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Waveforms on a Surface Transmission Line With an Inductive Load

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Abstract

The time dependence of the fields in a surface transmission-line simulator is calculated by using the usual approximations of conventional transmission-line theory. The line is excited by a step-function current generator at one end and loaded by an inductor at some other point. Waveforms are displayed for several ground conductivities, several distances from the source, and several values of load inductance.

Acknowledgment

The authors wish to thank Mr. R. W. Sassman and Mr. G. W. Carlisle, of Northrop, for their help in preparing this note. Thanks are also due to Captain Carl E. Baum for suggesting this problem.

I. Introduction

The possible effects of the electromagnetic pulse generated by a nuclear explosion must be determined. It is particularly important to find out how such a pulse will affect buried military installations. These structures are so complex that an approach to the problem based solely on computational methods is out of the question. On the other hand, actual nuclear tests are too rare to carry out all of the measurements necessary for a complete empirical study. These considerations have led to the development, over the last few years, of more and more sophisticated ways of artificially simulating the nuclear electromagnetic pulse over large areas.

During the later part of its life the real pulse varies slowly; this part of the pulse can be simulated by a buried parallel-plate transmission line.[1] During the earlier part of its life the real pulse varies rapidly; to simulate this portion it is necessary to resort to some other kind of structure, such as a surface transmission line.[2] Because of the complexity of the possible interactions, however, one must simulate the entire time history of the pulse in order to get an accurate picture of all the electromagnetic effects on buried equipment. Therefore, if the buried line and the surface line simulation techniques are indeed used, they must somehow be used simultaneously.

One way that the buried line and the surface line could be combined is shown schematically in figure 1. The source of energy in figure 1 feeds into a tapered transmission-line section leading to the surface line. For actual simulators, several sources in parallel are probably better than the one source indicated in the figure.[3] One plate of the buried line, made up of numerous vertical rods,[4] is inserted in the ground at approximately the position where the surface line starts. The second buried line plate is placed at the end of the surface line, and beyond that is a tapered termination section. In order to make efficient use of the slowly varying portion of the source energy, it is perhaps advisable[5] to place an inductive sheet at the position shown in figure 1. Such a sheet creates a low-impedance path for slowly varying currents and so avoids a great deal of dissipation in the load resistance, but it also disturbs the waveform of the electromagnetic fields within the surface line by generating undesired reflections. This note contains approximate curves of the field waveforms that can be expected if the simulation scheme shown in figure 1 is indeed used. By examining these curves it may be possible to decide if the waveform distortion for a given sheet inductance is acceptable.

It is impossible to analyze accurately the structure shown in figure 1. Therefore, taking the attitude of a man who looks for a lost dollar where the light is bright rather than where he dropped it, we will analyze a somewhat different problem -- the ordinary transmission line with three sections shown in figure 2. We will choose the parameters (Z's and γ's) of the transmission-line approximation to the problem according to our best judgment. (These parameters will be found to differ slightly from those used for similar analyses in the past.[2]) Nevertheless, a certain amount of blind faith is necessary in order to believe that the current and voltage on the transmission line of figure 2 represent the fields within the structure of figure 1.

In Section II we treat the special case where the entire line in figure 2 has a single set of parameters (a real Z and an imaginary γ). This case, where more can be done analytically than if the line were to have three sections and complex parameters, may be useful for purposes of comparison; it also may correspond to the real problem if the ground is highly conducting.

In Section III, waveforms are computed for the case where the transmission line parameters are complex. Hopefully, this case represents the real problem if the ground has a finite, and yet large, conductivity.

In Section IV some further remarks are made on the applicability of the conventional transmission-line approximation to the analysis of the structure shown in figure 1.

II. Lossless Line

Suppose that, in the conventional transmission line shown in figure 2, we set Z_1 equal to Z (Z being assumed in the present case to be a real, frequency-independent impedance) and also set $\gamma_1 = \gamma = \omega/c$, where ω is the Fourier-transform variable and $1/c = \sqrt{\mu_o \varepsilon_o}$. Suppose, in addition, that for the time being we allow the load inductor, L, to be in parallel with any real impedance Z_L rather than the line impedance as indicated in the figure. Then we may invoke conventional transmission-line theory and Fourier's integral theorem to write the following integral representation of the current due to a step-function current generator of strength I_o:

$$i(t,x) = \frac{I_o}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-i\omega t}}{-i\omega} \left[\frac{e^{i\omega x/c} + R\, e^{i\omega(2\ell - x)/c}}{1 + R\, e^{2i\omega\ell/c}} \right] d\omega \tag{1}$$

where

$$R = \frac{Z_L + i\omega L(1 - Z_L/Z)}{Z_L + i\omega L(1 + Z_L/Z)} .$$

Let us denote the transit time of the line by:

$$t_1 \equiv \ell/c$$

and the total time constant of the termination by:

$$t_2 \equiv L(Z + Z_L)/ZZ_L .$$

Further, let us define a pair of parameters characterizing the load through the equations:

$$g \equiv Z/Z_L$$

$$r \equiv t_1/t_2 \quad .$$

Finally, let us define a set of normalized variables by:

$$\xi \equiv x/\ell$$

$$\tau \equiv (t/t_1) - \xi$$

$$h(\tau,\xi) \equiv i(t,x)/I_o \quad .$$

Employing this notation, we may integrate equation (1) in the form:

$$h(\tau,\xi) = U(\tau) + \sum_{n=1}^{\infty} \left[F_n(\tau-2n+2\xi) - F_n(\tau-2n)\right] \quad , \qquad (2)$$

where $U(\tau)$ is the unit-step function and

$$F_n(\tau) = U(\tau)\left[1 - e^{-r\tau}\left(\frac{g-1}{g+1}\right)^n \sum_{m=0}^{n-1} \frac{(r\tau)^m}{m!} \sum_{s=m+1}^{n} \binom{n}{s}\left(\frac{2}{g-1}\right)^s\right] \quad . \qquad (3)$$

An examination of equation (3) will reveal that, unless g is unity (i.e. unless $Z_L = Z$), the current has undesired finite jumps at several values of time. Therefore, for the rest of this note we will assume that g is unity. With this value of g, the auxiliary functions, F_n, reduce to:

$$F_n(\tau) = U(\tau)\left[1 - e^{-r\tau} \sum_{m=0}^{n-1} \frac{(r\tau)^m}{m!}\right] \tag{4}$$

The representation of the normalized current given by equation (2) is well suited to numerical work; by using it and the analogous expression for the normalized voltage,

$$e(\tau,\xi) \equiv \frac{v(t,x)}{I_o Z} = U(\tau) + \sum_{n=1}^{\infty} (-)^n \left[F_n(\tau-2n+2\xi) + F_n(\tau-2n)\right] \quad ,$$

we obtained the data shown in graphical form in figures 3 through 10. The notation used to label these curves is the same as that given above.

One further quantity may be of interest: the maximum deviation of the normalized current from unity; it can be seen from the curves that this maximum occurs when τ is 2; so from equations (2) and (4) we can write

$$(\Delta h)_{max} = 1 - e^{-2r\xi} \quad .$$

This quantity is plotted as a function of r for various ξ's in figure 11; it is plotted as a function of $1/r$ for the same ξ's in figure 12.

III. Line With Lossy Section

Consider the situation depicted in figure 2 with $\ell_2 = 0$. The problem is to calculate the normalized current $h(\tau,\xi)$ and the normalized voltage $e(\tau,\xi)$ on the line when the current source at $x = 0$ is a step function in time. Direct application of the conventional transmission-line theory gives the following expressions that have already been arranged in the form best suitable for numerical computation:

$$
\begin{aligned}
e_1(\tau,\xi) &= \frac{K}{\sqrt{\pi\tau}} + \frac{1}{2\pi}\int_{-\infty}^{\infty} A_1(\omega)e^{-i\omega\tau}\, d\omega \\
h_1(\tau,\xi) &= U(\tau) + \frac{1}{2\pi}\int_{-\infty}^{\infty} B_1(\omega)e^{-i\omega\tau}\, d\omega
\end{aligned}
\tag{5}
$$

for $0 \le \xi \le (1 + \alpha)^{-1}$, and

$$
\begin{aligned}
e(\tau,\xi) &= \frac{K}{\sqrt{\pi\tau}} + \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\omega)e^{-i\omega\tau}\, d\omega \\
h(\tau,\xi) &= U(\tau) + \frac{1}{2\pi}\int_{-\infty}^{\infty} B(\omega)e^{-i\omega\tau}\, d\omega
\end{aligned}
\tag{6}
$$

for $1 \ge \xi \ge (1 + \alpha)^{-1}$. Here

$$
A_1 = \frac{i}{2\pi}\left\{\left(\frac{Z_i}{Z_1} + 1\right) + \left(\frac{Z_i}{Z_1} - 1\right)e^{-2i\omega\xi}\right\} - \frac{K}{\sqrt{-i\omega}}
$$

$$
B_1 = \frac{i}{2\omega}\left\{\left(\frac{Z_i}{Z_1} + 1\right) - \left(\frac{Z_i}{Z_1} - 1\right)e^{-2i\omega\xi}\right\} - \frac{i}{\omega}
$$

$$A = \frac{i}{4\omega}\left(\left(\frac{Z_i}{Z_1}+1\right)+\left(\frac{Z_i}{Z_1}-1\right)e^{-2ik_1\ell_1}\right)\times$$

$$\left(\left(1+\frac{Z/Z_1}{Z_s/Z_1}\right)e^{-i\omega f_-}+\left(1-\frac{Z/Z_1}{Z_s/Z_1}\right)e^{-i\omega f_+}\right)-\frac{K}{\sqrt{-i\omega}}$$

$$B = \frac{i}{4\omega}\left(\left(\frac{Z_i}{Z_1}+1\right)-\left(\frac{Z_i}{Z_1}-1\right)e^{-2ik_1\ell_1}\right)\times$$

$$\left(\left(1+\frac{Z_s/Z_1}{Z/Z_1}\right)e^{-i\omega f_-}+\left(1-\frac{Z_s/Z_1}{Z/Z_1}\right)e^{-i\omega f_+}\right)-\frac{i}{\omega}$$

$$K = \frac{\alpha}{1+\alpha}\,\frac{1}{\beta\sqrt{q}}$$

$$f_\pm = \left(\xi-\frac{1}{1+\alpha}\right)\left(1\pm\frac{\gamma}{\gamma_1}\right)$$

$$\frac{Z_i}{Z_1} = \frac{\left(Z_s/Z_1\right)-i\tan\left(k_1\ell_1\right)}{1-i\left(Z_s/Z_1\right)\tan\left(k_1\ell_1\right)}\quad,\qquad k_1\ell_1 = \frac{\omega}{1+\alpha}$$

$$\frac{Z_s}{Z_1} = \frac{Z}{Z_1}\,\frac{\left(Z_L/Z_1\right)-i\left(Z/Z_1\right)\tan(kd)}{\left(Z/Z_1\right)-i\left(Z_L/Z_1\right)\tan(kd)}\quad,\qquad kd = \frac{\alpha}{1+\alpha}\,\frac{\gamma}{\gamma_1}\,\omega$$

$$\frac{Z_L}{Z_1} = \frac{\omega}{\omega + 2ir}$$

$$\frac{Z}{Z_1} = \frac{\gamma}{\gamma_1} = \sqrt{1 + \frac{i}{\beta\omega}\frac{Z_g}{Z_o}}$$

$$\frac{Z_g}{Z_o} = \sqrt{\frac{\omega f(\omega)}{\omega\varepsilon_r + iq}} \quad ,$$

and f may take one of three forms:

$$\sqrt{1 + \frac{\omega}{\omega\varepsilon_r + iq}} \quad , \quad 1 \quad , \quad 1 + \frac{\omega\varepsilon_r}{iq} \quad .$$

Some straightforward, though tedious, calculations show that the integrands A_1 , B_1 , A , and B in (5) and (6) are well behaved over the entire range of integration. In particular, they have the following values at $\omega = 0$:

$$A_1 \to \frac{1}{r} + \frac{1}{1 + \alpha} - \xi$$

$$B_1 \to \xi$$

$$A \to \frac{1}{r}$$

$$B \to \frac{1}{1 + \alpha} \quad .$$

As ω approaches infinity, they tend to zero sufficiently rapidly that the integrals exist.

