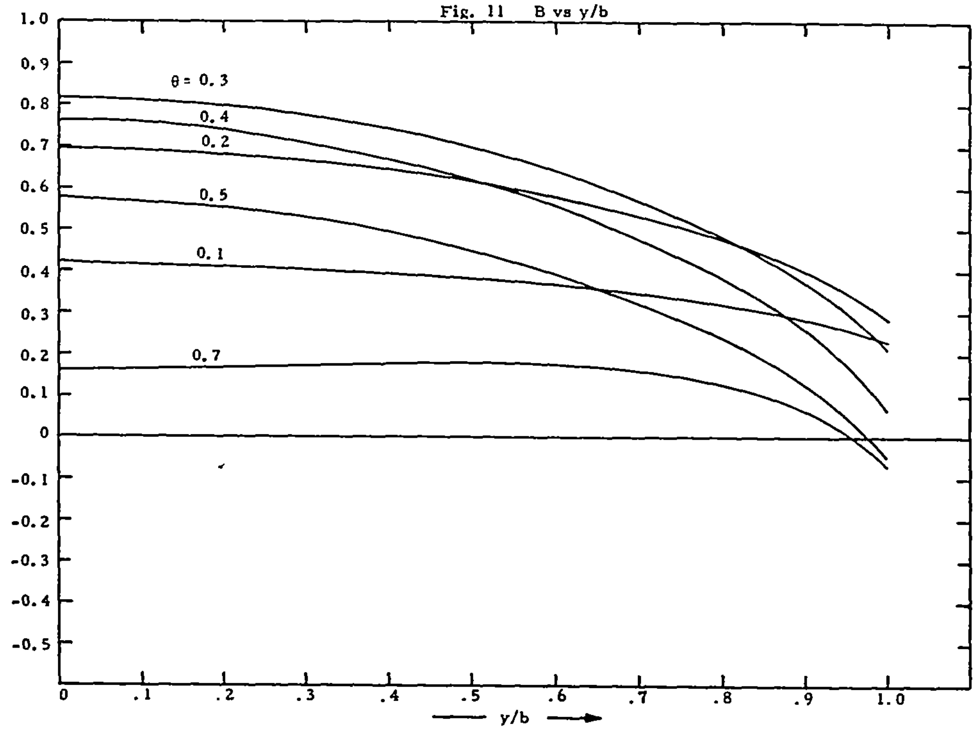


Fig. 12 G vs y/b

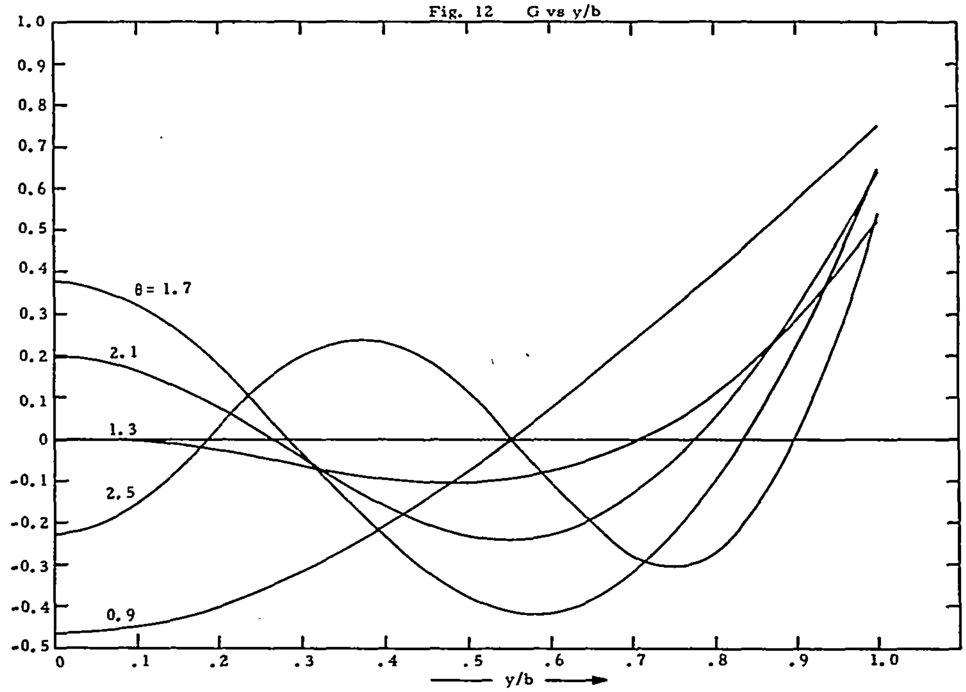


