Sensor and Simulation Notes

Note 531

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Combined Electric and Magnetic Dipoles for Mesoband Radiation, Part 2

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Abstract

This paper continues the discussion of the design of an antenna with collinear electric- and magnetic-dipole moments out of phase by approximately $\pi / 2$ (90°). Using a high-Q approximation, a switched oscillator is matched to the antenna for radiating an oscillatory pulse.

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1. Introduction

A recent paper introduced the concept of a balanced pair of collinear electric and magnetic dipoles [2]. It was shown that this could produce circular polarization in the far field. The calculations are based on a high-Q approximation, so that narrow-band (hypoband) (CW) concepts can be used. While polarization is normally a narrow-band concept, it can be generalized to the damped sinusoidal case [3, 7]. Matching this antenna to a switched oscillator [4], we consider the radiation from such a mesoband radiating source.

The details of the antenna are discussed in [2]. The basic geometry is indicated in Fig. 1.1 with a loop going around the electric-dipole axis. This loop (magnetic dipole) can be made in a fractional- (e.g., half-) turn geometry to suppress any associated transverse electric-dipole moment.

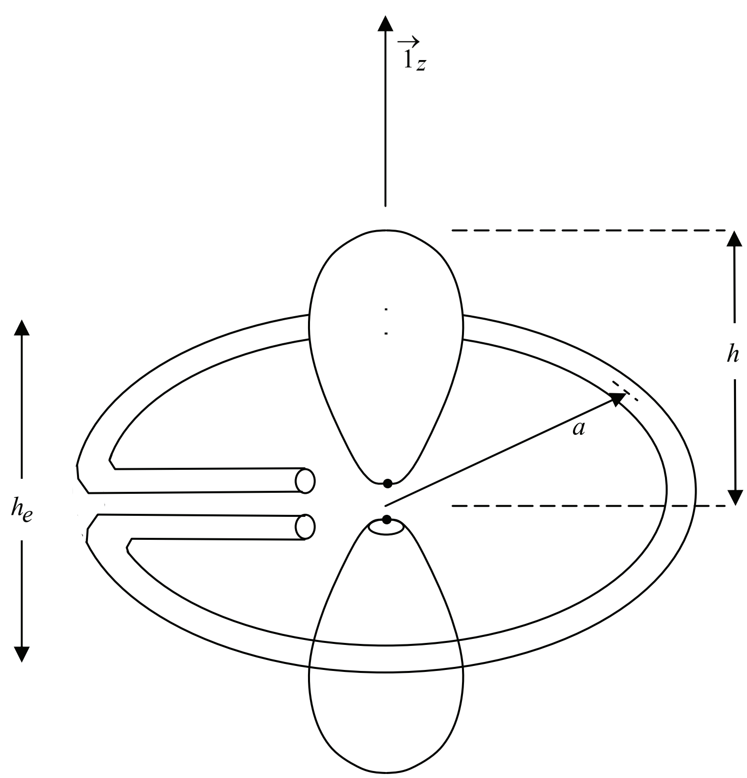


Fig. 1.1 Electric and Magnetic Dipoles

2. Balancing Electric and Magnetic Dipoles

As discussed in [2] we would like to make the electric- and magnetic- dipole moments approximately balanced as

$$\vec{\tilde{p}}(j\omega_0) = \pm j\frac{\vec{\tilde{m}}}{c}(j\omega_0) \;,\; \vec{\tilde{p}}(s) = \tilde{p}(s)\vec{1}_z \;,\; \vec{\tilde{m}}(s) = \tilde{m}(s)\vec{1}_z$$
$$\sim \;\equiv\; \text{two-sided Laplace transform over time, } t \tag{2.1}$$

while they are $\pi/2\;(90°)$ out of phase with each other to give an approximately circularly polarized for field. Each of these is related to sources as

$$\begin{aligned}
\vec{\tilde{p}}(s) &= \tilde{Q}(s)\vec{h}_e = C_e\vec{h}_e\tilde{V}_e(s) \\
\vec{\tilde{m}}(s) &= \tilde{I}_h(s)\vec{A}_h \\
\vec{h}_e &= h_e\vec{1}_z \;,\; \vec{A}_h = A_h\vec{1}_z \\
s &= \Omega + j\omega \equiv \text{Laplace-transform variable or complex frequency}
\end{aligned} \tag{2.2}$$