There are in total five parameters $(\alpha , \beta , \varepsilon_r , q , r)$ and two variables (τ , ξ) appearing in the above expressions. The geometrical parameters α and β are defined by

$$\alpha = \frac{d}{\ell_1} \quad , \quad \beta = \frac{h}{\ell_1 + d} \tag{7}$$

(h being the height of the top plate from the ground surface); the ground parameters, ε_r and q are defined by

$$\varepsilon_r = \frac{\varepsilon_1}{\varepsilon_o} \quad , \quad q = \frac{\sigma t_1}{\varepsilon_o} \quad ; \tag{8}$$

and the load parameter r is defined by

$$r = \frac{t_1 Z_1}{2L} \quad ; \tag{9}$$

where ε_1 and σ are the ground permittivity and conductivity and $t_1 = (\ell_1 + d)/c$. The variables τ and ξ are defined by

$$\tau = \frac{t - x/c}{t_1} \quad , \quad \xi = \frac{x}{\ell_1 + d} \quad . \tag{10}$$

Figures 13 through 20 illustrate the time behavior of e and h at several points on the line for $\alpha = 2$, $\beta = 0.04$, $\varepsilon_r = 10$, $q = 90\pi$, and $f = 1$. These numbers refer to the situation where $d = 50$ meters, $\ell_1 = 25$ meters, $h = 3$ meters, and $\sigma = 10^{-2}$ mhos per meter. Setting $f = 1$ is equivalent to replacing the effect of the ground by a surface impedance whose value is equal to the wave impedance of the ground. The implications of the three different forms for f mentioned previously will be discussed in a future note.

IV. Comments

There are two points, mentioned briefly before, that deserve further comment. The first point concerns the interpretation of the results of Sections II and III, under the assumption that the conventional transmission-line analogy is really applicable to the analysis of the actual simulator structure. From the waveform curves it can be concluded that, if one is interested mainly in the time dependence of the magnetic field (as opposed to the electric field), then the idea of inserting a sheet inductance as shown in figure 1 does indeed make sense. This conclusion is supported by the fact that there is relatively little distortion of the current waveform on the analogous conventional line if the time constant involving the inductance and the line impedance is at least as great as the transit time of the line. The case where the ground is perfectly conducting is, in a sense, a limiting case in that any ground conductivity reduces the load inductance required to make the distortion small. Whether such a high inductance can significantly reduce the energy required to inject the slowly varying portion of the pulse into the ground is a question that cannot be answered accurately until further calculations are performed.

The second point deserving comment is the basic question that arose in Section I: How appropriate is the conventional transmission-line analogy to the analysis of the real simulator? At very low frequencies $(\lambda >> \ell)$, there is no doubt that transmission line theory works; at high frequencies $(\lambda < h)$, transmission-line theory is irrelevent. Up to what frequency is the conventional transmission-line analogy valid? Is this limiting frequency high enough that the pulse waveforms we have calculated look like the real ones? These questions deserve a thorough study. Such a study is now being made; its results will be reported in a future note.

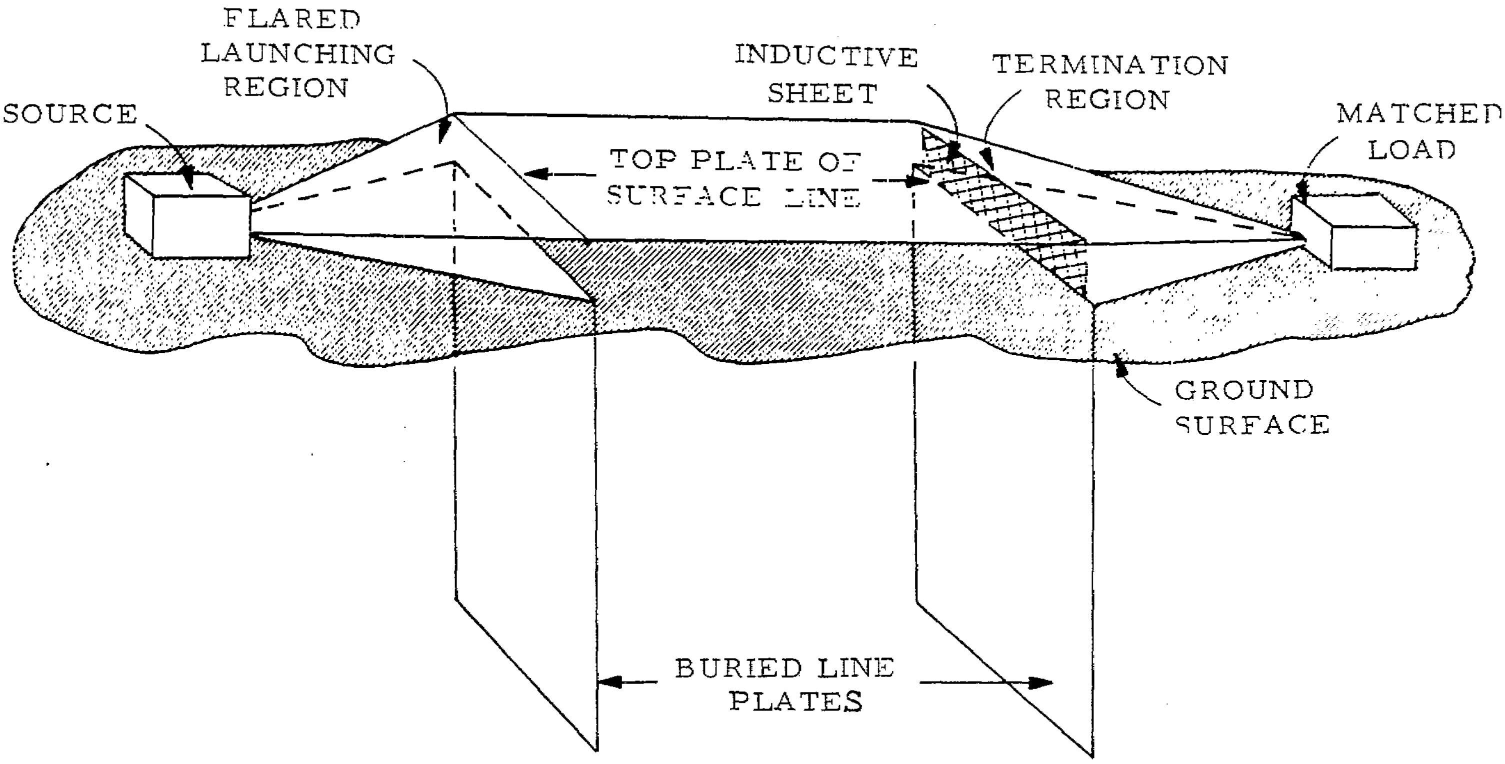

Figure 1. Combined surface and buried line simulator.

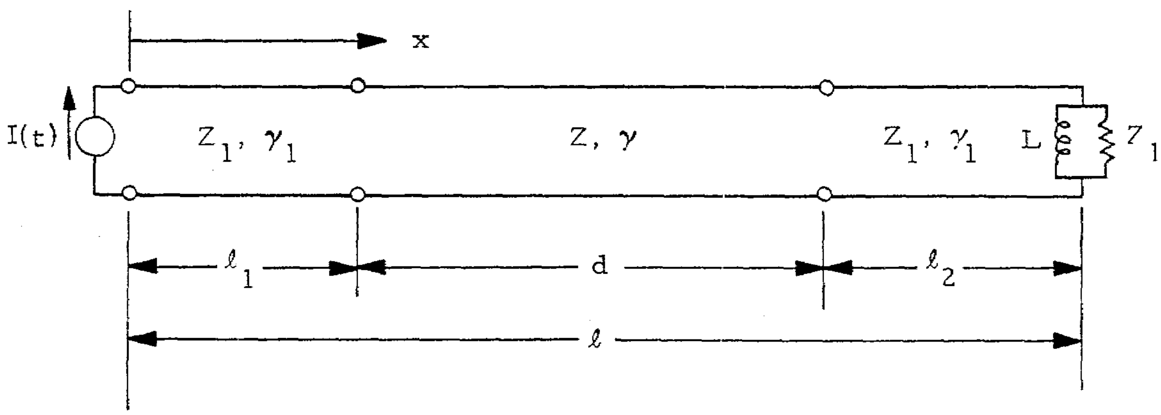

Figure 2. Transmission-line representation of combined simulator.

