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Matching Fast Electromagnetic Pulses into Dielectric Targets

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Abstract

This paper considers the use of fast electromagnetic pulses to diagnose the permittivities and thicknesses of layered dielectric media. The Brewster angle of the first layer plays an important role. For transmitting an intense short electromagnetic pulse, normal incidence, augmented by a special dielectric lens, is found to be optimal.

1 Introduction

Recent interest in diagnosing and treating biological targets, with fast (ns or less) electromagnetic pulses, has led to special focusing antenna designs [1–3]. These involve a prolate-spheroidal reflector and a spherically layered dielectric lens.

A previous paper [4] has indicated that there are some special techniques for matching pulsed radar signals into dielectric media and observing the scattered signals for the purpose of identifying buried targets (e.g., mines). The same concepts can also apply for biological targets (e.g., tumors) in or below the skin.

2 Brewster-Angle Diagnosis of Permittivity

As discussed in [4] one can orient an incident wave with vertical polarization such that at the Brewster angle there is no reflected wave from the surface of the semi-infinite dielectric medium (Fig. 2.1). The angles are related as:

$$\tan(\psi_{\rm iB}) = \left[\frac{\epsilon_{\rm r2}}{\epsilon_{\rm r1}}\right]^{1/2} = \left[\frac{\epsilon_2}{\epsilon_1}\right]^{1/2} = \cot(\psi_{\rm tB}) \tag{2.1}$$

$$\psi_{\rm iB} + \psi_{\rm tB} = \frac{\pi}{2} \text{ or } 90^\circ$$

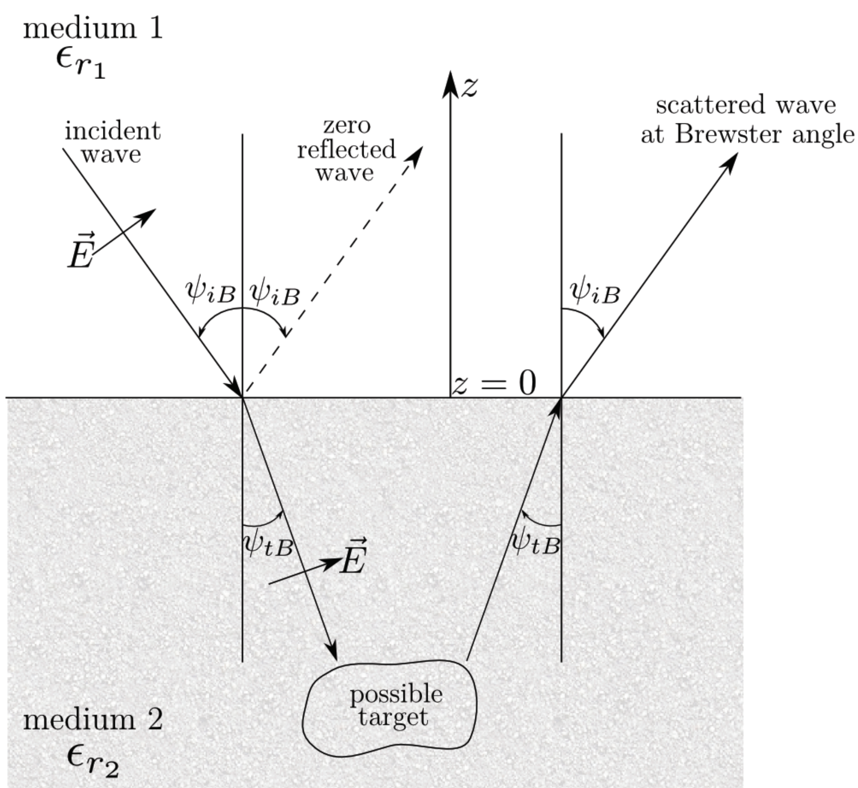

Figure 2.1: Brewster-angle passage of wave between two dielectric media for vertically polarized incident wave

Note that this assumes that we can neglect the media conductivities. If medium 1 is air, this is an excellent approximation. For medium 2, such as tissue or soil, one is more concerned with the conductivity. At the Brewster angle this then gives a minimum in the scattered wave, instead of a zero. Our concern here is fast pulses (100 ps or so) for which the important frequencies of interest are above

$$\omega_r = \frac{1}{t_r} = \frac{\sigma}{\epsilon} = \text{relaxation frequency} = [\text{relaxation time}]^{-1} \tag{2.2}$$

Of course this depends on the details of $\tilde{\sigma}(\omega)$ and $\tilde{\epsilon}(\omega)$ (or their combination as a single complex quantity).

