NOTE 51

MINIMIZATION OF INDUCED CURRENTS BY IMPEDANCE LOADING

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R.W. Latham
K.S.H. Lee
Northrup Corporate Laboratories

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R. W. Latham and K. S. H. Lee
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Pasadena, California 91101

Abstract

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ABSTRACT

The problem of minimizing the induced currents on an axially symmetric body by impedance loading is formulated by means of the Lorentz reciprocity theorem. The currents in the gap where the tangential electric field is non-vanishing are carefully defined and critically examined from the viewpoint of field theory.

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I. Introduction

In any electromagnetic field measurements it is necessary to minimize the interaction of the sensor platform with the field to be measured. The platform may take the form of a rocket which will simply be referred to as "antenna" throughout this note. Minimizing such interaction is tantamount to damping the induced currents on the antenna. One way to achieve this is to load the antenna with various appropriate impedances. In practice, the load impedance required for a given purpose may be realized by cutting a slot around the antenna with various appropriate materials filling the slot (Figure 1). This has the effect of increasing the electrical length of an otherwise unloaded antenna and thereby of offsetting the resonance condition for an incident wave of given frequency.

Although the idea of minimizing the induced currents by impedance loading is intuitively simple from an engineer's point of view, it is by no means straightforward if one tries to formulate this idea into a satisfactory theory based on Maxwell's equations. The present note is an attempt to provide such a theory deduced solely from field-theoretic considerations. Only from such considerations can various concepts used in circuit theory be clearly understood. Thus, the problem of minimizing the induced currents by impedance loading will be treated as a boundary-value problem in which impedance is no more than a derived concept, i.e., a quantity defined in terms of some basic quantities, namely the fields. Also, the problem will be formulated in accord with two requirements: (1) no major modifications will be needed of the presently available computer codes for calculating the induced currents on an unloaded cylinder[1,2], and (2) the parameter Z_L , the so-called load impedance whose value may be at one's disposal, should not appear in the intricate part of the computer codes to be developed. The requirement (1) will restrict our consideration to the case of axially symmetric antennas where the total axial current is not coupled to the other component. The requirement (2) is quite desirable, since one wishes to study the variation of the induced current with Z_L without having to solve some complicated integral equations for each value of Z_L.

The best approach to the problem is found in the Lorentz reciprocity theorem which enables one to obtain the solution of the reception problem directly from those of the transmission and parasite problems. The parasite problem is merely a special case of the reception problem and is immediately obtainable from the latter when Z_L is equal to zero. The transmission problem is defined to be one in which Maxwell's equations are to be solved outside the antenna and subject to the radiation condition at infinity for a specified distribution of the tangential electric field over a circumferential "gap" on the antenna (Figure 1). Whether or not such a distribution of electric field over the "gap" can be realized in practice is irrelevant here, since our transmission problem is defined to be a purely mathematical problem.

The motivation of our approach to the problem actually stems from the paper of Stevenson[3] in which the principle of superposition is used to relate the solution of the reception problem to those of the transmission and parasite problems. In Stevenson's approach, however, the distribution of the tangential electric field over the "gap" in the reception problem is tacitly assumed to be similar to that in the transmission problem. While this would be the case if the "gap" were narrow enough and the antenna thin enough, it is not at all obvious that his assumption would remain valid in a more general case. The approach adopted in the present note avoids this somewhat moot question and requires less stringent assumptions, although both approaches lead to the same results; in this sense the present approach is considered more general and more satisfactory.

In this note only the mathematical formulation of the problem is presented; numerical results for the problem will be reported in a future note.

Throughout the following discussion, the time-harmonic factor $e^{-i\omega t}$ will be suppressed and surrounding medium outside the antenna is assumed to be a vacuum.

II. The Reception Problem and The Reciprocity Theorem

Let $(\underline{E}^r, \underline{H}^r)$ and $(\underline{E}^t, \underline{H}^t)$ be the electromagnetic fields oscillating at the same frequency ω and having no singularities within the volume V bounded by the surfaces S_a and S_∞ (Figure 1). The superscript r (or t) on a quantity denotes that that quantity is associated with the reception (or the transmission) problem. Then, the Lorentz reciprocity theorem gives

$$\int_{S_a+S_\infty} \left(\underline{E}^t \times \underline{H}^r - \underline{E}^r \times \underline{H}^t\right) \cdot \underline{n} dA = 0 \quad , \tag{1}$$