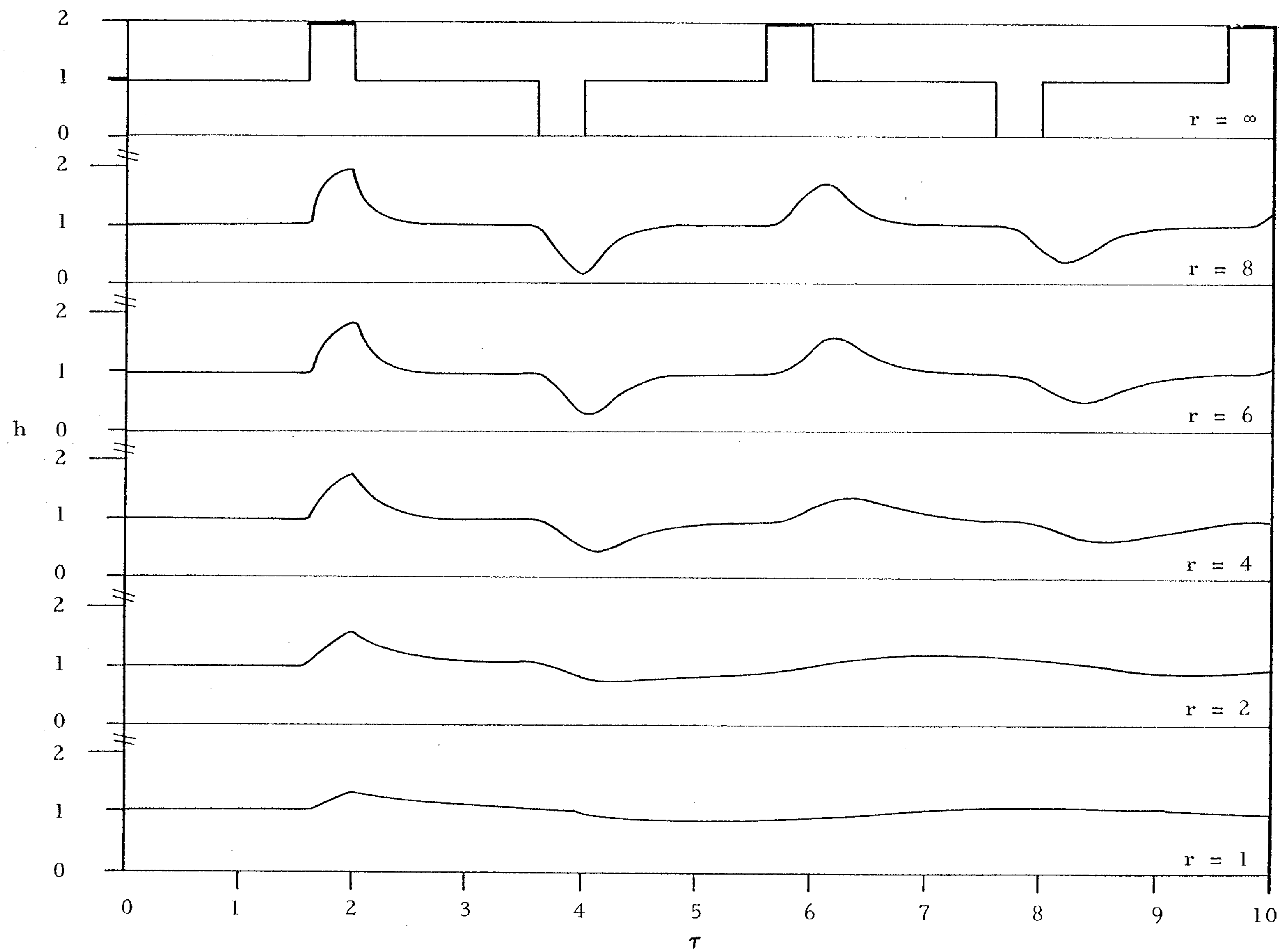

Figure 3. h vs. τ at $\xi = 0.2$ (uniform line).

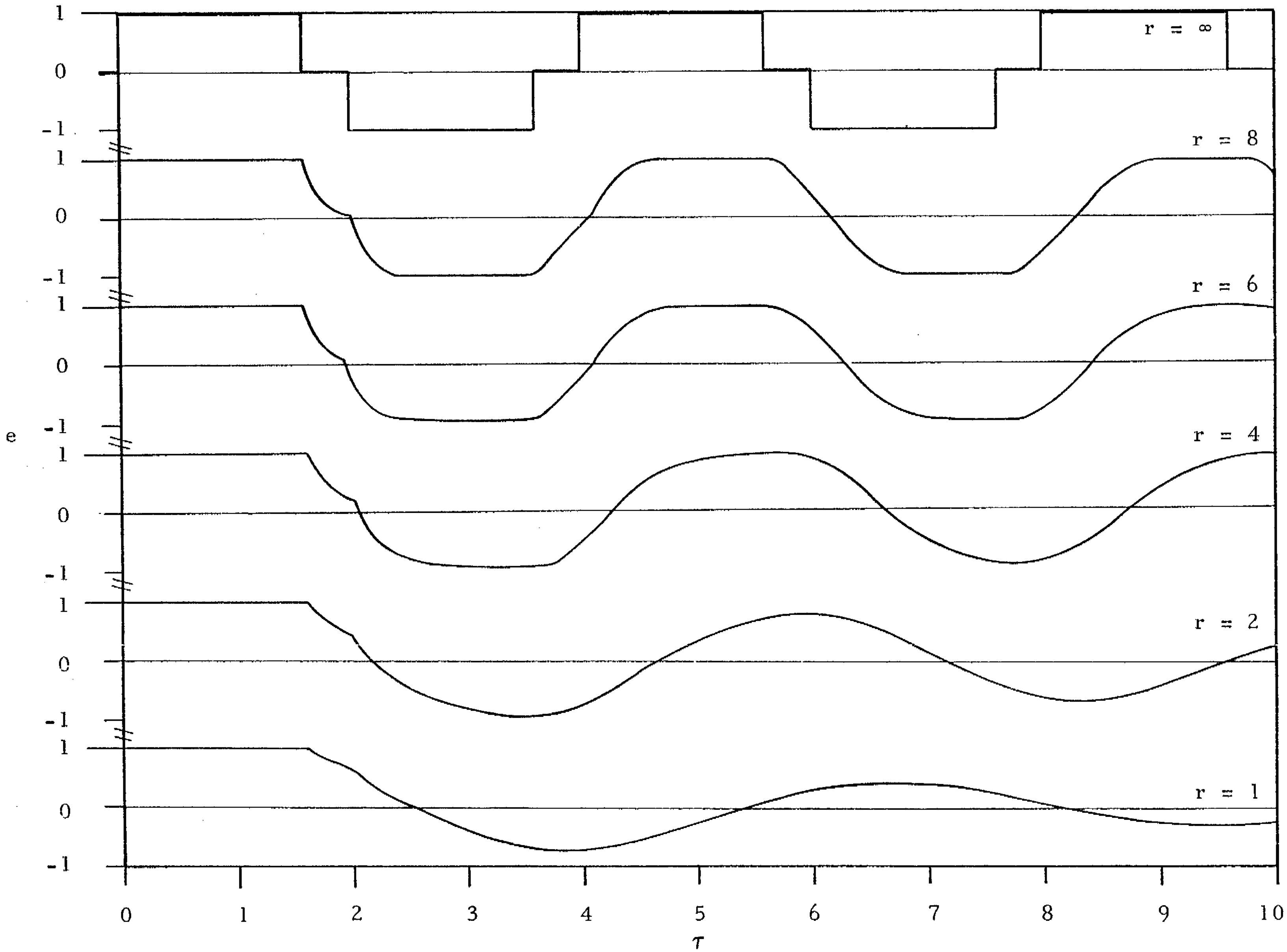

Figure 4. e vs. τ at ξ = 0.2 (uniform line).

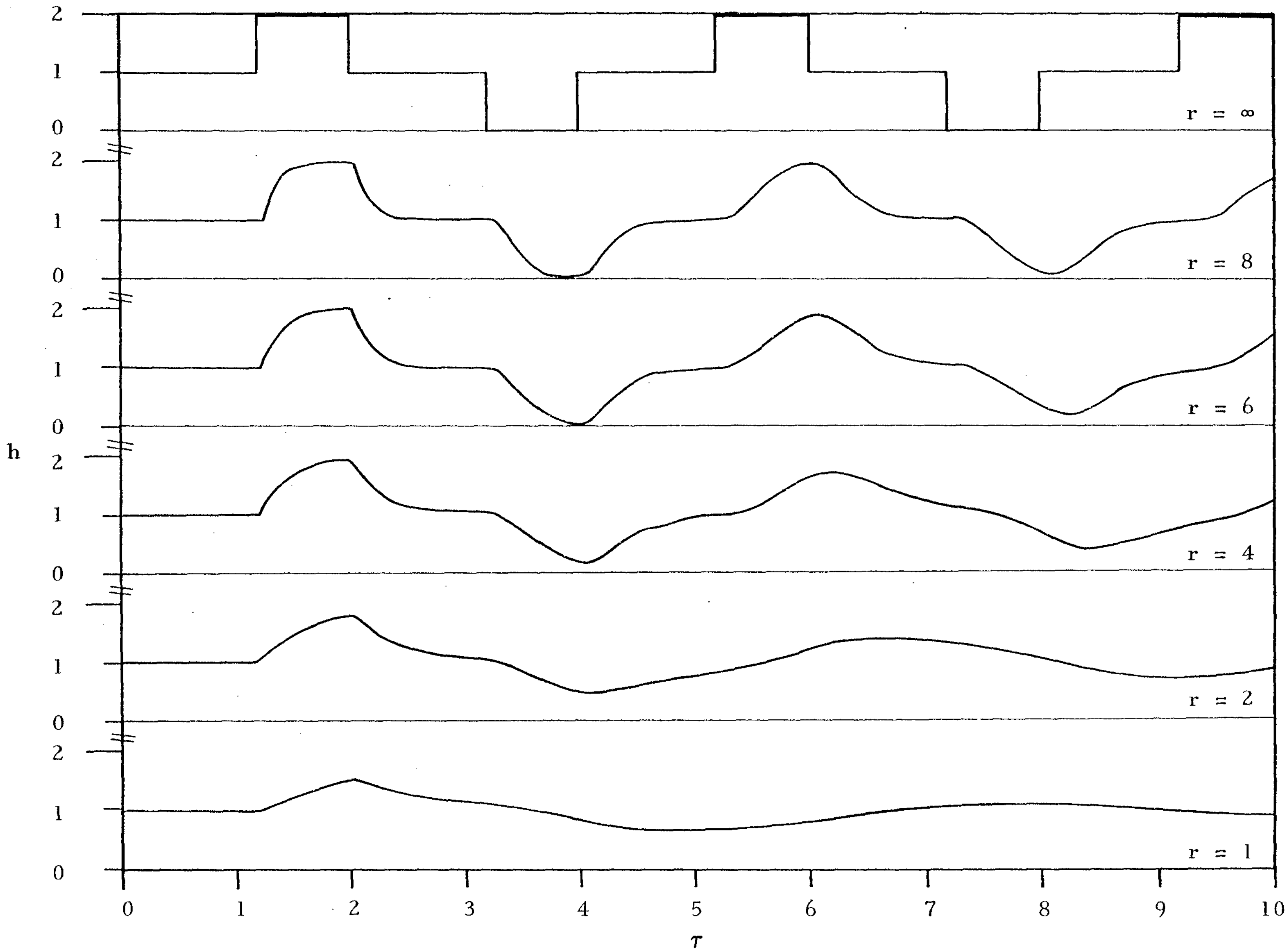

Figure 5. h vs. τ at $\xi = 0.4$ (uniform line).

