Sensor and Simulation Notes

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Increasing Lens-Medium Permittivity Over Target-Medium Permittivity to Increase Electric Field and Decrease Spot Size at Target

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Abstract

Analytical calculations to increase the fields and decrease the focal spot size of a prolate-spheroidal IRA by increasing the lens-medium permittivity over the target-medium permittivity are discussed.

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1. Introduction

In designing a prolate-spheroidal Impulse–radiating antenna (IRA) for concentrating the fields on a small biological target, various analytical results have been achieved [1,2].

In particular, it has been shown that the spot size is limited by the pulse width of the impulsive term, which, in turn, is limited by the rise time (t_{mr}) of the driving waveform of the source (at the first of two focii). From [1] we have an approximation for the radius of the spot size as

$$
\begin{aligned}
&\Delta\Psi = \frac{a}{b} c t_\delta , \\
&t_\delta = \text{pulse width} \cong t_{mr} , \\
&a = \text{major radius (say 0.5 meters)}, \\
&b = \text{minor radius (say 0.4 meters)}, \\
&z_0 = \left[a^2 - b^2\right] = \text{distance from center to each focus (say 0.3 meters)}.
\end{aligned}
\tag{1.1}
$$

These are only rough numbers, but they will do for comparison with the lens results.

1.1. Example Case without Lens

One can approach this form a different direction. Suppose we consider the power in the impulsive term. This comes first from the power launched from the first focus, for which we have

$$
\begin{aligned}
&P_{source} = \frac{4V_0^2}{Z_c} , \\
&Z_c \equiv 200\,\Omega \text{ (for four feed arms, full symmetrical feed)} \\
&2V_0 \equiv \text{full voltage driving antenna} \\
&V_0 \equiv 100\,\text{kV(assumption)} \\
&t_{mr} \equiv 100\,\text{ps (assumption)}
\end{aligned}
\tag{1.2}
$$

The power from the source is then

$$P_{source} \cong 0.2\,GW \tag{1.3}$$

The reflector reflects half of this toward the second focus [3] giving

$$P_{refl} = \frac{P_{source}}{2} = 2\frac{V_0}{Z_c} \cong 0.1\,GW \tag{1.4}$$

This is seen from the property of a self-reciprocal antenna, noting that half the current (but the full voltage) is associated with the fields outside the aperture. This is a price that one pays for a TEM-mode field distribution (used for a nondispersive EM pulse).

We can combine the above with the estimated spot size to obtain an average power density (over the spot size) at the target as

$$
\begin{aligned}
&\Delta\Psi = 3.75\,cm \\
&P_{target} \cong \frac{P_{refl}}{\pi[\Delta\Psi]^2} \cong 22.6\,GW/m^2 \\
&\quad = \frac{E_{target}^2}{2Z_0} + \frac{Z_0 H_{target}^2}{2} = \frac{E_{target}^2}{Z_0} \\
&E_{target} \cong \left[Z_0 P_{target}\right]^{1/2} = 2.3\,MV/m
\end{aligned}
\tag{1.5}
$$

This is some kind of average, nothing that the field should be somewhat larger in the center of the spot. However, this is also an overestimate since it does not account for the fields for $\Psi > \Delta\Psi$, i.e., outside the spot size.

A more accurate estimate is obtained from [1]. In this case the results are based on a two-arm IRA ($400\,\Omega$). The results are increased for four arms at 60^0 by 1.606 [2]. So we have from [1]

$$
\begin{aligned}
&E_\delta = 1.606 \frac{V_0}{\pi f_g c} \frac{b}{a} \\
&f_g = 400/377 = 1.06 \\
&E_\delta = 1.28\;10^{-4}\,Vs/m \,(\text{area of impulse}) \\
&E_{peak} = E_\delta / t_{mr} = 1.28\,MV/m
\end{aligned}
\tag{1.6}
$$

which is lower than in (1.5), as it should be. This approximately agrees with the experimental results, noting the various losses in the experiment (cable, losses, etc.). Note that this does not include the contribution of the prepulse and postpulse which are discussed in [2].