where $\underline{n}$ is the unit normal pointing into V . Since $\underline{n} \times \underline{E}^r$ and $\underline{n} \times \underline{E}^t$ vanish on S_a except in the gap, equation (1) becomes

$$\int_{gap} \left(\underline{E}^t \times \underline{H}^r - \underline{E}^r \times \underline{H}^t\right) \cdot \underline{n} dA = \int_{S_\infty} \left(\underline{E}^t \times \underline{H}^r - \underline{E}^r \times \underline{H}^t\right) \cdot \underline{e}_r dA \quad , \tag{2}$$

where $\underline{e}_r$ is the outward, radial, unit vector. On the spherical surface (S_∞) at infinity one may write

$$\begin{aligned} \underline{E}^t(\underline{r}) &= \underline{E}^{rad}(\theta,\phi) \frac{e^{ikr}}{kr} \\ \underline{H}^t(\underline{r}) &= \frac{1}{Z_o} \underline{e}_r \times \underline{E}^{rad} \frac{e^{ikr}}{kr} \end{aligned} \tag{3}$$

and

$$\begin{aligned} \underline{E}^r(\underline{r}) &= \underline{E}^{inc} e^{ikr(\underline{e}_r \cdot \underline{n}_o)} + \underline{F}(\theta,\phi) \frac{e^{ikr}}{kr} \\ \underline{H}^r(\underline{r}) &= \frac{1}{Z_o} \underline{n}_o \times \underline{E}^{inc} e^{ikr(\underline{e}_r \cdot \underline{n}_o)} + \frac{1}{Z_o} \underline{e}_r \times \underline{F} \frac{e^{ikr}}{kr} \quad . \end{aligned} \tag{4}$$

Here Z_o is the free-space wave impedance and $\underline{n}_o$ is the unit vector in the direction of incident Poynting's vector, the incident wave being assumed to be a homogeneous plane wave oscillating at the frequency ω .

Substituting (3) and (4) into the right-hand side of (2) one can show by the method of stationary phase that [4]

$$\int_{\text{gap}} \left(\underline{E}^t \times \underline{H}^r - \underline{E}^r \times \underline{H}^t\right) \cdot \underline{n} dA = - i \frac{\lambda^2}{\pi Z_o} \underline{E}^{inc}(\theta_o,\phi_o) \cdot \underline{E}^{rad}(\theta_o,\phi_o) \quad , \quad (5)$$

where λ is the wave length of the incident wave.

Let us now examine the left-hand side of (5). Since the distribution of the tangential electric field over the gap in the transmission problem can be specified at our disposal, we take the electric field over the gap to have only one component independent of ϕ , i.e.,

$$\underline{E}^t = \underline{e}_s E_s^t(s) \quad (6)$$

and, because of rotational symmetry in the antenna, the tangential magnetic field over the gap likewise assumes the form

$$\underline{H}^t = \underline{e}_\phi H_\phi^t(s) \quad . \quad (7)$$

The directions of $(\underline{E}^t,\underline{H}^t)$ must be defined in such a way that the corresponding Poynting's vector has a positive outward component, whereas the Poynting vector corresponding to $(\underline{E}^r,\underline{H}^r)$ must have a positive inward component at the gap. This is clearly illustrated in figure 2 where is also indicated the convention usually adopted to define the directions of voltage rise and current flow.

Insertion of (6) and (7) in the left-hand side of (5) gives (Figures 1 and 2).

$$\int_{-\Delta}^{\Delta} I_r(s)E_s^t(s)ds + \int_{-\Delta}^{\Delta} I_t(s)\langle E_s^r(s)\rangle ds = -i\frac{\lambda^2}{\pi Z_o}\underline{E}^{inc}(\theta_o,\phi_o)\cdot\underline{E}^{rad}(\theta_o,\phi_o) \quad , \quad (8)$$

where

$$\langle E_s^r(s)\rangle = \frac{1}{2\pi}\int_0^{2\pi} E_s^r(s,\phi)d\phi$$

$$I_r(s) = \rho(s)\int_0^{2\pi} H_\phi^r(s,\phi)d\phi$$

$$I_t(s) = \rho(s)\int_0^{2\pi} H_\phi^t(s)d\phi = 2\pi\rho H_\phi^t \quad .$$

We now define the voltage, the current, and the impedance by

$$V_t = Z_T\overline{I}_t = -\int_{-\Delta}^{\Delta} E_s^t(s)ds \quad ,$$

$$V_r = Z_L\overline{I}_r = -\int_{-\Delta}^{\Delta} \langle E_s^r(s)\rangle ds \quad . \qquad (9)$$

