Sensor and Simulation Notes

Note 537

April 2009

Scaling Relationships for Electromagnetic Parameters for Focusing Graded Dielectric Lenses

Carl E. Baum, Serhat Altunc and Prashanth Kumar
University of New Mexico
Department of Electrical and Computer Engineering
Albuquerque New Mexico 87131

Abstract

This paper establishes scaling relationships for electric field, displacement, and magnetic field for focusing graded dielectric lenses. This accompanies the reduction in spot size. Examples are given in tabular form.

1. Introduction

An earlier paper [1] gave some example scaling relationships for spot size, power density and electric field. Here, we extend this to the displacement current density and magnetic parameters as well.

Figure 1.1 gives the lens geometry. An incoming spherical electromagnetic pulse is incident on the lens at the outer radius r_{max}. It propagates through a smoothly varying $\varepsilon_r(r)$ beginning at $\varepsilon_r(r_{max}) = 1$ to a larger $\varepsilon_r(r_{min}) = \varepsilon_{r\,max}$. The wave is assumed short enough in time t_δ (rise time) so that the droop in the pulse [2] t_d is

$$0 < t_\delta << t_d \tag{1.1}$$

and can be neglected. In addition, it is assumed that the steps in $\varepsilon_r(r)$ as r is decreased are small enough that a set of spherical layers has enough layers to approximate the continuous case [2].

A fundamental approximation concerns the number of spatial pulse widths in the dielectric along a sphere of constant radius r. As in Fig. 1.1 let the significant portion of the incident wave be concentrated in a circular cone of angle ψ_0 from the axis. In this region we need many pulse widths (like wavelengths if we are dealing with single frequency) extending over this domain of angular diameter $2\psi_0$. In this case we can calculate the wave propagation into smaller r as though it were a plane wave. Another way to look at this is as power conservation as the wave enters a smaller and smaller cross section (diameter $\cong 2r\psi_0$) or area $\cong \pi(r\psi_0)^2$. We are, of course, assuming negligible loss and dispersion in the dielectric for these calculations to apply.

The functional form of ε_r is [2,3]

$$\begin{aligned} \varepsilon_r &= \left[\frac{r_{max}}{r}\right], \quad r_{min} \le r \le r_{max} \\ \varepsilon_{r\,max} &= \left[\frac{r_{max}}{r_{min}}\right], \quad 0 \le r \le r_{min} \end{aligned} \tag{1.2}$$

in its continuous form. On leaving r_{max} into the focusing region the pulse goes to a minimum spot size which we can estimate from [4]. The radius before inserting the lens is

$$\Delta\Psi_0 = \frac{a}{2b} c t_\Psi = \frac{a}{b} c t_\delta \tag{1.3}$$

where a and b are the radii of the prolate-spheroid ($a < b$), c is the speed of light in free space, and t_Ψ is the pulse width with respect to Ψ. For reference we can have an example [5]

$$\frac{a}{b} = \frac{5}{4} = 1.25$$
$$t_\delta = 100\,\mathrm{ps}, \quad t_\Psi = 2\,t_\delta = 200\,\mathrm{ps} \tag{1.4}$$
$$\Delta\Psi_0 = 3.75\,\mathrm{cm}$$

We have a scaling parameter as $\varepsilon_{r\max}^{-1/2}$ for the spot size. Note that at a given r the number of pulse widths is given by

$$\frac{\upsilon}{\upsilon_0} \cong r\,\varepsilon_r^{-12}(r) \cong 1 \tag{1.5}$$
$$\upsilon_0 \equiv \text{number of pulse widths at } r_{\max}$$