2. Addition of a Lens

Now as indicated in Fig. 2.1, let us add a lens to increase the fields on the target by decreasing the spot size and better matching the wave into the target (where $\varepsilon > \varepsilon_0$). As discussed in [4] we ideally have a relative dielectric constant in the lens as

$$
\varepsilon_r(\mathrm{r}) = \left[\frac{r_{max}}{r}\right]^2 \tag{2.1}
$$

This has the electric field increase like [5]

$$g_E = \varepsilon_r^{1/4}(r_{min}) \tag{2.2}$$

Noting that the wavelength in the lens is proportional to $\varepsilon_r^{-1/2}$, the spot size scales the same way. Note that the power is concentrated in a smaller cross section area (proportional to ε_r^{-1}), neglecting lens losses (radiation to sides and discretization of lens into spherical layers of constant ε_r). After such losses, numerical calculations indicate a reduced field-transmission factor of about [6,7]

$$T_{loss} = 0.9 \text{ or so} \tag{2.3}$$

for a relatively large lens (large r_{max}).

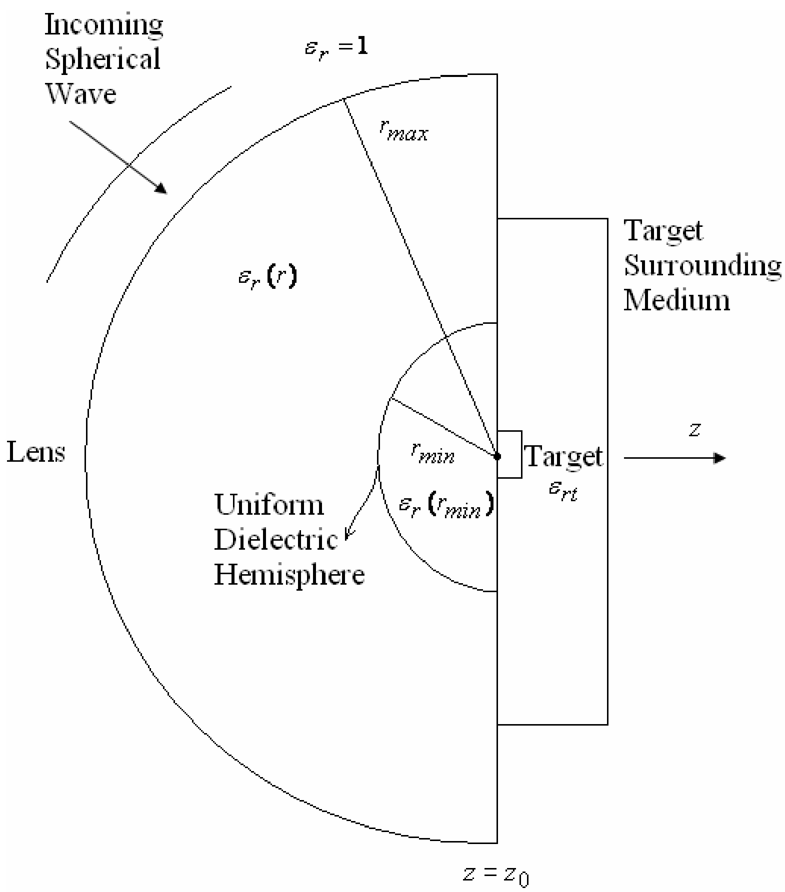

Figure 2.1 Lens Concentrating Fields to Second Focus of Prolate-Spheroidal IRA

As an example we might take

$$
\begin{aligned}
&\varepsilon_r(\mathrm{r_{min}}) = \varepsilon_r(\mathrm{r_{max}}) = 81\,(\text{water}),\\
&\varepsilon_r^{1/4}(\mathrm{r_{max}}) = 3,
\end{aligned}
\tag{2.4}
$$

which would triple the field, except for the losses. Increasing the field by about 2.7 gives (from (1.6))

