

Sensor and Simulation Notes

Note 535

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## Terahertz Antennas and Oscillators Including Skin-Effect Losses

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### Abstract

Good conductors have significant skin-effect losses at high frequencies. This paper estimates their impact, and techniques to mitigate them.

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## 1. Introduction

As we go to higher frequencies approaching the THz regime we encounter some special design problems associated with planar geometries [4]. This applies to the antennas and the switched oscillators which can drive them to produce mesoband radiated pulses.

At these high frequencies, one of the problems can be losses in the conductors due to skin effect. This will impact the choice and geometry of conductors and dielectrics.

## 2. Skin-Effect Losses

Skin effect is a classical phenomenon by which imperfect conductors have a resistive and inductive surface impedance associated with the exponential decay of the current density as one looks into the conductor surface. This applies for frequencies high enough that the skin depth is much less than the conductor thickness.

The skin depth for a good conductor is

$$\delta_s = \left[ \frac{2}{\omega \mu \sigma} \right]^{1/2} = \left[ \frac{1}{\pi f \mu_0 \sigma} \right]^{1/2} \text{ for } \mu = \mu_0 \quad (2.1)$$

giving a surface impedance [8]

$$Z_s = [1+j][\sigma \delta_s]^{-1} = [1+j] \left[ \frac{\pi f \mu_0}{\sigma} \right]^{1/2} = [1+j] 2\pi \left[ \frac{10^{-7} f}{\sigma} \right]^{1/2} \quad (2.2)$$

This can be written as

$$\begin{aligned} Z_s &= R_s + j\omega L_s \\ R_s &= 2\pi \left[ \frac{10^{-7} f}{\sigma} \right]^{1/2} \end{aligned} \quad (2.3)$$

with the latter accounting for the resistive losses. For a conductor of length  $\ell$  and width  $w$  this gives a resistance

$$R = \frac{\ell}{w} R_s \quad (2.4)$$

which can be compared to other impedances in our antenna/oscillator system.

Typical conductivities are given in Table 2.1

Table 2.1. Conductivities of Some Important Metals.

Material	$\sigma$ (Conductivity S/m) at 20°C
Aluminum	$3.8 \times 10^7$
Copper	$5.8 \times 10^7$
Gold	$4.1 \times 10^7$
Silver	$6.2 \times 10^7$

These, in turn, give surface resistances as in Table 2.2.

Table 2.2. Surface Resistance of Some Important Metals.

Material	$R_s$ ( $\Omega$ ) at 100 GHz	$R_s$ ( $\Omega$ ) at 1 THz
Aluminum	0.101	0.32
Copper	0.083	0.26
Gold	0.098	0.31
Silver	0.080	0.25

As we can see, these resistances are rather small. However, we need to multiply them by some geometric factor to obtain an appropriate resistance to compare to other impedances in the antenna/oscillator system. Typically we might multiply by a factor  $\ell/w$  where

$\ell$  = some characteristic length (in direction of current flow)

$w$   $\equiv$  some characteristic width (in direction transverse to current flow, but parallel to surface) (2.5)

$f_s = \frac{\ell}{w} \equiv$  geometric factor for skin effect

Typical values of  $f_s$  might be of order ten, giving a few ohms for R. For the antenna (resonant) this may not be a big problem. However, for the switched oscillator the transmission-line characteristic impedance  $Z_c$  may be rather low, but may have to be limited to some factor greater than R depending on the desired oscillator Q.

### 3. Edge Losses

As discussed in [5], sharp (knife) edges can greatly increase the skin-effect losses. This is associated with the  $x^{-1/2}$  singularity in the surface current density,  $J_s$ , as one approaches the edge where  $x$  is the distance from the edge. Then  $J_s^2$  is proportional to  $x^{-1}$ , which when integrated to give power loss, gives a logarithmic singularity.

The remedy, of course, is to avoid sharp edges, such as by using rollups [6]. This has the additional benefit of reducing the electric field maxima and allowing higher voltage and power operation.