This requires that $\tilde{V}_e$ and $\tilde{I}_h$ be $\pi/2(90°)$ out of phase for

$$\begin{aligned}
s_0 &= j\omega_0 \;,\; \omega_0 = 2\pi f_0 \\
f_0 &= \text{approximate resonance frequency}
\end{aligned} \tag{2.3}$$

As we shall see later the resonance is moved into the left-half s-plane as

$$s_0 = \Omega_0 + j\omega_0 \;,\; \Omega_0 < 0 \tag{2.4}$$

due to radiation losses. With

$$\vec{p}(t) \;,\; \vec{m}(t) \propto e^{s_o t}u(t) = e^{\Omega_0 t}e^{j\omega_0 t}u(t) \tag{2.5}$$

we have the commonly used quality factor [4]

$$Q = \pi n$$

$$n = \text{number of cycles to decay to } e^{-1} \tag{2.6}$$

$$= \frac{f_0}{|\Omega_0|} = \frac{\omega_0}{2\pi|\Omega_0|}$$

For the first part of our analysis we can neglect the shift into the left half plane as a high-Q approximation. Then we can correct for this shift based on energy considerations.

So let us choose some f_0 and see what is required to balance the dipole moments as in (2.1). We are assuming that the antenna is electrically small so that

$$K_0\ell = \frac{\omega_0}{c}\ell << 1$$

$$\ell = \text{some characteristic length} \tag{2.7}$$

This allows our approximation that the fields are dominantly described by the dipole terms.

The effective height h_e of the electric dipole is roughly h (as in Fig. 1.1) where $2h$ is the end-to-end dimension. The resonant length is then roughly $2h$ (half wavelength). So we need

$$\lambda_0 >> 4h \simeq 2 \text{ [length]} \tag{2.8}$$

In this case we can still approximately think of an electric dipole, but the input impedance is resistive, not capacitive.

Similar considerations apply to the magnetic dipole (loop). For a single-turn loop of radius a, electrically small means

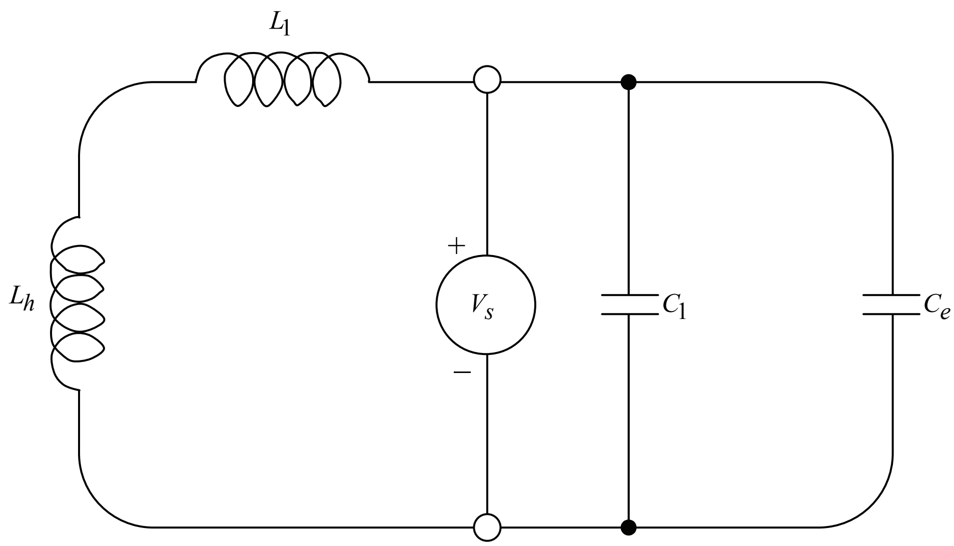
$$\lambda_0 >> 2\pi a \simeq \text{circumference} \tag{2.9}$$

not counting the leads to the source. In this case there is a strong electric-dipole term which we need to avoid. So the restriction (2.9) is important. As discussed in [2] we can use a segmented loop. In particular a half-turn (N=1/2) loop suppresses the aforementioned resonance and the associated electric-dipole moment.