As discussed in [4] one can look for scattered fields from a buried target by also using the Brewster angle (vertical polarization) for the scattered field passing through the interface (no reflection). As illustrated in [[4], Fig. 3.2] the measurement of the scattered field need not be in a coplanar arrangement. The measurement location can be rotated about the z axis to minimize the scattering of the incident wave into the measurement antenna, as well as to obtain more information by measurement at various azimuthal angles.

Now medium 1 need not be free space. It can be the last part (focusing region) of a spherically layered dielectric lens as in [3]. Then as in Fig. 2.2 the appropriate ϵ_{r1} is that of the last portion of the focusing lens. This merely changes the angles as in (2.1). Note that as

$$\epsilon_{\mathrm{r1}} \to \epsilon_{\mathrm{r2}} \tag{2.3}$$

we have

$$\psi_{\mathrm{iB}} \to \frac{\pi}{4}, \ \psi_{\mathrm{tB}} \to \frac{\pi}{4} \tag{2.4}$$

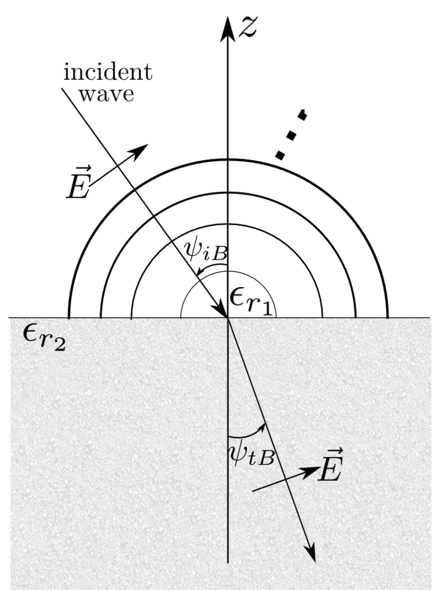

Figure 2.2: Brewster-angle from lens into another dielectric medium

Of course, for this special case the wave passes from medium 1 to medium 2 with no reflection at any angle. However, for

$$\epsilon_{r2} > \epsilon_{r1} \tag{2.5}$$

we have

$$\frac{\pi}{4} < \psi_{iB} < \frac{\pi}{2} \tag{2.6}$$

So, as ϵ_{r1} increases toward ϵ_{r2}, the Brewster angle for incidence decreases toward $\pi/4$ (45°).

With the introduction of the lens, however, there is a problem with the scattered wave from a buried target. If the target is buried at a significant depth, then the scattered wave entering the lens is not centered at the focal point (at the media interface). For such deeply buried targets one can still look at the scattered wave back in the direction of the incident wave (reciprocity). However, one may wish to look in a nearby direction to avoid the large incident radar pulse in the receive antenna (as in Fig. 3.2 in [4]).

An alternate approach would be to cut away a portion of the lens near the interface, so as to move the focal point into the ϵ_{r2} medium.

3 Diagnosis of Second Dielectric Layer

By using a narrow pulse to minimize the scattering from the first dielectric interface (e.g., air to skin) we have found the Brewster angle for this interface. From (2.1), and knowing ϵ_{r1} (e.g., air or lens), we know ϵ_{r2}. If the first layer has thickness h, we can estimate the round trip travel as $2h/v$ with a correction for ψ_{tB} and with

$$\begin{aligned} v &= \epsilon_{r2}^{-1/2} c \\ c &= [\mu_0 \epsilon_0]^{1/2} \end{aligned} \tag{3.1}$$

(See Fig. 3.1). So our pulse or at least its leading edge, should have a width less than this.