In the present context of tiny antennas, circular cylindrical rollups may be difficult to construct. As illustrated in Fig. 3.1, one can use “thick” conducting strips which avoid a  $360^\circ$  edge opening, reducing it to  $270^\circ$  for a rectangular edge, thereby reducing the order of the singularity in  $J_s$ . Even better, one may try to round the edges.

### 4. Antenna and Oscillator Losses

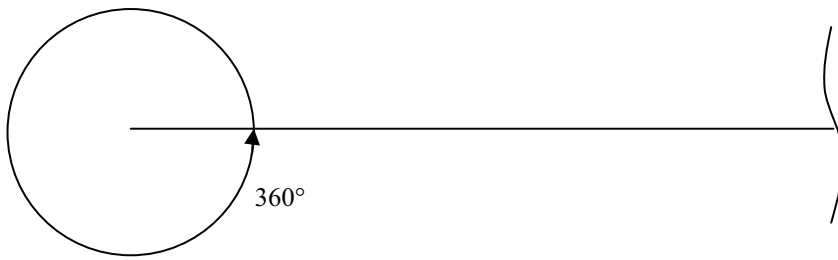
As in Fig. 4.1, consider a simple antenna/oscillator geometry. The antenna conductor is placed  $\lambda/4$  (in the dielectric) away from a conducting ground plane to make the image antenna add to the radiation in the forward direction.

We would normally have

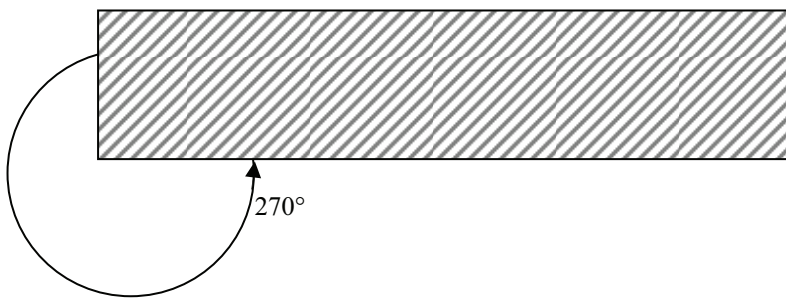
$$\begin{aligned}
 0 &< w_a \ll \ell_a \\
 w_a &\equiv \text{antenna width} \\
 \ell_a &\equiv \text{antenna length}
 \end{aligned}
 \tag{4.1}$$

For a high input impedance to the antenna we would have [1]

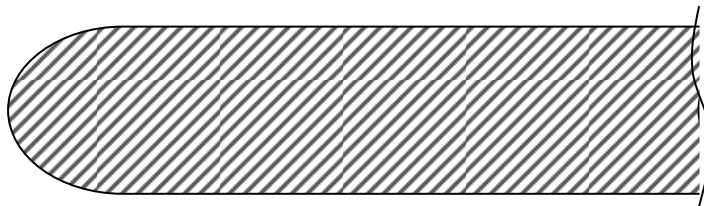
$$\ell_a \approx \frac{\lambda}{2}
 \tag{4.2}$$