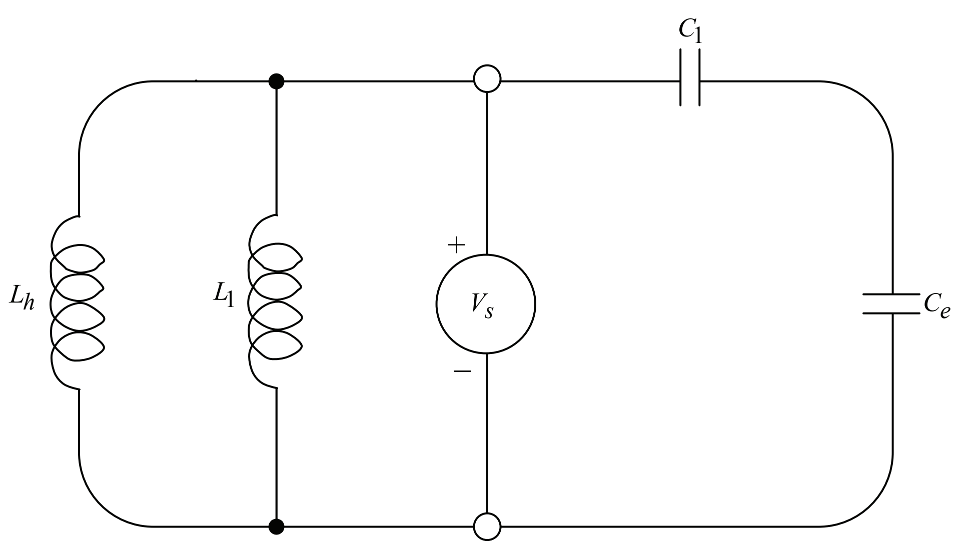
Recognizing our size limitations, then let us balance the dipole moments for some selected f_0. We need

$$
\begin{aligned}
\tilde{p}(j\omega_0) &= \pm j\frac{\tilde{m}(j\omega_0)}{c} \\
|\tilde{p}(j\omega_0)| &= C_e h_e \; |\tilde{V}_e(j\omega_0)| \\
|\tilde{m}(j\omega_0)| &= A_h \; |\tilde{I}_h(j\omega_0)|
\end{aligned}
\tag{2.10}
$$

For this we need to consider the equivalent circuits as in Fig. 2.1.



A. Addition of series L_1 and parallel C_1



B. Addition of parallel L_1 and series C_1

Fig 2.1 Equivalent Circuits for Combined Dipoles

In Fig. 2.1A we have an equivalent circuit in which the loop inductance is in series with L_1, and the electric-dipole capacitance is in parallel with C_1. The resonance frequency is now

$$\begin{aligned} \omega_0 &= [\mathrm{L\,C}]^{-1/2} = k_0 c \\ \mathrm{L} &= L_h + L_1 \;,\quad C = C_e + C_1 \end{aligned} \tag{2.11}$$

The previous formulas in [2] can now be carried forward with the above substitution. Note now that the current I_h still flows through L_h as well as L_1, and the voltage across C_e is also across C_1. The dipole moments are, however, only associated with L_h and C_e. This gives some flexibility in the design to match the dipole moments.

In this case let us adjust L_h and C_e to match the dipole moments for some chosen ω_0. We need for this

$$\begin{aligned} &\left|\tilde{I}_h(j\omega_0)\right|\frac{A_h}{c} = \left|\tilde{V}_e(j\omega_0)\right|C_e h_e && \text{(dipole moments)} \\ &\frac{\left|\tilde{V}_e(j\omega_0)\right|}{\left\|\tilde{I}_h(j\omega_0)\right\|} = \omega_0\left[L_h + L_1\right] && \text{(inductor impedance)} \\ &\frac{A_h}{c} = \omega_0\left[L_h + L_1\right]C_e h_e = \left[\frac{L_h + L_1}{C_e + C_1}\right]^{1/2} C_e h_e && \\ &\frac{A_h}{h_e}k_0 = \frac{C_e}{C_e + C_1} \le 1 && \text{(dimentionless)} \\ &k_0 = \frac{\omega_0}{c} = \lambda\!\!\!^{-}{}_0^{-1} && \text{(radian wavelength)} \end{aligned} \tag{2.12}$$

For given h_e and A_h, then k_0 must be *small* enough ($\lambda\!\!\!^{-}{}_0$ large enough) to achieve the above inequality. This in turn gives constraints on C_1 and L_1. So one key to the design is to make the frequency low enough, not only to make the antenna electrically small, but also to achieve the dipole matching.