Fig. 13 B vs y/b

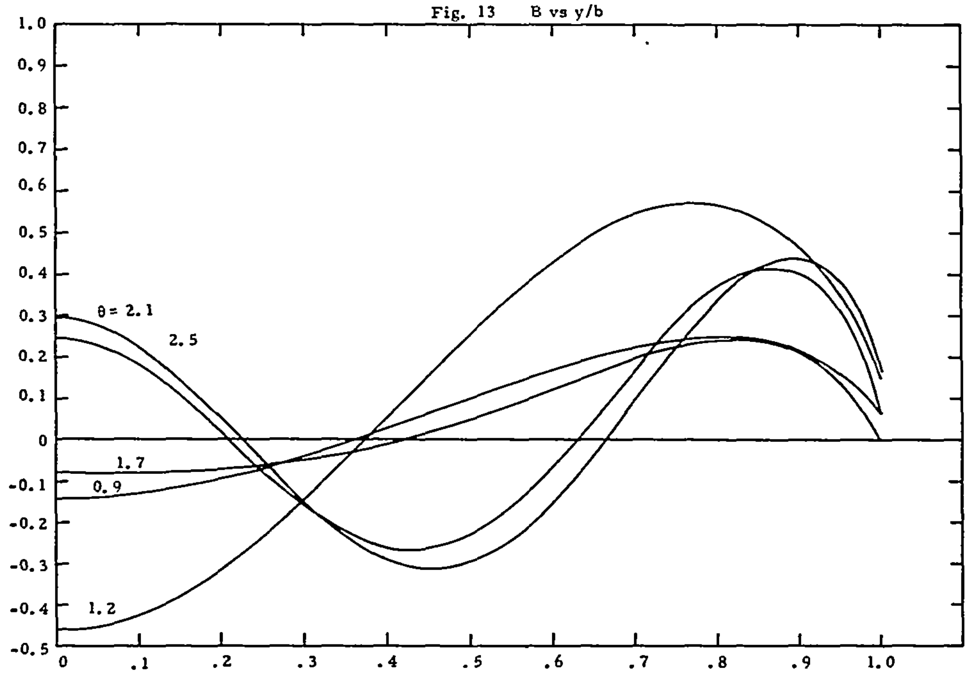


Fig. 14 G vs y/b

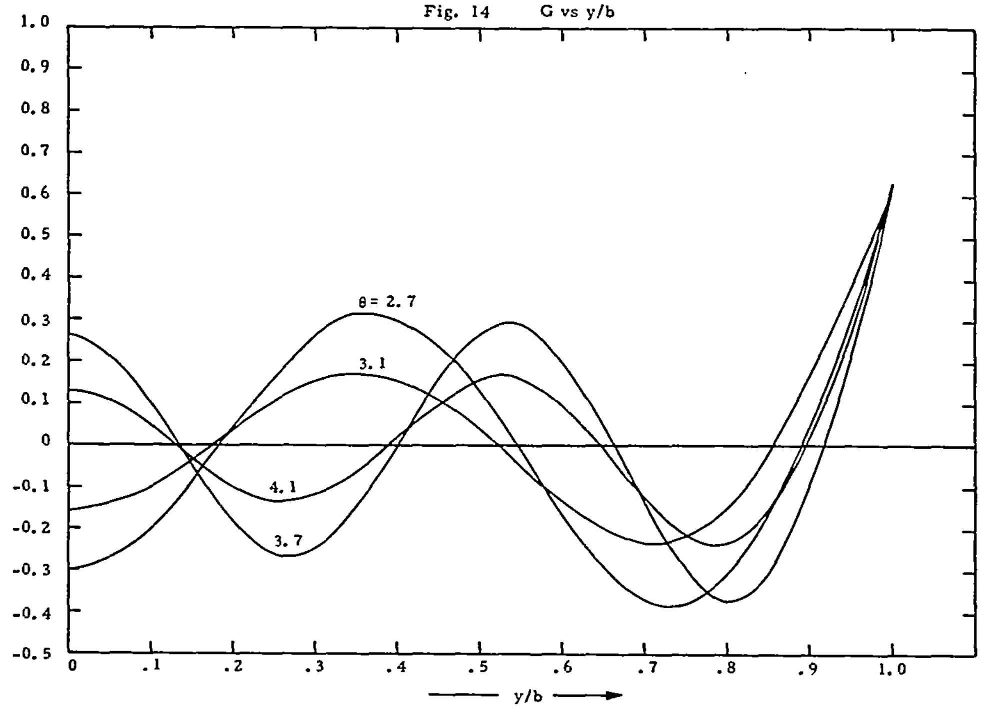