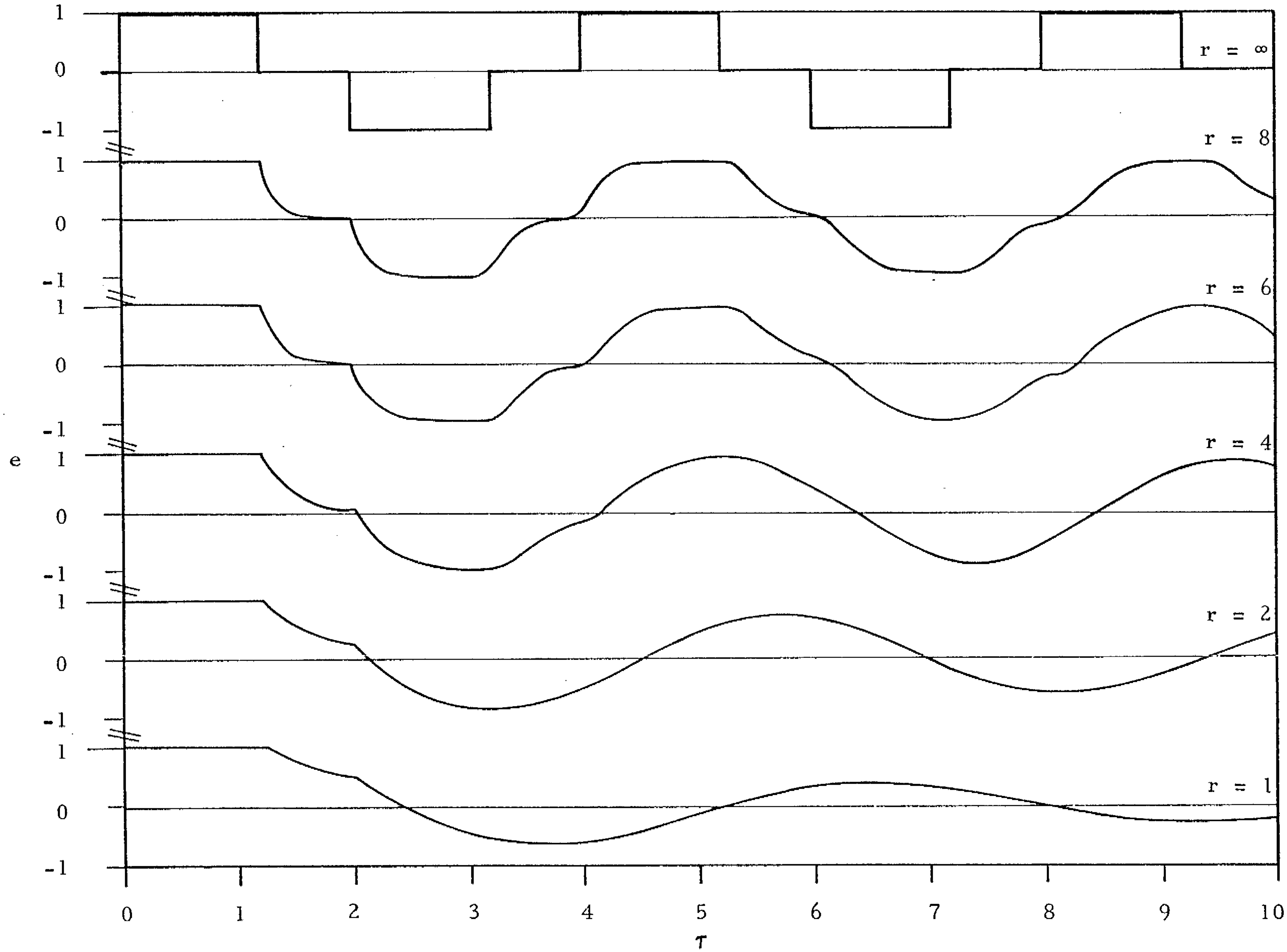

Figure 6. e vs. τ at $\xi = 0.4$ (uniform line).

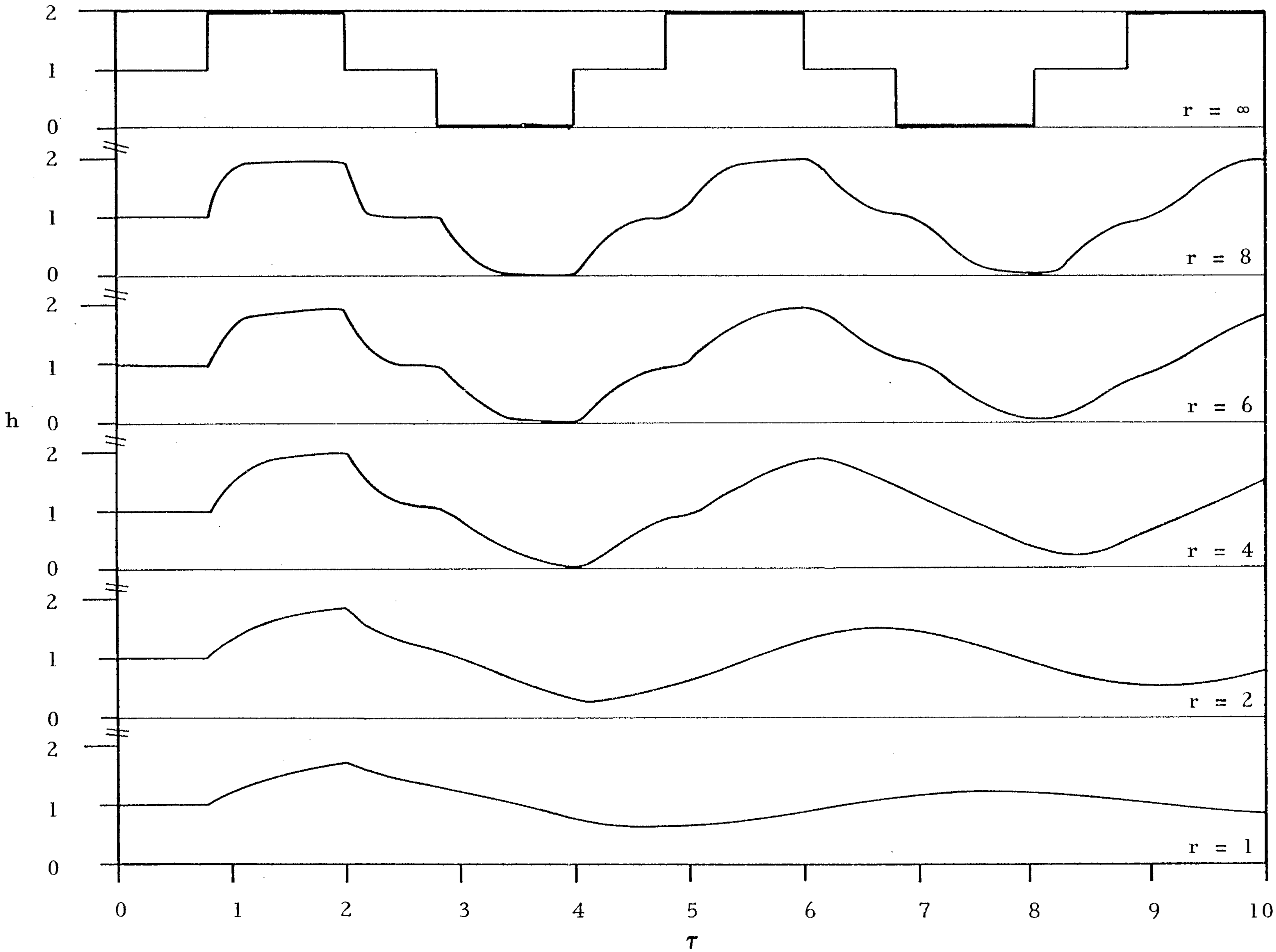

Figure 7. h vs. τ at $\xi = 0.6$ (uniform line).

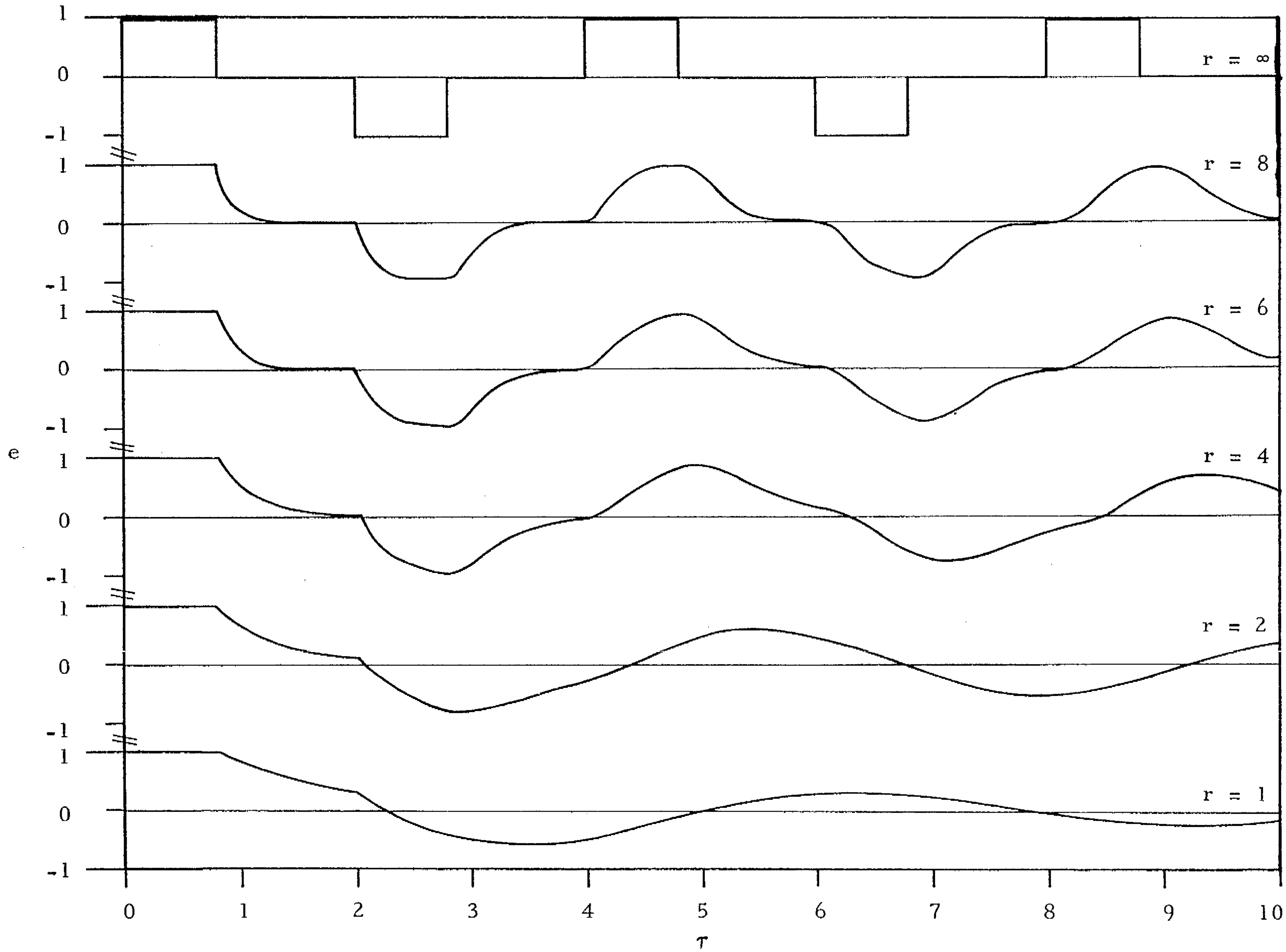

Figure 8. e vs. τ at $\xi = 0.6$ (uniform line).