A. Sharp edge

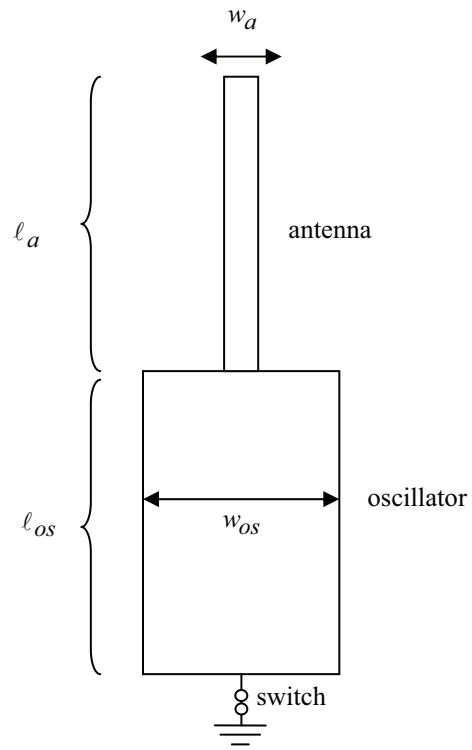


B. Rectangular edge

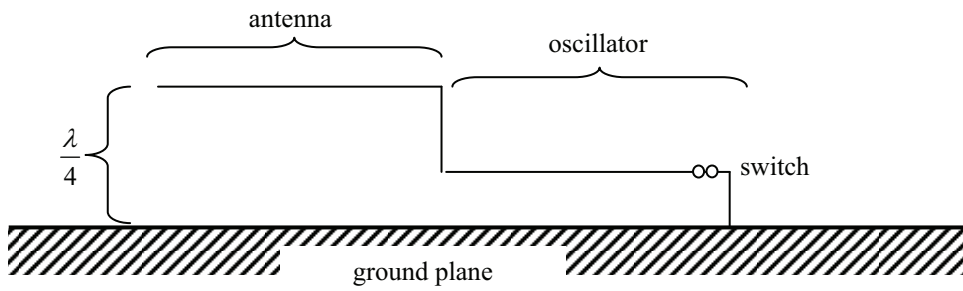


C. Rounded edge

Fig. 3.1 Avoiding Sharp Conductor Edges



A. Front view



B. Side view

Fig. 4.1 Antenna and Switched Oscillator

with

$$\frac{\ell_a}{w_a} \simeq 10 \quad (4.3)$$

and an antenna input impedance in the  $100 \Omega$  ballpark, the few ohms of skin losses should not be significant.

As indicated in Fig. 4.1B, the oscillator conductor is spaced less than  $\lambda/4$  from the ground plane to minimize radiation here. In Fig. 4.2A we show what might be a cross section of such an oscillator. Note that the conductor at potential  $V$  has

$$\begin{aligned} \Delta_{os} &= \text{thickness} \\ w_{os} &= \text{width} \\ h_{os} &= \text{spacing from ground plane} \end{aligned} \quad (4.4)$$

with rounded edges (if practical). The characteristic impedance of the transmission line is given by

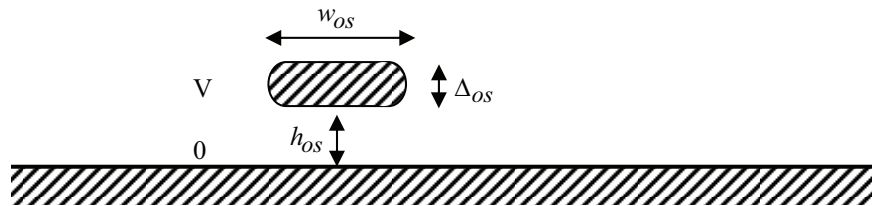
$$\begin{aligned} Z_c &\equiv \text{characteristic impedance} \\ &\simeq \frac{h_{os}}{w_{os}} Z_w \\ Z_w &\equiv \text{wave impedance of dielectric medium} \\ &= \left[ \frac{\mu_0}{\epsilon} \right]^{1/2} \end{aligned} \quad (4.5)$$

for large  $w_{os} / h_{os}$ . For low  $Z_c$  we might have

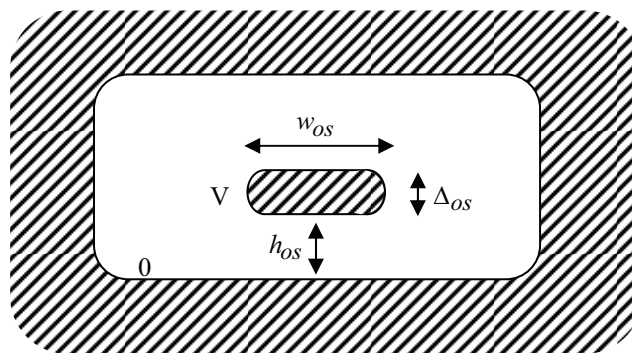
$$\frac{h_{os}}{w_{os}} \simeq 0.1 \quad (4.6)$$