Fig. 15 B vs y/b

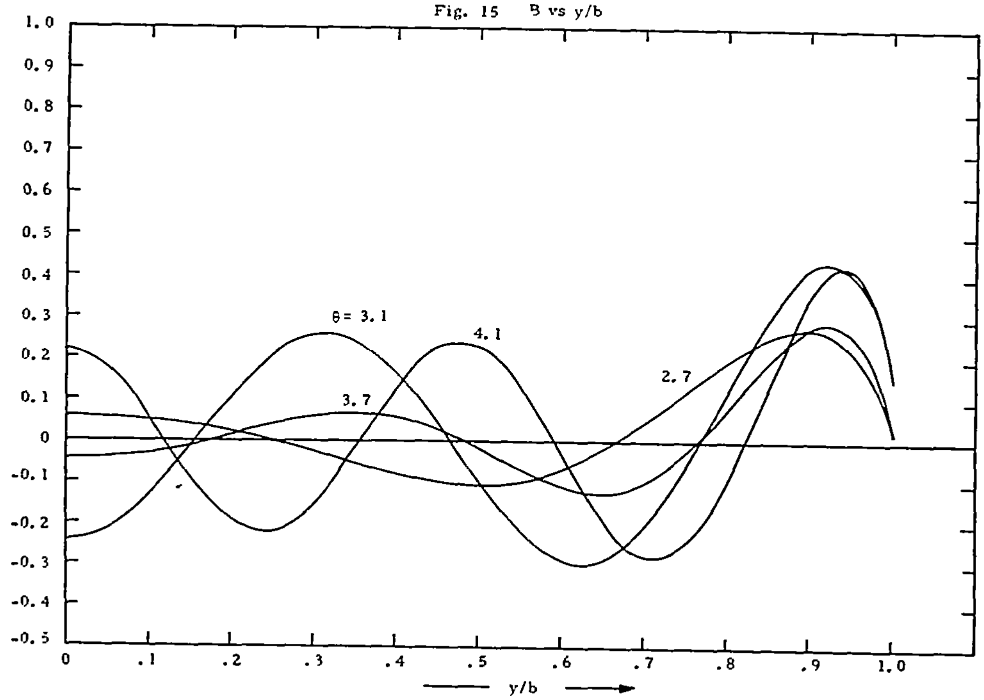


Fig. 16 G vs y/b