Another approach is indicated in Fig. 2.1B with L_2 in parallel with L_h and C_2 in series with C_e. In this case the voltage on the electric dipole is

$$\tilde{V}_e(j\omega_0) = \frac{C_2}{C_e + C_2}\tilde{V}_s(j\omega_0) \tag{2.13}$$

The current in the loop is

$$\tilde{I}_h(j\omega_0) = j\omega_0 \, L_h \, \tilde{V}_s(j\omega_0) \tag{2.14}$$

The resonant frequency is

$$\begin{aligned} \omega_0 &= [\mathrm{L\,C}]^{-1/2} \\ \mathrm{L} &= \left[L_e^{-1} + L_1^{-1}\right]^{-1} \\ C &= \left[C_e^{-1} + C_1^{-1}\right]^{-1} \end{aligned} \tag{2.15}$$

Now we need

$$\begin{aligned} \left|\tilde{I}_h(j\omega_0)\right| \frac{A_h}{c} &= \left|\tilde{V}_e(j\omega_0)\right| C_e h_e && \text{(dipole moments)} \\ \frac{\left|\tilde{V}_s(j\omega_0)\right|}{\left|\tilde{I}_h(j\omega_0)\right|} &= \omega_0 \, L_h && \text{(antenna inductive impedance)} \\ \frac{\left|\tilde{V}_e(j\omega_0)\right|}{\left|\tilde{I}_h(j\omega_0)\right|} &= \omega_0 \frac{C_2}{C_e + C_2} \, L_h && \text{(capacitive divider)} \\ \frac{A_h}{c} &= \omega_0 \frac{C_e C_2}{C_e + C_2} \, L_h \, h_e = \frac{1}{\omega_0} \frac{L_h + L_2}{L_2} h_e \\ \frac{A_h}{h_e} k_0 &= \frac{L_h + L_2}{L_2} \geq 1 && \text{(dimensionless)} \end{aligned} \tag{2.16}$$

Again for given h_e and A_h, then k_0, must be *large* enough ($\lambda\!\!\!^{-}_0$ small enough) to achieve the above inequality. However, the antenna must be electrically small to achieve dipolar operation, placing additional constraints.

So there are various tuning networks that one can use to match the dipole moments for a given resonance frequency. The foregoing are only examples.

3. Radiated Fields and Power

From [2] we have the far fields (r^{-1}) from the dipoles as

$$
\begin{aligned}
\vec{\tilde{E}}_f(\vec{r},s) &= e^{-\gamma r}\frac{s^2}{4\pi r}\left[-\mu_0 \overleftrightarrow{1}_r \cdot \vec{1}_z\ \tilde{p}(s) + \frac{\mu_0}{c}\vec{1}_r \times \vec{1}_z\ \tilde{m}(s)\right] \\
\vec{\tilde{H}}_f(\vec{r},s) &= e^{-\gamma r}\frac{s^2}{4\pi r}\left[-\frac{1}{c^2}\overleftrightarrow{1}_r \cdot \vec{1}_z\ \tilde{m}(s) - \frac{1}{c}\vec{1}_r \times \vec{1}_z\ \tilde{p}(s)\right] \\
\overleftrightarrow{1}_r &\equiv \overleftrightarrow{1} - \vec{1}_r\vec{1}_r = \vec{1}_\theta\vec{1}_\theta + \vec{1}_\phi\vec{1}_\phi \\
\left|\tilde{p}(s)\right| &= \frac{1}{c}\left|\tilde{m}(s)\right| \\
\gamma &= \frac{s}{c}
\end{aligned}
\tag{3.1}
$$

The radiated power density at the resonance frequency is

$$
\begin{aligned}
\tilde{P}(\vec{1}_r, j\omega_0)\vec{1}_r &= \frac{1}{2}\vec{\tilde{E}}_f(\vec{r}, j\omega_0) \times \vec{\tilde{H}}_f(\vec{r}, -j\omega_0) \\
&= \frac{\omega_0^4}{32\pi^2 r^2}\frac{\mu_0}{c}\sin^2(\theta)\vec{1}_r\left[\vec{\tilde{p}}(j\omega_0) \cdot \vec{\tilde{p}}(-j\omega_0) + \frac{\vec{\tilde{m}}}{c}(j\omega_0) \cdot \frac{\vec{\tilde{m}}}{c}(-j\omega_0)\right]
\end{aligned}
\tag{3.2}
$$

