Sensor and Simulation Notes

Note 58

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Positioning of Rods for Uniform TEM Waves

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Abstract

Conducting rods can be used to form a transmission line that supports a wave which closely resembles a uniform TEM wave. The problem of positioning these rods is considered, and the positions of the rods for symmetrical six and eight-rod transmission lines are determined. The impedances of these transmission lines are computed.

I. Introduction

The effect on buried structures due to the low-frequency content of an electromagnetic pulse emanating from a nuclear burst can be simulated by a buried parallel-plate transmission line[1]. The purpose of the transmission line is to transport a uniform TEM wave to a test body which is located between the parallel plates. For the sake of generality it should be remarked that the medium in which the transmission line is immersed, and hence the concern of this note, is not limited to earth; instead the medium can be any isotropic, homogeneous, linear medium.

In a previous note, which deals with a related topic, the charge and field distributions have been computed for the case of a cylinderical test body with a diameter that is appreciable compared to the spacing between the parallel plates[2]; these distributions are compared with those for the case in which the diameter of the test body is negligible.

From the standpoint of constructing a parallel-plate transmission line it is convenient to approximate the parallel plates by two correspondingly parallel linear-arrays composed of uniformly spaced conducting rods. If the arrays extend to infinity, containing an infinite number of rods, and if the two arrays are spaced far enough apart, the fields midway between the two arrays closely approximate those of a uniform TEM wave. The arrays, of course, cannot extend to infinity; they must be truncated and contain only a finite number of rods. This truncation causes the fields to distort from the uniform TEM distribution that is desired.

The problem with which the present note is concerned is that of positioning a finite number of conducting rods to form a transmission line which more nearly produces a uniform TEM distribution than a transmission line formed by two truncated, parallel linear-arrays containing the same finite number of rods. As might be expected, it turns out that the rods should be positioned in arrays which are symmetrical about the origin as shown in figure 1. The problem of positioning four rods was considered in a previous note[3].

II. Positioning of Rods

Figure 1 shows the cross-section of a transmission line formed by conducting rods which are arranged symmetrically about the origin and which extend to infinity along the z-axis. In order for the structure to be operated conveniently as a transmission line the potential difference between the upper rods $(y > 0)$ and the lower rods $(y < 0)$ must be uniform; i.e., all of the upper rods must be at one potential and all of the lower rods must be at the same magnitude of potential with opposite algebraic sign. Hence requiring that the structure operate as a transmission line imposes a restraint on the potentials of the rods.

As nearly as possible it is desired that the fields on the transmission line in figure 1 match those of a uniform TEM wave which has an electric field with only a y-component and whose magnitude is constant over the cross-section. If the rods are positioned symmetrically about the origin, the x-component of the electric field tends to cancel out in the vicinity of the origin, giving a y-component only. The magnitude of E_y can be made uniform in the vicinity of the origin by representing E_y in terms of a Taylor expansion in x about the origin and by forcing as many derivatives as possible to vanish at the origin.

An expansion in circular harmonics of the potential of a line charge q_i, whose coordinates are r_i and θ_i, is[4]

$$\phi_i = \frac{q_i}{2\pi\varepsilon}\left[\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{r}{r_i}\right)^n (\cos n\theta_i \cos n\theta_i + \sin n\theta_i \sin n\theta) - \ln r_i\right] \tag{1}$$

where $r < r_i$. Along the x-axis the y-component of the electric field due to this line charge is

$$E_y^i\Big|_{y=0} = \frac{q_i}{2\pi\varepsilon}\sum_{n=1}^{\infty} \frac{x^{n-1}}{r_i^n} \sin n\theta_i \quad . \tag{2}$$

The m-th partial derivative of E_y with respect to x evaluated at the origin is

$$\left.\frac{\partial^m E_y^i}{\partial x^m}\right|_{\substack{x=0\\y=0}} = \frac{q_i}{2\pi\varepsilon}\left[\frac{m!}{r_i^{m+1}}\sin(m+1)\theta_i\right] \qquad m = 1,2,3... \tag{3}$$