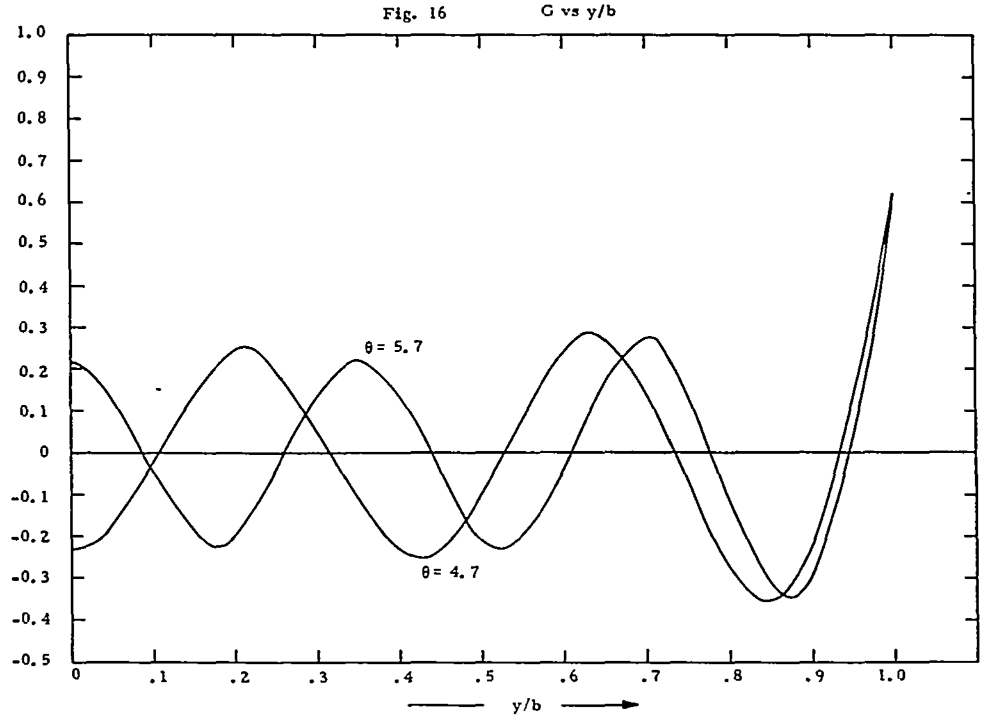


Fig. 17 B vs y/b

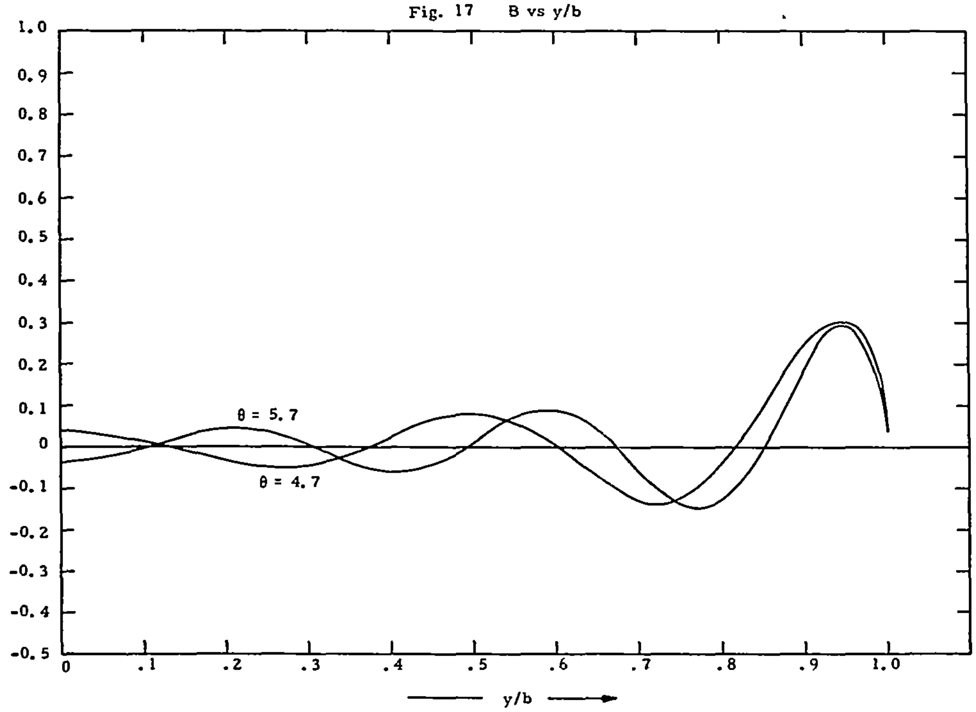