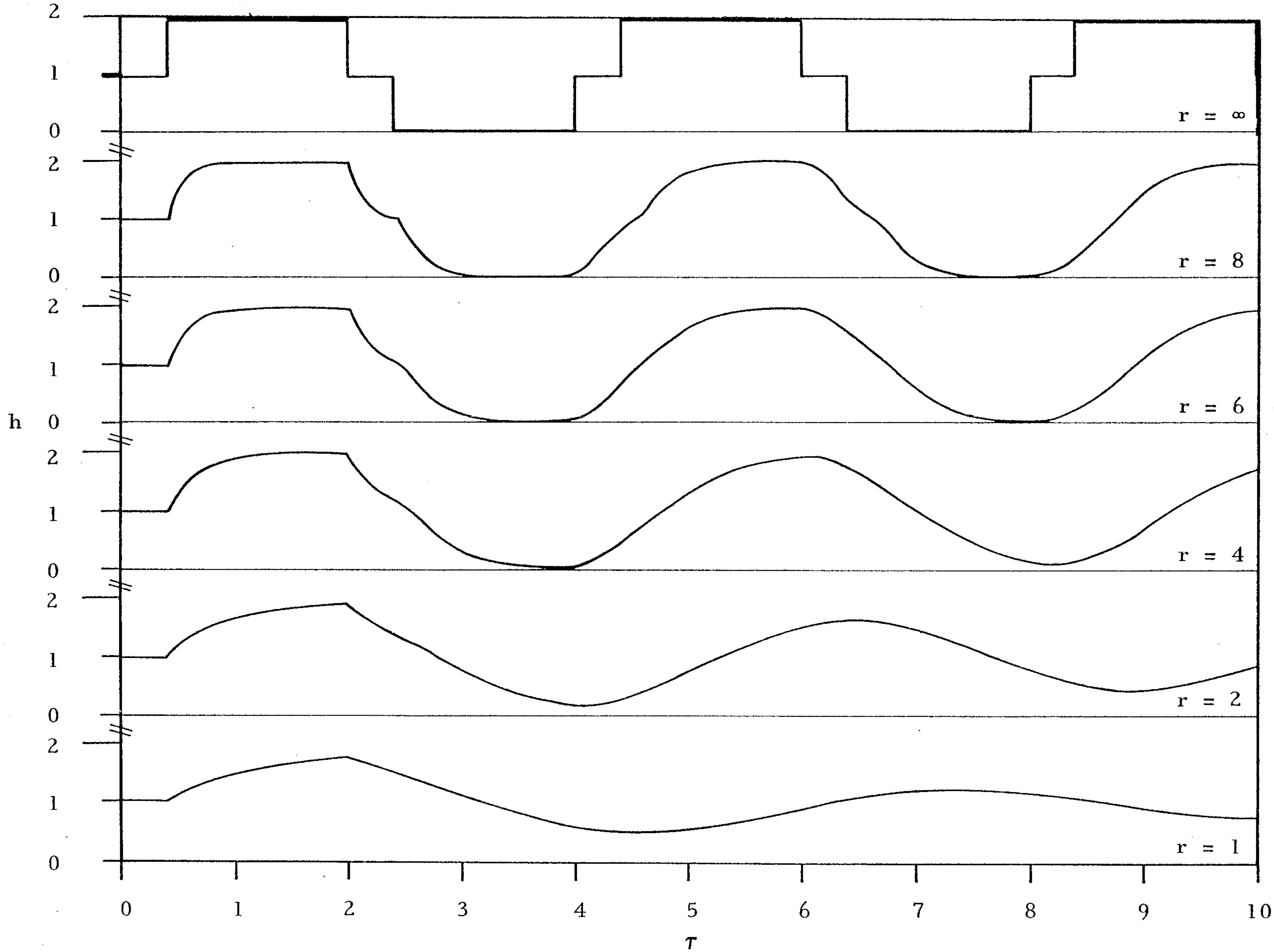

Figure 9. h vs. τ at $\xi = 0.8$ (uniform line).

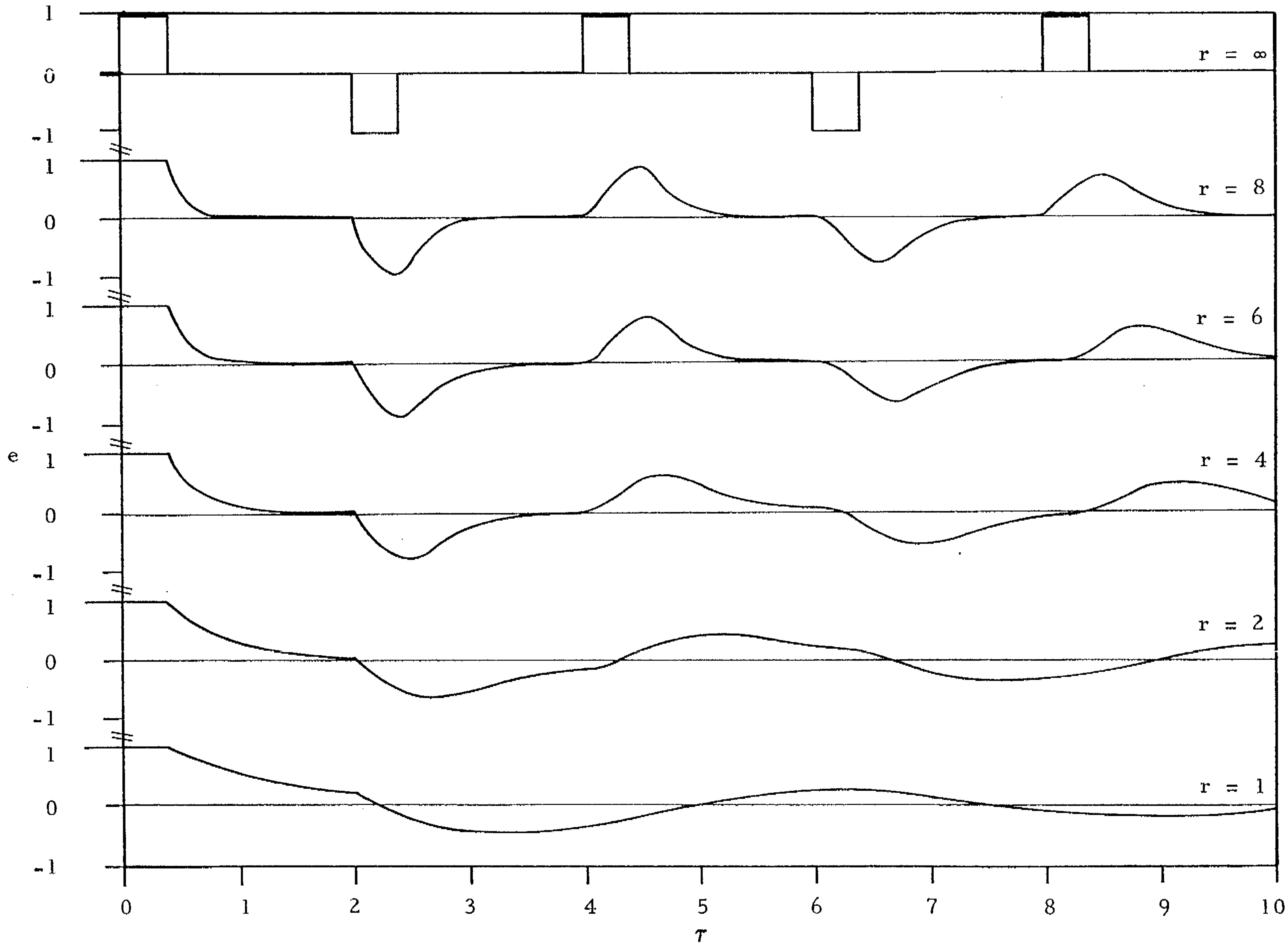

Figure 10. e vs. τ at ξ = 0.8 (uniform line).

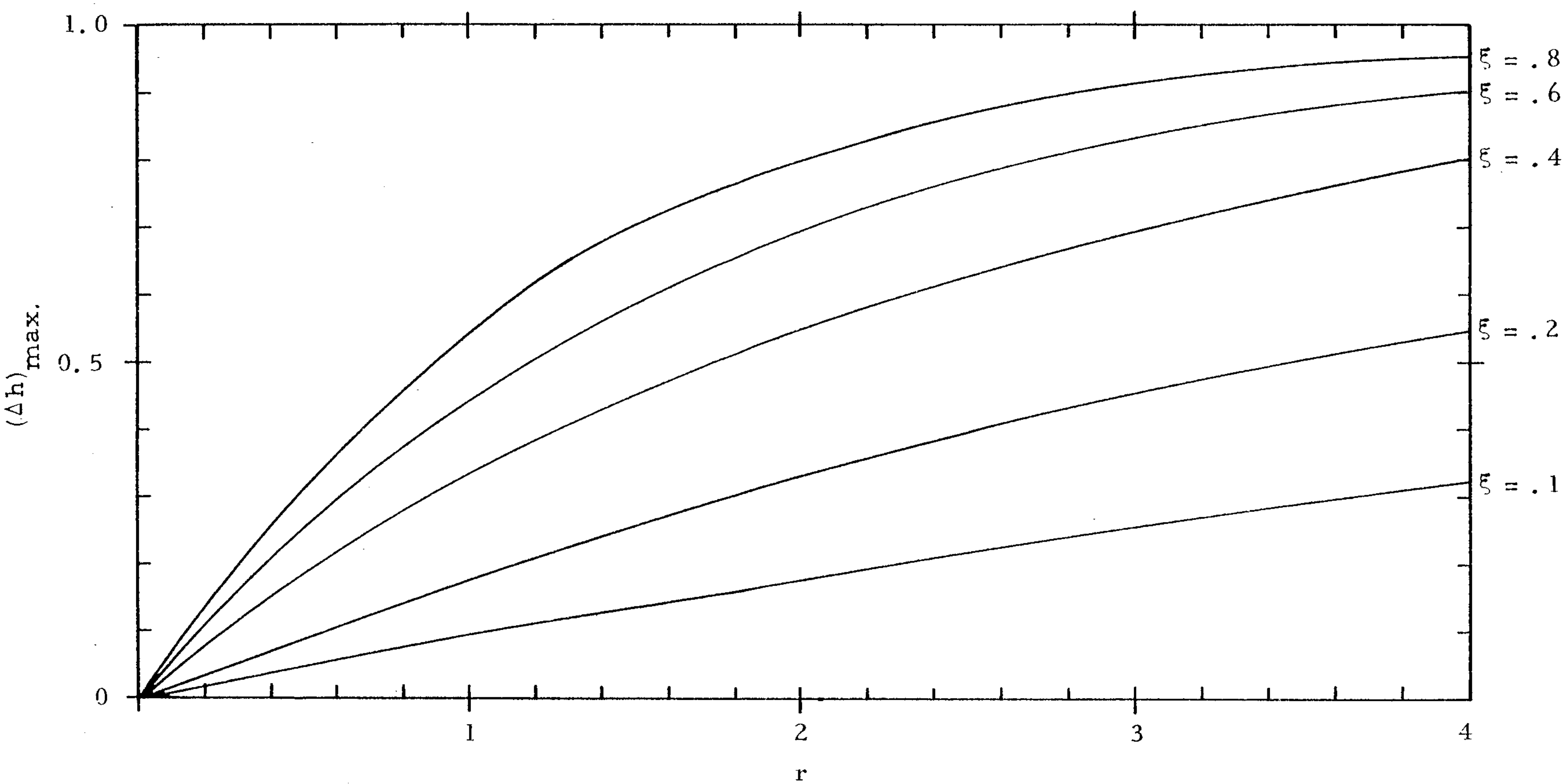

Figure 11. Maximum deviation of h.