Assuming that $\epsilon_{r3} > \epsilon_{r2}$, then $\psi_{t3} < \psi_{tB}$, and conversely. In any event there is a reflected wave from the second planar interface (between ϵ_{r2} and ϵ_{r3}), at the angle ψ_{tB}. In turn it passes through the first interface with no reflection at the angle ψ_{iB}. The reflection coefficient at the second interface is [5]

$$R_2 = \frac{\cos\psi_{tB} - \left[\frac{\epsilon_{r2}}{\epsilon_{r3}}\right]^{1/2} \left[1 - \frac{\epsilon_{r2}}{\epsilon_{r3}} \sin^2(\psi_{tB})\right]^{1/2}}{\cos\psi_{tB} + \left[\frac{\epsilon_{r2}}{\epsilon_{r3}}\right]^{1/2} \left[1 - \frac{\epsilon_{r2}}{\epsilon_{r3}} \sin^2(\psi_{tB})\right]^{1/2}} \tag{3.2}$$

This is solved as

$$\left[\frac{\epsilon_{r2}}{\epsilon_{r3}}\right]^{1/2} \left[1 - \frac{\epsilon_{r2}}{\epsilon_{r3}} \sin^2(\psi_{tB})\right]^{1/2} = \frac{1 - R_2}{1 + R_2} \cos\psi_{tB} \equiv X \tag{3.3}$$

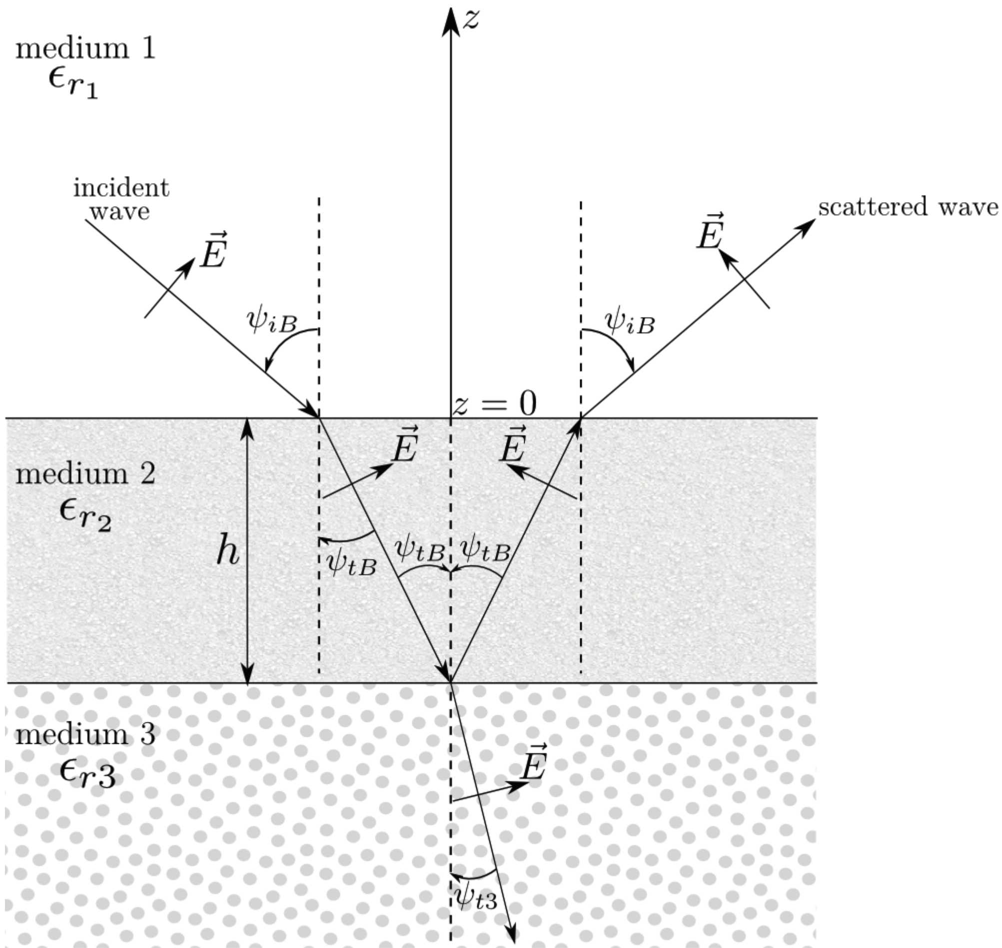

Figure 3.1: Passage of incident wave at Brewster angle of first interface to diagnose second dielectric layer

where X is known. This gives a quadratic equation in $\epsilon_{\rm r2}/\epsilon_{\rm r3}$ as

$$\left[\frac{\epsilon_{\rm r2}}{\epsilon_{\rm r3}}\right]^2 \sin^2(\psi_{\rm tB}) - \frac{\epsilon_{\rm r2}}{\epsilon_{\rm r3}} + X^2 = 0 \tag{3.4}$$

with the solution

$$\frac{\epsilon_{\rm r2}}{\epsilon_{\rm r3}} = \frac{1 \pm [1 - 4\sin^2(\psi_{\rm tB})X^2]^{1/2}}{2\sin^2(\psi_{\rm tB})} \tag{3.5}$$