Fig. 18 G vs y/b

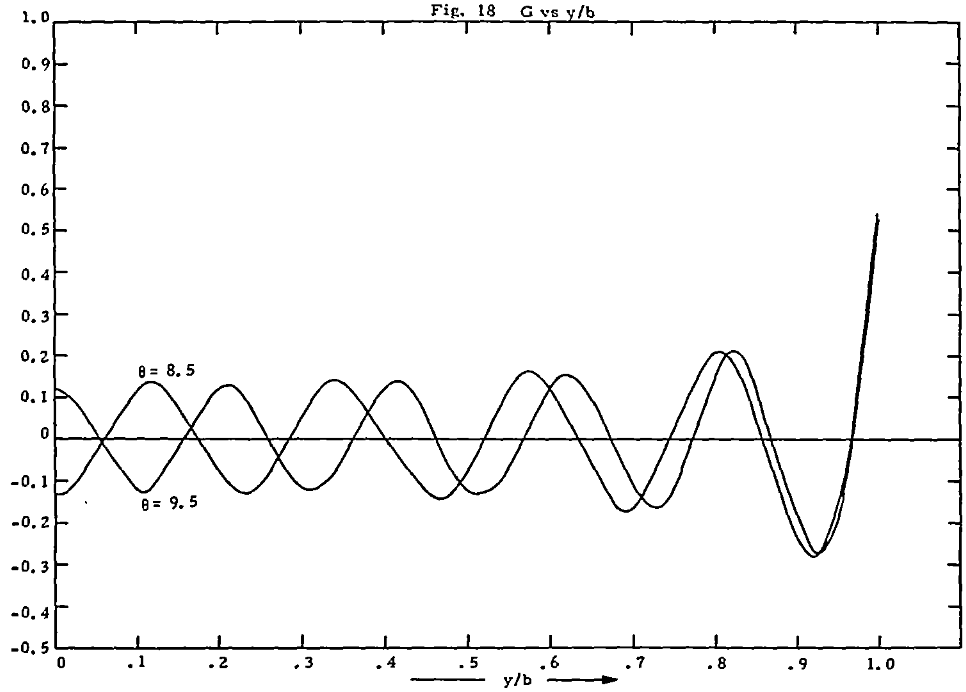
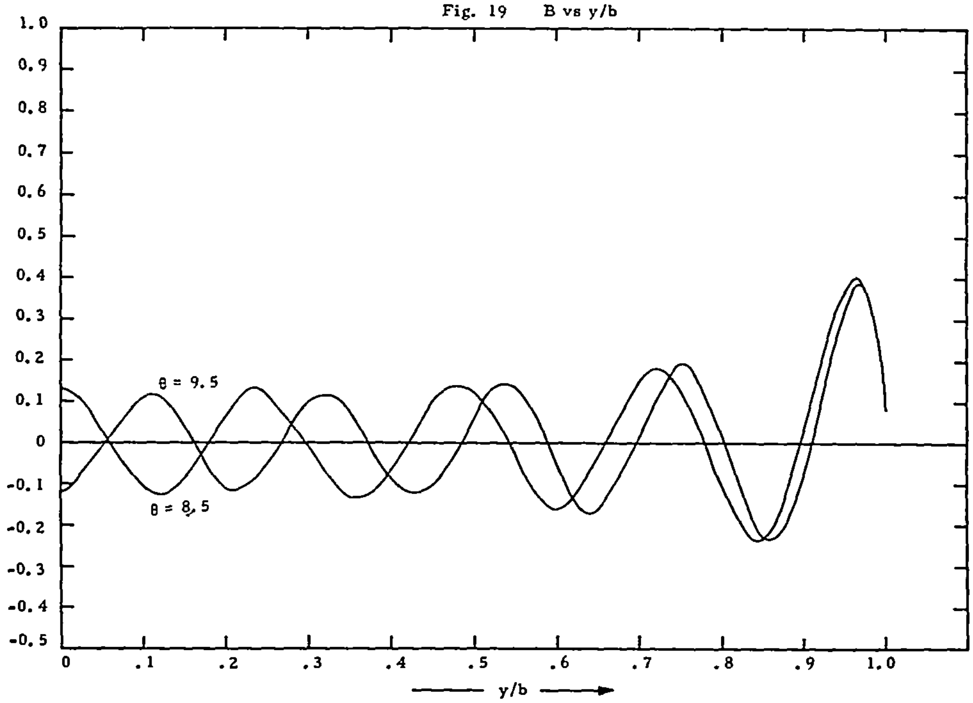