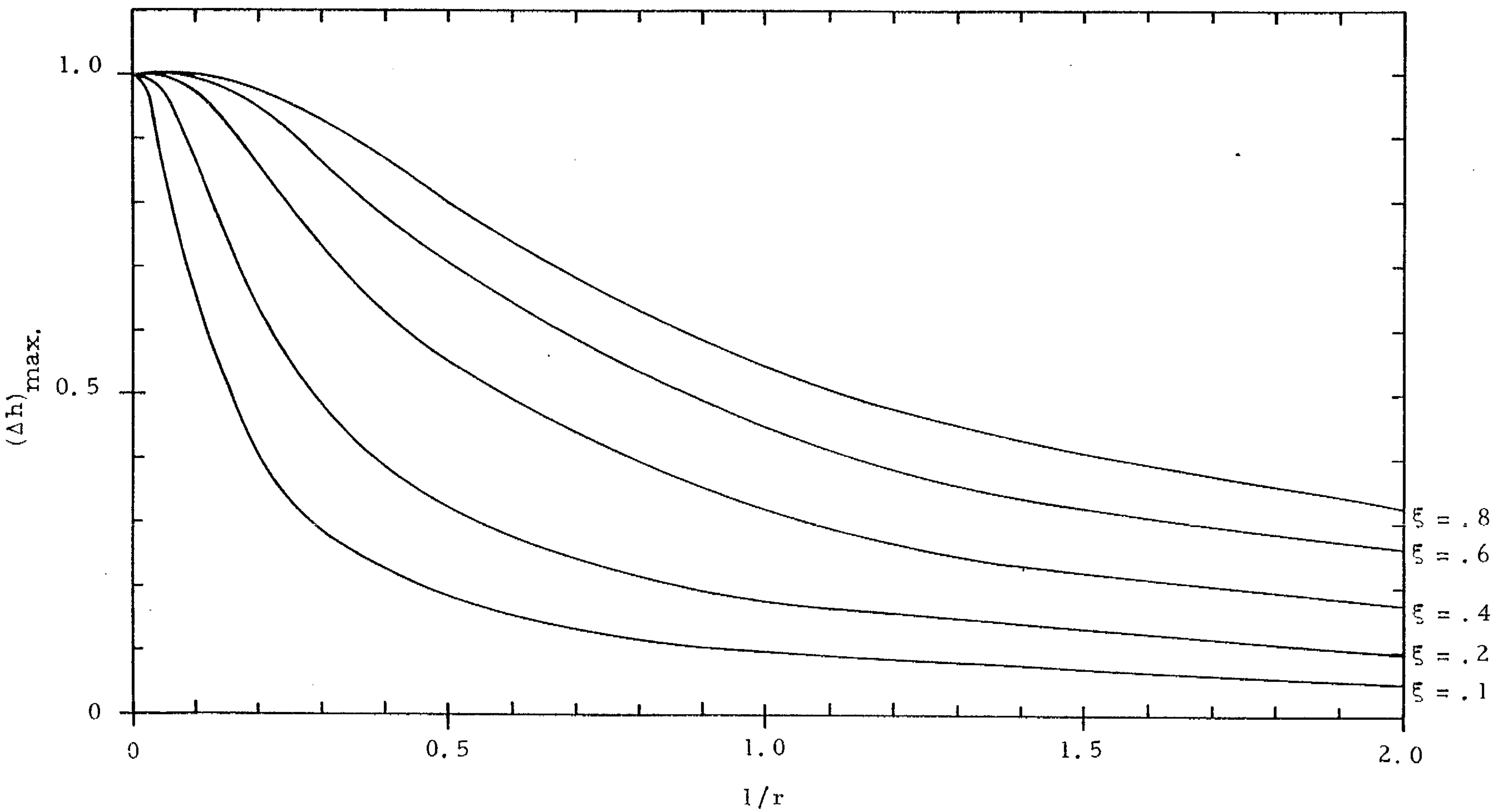

Figure 12. Maximum deviation of h.

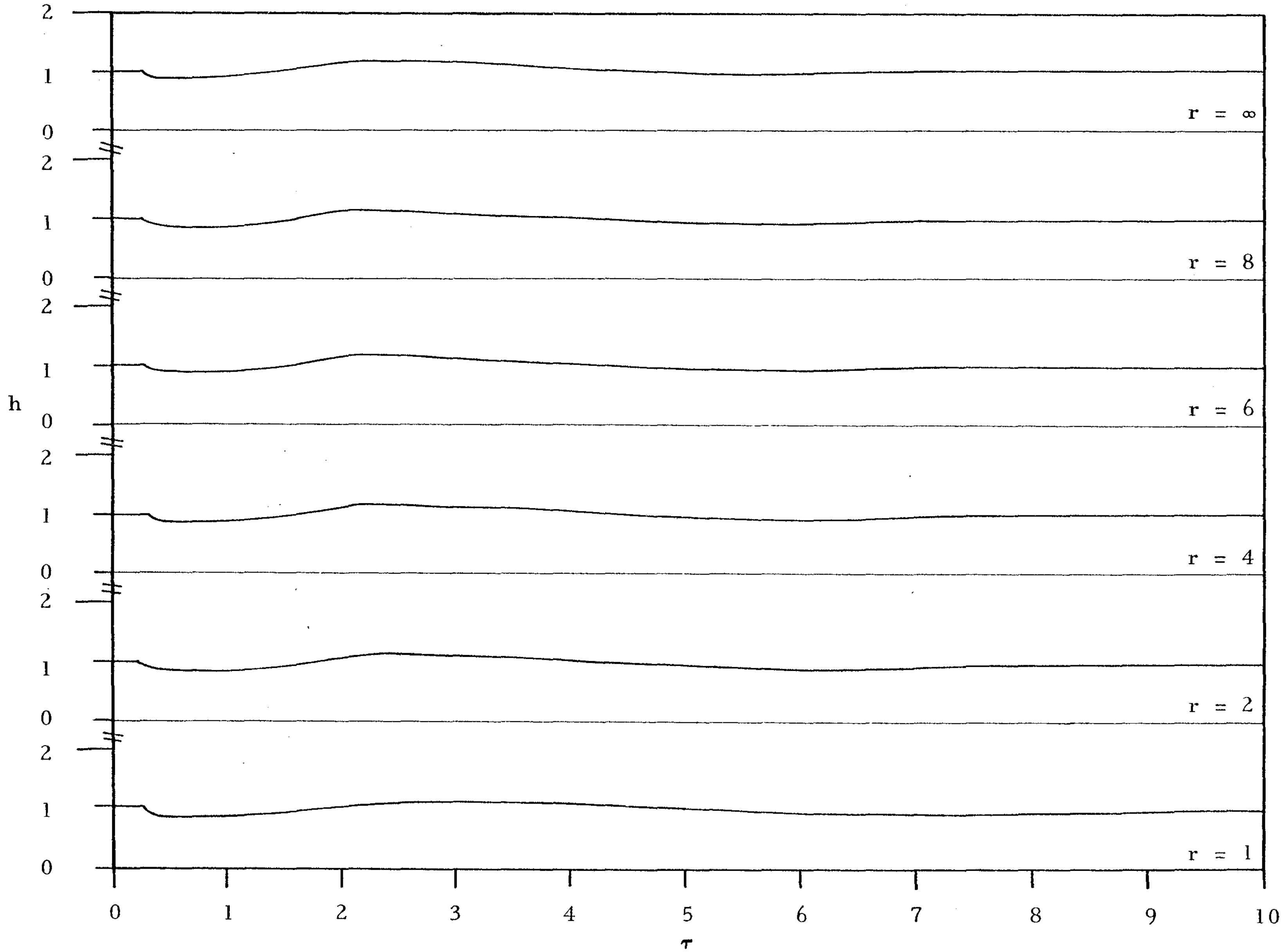

Figure 13. h vs. τ at $\xi = 0.2$ (lossy line).

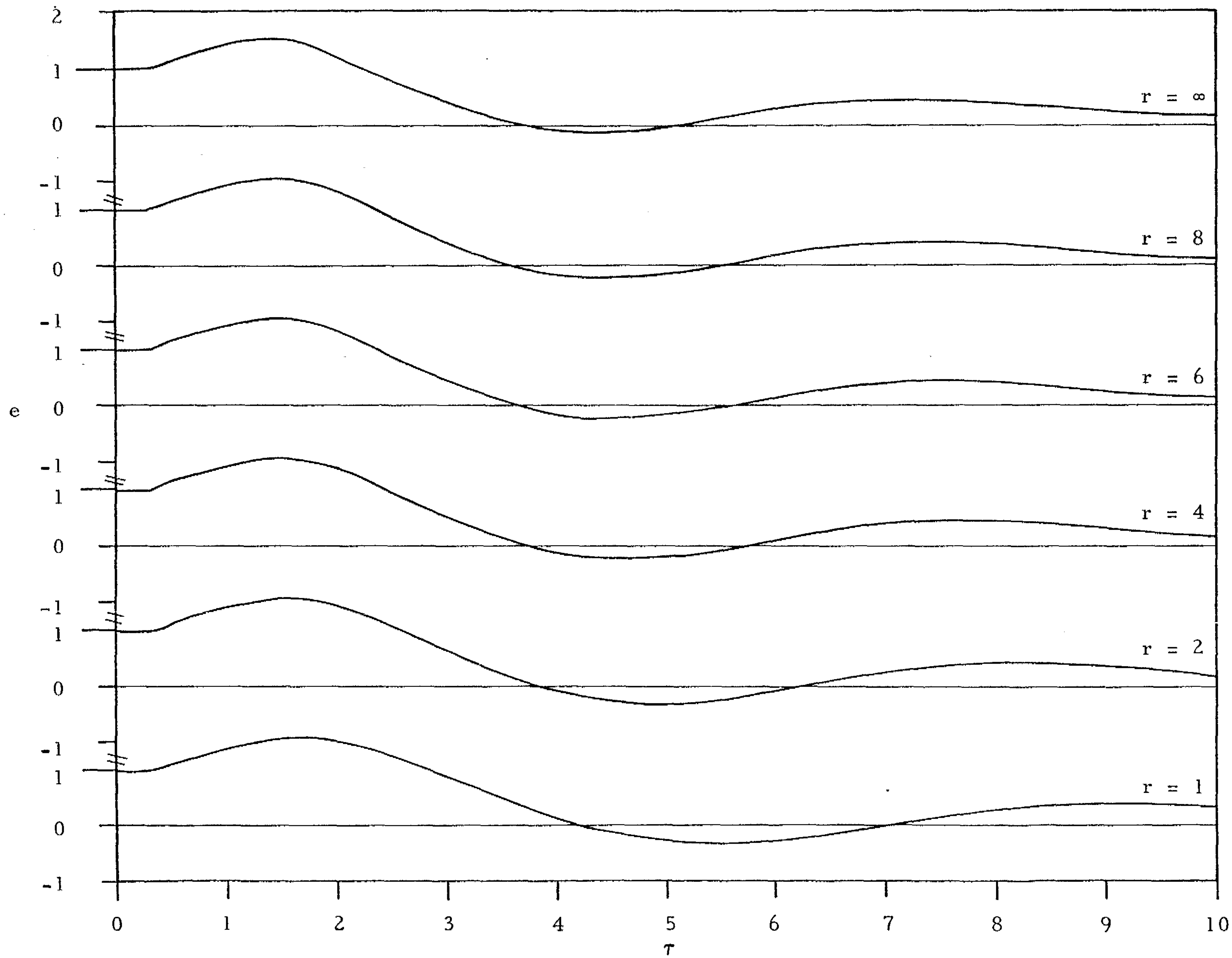

Figure 14. e vs. τ at $\xi = 0.2$ (lossy line).