For a solution to exist (real reflection) we need (from (3.2))

$$\begin{aligned} &\frac{\epsilon_{\rm r2}}{\epsilon_{\rm r3}} \sin^2(\psi_{\rm tB}) \leq 1 \\ &\Rightarrow \sin^2(\psi_{\rm tB}) \leq \frac{\epsilon_{\rm r3}}{\epsilon_{\rm r2}} \end{aligned} \tag{3.6}$$

For $\epsilon_{r3} > \epsilon_{r2}$ this is easily satisfied. From (2.1) we have

$$\frac{\epsilon_{r2}}{\epsilon_{r1}} = \cot^2(\psi_{tB}) = \csc^2(\psi_{tB})[1 - \sin^2(\psi_{tB})] = \csc^2(\psi_{tB}) - 1 \tag{3.7}$$

$$\sin^2(\psi_{tB}) = \left[\frac{\epsilon_{r2}}{\epsilon_{r1}} + 1\right]^{-1} \leq \frac{\epsilon_{r3}}{\epsilon_{r2}}$$

So we need

$$\frac{\epsilon_{r3}}{\epsilon_{r2}} \geq \left[\frac{\epsilon_{r2}}{\epsilon_{r1}} + 1\right]^{-1} \tag{3.8}$$

$$\Rightarrow \epsilon_{r3} \geq \left[\frac{1}{\epsilon_{r1}} + \frac{1}{\epsilon_{r2}}\right]^{-1}$$

For $\epsilon_{r3} \geq \epsilon_{r1}$ this is easily achieved.

For calibration of the reflection measurement (allowing for beam divergence) we can first place a thin metal sheet on the $z = 0$ plane. Then the measured reflection can be normalized by division by the amplitude of the reflection from the conducting sheet. One can think of this as a special kind of TDR (time-domain reflectometer).

4 Matching High-Amplitude Narrow Pulses into Biological Targets

The foregoing section 2 is also an introduction to the problem of matching fast, high-amplitude pulses into biological targets. With a biological target such as skin over other tissues (perhaps including a tumor), we can have a situation as in Fig. 4.1. This shows the general case with incident and reflected beams (ray-optics approximation) coming from medium 1 (such as air or a lens dielectric) into medium 2 (biological entity). This is the general case.

Letting (3.2) apply to normal incidence we have

$$\psi_i = \psi_t = 0$$

$$R_1 = \frac{1 - \left[\frac{\epsilon_{r1}}{\epsilon_{r2}}\right]^{1/2}}{1 + \left[\frac{\epsilon_{r1}}{\epsilon_{r2}}\right]^{1/2}} \text{ (reflection from interface)} \tag{4.1}$$

$$T_1 = 1 - R_1 = \frac{2\left[\frac{\epsilon_{r1}}{\epsilon_{r2}}\right]^{1/2}}{1 + \left[\frac{\epsilon_{r1}}{\epsilon_{r2}}\right]^{1/2}} \text{ (transmission of electric field through interface)}$$

For $\epsilon_{r2} > \epsilon_{r1}$ we have

$$T_1 < 1 \tag{4.2}$$

With normal incidence the beam diameter is unchanged. The electric field is reduced by the factor T_1 (while the displacement vector and the magnetic field are increased).