giving a  $Z_c$  in the  $20 \Omega$  range for a  $200 \Omega$  wave impedance. To lower the skin-effect losses we would like  $w_{os}$  large, but it needs to be small compared to  $\ell_{os}$  (say 0.5 or so). This limits the skin resistance discussed in Section 2.

Figure 4.2B shows a possible geometry for an enclosed switched oscillator. For large  $w_{os} / h_{os}$  this roughly halves  $Z_c$  for a given  $w_{os} / h_{os}$ . However, the construction may be much more difficult.



A. Single ground plane



B. Flat coaxial structure

Fig. 4.2 Oscillator Cross Section



Noting the restriction on  $w_{os} (\ll \ell_{os})$  for a transmission-line approximation, one might try what might be called a double switched oscillator as in Fig. 4.3. This has been previously discussed in [7]. This is a two-dimensional form, as compared to the two- and three-dimensional forms as in [7]. Now as long as

$$w_2 \gg h_{os} \tag{4.7}$$

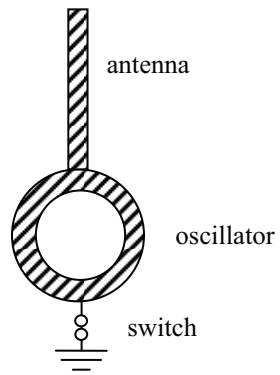
we can think of this as two parallel transmission lines, not coupled to each other, except at the ends. This is another way to lower the effective  $Z_c$ .

## 5. Concluding Remarks

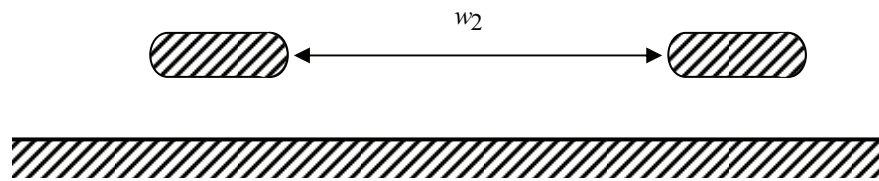
So now we can see the impact of skin-effect losses. This is most significant in low-characteristic-impedance switched oscillators. However, various design considerations mitigate the impact of skin resistance. While our illustrations here are for single-ended oscillators (and antennas), the results readily extend to differential switched oscillators [2, 3].

## References

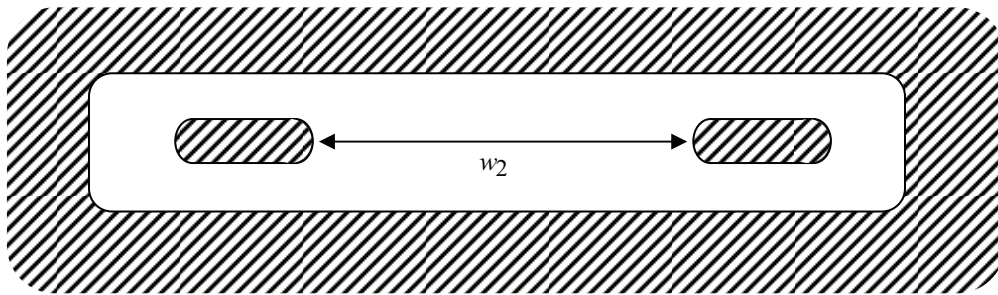
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A. Front view



B. Cross section: single ground plane



C. Cross section: double ground plane

Fig. 4.3 Double Switched Oscillator