The m-th partial derivative of the electric field E_y^i due to a set of four line charges $q_i, q_i, -q_i$ and $-q_i$ that are located symmetrically at (r_i, θ_i), $(r_i, \pi-\theta_i)$, $(r_i, \pi+\theta_i)$ and $(r_i, 2\pi-\theta_i)$, respectively, is

$$\left.\frac{\partial^m E_y^i}{\partial x^m}\right|_{\substack{x=0\\y=0}} = \frac{4q_i}{2\pi\varepsilon}\left[\frac{m!}{r_i^{m+1}}\sin(m+1)\theta_i\right] \tag{4}$$

for even values of m and identically zero for odd values of m. From (4) it is evident that for the case of four conducting rods arranged symmetrically about the origin, the first three partial derivatives are zero at the origin if $\theta_i = \pi/3$ which is the result reported in reference 3. The m-th partial derivative for the special case of two line charges q_o and $-q_o$ at $(r_o, \pi/2)$ and $(r_o, 3\pi/2)$, respectively, is

$$\left.\frac{\partial^m E_y}{\partial x^m}\right|_{\substack{x=0\\y=0}} = \frac{2q_o}{2\pi\varepsilon}\left[\frac{m!}{r_o^{m+1}}(-1)^{m/2}\right] \tag{5}$$

for even values of m and identically zero for odd values of m.

The conducting rods can be arranged symmetrically with or without rods being placed on the y-axis. For the case of 4K rods arranged symmetrically in K sets of four rods with no rods on the y-axis, (4) gives the m-th derivative of E_y which, if set equal to zero, is

$$\sum_{i=1}^{K} \frac{q_i}{r_i^{m+1}} \sin(m+1)\theta_i = 0 \quad . \tag{6}$$

For the case of $4K + 2$ rods arranged symmetrically in K sets of four rods plus a pair of rods on the y-axis, (4) and (5) give

$$\frac{q_o}{r_o^{m+1}} (-1)^{m/2} + 2 \sum_{i=1}^{K} \frac{q_i}{r_i^{m+1}} \sin(m+1)\theta_i = 0 \tag{7}$$

for the m-th derivative equated to zero.

That the structure operate as a transmission line imposes an aforementioned restraint on the potentials of the rods. Due to symmetry the potential at each rod of a symmetrical set, which might be part of a larger symmetrical arrangement, satisfies the restraint; e.g., if the potential of the rods at (r_i , θ_i) and $(r_i , \pi-\theta_i)$ is ϕ_i , the potential of the rods at $(r_i , \pi+\theta_i)$ and $(r_i , 2\pi-\theta_i)$ is $-\phi_i$. Thus the restraint on the potentials is met by requiring

$$\phi_o = \phi_1 = \ldots = \phi_i = \ldots = \phi_K \tag{8}$$

where these potentials pertain to the potentials of the rods at $(r_o , \pi/2)$, (r_1 , θ_1) , ... (r_i , θ_i) , ... (r_K , θ_K) , respectively. Clearly for an arrangement of $4K$ rods, which has no rods on the y-axis, ϕ_o is omitted from (8).

III. Impedance

The impedance of a multiple-rod transmission line can be written as

$$Z_L = f_g Z_o \tag{9}$$

where f_g is a geometrical factor given by

$$f_g = \frac{\varepsilon V}{Q} \tag{10}$$

and Z_o is the wave impedance of the medium in which the rods are immersed[5]. In (10) ε is the permittivity of the medium, V is the potential difference between the upper and lower rods, and Q is the sum of the line charge of all of the rods either above or below $y=0$.

Values of f_g have been computed for the transmission lines considered in this note and are listed in Tables 1 and 2.