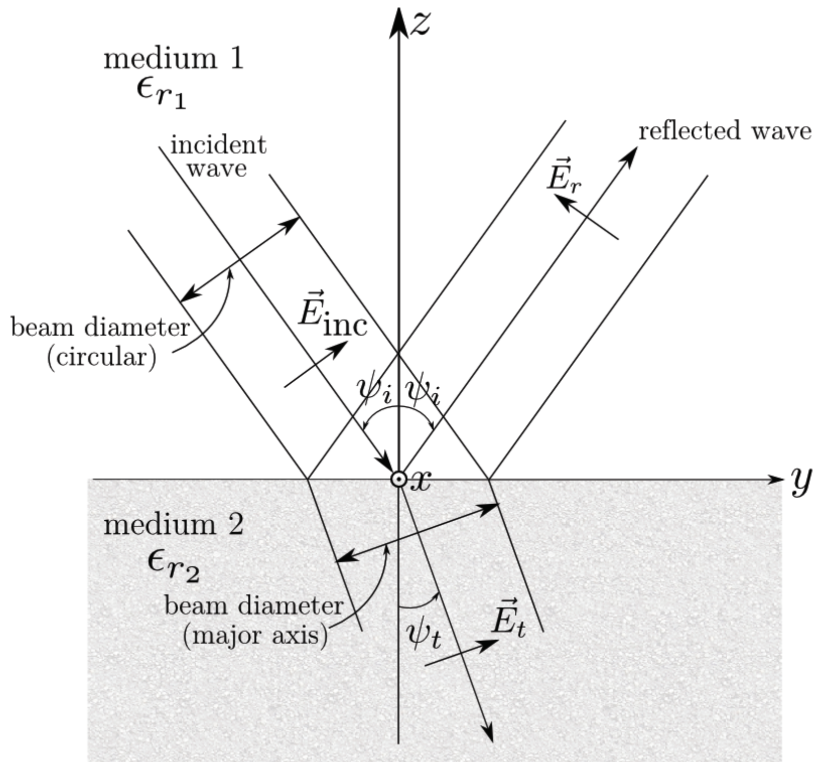

Figure 4.1: Passage of beam through dielectric interface

Next let (3.2) apply to the Brewster angle as in (2.1) giving

$$\tan(\psi_{\rm tB}) = \left[\frac{\epsilon_{\rm r2}}{\epsilon_{\rm r1}}\right]^{1/2} = \cot(\psi_{\rm tB}) \tag{4.3}$$

Now the electric field is transmitted in a form to match the tangential electric fields at the boundary as

$$\begin{aligned} E_1\cos(\psi_{\rm iB}) &= E_2\cos(\psi_{\rm tB}) \\ \cos(\psi_{\rm iB}) &= [1+\tan^2(\psi_{\rm iB})]^{-1/2} = \left[1+\frac{\epsilon_{\rm r2}}{\epsilon_{\rm r1}}\right]^{-1/2} \\ \cos(\psi_{\rm tB}) &= \cot(\psi_{\rm tB})[1+\cot^2(\psi_{\rm tB})]^{-1/2} = \left[\frac{\epsilon_{\rm r2}}{\epsilon_{\rm r1}}\right]^{1/2}\left[1+\frac{\epsilon_{\rm r2}}{\epsilon_{\rm r1}}\right]^{-1/2} \\ T_{1B} &\equiv \frac{E_2}{E_1} = \left[\frac{\epsilon_{\rm r1}}{\epsilon_{\rm r2}}\right]^{1/2} \end{aligned} \tag{4.4}$$

For $\epsilon_{\rm r2} > \epsilon_{\rm r1}$ we have

$$T_{1B} < 1 \tag{4.5}$$

Comparing the two results we have

$$\frac{T_{1B}}{T_1} = \frac{1}{2}\left[1+\left[\frac{\epsilon_{\rm r1}}{\epsilon_{\rm r2}}\right]^{1/2}\right], \quad \frac{1}{2} < \frac{T_{1B}}{T_1} < 1 \text{ for } \epsilon_{\rm r2} > \epsilon_{\rm r1} \tag{4.6}$$

This gives that normal incidence results in a larger electric field in medium 2. This is also associated with the larger beam diameter (in near y-direction, same size in x-direction) in the Brewster-angle case.

The closer $\epsilon_{\rm r1}$ can approach $\epsilon_{\rm r2}$, then both T_1 and T_{1B} approach unity. This is an added benefit of a lens with $\epsilon_r > 1$ in its layer nearest the target.

For the case of multiple dielectric layers the conclusion can be different. For example, for a third medium with $\epsilon_{\rm r3} = \epsilon_{\rm r1}$ the Brewster angle gives a net transmission of unity, exceeding the normal-incidence case.

5 Concluding Remarks

In matching electromagnetic pulses into dielectric media, the Brewster angle is an important concept. It can be used to aid in measuring the dielectric constants and thicknesses of the first few layers of a layered dielectric medium. However, from a pulse-amplitude point of view, normal incidence is a little better.

References

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