Here $\overline{I}_t$ and $\overline{I}_r$ are the average currents in the gap whose precise definitions will soon be given. Writing

$$E_s^t(s) = -\frac{V_t}{2\Delta}f_t(s) \quad , \quad \langle E_s^r(s)\rangle = -\frac{V_r}{2\Delta}f_r(s) \qquad (10)$$

we obtain from (9) the following integral relations for the distribution functions $f_t(s)$ and $f_r(s)$:

$$\frac{1}{2\Delta}\int_{-\Delta}^{\Delta} f_t(s)ds = 1 \quad , \quad \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} f_r(s)ds = 1 \quad . \tag{11}$$

Substitution of (10) into (8) gives

$$V_t\bar{I}_r + V_r\bar{I}_t = i\,\frac{\lambda^2}{\pi Z_o}\,\underline{E}^{inc}\cdot\underline{E}^{rad} \quad , \tag{12}$$

where as well as in (9)

$$\bar{I}_t = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} I_t(s)f_r(s)ds \quad ,$$

$$\bar{I}_r = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} I_r(s)f_t(s)ds \quad , \tag{13}$$

that is to say, $\bar{I}_t$ and $\bar{I}_r$ are the average currents weighted respectively by the distributions of $\langle E_s^r\rangle$ and E_s^t .

The right-hand side of (12) can be expressed directly in terms of the induced current $\bar{I}_p$ in the parasite problem. Setting $V_r = 0$ in (12) one immediately obtains, with $\bar{I}_r$ replaced by $\bar{I}_p$,

$$\bar{I}_p = \frac{i\lambda^2}{\pi Z_o V_t}\,\underline{E}^{inc}\cdot\underline{E}^{rad} \quad . \tag{14}$$

With the aid of (9) and (14) we find from (12) that the induced current in the reception problem is given by

$$\bar{I}_r = \frac{Z_T}{Z_T + Z_L}\,\bar{I}_p \tag{15}$$

at the gap. Of course, there is no gap in the parasite problem, but we still speak of "the current at the gap" simply for reason of clarity.

Before proceeding any further let us discuss the definitions (13) for the average currents $\bar{I}_t$ and $\bar{I}_r$. When the weighting functions $f_t(s)$ and $f_r(s)$ are not the same, two undesirable features will arise, viz. (1) the lack of uniformity in the definition of averaging and (2) the current in the reception (transmission) problem weighted by the distribution of the tangential electric field over the gap in the transmission (reception) problem rather than by that in the reception (transmission) problem. Unfortunately, these undesirable features are inevitable if one insists on maximum generality in the formulation of the reception problem. However, due to the nature of the conditions (9) which specify only the integrals of the tangential electric fields over the gap, f_t and f_r may take many forms and yet satisfy the integral relations (11). In fact, one can take f_t equal to f_r without violating (11) and at the same time the aforementioned undesirable features can be avoided entirely. Hence we shall assume the distribution functions f_r and f_t to be the same in the following discussions. The simplest and the most useful is the uniform distribution. The other widely used distribution is the delta function which, however, leads to the difficulty of singularity.

We can now proceed to find, by the principle of superposition, the current $I_r(s)$ at some point on the antenna rather than at the gap. Thus, we write

$$I_r(s) = I_p(s) + \alpha I_t(s) \qquad , \tag{16}$$

α being a constant for all values of s ranging over the antenna's surface. First, we average (16) over the gap to obtain the value for α with the aid of (15). Then, substituting the value of α thus obtained into (16) we have

$$\frac{I_r(s)}{\bar{I}_r} = \frac{I_p(s)}{\bar{I}_p} - \frac{Z_L}{Z_T}\left[\frac{I_t(s)}{\bar{I}_t} - \frac{I_p(s)}{\bar{I}_p}\right] \qquad . \tag{17}$$

Equation (17) is as far as one can get from the Lorentz reciprocity theorem and the principle of superposition with f_t taken equal to f_r. Let us recall that f_r is the distribution of $\langle E_s^r \rangle$ while the other possible tangential component E_ϕ^r does not enter the formulation because of our choice (6) for E_s^t. Thus, taking f_t equal to f_r is not in general the same as assuming the distributions of the tangential electric fields over the gap to be similar in the transmission and the reception problems. In fact, no assumption is made by equating f_t to f_r, since f_t can be chosen freely. Hence equation (17) can be regarded as exact.