IV. Symmetrical Six-rod Transmission Line

A six-rod transmission line is shown in figure 2. For the case of six rods, which are arranged in a set of four rods plus two rods on the y-axis, it is possible to make the first five derivatives zero. In general for the case of $4K + 2$ rods it is possible to make the first $4K + 1$ derivatives vanish.

If the line charge is normalized with respect to q_o and the distances are normalized with respect to r_o, (7) gives

$$\begin{aligned} -1 + \frac{2q_1}{r_1^3} \sin 3\theta_1 &= 0 \\ 1 + \frac{2q_1}{r_1^5} \sin 5\theta_1 &= 0 \end{aligned} \tag{11}$$

and (8) requires

$$\phi_o = \phi_1 \tag{12}$$

giving three equations in terms of three independent variables θ_1, r_1 and q_1.

The potential at the surface of a rod centered at (r_i, θ_i) with radius a and line charge q_i has the form

$$\phi_i = S_i + M_i \tag{13}$$

where $S_i = q_i \ln a$ is a "self-potential" due to the line charge at the center of the rod and M_i is a "mutual-potential" due to all of the other line charges.

Since a is very small compared to the spacing of any pair of rods, S_1 is the dominate term. If all of the rods have the same radius and if (12) is to be satisfied, in the first approximation it is reasonable to compute first-order values for r_1 and θ_1 by means of (11) with $q_1 = q_o = 1$. Substitution of these values for r_1 and θ_1 into (12) gives a second-order value for q_1 which, in turn, if substituted back into (11) gives second-order values for r_1 and θ_1 and hence establishes an iterative procedure.

A summary of data which are pertinent to a six-rod transmission line is presented in Table 1. In figures 4 and 5 E_y is plotted versus x/r_{min} and y/r_{min} , respectively, where $r_{min} = r_o$ is the distance from the origin to the nearest conducting rod. These figures show that E_y is virtually uniform along both the x and y axes for values of x/r_o and y/r_o less than 0.5. At $x/r_o = 0.7$ E_y has dropped off less than four percent. From these considerations it is reasonable to state that E_y is uniform in the region $r \leq 0.5\ r_o$.

V. Symmetrical Eight-rod Transmission Line

An eight-rod transmission line is shown in figure 3. For the case of eight rods, arranged in two sets of four, it is possible to make the first seven derivatives zero. In general for the case of $4K$ rods it is possible to make the first $4K-1$ derivatives vanish.

Normalized with respect to q_1 and r_1, (6) gives

$$
\begin{aligned}
\sin 3\theta_1 + \frac{\sin 3\theta_2}{r_2^3} &= 0 \\
\sin 5\theta_1 + \frac{\sin 5\theta_2}{r_2^5} &= 0 \qquad (14) \\
\sin 7\theta_1 + \frac{\sin 7\theta_2}{r_2^7} &= 0
\end{aligned}
$$

and (8) requires

$$\phi_1 = \phi_2 \quad . \qquad (15)$$

These equations can be solved by using the iterative procedure outlined previously for the six-rod transmission line.

Data for an eight-rod transmission line are listed in Table 2. Figures 4 and 5 show curves of E_y versus x/r_{min} and y/r_{min} where $r_{min} = r_1$ for the case of an eight-rod transmission line. These curves show that E_y is virtually uniform for x/r_1 and y/r_1 less than 0.6 and suggest that E_y is uniform within the region $r \leq 0.6\ r_1$.

VI. Concluding Remarks

Although an eight-rod transmission line provides a uniform TEM wave over a fairly large area, transmission lines containing more than eight rods may be needed in practical application due to power-density requirements. Equations for transmission lines with more than eight rods can be obtained easily from the preceding development; however, these equations are not readily solvable. A systematic computer solution is not evident -- at least not to this investigator. Nevertheless it should be remarked that, if the need warranted sufficient time and effort, solutions probably could be obtained.