With the balanced-dipole condition we have

$$
\tilde{P}(\vec{1}_r, j\omega_0)\vec{1}_r \equiv \frac{\omega_0^4}{16\pi^2 r^2}\frac{\mu_0}{c}\sin^2(\theta)\left|\vec{\tilde{p}}(j\omega_0)\right|^2\vec{1}_r
\tag{3.3}
$$

With the high-Q resonance condition (small $|\operatorname{Re}(s_0)|$) we have the energy radiated in one cycle as

$$
\begin{aligned}
U_1 &= 4t_r \int_{\substack{unit\\ sphere}} r^2 \tilde{P}(\vec{1}_r, j\omega_0)\,dS \\
&= t_r\frac{\omega_0^4}{4\pi^2}\frac{\mu_0}{c}\left|\vec{\tilde{p}}(j\omega_0)\right|^2 \int_{\substack{unit\\ sphere}} \sin^3(\theta)\,d\theta\,d\phi \\
&= \frac{t_r}{2\pi}\frac{\mu_0}{c}\left|\vec{\tilde{p}}(j\omega_0)\right|^2 \int_0^\pi \sin^3(\theta)\,d\theta \\
&= \frac{2}{3\pi}\ t_r\omega_0^4\frac{\mu_0}{c}\left|\vec{\tilde{p}}(j\omega_0)\right|^2
\end{aligned}
$$

$t_r \equiv$ transit line of switched oscillator (next section) (3.4)

$= 1/4$ period

It is this energy loss to radiation which will lower the Q of the oscillation.

4. Available Energy From Oscillator

The stored energy in the switched oscillator is [4]

$$U_0 = \frac{1}{2} C_{os} V_0^2 = \frac{1}{2} \frac{t_r}{Z_c} V_0^2$$

$$t_r = \frac{T_{os}}{4} = \frac{1}{4 f_0} \equiv \text{transit time}$$

T_0 = period , $t_r \equiv$ transit time

$Z_c \equiv$ characteristic impedance of transmission line

$$= f_g Z_w = f_g \left[\frac{\mu_{os}}{\varepsilon_{os}} \right]^{1/2}$$

f_g = geometric factor (4.1)

Z_w = wave impedance in oscillator medium

As in Fig. 4.1, the oscillator is connected to the antenna which may include tuning elements (as in Fig. 2.1) to give some net L and C. We need to know the voltage across C_e (for the electric dipole) and the current through L_h (for the magnetic dipole).

A first observation concerns a nonideal aspect of the switched oscillator, namely the square-wave nature of the waveform produced on closing the switch. Suppose we remove the load (open circuit the oscillator). The waveform at the output is a square wave of amplitude $2V_0$ with period T_0. Not all the energy in this oscillation is at the frequency f_0, but includes components at $3f_0$, $5f_0$, ... The total energy in all those components is U_0. Note that there is a blocking capacitor with

$$C_b >> C, C_{os} \tag{4.2}$$

There is energy also stored here which we are not considering for present purposes.