What remains to be solved now is the transmission problem the solution of which will enable us to determine not only the solution of the parasite problem by (14) but also the solution of the reception problem by (15) and (17).

III. The Transmission and The Parasite Problems

We now go on to solve the transmission problem. Our point of departure is the representation of $\underline{H}$ at an interior point $\underline{r}$ in a source-free region V bounded by a regular surface S in terms of the values of $\underline{E}$ and $\underline{H}$ on S, viz.[5]

$$\underline{H}(\underline{r}) = -\int_S \left[i\omega\varepsilon(\underline{n}' \times \underline{E})G - (\underline{n}' \times \underline{H}) \times \nabla' G - (\underline{n}' \cdot \underline{H})\nabla' G \right] dS' \quad , \qquad (18)$$

where

$$G(\underline{r},\underline{r}') = \frac{1}{4\pi} \frac{e^{ik\sqrt{(z-z')^2 + \rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi-\phi')}}}{\sqrt{(z-z')^2 + \rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi-\phi')}} \quad ; \qquad (19)$$

S is the surface enclosing the antenna when no sources are present at infinity; $\underline{n}'$ is the outward unit normal to S (See figure 1).

Let us first multiply (18) scalarly by the unit vector $\underline{e}_\phi$ and by the axial distance ρ. Then, integrating the resulting equation with respect to ϕ from 0 to 2π we obtain, after bringing $\underline{r}$ onto the surface S,

$$\frac{1}{2} I_t(s) + \int_{s_1}^{s_2} K(s,s') I_t(s') ds' = \int_{s_1}^{s_2} Y(s,s') \langle E_s^t(s') \rangle \, ds' \quad , \qquad (20)$$

where, as before, I_t and $\langle E_s^t \rangle$ are defined by

$$I_t(s) = \rho(s) \int_0^{2\pi} H_\phi(s,\phi) d\phi$$

$$\langle E_s^t \rangle = \frac{1}{2\pi} \int_0^{2\pi} E_s^t(s,\phi) d\phi \quad .$$

Moreover, Y and K are given by

$$Y(s,s') = 2\pi i\omega\varepsilon\rho\rho' \int_0^{2\pi} \cos\psi \, G(\psi) d\psi \tag{21}$$

$$K(s,s') = -\rho \int_0^{2\pi} \left\{ \cos\psi \frac{\partial G}{\partial n'} - \sin\chi \sin\psi \frac{1}{\rho'} \frac{\partial G}{\partial \psi} \right\} d\psi \quad , \tag{22}$$

where $\psi = \phi - \phi'$, $\cos\chi = \underline{n} \cdot \underline{e}_z$, $\rho = \rho(s)$, $\rho' = \rho'(s')$, etc.. In the case of a circular cylinder of radius a equations (21) and (22) reduce to, respectively,

$$Y(z - z') = \frac{2ika^2}{Z_o} \int_0^{\pi/2} \cos 2\phi \frac{e^{ik\sqrt{(z-z')^2 + 4a^2 \sin^2\phi}}}{\sqrt{(z-z')^2 + 4a^2 \sin^2\phi}} d\phi \tag{23}$$

$$K(z - z') \doteq \frac{a}{2\pi} \frac{\partial}{\partial a} \int_0^{\pi/2} \frac{e^{ik\sqrt{(z-z')^2 + 4a^2 \sin^2\phi}}}{\sqrt{(z-z')^2 + 4a^2 \sin^2\phi}} d\phi \quad . \tag{24}$$

Since $\langle E_s^t \rangle$ is zero on the antenna's surface except in the gap where it is given by the first equation of (10), equation (20) becomes, with f replacing f_t and f_r ,

$$\frac{1}{2} I_t(s) + \int_{s_1}^{s_2} K(s,s') I_t(s') ds' = -\frac{V_t}{2\Delta} \int_{-\Delta}^{\Delta} Y(s,s') f(s') ds' \tag{25}$$

For a given f equation (25) can readily be solved for $I_t(s)$ with the aid of a computer and then Z_T can be computed from (9).