Acknowledgement

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References

1. Carl E. Baum, Sensor and Simulation Note 22, "A Transmission-Line EMP Simulation Technique for Buried Structures," June 1966.
2. R. W. Latham, Sensor and Simulation Note 55, "Interaction Between a Cylinderical Test Body and a Parallel Plate Simulator," May 1968.
3. Carl E. Baum, Sensor and Simulation Note 27, "Impedances and Field Distributions for Symmetrical Two Wire and Four Wire Transmission Line Simulators," October 1966.
4. W. R. Smythe, <u>Static and Dynamic Electricity</u>, 2nd ed, 1950, p. 65.
5. Carl E. Baum, Sensor and Simulation Note 21, "Impedances and Field Distributions for Parallel Plate Transmission Line Simulators," June 1966.

TABLE 1. SYMMETRICAL SIX-ROD TRANSMISSION LINE

$\ln(a/r_o)$	a/r_o	x_1/r_o	y_1/r_o	q_o/q_1	f_g	r_1/r_o	θ_1 (DEGREES)
$-\infty$	0.00	.752	.759	1.000		1.092	46.5
-9	1.234×10^{-4}	.773	.823	1.140	1.120	1.130	46.8
-8	3.355×10^{-4}	.776	.829	1.160	1.015	1.135	46.9
-7	9.119×10^{-4}	.779	.835	1.185	.911	1.142	47.0
-6	2.479×10^{-3}	.783	.844	1.225	.800	1.151	47.1
-5	6.738×10^{-3}	.789	.858	1.285	.676	1.166	47.4
-4	1.832×10^{-2}	.799	.875	1.385	.586	1.187	47.6

TABLE 2. SYMMETRICAL EIGHT-ROD TRANSMISSION LINE

$\ln(a/r_1)$	a/r_1	x_1/r_1	y_1/r_1	x_2/r_1	y_2/r_1	q_2/q_1	f_g	r_2/r_1	θ_1 (DEGREES)	θ_2 (DEGREES)
$-\infty$	0.000	.287	.958	.880	.696	1.000		1.122	73.4	38.3
−9	1.234×10^{-4}	.279	.960	.904	.738	1.165	.940	1.167	73.8	39.2
−8	3.355×10^{-4}	.279	.960	.905	.740	1.186	.862	1.169	73.8	39.3
−7	9.119×10^{-4}	.277	.961	.906	.745	1.220	.782	1.173	73.9	39.4
−6	2.479×10^{-3}	.277	.961	.914	.754	1.267	.712	1.185	73.9	39.5
−5	6.738×10^{-3}	.274	.962	.920	.770	1.337	.620	1.200	74.1	39.9
−4	1.832×10^{-2}	.271	.963	.931	.796	1.45	.538	1.225	74.3	40.5