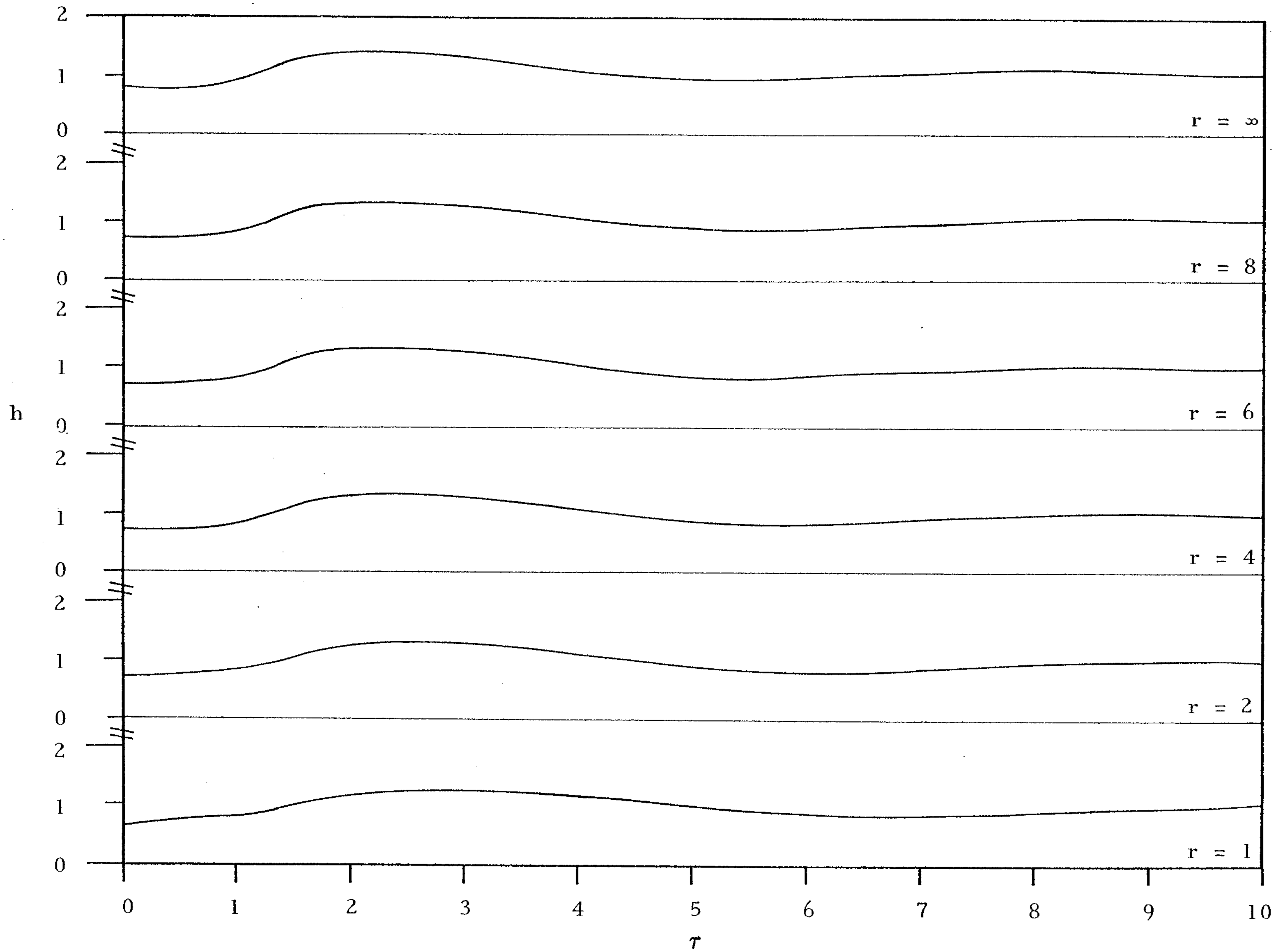

Figure 15. h vs. τ at $\xi = 0.4$ (lossy line).

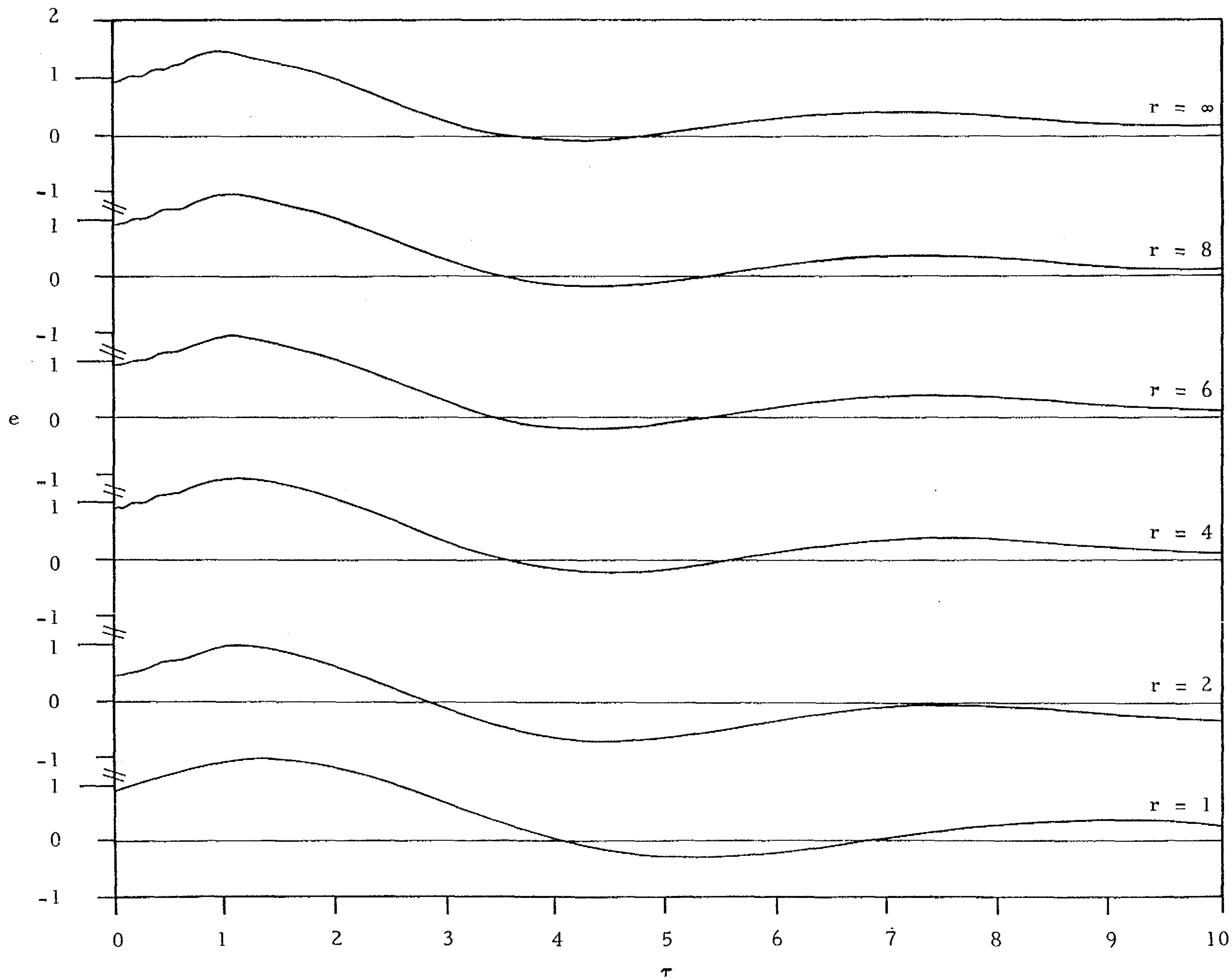

Figure 16. e vs. τ at $\xi = 0.4$ (lossy line).

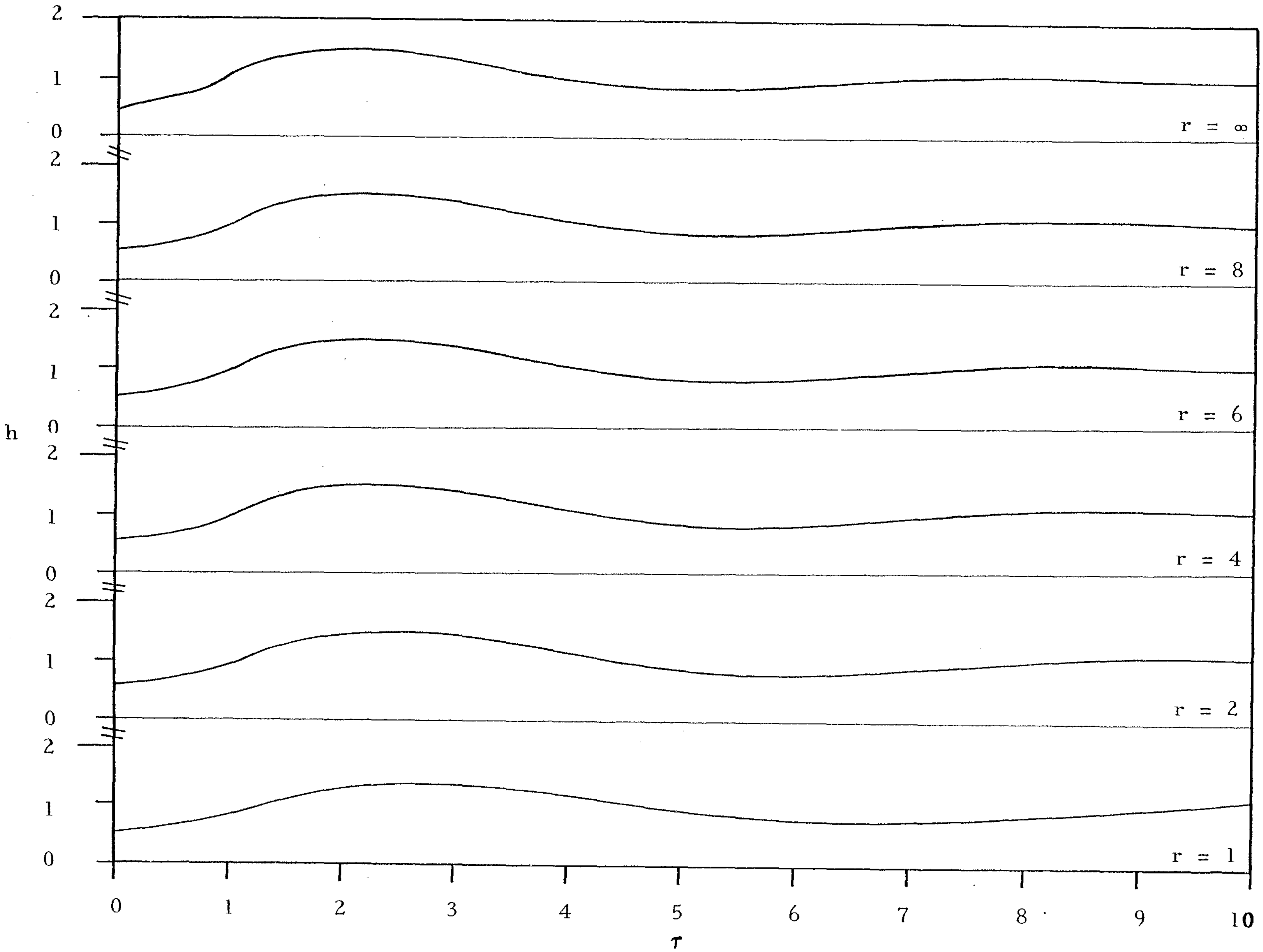

Figure 17. h vs. τ at $\xi = 0.6$ (lossy line).