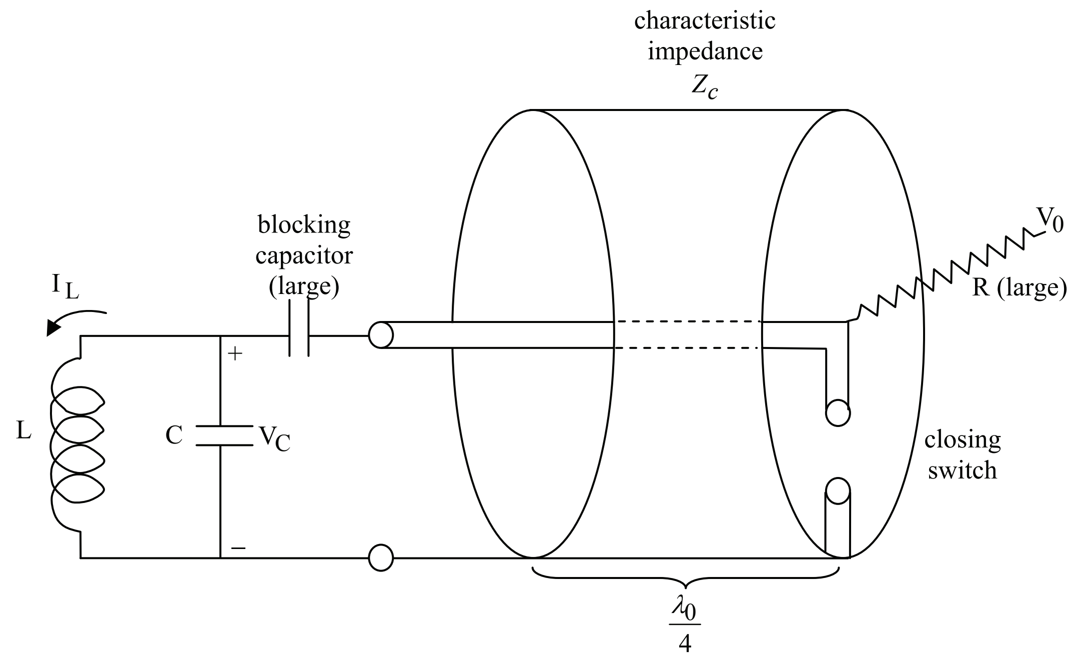


Fig. 4.1 Switched Oscillator Connected to Resonant Antenna/Load.

After the switch closes, there are various ways to evaluate the energy in the primary resonance, depending on which time one uses for the evaluation. If we make a Fourier analysis of the energy components, we can divide the energy appropriately. At $t = 0$ (just after the switch closure) we still have V_0 on the center conductor. This gives an amplitude for the f_0 resonance as

$$\begin{aligned} V_s &= \frac{\int_0^{\frac{\lambda_0}{4}} V_0 \cos\left(\frac{\pi}{2} z \frac{4}{\lambda_0}\right) dz}{\int_0^{\frac{\lambda_0}{4}} \cos^2\left(\frac{\pi}{2} z \frac{4}{\lambda_0}\right) dz} \\ &= \frac{8}{\lambda_0} V_0 \int_0^{\frac{\lambda_0}{4}} \cos\left(\frac{\pi}{2} z \frac{4}{\lambda_0}\right) dz \\ &= \frac{8}{\lambda_0} V_0 \frac{2}{\pi} \frac{\lambda_0}{4} \sin\left(\frac{\pi}{2} z \frac{4}{\lambda_0}\right)\Bigg|_0^{\frac{\lambda_0}{4}} \\ &= \frac{4}{\pi} V_0 \end{aligned} \tag{4.3}$$

The energy available at f_0 is then

$$
\begin{aligned}
U_{os} &= \frac{C_{os}}{2}\left[\frac{4}{\pi}V_0\right]^2 \int_0^{\frac{\lambda_0}{4}} \cos^2\left(\frac{\pi}{2}z\frac{4}{\lambda_0}\right)dz \\
&= \frac{1}{2}\frac{t_r}{Z_c}\left[\frac{4}{\pi}V_0\right]^2 \frac{4}{\lambda_0}\frac{\lambda_0}{8} \\
&= \frac{4}{\pi^2}\frac{t_r}{Z_c}V_0^2 = \frac{4}{\pi^2}C_{os}V_0^2 = \frac{8}{\pi^2}U_0
\end{aligned}
\tag{4.4}
$$

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Comparing to (4.1) we can see that about 80 percent of the stored energy is available for the first resonance.

5. Q of System

Allowing for the radiation losses (Section 3) and the available energy we can establish Q from the number of cycles for the oscillation amplitude to decay to e^{-1}, or energy to decay to e^{-2} [4, 6]. After one cycle we have the amplitude as

$$
\left[\frac{U_{os}-U_1}{U_{os}}\right]^{1/2} = 1 - \frac{U_1}{2U_{os}} + \mathrm{O}\left(\left[\frac{U_1}{U_{os}}\right]^2\right) \tag{5.1}
$$

Comparing to an exponential decay gives

$$
\begin{aligned}
&e^{-\frac{U_1}{2U_{os}}\frac{t}{T}} = e^{-\frac{U_1}{2U_{os}}n} \simeq 1 - \frac{U_1}{2\,U_{os}}n \\
&T \equiv \text{period of oscillation} \\
&n = \text{number of cycles} \\
&Q = \pi n
\end{aligned}
\tag{5.2}
$$