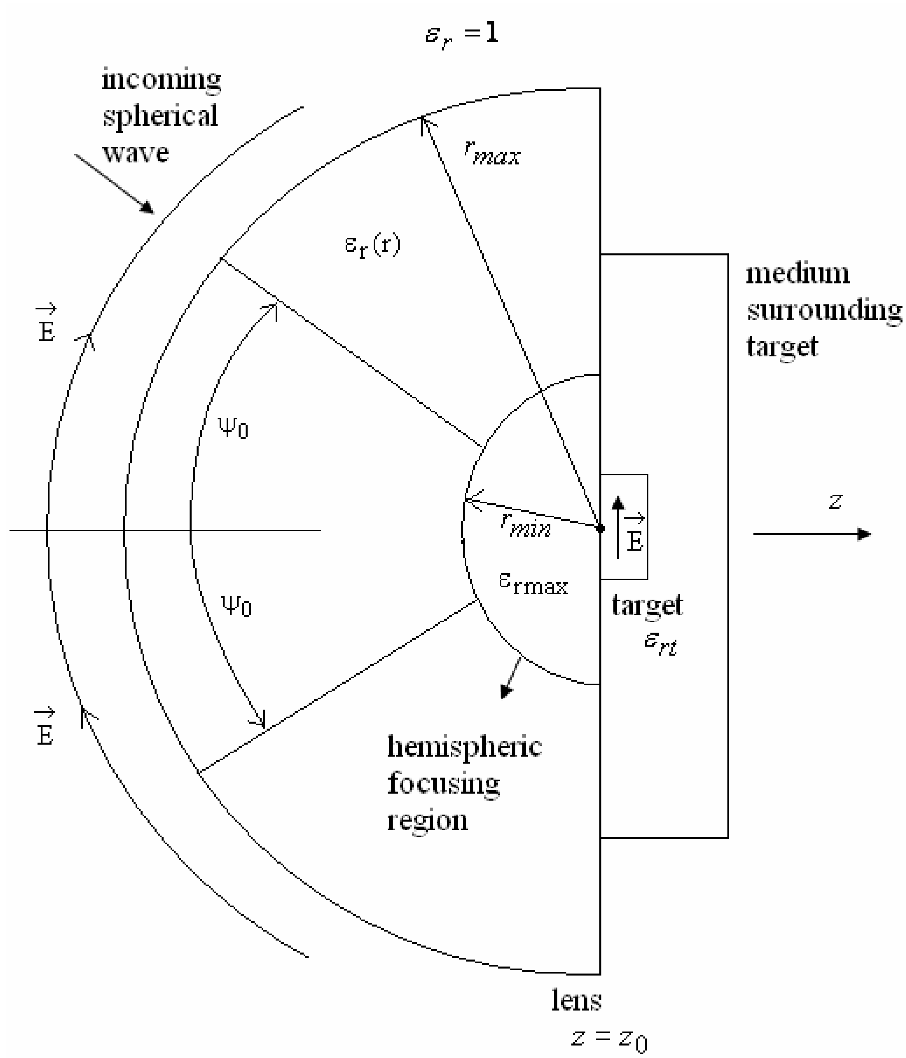

Figure 1.1 Lens Concentrating an Inward-Propagating Spherical Pulse

2. Ideal Scaling Relationships at Focus

We can now give several scaling relationships for the various electromagnetic parameters in summary form.

2.1 Spot radius

$$\frac{\Delta\Psi(r_{min})}{\Delta\Psi(\text{no lens})} = \frac{r_{min}}{r_{max}} = \varepsilon_{r\,max}^{-1/2} \tag{2.1}$$

2.2 Spot area

$$\frac{A(r_{min})}{A(\text{no lens})} = \left(\frac{r_{min}}{r_{max}}\right) = \varepsilon_{r\,max}^{-1} \tag{2.2}$$

2.3 Power density in spot

$$\frac{P(r_{min})}{P(r_{max})} = \frac{A(r_{min})}{A(r_{max})} = \varepsilon_{r\,max} \tag{2.3}$$

$$-\vec{1}_r \times \vec{E}(r) = Z_w\,\vec{H}(r)$$

$$E(r) = \sqrt{\frac{\mu}{\varepsilon(r)}}H(r) = \varepsilon(r)^{-1/2} Z_0 H(r)$$

$$P(r) = -\vec{1}_r \bullet \left[\vec{E}(r) \times \vec{H}(r)\right]$$

$$= E(r)H(r) = \varepsilon(r)^{-1/2} Z_0\, H(r)^2$$

$$= \frac{\varepsilon(r)^{1/2}}{Z_0} E(r)^2$$

$$\frac{P(r_{min})}{P(r_{max})} = \varepsilon_{r\,max}^{1/2}\frac{E^2(r_{min})}{E^2(r_{max})} = \varepsilon_{r\,max}^{-1/2}\frac{H^2(r_{min})}{H^2(r_{max})}$$

2.4 Electric field enhancement

$$\frac{E(r_{min})}{E(r_{max})} = \varepsilon_{r\,max}^{1/4} \tag{2.4}$$

2.5 Displacement (electric flux density) enhancement

$$\frac{D(r_{min})}{D(r_{max})} = \varepsilon_r \frac{E(r_{min})}{E(r_{max})} = \varepsilon_{r\,max}^{5/4} \tag{2.5}$$

2.6 Magnetic field enhancement

$$\frac{H(r_{min})}{H(r_{max})} = \varepsilon_{r\,max}^{3/4} \tag{2.6}$$

2.7 Displacement (electric flux density) enhancement

$$\frac{B(r_{min})}{B(r_{max})} = \varepsilon_{r\,max}^{3/4} \tag{2.7}$$

2.8 Some implications

What we can see here the great increase in the displacement associated with ε_r. The time derivative of this is a current density in the lens and in the tissue. So the incident wave gives the electromagnetic parameters at the focus and they are presented in Table 2.1.

Table 2.1 Electromagnetic parameters at lens focus

ε_r	9	81
electric enhancement	1.73	3
displacement enhancement	15.6	243
magnetic enhancement	5.2	27
relative wave impedance	1/3	1/9
relative spot size at focus	1/3	1/9
relative spot area at focus	1/9	1/81

Recalling our example in (1.4) we have focal spot parameters in Table 2.2.