Fig. 19 B vs y/b



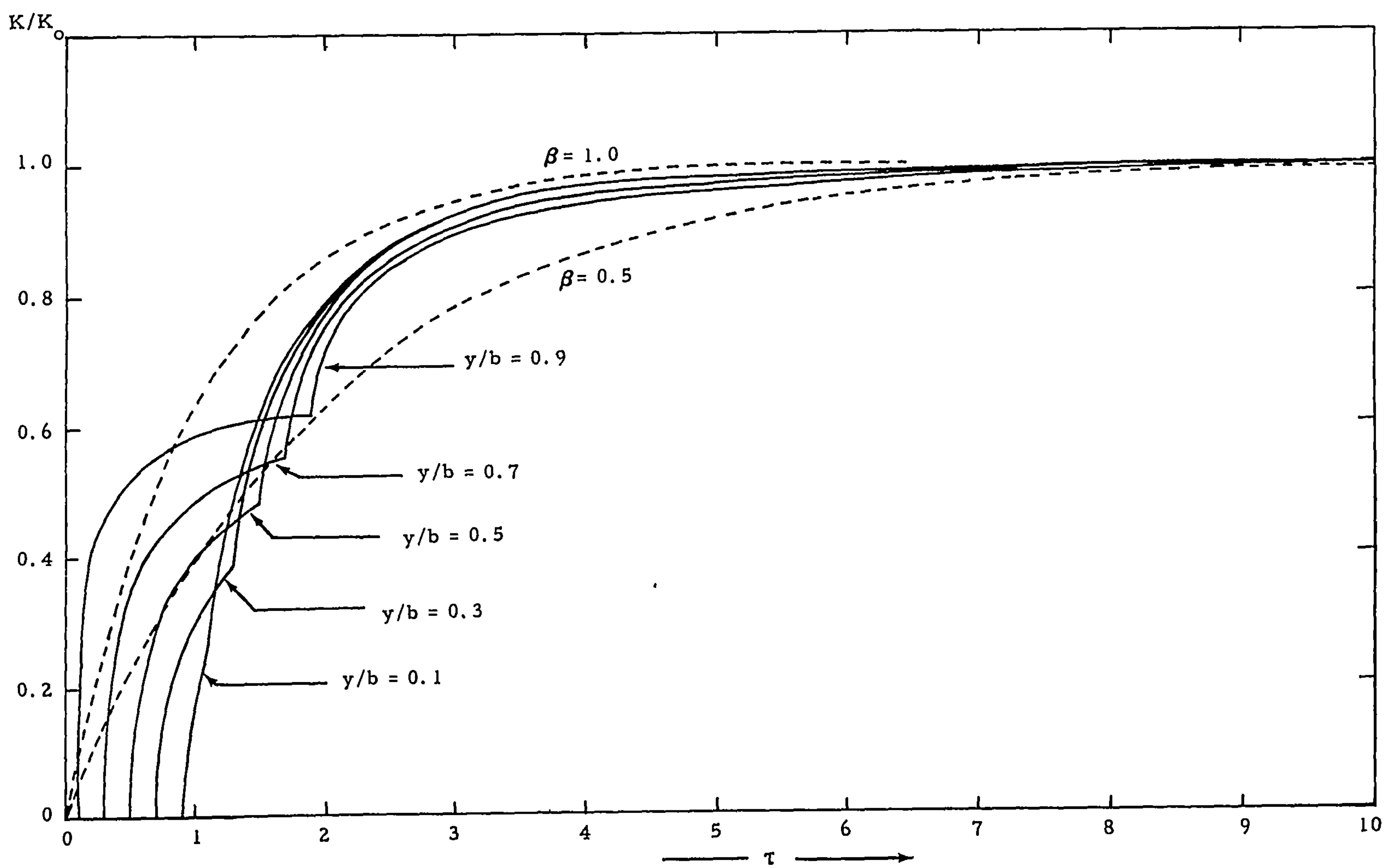


Figure 20. Sheet current vs. time.

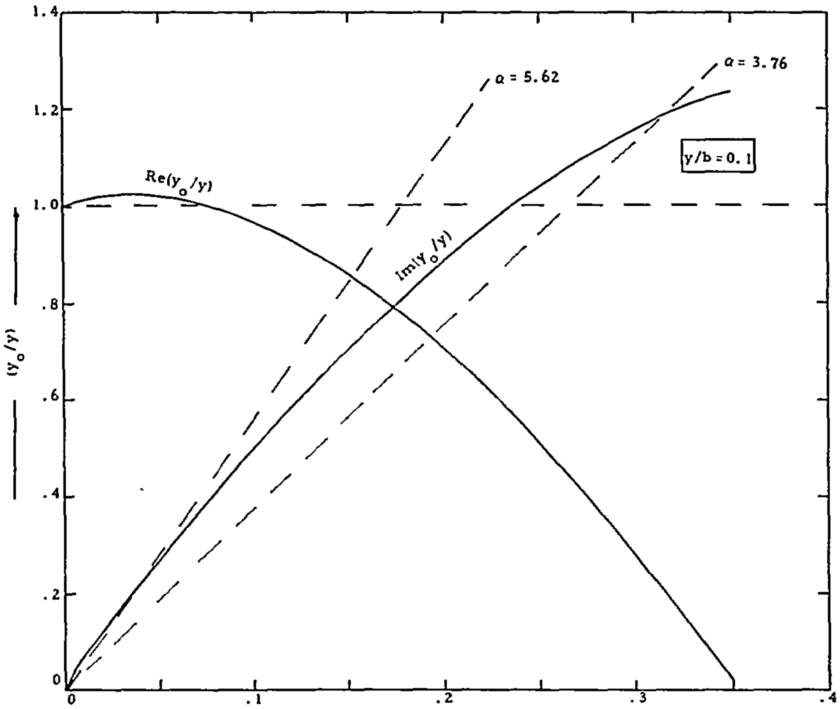


Figure 21. Sheet impedance vs. frequency.