Applying this to the present problem gives (for the circuit in Fig. 2.1A)

$$
\begin{aligned}
n = \frac{Q}{\pi} &= 2\frac{U_{os}}{U_1} \\
&= \frac{8}{\pi^2}\frac{t_r}{Z_c}\,V_0^2\,\frac{3\pi}{2t_r}\,\omega_0^{-4}\,\frac{c}{\mu_0}\,|\,\vec{\tilde{p}}(j\omega_0)\,|^{-2} \\
&= \frac{12}{\pi}\,\frac{c}{Z_c\mu_0}\,\omega_0^{-4}\,\frac{V_0^2}{|\,\vec{\tilde{p}}(j\omega_0)\,|^{-2}} \\
&= \frac{12}{\pi}\,\frac{c}{Z_c\mu_0}\,\omega_0^{-4}\,C_e^{-2}h_e^{-2}\left[\frac{V_0}{V_e}\right]^2
\end{aligned}
$$

$$= \frac{12}{\pi} \frac{c}{Z_c \mu_0} \left[\frac{\lambda\!\!\!^{-}_0}{c} \right]^4 C_e^{-2} h_e^{-2} \left[\frac{V_0}{V_e} \right]^2$$
$$= \frac{12}{\pi} \frac{Z_0}{Z_c} \lambda\!\!\!^{-\,4}_0 \left[\frac{\varepsilon_0}{C_e} \right]^2 h_e^{-2} \left[\frac{V_0}{V_e} \right]^2 \tag{5.3}$$

Now we can see the relative contributions of various factors. For an oscillator with free-space constitutive parameters we have

$$\frac{Z_0}{Z_c} = f_g^{-1} \text{ (transmission-line geometric factor, dimensionless)} \tag{5.4}$$

Typical numbers might have Z_c a few ohms with f_g of order 10^{-2}. Even smaller numbers might be possible [5]. Large n (high Q, longer pulses) is achieved with small f_g, large $\lambda\!\!\!^{-}_0$ (low frequencies), and small $C_e h_e$. V_0 / V_s is about $\pi / 4$ for the circuit in Fig. 2.1A, but is increased for the circuit in Fig. 2.1B due to the capacitive divider.

6. Tradeoffs

Neglecting other (resistive) losses all the energy stored in the first oscillator resonance is radiated. It is just a question of trading off amplitude and pulse width (number of cycles). So we can regard

$$\left[E_f r \right]^2 n \equiv \text{conserved parameter} \tag{6.1}$$

So the time-domain peak field is proportional to $n^{-1/2}$ (or $Q^{-1/2}$). Large fields give shorter pulse widths and conversely.

In frequency domain we can think of all the energy being radiated in a band of frequencies proportional to Q^{-1} (or n^{-1}). The amplitude (proportional to square root of energy density) then is proportional to $n^{1/2}$ (or $Q^{1/2}$). So we have a tradeoff between spectral width and peak energy density at the center frequency.

7. Concluding Remarks

Building on previous results [2], this paper has extended the design considerations for combined electric and magnetic dipoles producing approximately circular, damped sinusoidal radiation. This includes networks to balance the dipole moments, and tradeoff between radiated field amplitudes and pulse width for a given resonance frequency. An example calculation is given in the appendix.

Appendix A. Example Calculation

To give the reader some feel for the interplay of the various design parameters, let us consider a hypothetical case. Use the equivalent circuit in Fig. 2.1A for tuning the device.

A.1. Electric-dipole parameters

Let

$$\begin{aligned} h_e &= 0.5\, m \\ C_e &= 30\ pf \end{aligned} \tag{A.1}$$

This corresponds roughly to an antenna length of 1 m. Using an ACD kind of design [1] the feed point looks roughly like a 100 Ω bicone.