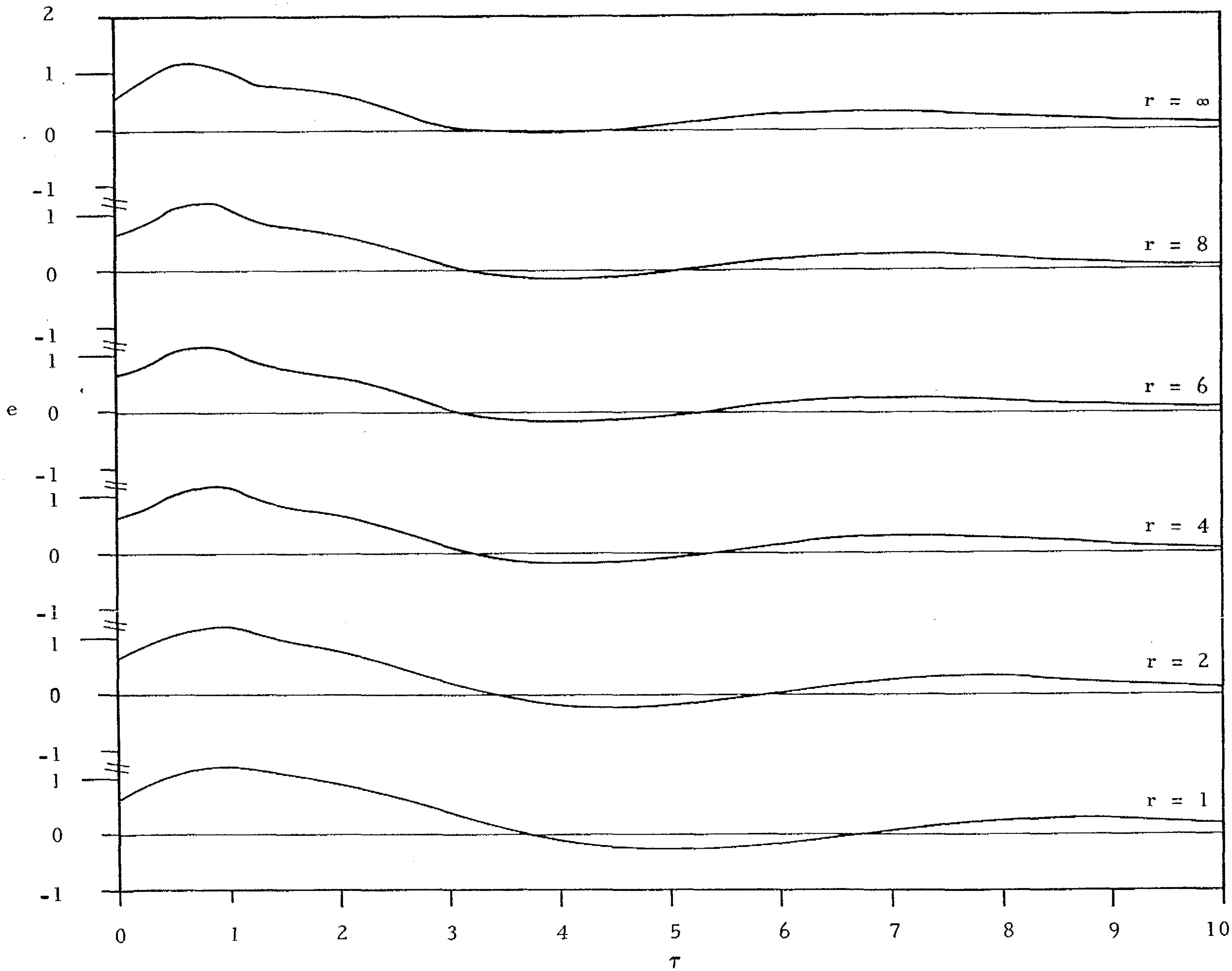

Figure 18. e vs. τ at $\xi = 0.6$ (lossy line).

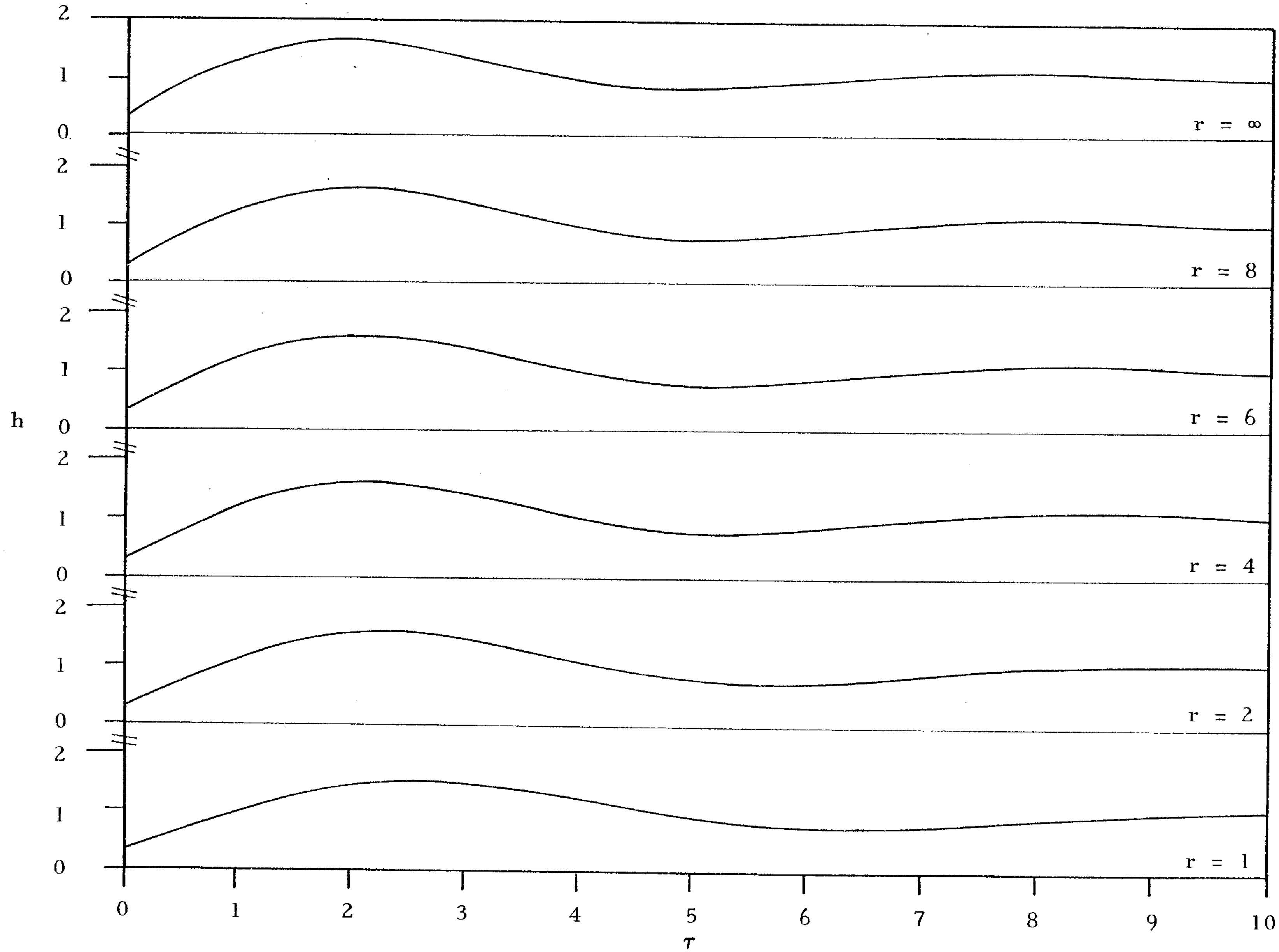

Figure 19. h vs. τ at $\xi = 0.8$ (lossy line).

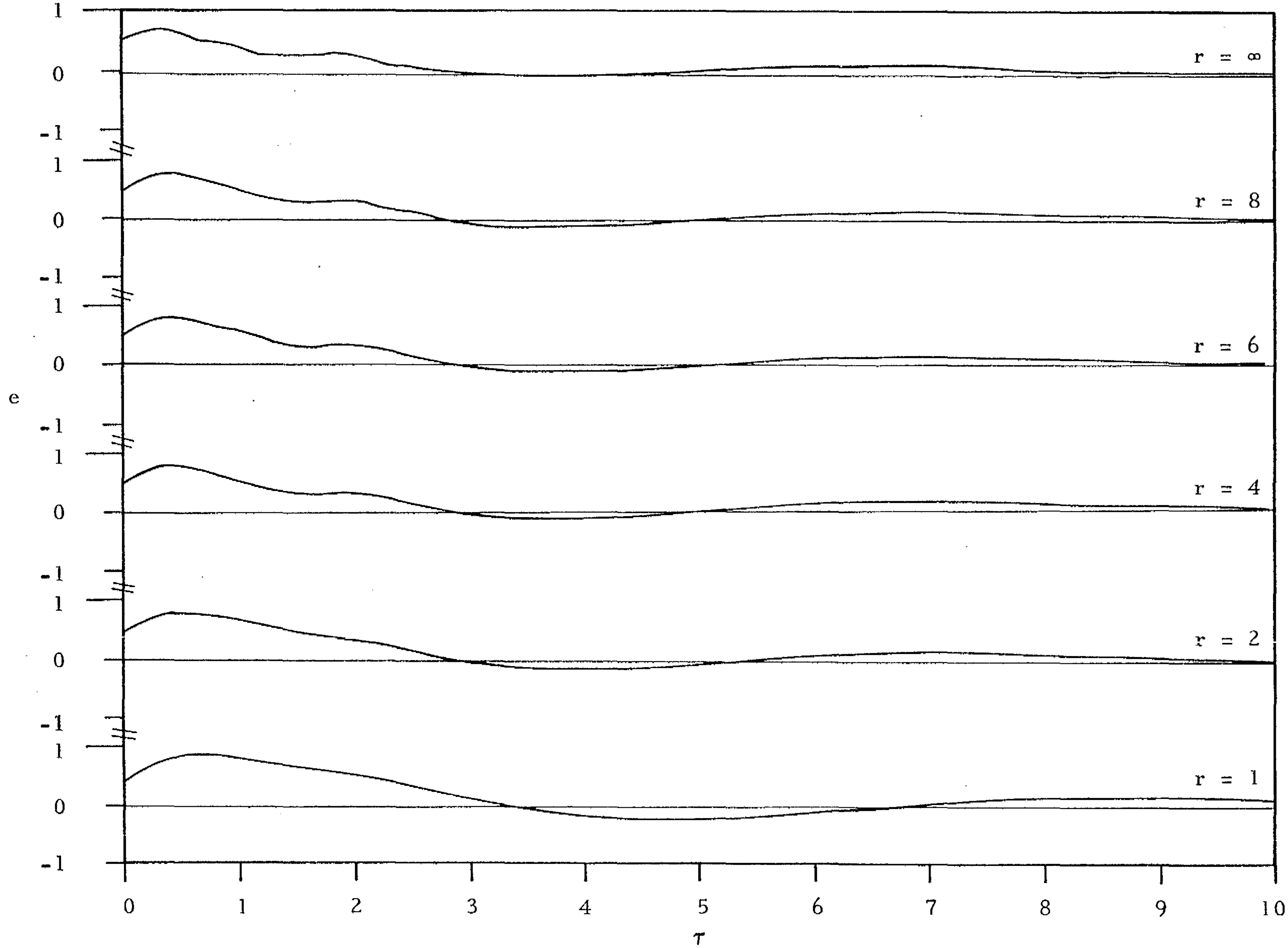

Figure 20. e vs. τ at $\xi = 0.8$ (lossy line).

References

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3. Guy W. Carlisle, Sensor and Simulation Note 54, "Matching the Impedance of Multiple Transitions to a Parallel-Plate Transmission Line", April 1968.
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