In the parasite problem an additional term $\underline{H}^{inc}$, the magnetic intensity vector of the incident wave, will appear in the right-hand side of (18), viz.

$$\underline{H}(\underline{r}) = \underline{H}^{inc}(\underline{r}) + \int_{S_a} \left\{ (\underline{n}' \times \underline{H}) \times \nabla' G + (\underline{n}' \cdot \underline{H}) \nabla' G \right\} dS' \quad . \tag{26}$$

Following the same procedure as in the transmission problem we have

$$\frac{1}{2} I_p(s) + \int_{s_1}^{s_2} K(s,s')I_p(s')ds' = I^{inc}(s) \quad . \tag{27}$$

Equation (27) has been programmed and solved for the case of a circular cylinder[1,2] .

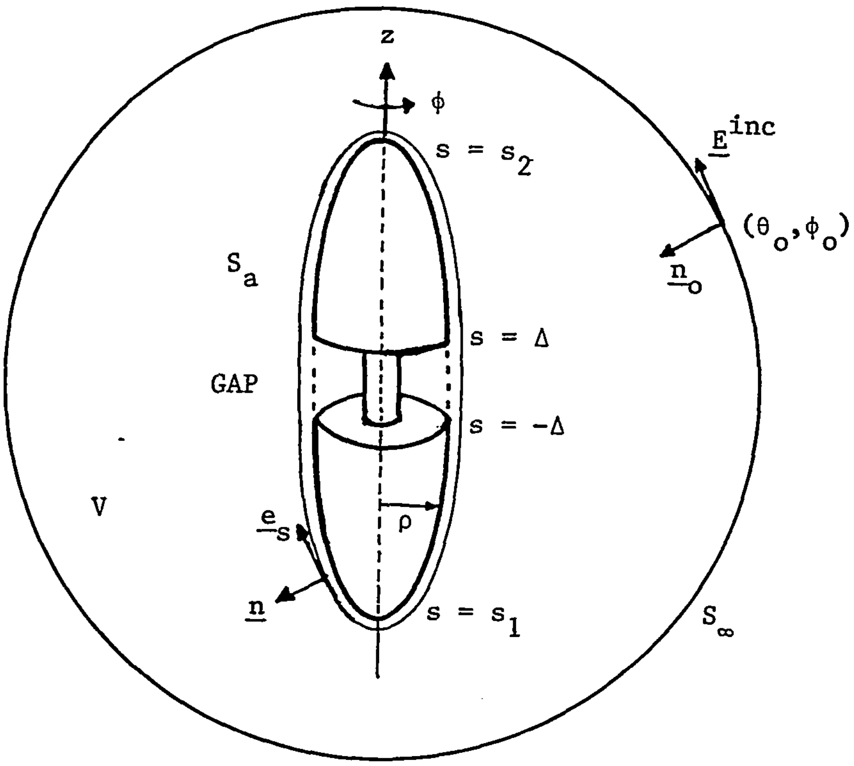

Figure 1: The geometry of the reception problem.

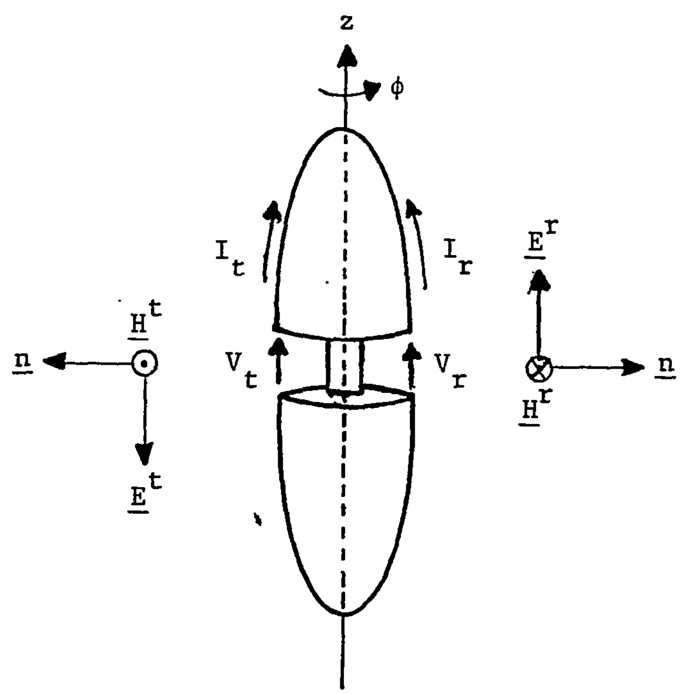

Figure 2: The convention of voltage rise and current flow.

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