$$
E_{peak} \cong 3.5\,MV/m \tag{2.5}
$$

a significant increase. At the same time the spot size in (1.5) is reduced to

$$
\Delta\Psi \cong 0.375/\varepsilon_r^{1/2} \cong 4.2\,mm \tag{2.6}
$$

The peak power density in (1.6) is

$$
P_{peak\,0} = \frac{E_{peak}^2}{Z_0} \cong 4.3\ GW/m^2 \tag{2.7}
$$

This is increased in our example to

$$
\begin{aligned}
P_{peak\,1} &= T_{loss}^2\left[\Delta\Psi\ ratio\right]^{-2} P_{peak\,0}\\
&\cong T_{loss}^2\,\varepsilon_r P_{peak\,0} \cong 0.3\ TW/m^2
\end{aligned}
\tag{2.8}
$$

Thus the lens can give us a significantly increased performance.

3. Lens Matched to Target Medium

Our first-case option is to make the medium surrounding the target matched in permittivity to the last portion of the lens, i.e., at

$$
\varepsilon_{rt} = \varepsilon_{r\,max} = \varepsilon_r(\mathrm{r_{min}}) \tag{3.1}
$$

In this case, the calculations in the previous section apply directly the fields incident on the target.

4. Step up of Field Going Into Target Medium

Suppose now that the target medium characterized by

$$\varepsilon_{rt} < \varepsilon_{r\,max} = \varepsilon_r(\mathrm{r_{min}}) \tag{4.1}$$

What should we expect to happen?

If we think of a plane wave propagating from one medium (the lens) into a second medium (the target medium) there will be a step up of the electric field as

$$\begin{aligned} T_{et} &= \frac{2Z_{wt}}{Z_{wlens} + Z_{wt}} = \frac{2\ \varepsilon_t^{-1/2}}{\varepsilon_{r\,max}^{-1/2} + \ \varepsilon_t^{-1/2}} \\ &= \frac{2}{1 + \left(\varepsilon_t / \varepsilon_{r\,max}\right)^{1/2}} \end{aligned} \tag{4.2}$$

$$Z_w = \left[\mu_0 / \varepsilon\right]^{1/2} \equiv \text{wave impedance in medium (second subscript)}$$

From (4.1) this implies

$$1 < T_{et} < 2 \tag{4.3}$$

For Example, suppose we have

$$\begin{aligned} &\varepsilon_{r\,max} = 81,\ \varepsilon_t = 25 \\ &T_{et} = 1.3, \end{aligned} \tag{4.4}$$

And (2.5) is changed to

$$E_{peak} \cong 4.5\,MV/m \tag{4.5}$$

Let us compare to the case of $\varepsilon_{r\,max_1} = 25$ with $\varepsilon_{r\,max_2} = 81$ and calculate the gain

$$gain = \frac{\varepsilon_{r\,max\,2}\, T_{et} T_{loss}}{\varepsilon_{r\,max\,1}\, T_{loss}} = 1.7 \tag{4.6}$$

We have a gain of 1.7 by using $\varepsilon_{r\,max_2} = 81$ instead of $\varepsilon_{r\,max_1} = 25$.

Of course, this is not an accurate estimate, but it does indicate an increase in the electric field. The reflection of the wave at the interface to the target medium does, however, reduce the power density at the target from (2.8), some of the power being reflected.

The spot size at the beginning of the target medium should be about the same as in (2.6). However, as the wave enters the target medium ($z > z_0$) the wave should diverge (radially expand) more rapidly away from the focal point. This is associated with the fact that the spot size in the target medium (were $\varepsilon_{r\,max}$ matched to this) would be somewhat larger.

5. Concluding Remarks

We can now see some benefit in using large relative-dielectric constants in our lens design. There may be some benefit in making $\varepsilon_{r\,max} > \varepsilon_{rt}$. This decreases the spot size and increases the electric field at the target.

On the other hand, as we go to larger ε_{rt} we may be concerned with dispersion in such media which could lower the electric field at the target. This becomes an experimental question and our experiments can give us more information about the dispersion.

References

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