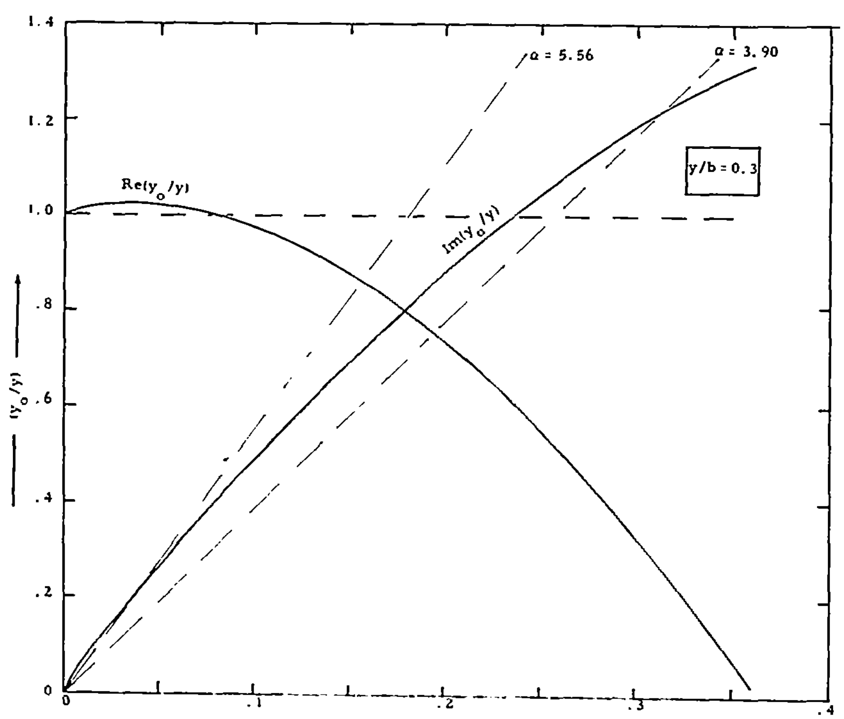


Figure 22

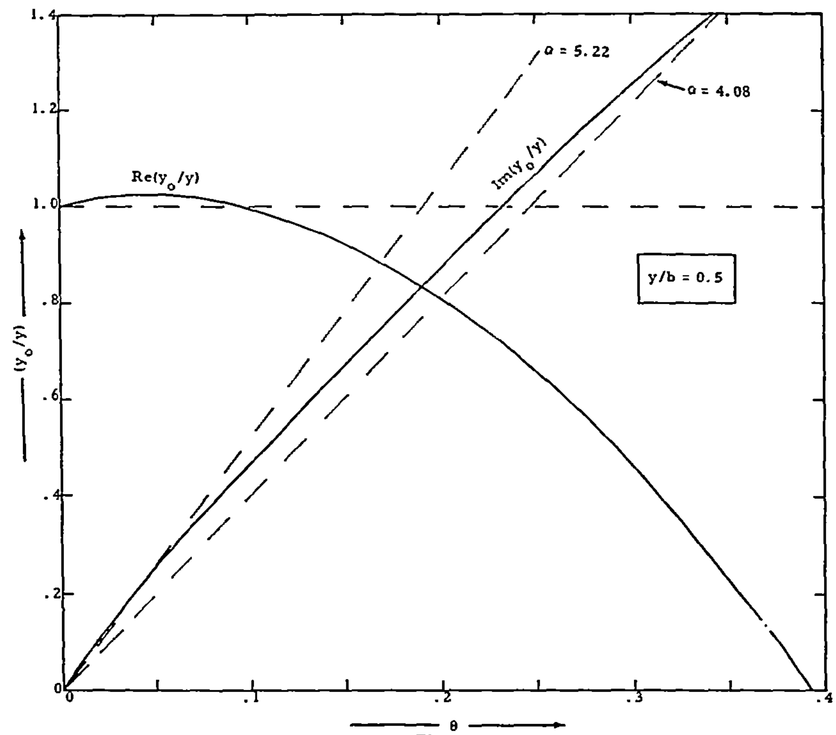


Figure 23.