A.2. Magnetic-dipole parameters

Choosing, as previously discussed for suppressing an electric-dipole contribution,

$$N = \frac{1}{2} \text{ turn loop} \tag{A.2}$$

We then choose

$a = 0.5\ m$ radius (1 m diameter)

$$\begin{aligned} A_h &= \frac{\pi}{2} a^2 \simeq 0.39\, m^2 \\ \frac{b}{a} &\simeq 0.25 = \frac{\text{minor radius}}{\text{major radius}} \\ L_h &\simeq N^2 \mu_0\ a \left[\ell n\left(\frac{8a}{b} \right) - 2 \right] \\ &\simeq 0.23\ \mu H \end{aligned} \tag{A.3}$$

A.3 Matching dipole moments

From (2.11) we have

$$\frac{A_h}{c} = \left[\frac{L_h}{C_e} \right]^{1/2} \left[\frac{1 + \frac{L_1}{L_h}}{1 + \frac{C_1}{C_e}} \right]^{1/2} C_e h_e$$

$$\begin{aligned} \frac{1 + \frac{L_1}{L_h}}{1 + \frac{C_1}{C_e}} &= \left[\frac{A_h}{c\, C_e h_e} \right]^2 \frac{C_e}{L_h} = \left[\frac{A_h}{h_e} \right]^2 \frac{1}{c^2 C_e L_h} \\ &\simeq 0.98 \end{aligned} \tag{A.4}$$

By serendipity we have found a case in which extra inductance and/or capacitance is not required. However, one could make the above more precise if desired. Furthermore parasitic inductance and capacitance (e.g., leads) has not been included.

A.4 Resonant frequency

From (2.10) we have

$$\begin{aligned} \omega_0 &= \left[L_h C_e\right]^{-1/2} \simeq 0.38 \times 10^9 \ s^{-1} \\ f_0 &= \frac{\omega_0}{2\pi} \simeq 61 \ \text{MHz} \\ \lambda_0 &= \frac{c}{f_0} \simeq 4.9 \, m \\ \lambda\!\!\!^{-}_0 &= \frac{\lambda_0}{2\pi} \simeq 0.78 \, m \end{aligned} \tag{A.5}$$

The resonant half wavelength of 2.5 m is considerably larger than the approximately 1 m length at the electric dipole. Similar considerations concern this 2.5 m as compared to the length of 1.6 m for the half loop. So the structure can be approximately considered as electrically small. If a lower frequency is desired for the given antenna dimensions, this can be accomplished by adding L_1 and C_1 .

A.5 Switched oscillator

For the switched oscillator, we have (for matching the resonant frequency)

$$t_r = \frac{1}{4 f_0} \simeq 4.1 \text{ ns} \tag{A.6}$$

For a free-space dielectric this is rather long, but can be shortened by use of dielectric and/or a folded geometry. In addition we need the transmission-line characteristic impedance which we take as

$$Z_c = 4 \, \Omega \tag{A.7}$$

For the present example. For energy let us take

$$\begin{aligned} V_0 &= 100 \, kV \\ U_0 &= \frac{1}{2} \frac{t_r}{Z_c} V_0^2 \simeq 5.1 \text{ J} \\ V_s &= \frac{4}{\pi} V_0 \simeq 127 \text{ kV} \\ U_{os} &= \frac{4}{\pi^2} \frac{t_r}{Z_c} V_0^2 \simeq 4.2 \text{ J} \end{aligned} \tag{A.8}$$

A.6 System Q

From (5.3) we have

$$n = \frac{Q}{\pi} = \frac{12}{\pi} \frac{Z_0}{Z_c} \lambda_0^4 \left[\frac{\varepsilon_0}{C_e}\right]^2 h_e^{-2} \left[\frac{V_0}{V_s}\right]^2 \simeq 29 \qquad \text{(A.9)}$$

The high Q is due to the fact that we are operating below the antenna resonance (electrically small antennas) with a low-impedance switched oscillator. This example then approaches a hypoband (narrow band) radiator with all the available energy concentrated near f_0. The Q can be lowered by increasing Z_c (and perhaps thereby allowing a larger V_0 and associated far field.

A.8 Radiated fields

The electric-dipole moment at early times is approximately

$$p \simeq C_e h_e V_s = 1.9\ \mu Cm$$
$$r\ E_f \simeq \frac{\omega_0^2 \mu_0 p}{4\pi} \simeq 27\ \text{kV} \quad \text{(from 3.1)} \qquad \text{(A.10)}$$

A.9 Remarks

The present is but one example. Many more can be imagined. The present example can also be scaled according to the usual rules of electrodynamic scaling (e.g., multiplying linear dimensions by some constant).

Since there are various approximations in the present analysis, one can resort to more detailed calculations and/or construct scale models to obtain more accurate results.

References

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