Table 2.2 Example focal spot dimensions

ε_r	9	9	81
$\Delta\Psi$ (spot radius)	3.75 cm	1.25 cm	4.2 mm
$\pi\Delta\Psi^2$ (spot area)	44.2 cm^2	4.9 cm^2	.55 cm^2

3. Matching from Focus Medium to Target Medium

As discussed in [1] as the waves go from the focus region to the target there may be a discontinuity in ε_r. There is a transmission coefficient which expresses the change of the fields (up or down) in crossing this boundary. We estimate this based on plane-wave formulae. If

$$\varepsilon_{r\,max} = \varepsilon_{r\,t} \tag{3.1}$$

then the results in Section 2 directly can be applied.

The electric field transmission coefficient is

$$T_{et} = \frac{2\,\varepsilon_{r\,t}^{-1/2}}{\varepsilon_{r\,max}^{-1/2} + \varepsilon_{r\,t}^{-1/2}} = \frac{2\,\varepsilon_{r\,t}^{-1/2}}{1+\left[\frac{\varepsilon_{r\,t}}{\varepsilon_{r\,max}}\right]^{1/2}} \tag{3.2}$$

The displacement transmission is then

$$T_{d\,t} = \frac{\varepsilon_{r\,t}}{\varepsilon_{r\,max}} T_{et} = \frac{\varepsilon_{r\,t}}{\varepsilon_{r\,max}} \frac{2}{1+\left[\frac{\varepsilon_{r\,t}}{\varepsilon_{r\,max}}\right]^{1/2}} \tag{3.3}$$

The magnetic transmission is

$$T_{h\,t} = T_{b\,t} = \frac{Z_{w\,t}}{Z_{w\,max}} T_{et} = \left[\frac{\varepsilon_{r\,max}}{\varepsilon_{r\,t}}\right]^{1/2} T_{et} = \frac{2}{1+\left[\frac{\varepsilon_{r\,max}}{\varepsilon_{r\,t}}\right]^{1/2}} \tag{3.4}$$

Combining these factors gives Table 3.1. As we can see there is a large increase in the displacement current (and associated displacement current density).

Table 3.1 Total Parameter Enhancement Including Mismatch into Target

ε_{rt} (target)	40.5		81		
$\varepsilon_{r\,max}$ (lens)	9	81	9	81	
$\frac{E(r_{min})}{E(r_{max})}$	1.73	3	1.73	3	in focal region of lens
$\frac{D(r_{min})}{D(r_{max})}$	15.6	243	15.6	243	
$\frac{H(r_{min})}{H(r_{max})} = \frac{B(r_{min})}{B(r_{max})}$	5.2	27	5.2	27	
T_{et}	0.32	1.17	0.5	1	transmission to target medium
T_{dt}	1.44	0.59	4.5	1	
T_{ht}	1.36	0.83	1.5	1	
$\frac{E_t}{E(r_{max})}$	0.55	3.51	0.87	3	in target medium
$\frac{D_t}{D(r_{max})}$	22.5	143	70	243	
$\frac{H_t}{H(r_{max})} = \frac{B_t}{B(r_{max})}$	7.1	22.4	7.8	27	

4. Concluding Remarks

Here we can appreciate the benefits of a high-dielectric-constant lens, provided we can neglect loss and dispersion. Most notably we can see a large increase in the electric displacement (and displacement current density) in the target medium for exposing biological targets. In addition the focal spot size is significantly decreased. This can be useful when exposing small targets, such as melanomas.

The reader can consult [1] for additional information, including some typical numbers for electric field if the prolate-spheroidal IRA is driven by 200 kV.

References

1. S. Altunc, C. E. Baum, C. G. Christodoulou and E. Schamiloglu "Increasing Lens-Medium Permittivity Over Target-Medium Permittivity to Increase Electric Field and Decrease Spot Size at Target, "Sensor and Simulation Note 529, June 2008.
2. S. Altunc, C. E. Baum, C. G. Christodoulou and E. Schamiloglu "Lens Design for a Prolate-Spheroidal Impulse Radiating Antenna (IRA), "Sensor and Simulation Note 525, October 2007.
3. S. Altunc, C. E. Baum, C. G. Christodoulou and E. Schamiloglu "Lens Design with Constant Wavelength to Cross Section Ratio as ε_r Varies: a Log Periodic Lens, "EM Implosion Memos 19, February 2008.
4. Baum, C.E. , Focal Waveform of a Prolate-Spheroidal Impulse-Radiating Antenna (IRA), Radio Sci. , Vol. 42, RS6S27, 2007.
5. S. Altunc, C. E. Baum, C. G. Christodoulou, E. Schamiloglu, and G. Buchenauer, "Focal waveforms for various source waveforms driving a prolate-spheroidal impulse radiating antenna," Radio Sci., Feb. 2008, Vol. 43, RS003775, 2008.