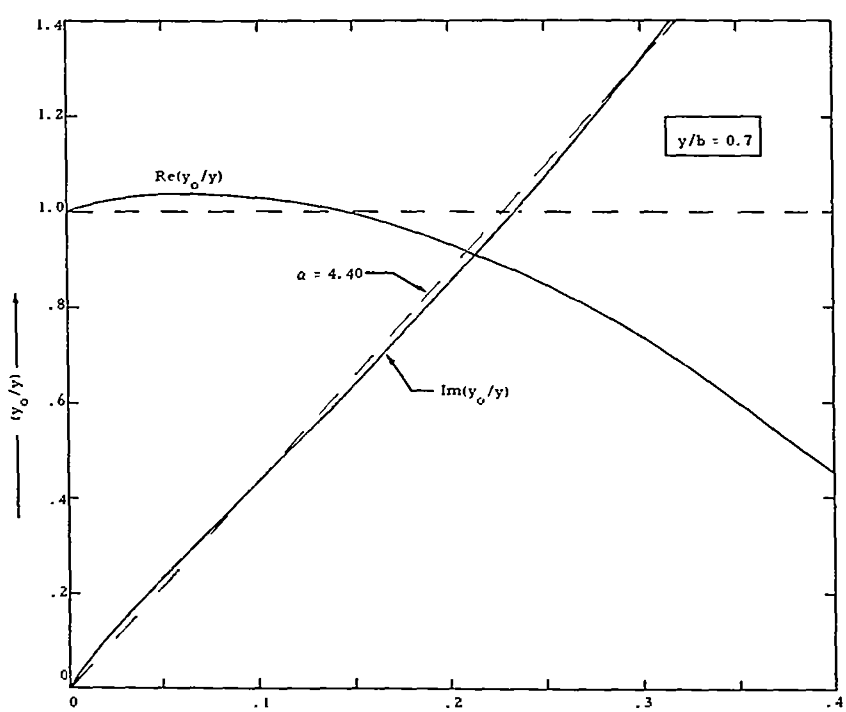


Figure 24.

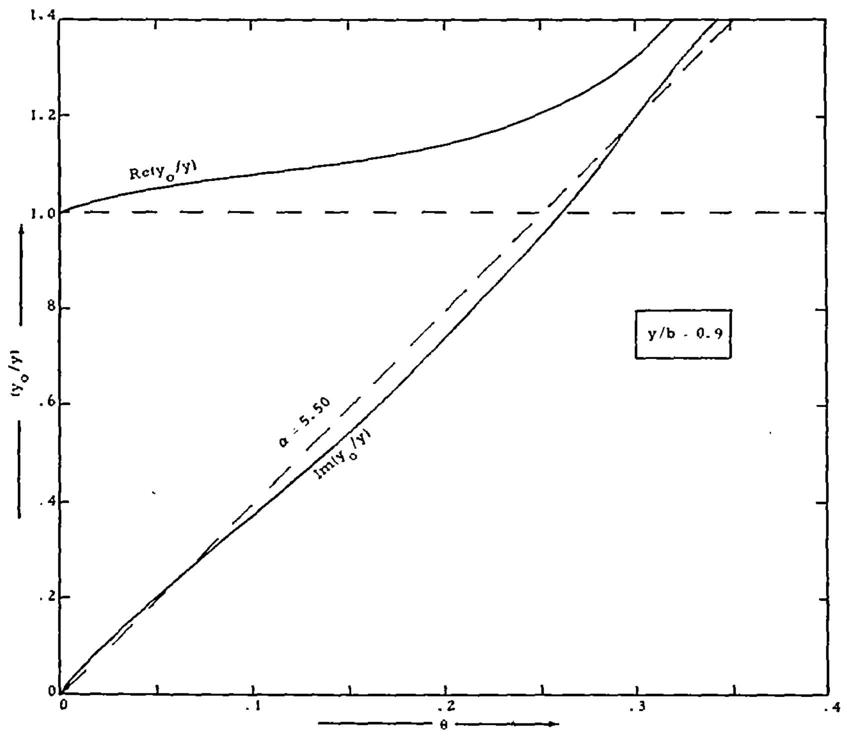


Figure 25

References

1. Baum, Carl E., Sensor and Simulation Note 53, "Admittance sheets for terminating high-frequency transmission lines," April 1968.

2. Noble, B., Methods based on the Wiener-Hopf technique for the solutions of partial differential equations, Pergamon Press, 1958.

3. Jones, D. S., "Diffraction by a thick semi-infinite plate," Proc. Roy. Soc., A 217, pp. 153-175, 1953.

4. Abromowitz, M. and I. A. Stegun, Editors, Handbook of Mathematical Functions, National Bureau of Standards, AMS-55, p. 492, 1964.

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