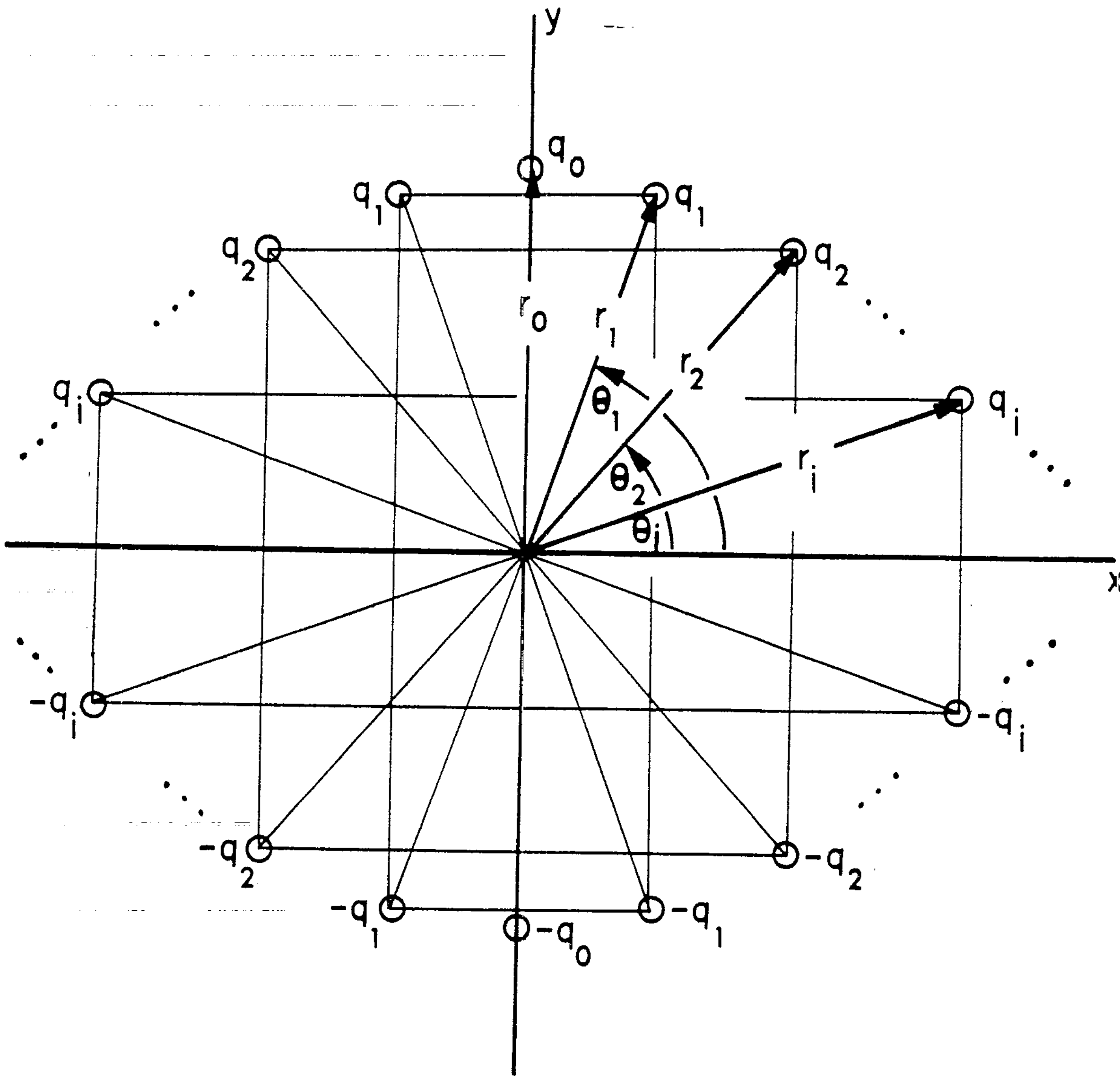

Figure 1. Cross-section of a Multiple-Rod Transmission Line.

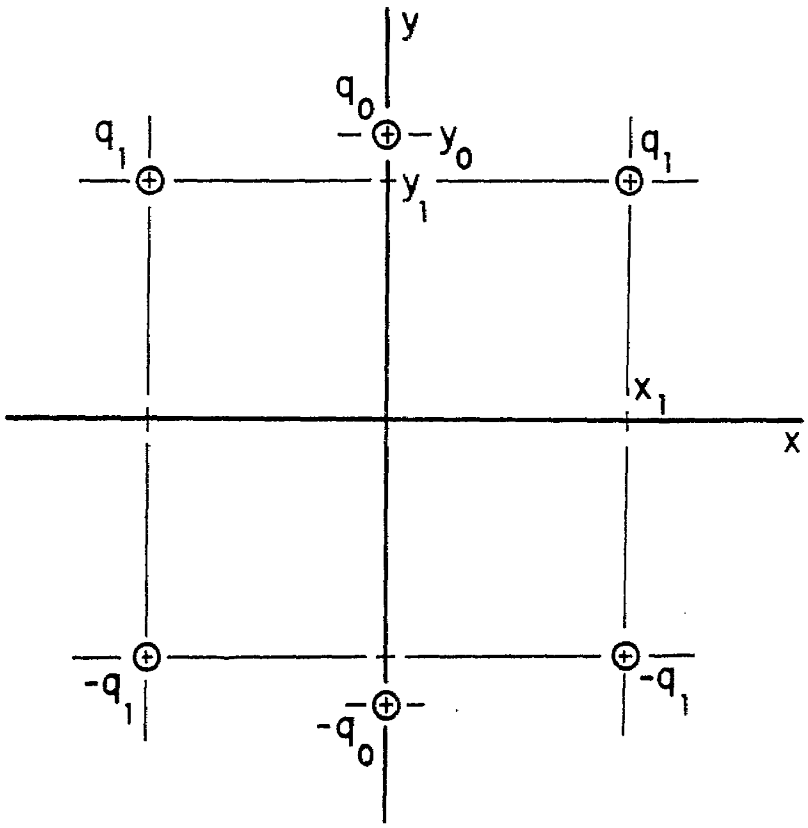

Figure 2. Symmetrical Six-Rod Transmission Line.

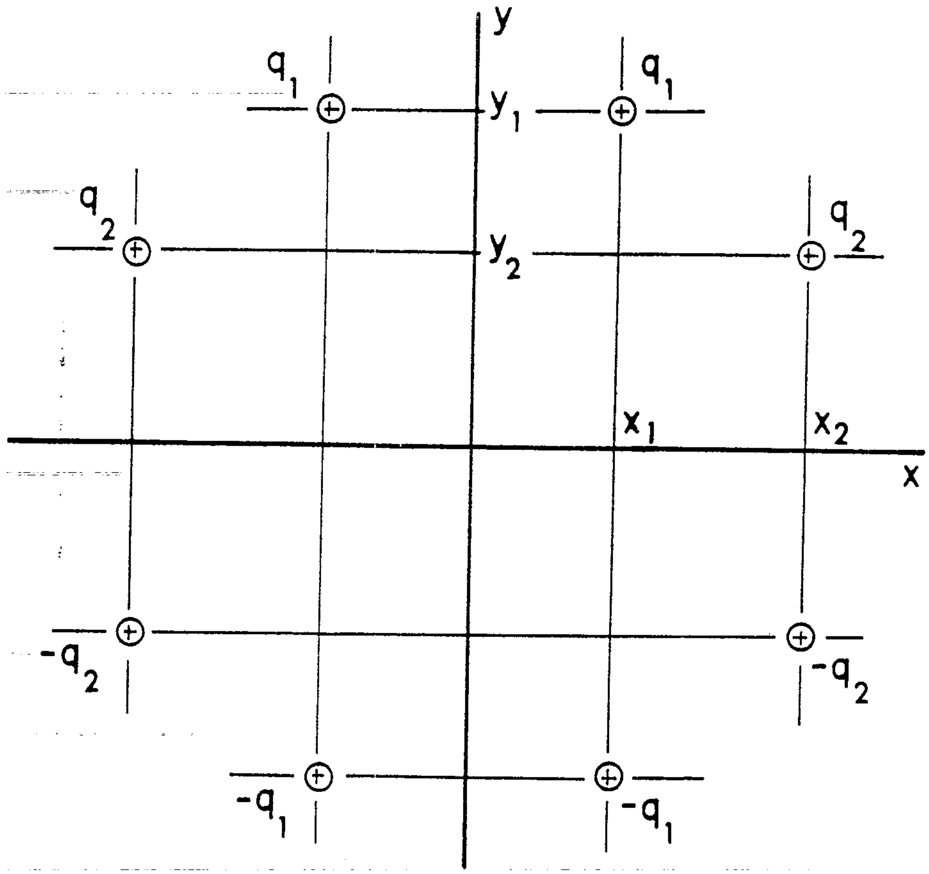

Figure 3. Symmetrical Eight-Rod Transmission Line.

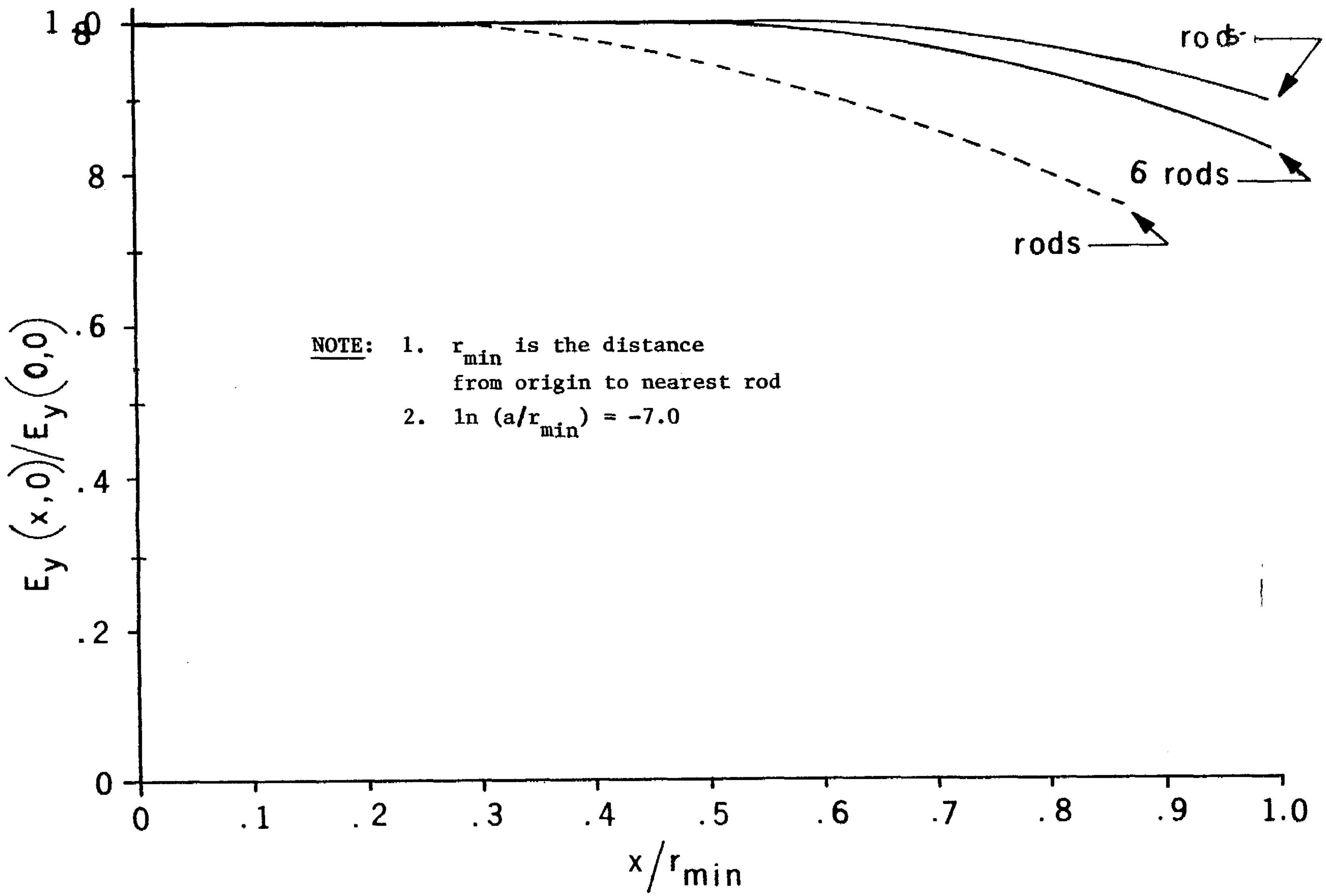

Figure 4. Normalized E_y Along the X-Axis for Symmetrical Multiple-Rod Transmission Lines.

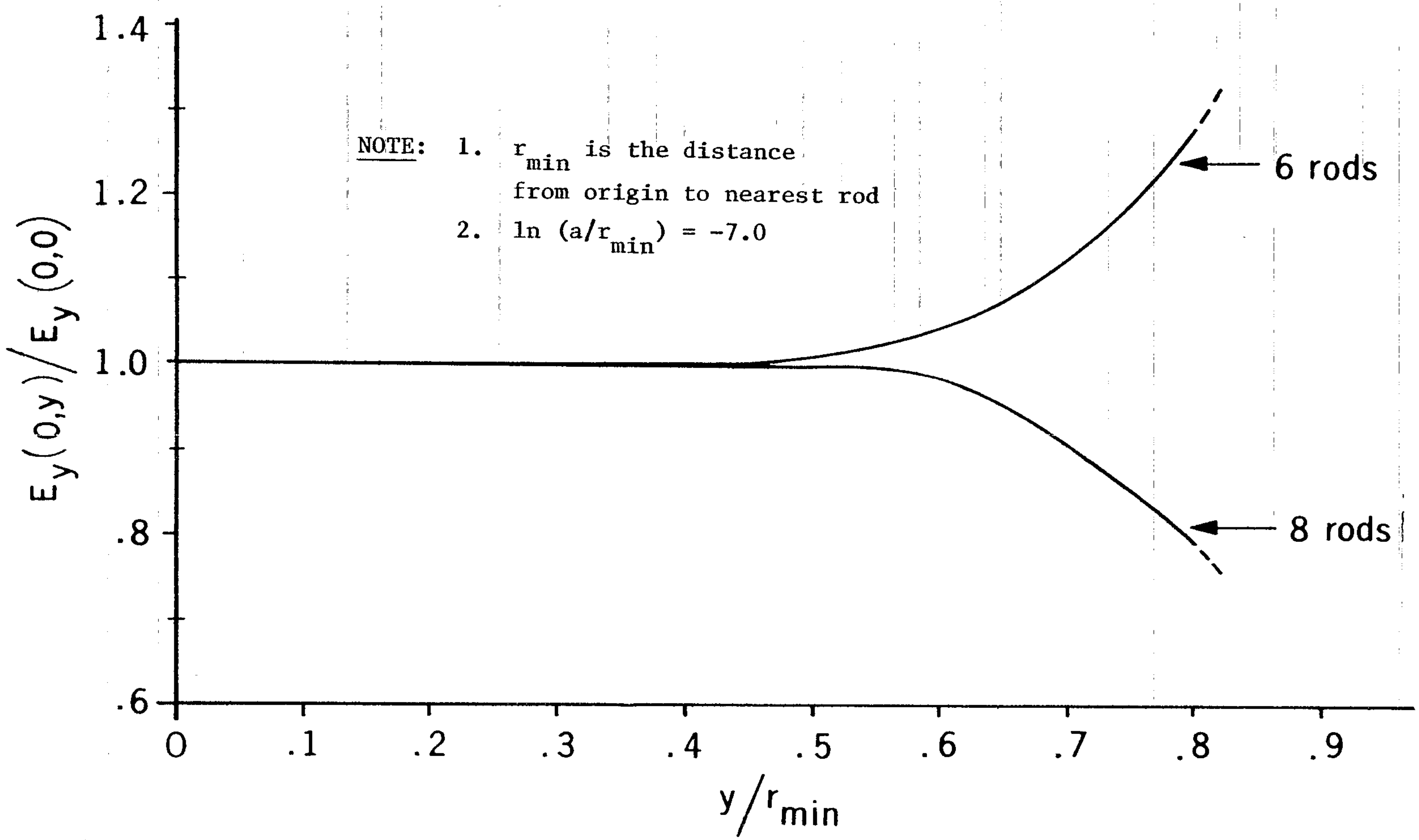

Figure 5. Normalized E_y Along the Y-Axis for Symmetrical Multiple-Rod Transmission Lines.