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The Buried-Transmission-Line Simulator Driven by Multiple Capacitive Sources

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Abstract

The buried transmission line is combined with several capacitive energy sources. By appropriate choice of the capacitances, charging voltages, and switch closure times, the resulting waveforms for the electromagnetic fields can be shaped with considerable flexibility. In some cases it is also advantageous to switch a short across the buried transmission line at an appropriate time. Pulse shapes are calculated for up to three capacitors plus an electrical short chosen to give a magnetic-field waveform which is roughly flat after the initial rise and until the final decay.

Foreword

The graphs of the pulse shapes are grouped at the ends of the appropriate sections. We would like to thank Mr. Robert Myers, Mr. Ralph Powell, Mr. Ronald Thompson, and Mr. John N. Wood for the numerical calculations and the resulting graphs.

Contents

I. Introduction

Two previous notes have discussed the buried transmission line.[1,2] The first note considered the response characteristics in the frequency domain and the response to a step-function magnetic field at the ground surface. The second note considered the pulse shapes for the electromagnetic fields produced by the discharge of a capacitor through a damping resistor into the buried transmission line. The transmission line was considered both as effectively of infinite length and as in the open circuited configuration. The use of a resistance in series with the capacitive energy source can dampen oscillations and give some flexibility in shaping the waveforms. However, energy is dissipated in such a resistance giving a loss in efficiency. In some cases it may be desirable to minimize such energy losses because of the large amount of energy required for a single pulse.

One way to shape the pulse is to use multiple capacitive energy sources which are switched onto the load (the buried transmission line) at different times. As illustrated in figure 1 there are N-1 capacitors each of a capacitance, C_k, charged to an initial voltage, V_k. There are N switches each of which closes at a time, t_k. There are various ways of including the switches in the circuit and two of these are illustrated in figure 1. Since we constrain that

$$t_{k+1} > t_k \tag{1}$$

and

$$t_1 = 0 \tag{2}$$

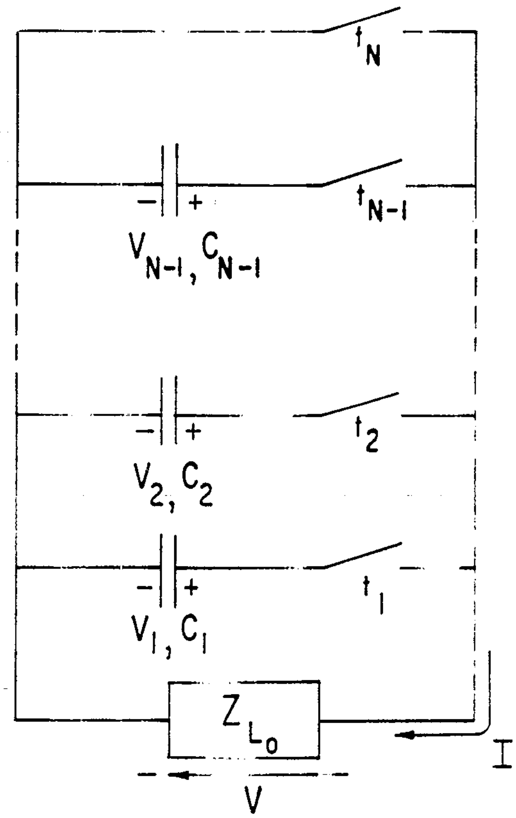
then these two equivalent circuits give the same results. However, one arrangement may be preferable to another for various reasons such as high voltage standoff and switch resistance.

The parameters of the first capacitor, V_1 and C_1, are chosen to give a certain rise time and peak amplitude to the current, I, into the load. We constrain that

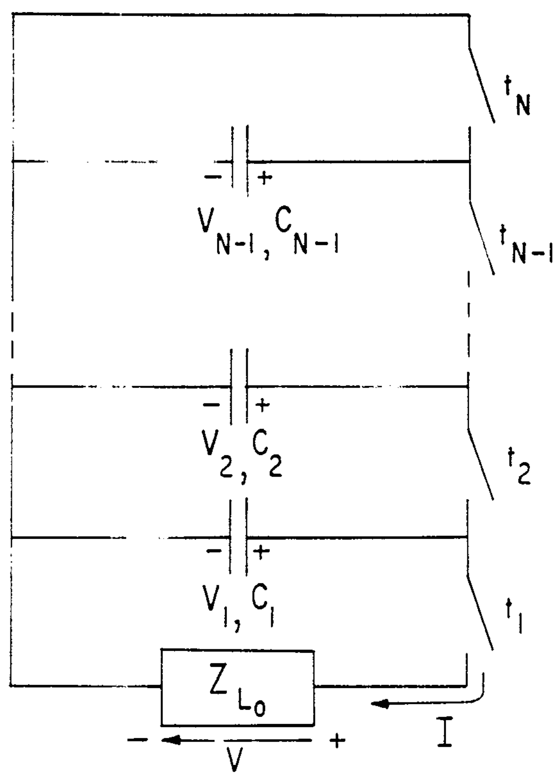
$$0 < V_{k+1} < V_k \tag{3}$$

The time, t_2, for the second switch to be closed is the time when the voltage, V, on the load drops from V_1 to V_2 (due to the partial discharge of C_1). By closing the switch the net capacitance of the source is increased. This process is repeated, the kth switch being closed when V has dropped to V_k. When V has dropped to zero the Nth switch is closed,

1. Lt Carl E. Baum, Sensor and Simulation Note XXII, A Transmission Line EMP Simulation Technique for Buried Structures, June 1966.
2. Capt Carl E. Baum, Sensor and Simulation Note XLIV, The Capacitor Driven, Open Circuited, Buried-Transmission-Line Simulator, June 1967.



A. SWITCHES IN PARALLEL



B. SWITCHES IN SERIES

FIGURE 1. EQUIVALENT CIRCUITS FOR THE BURIED-TRANSMISSION-LINE SIMULATOR DRIVEN BY MULTIPLE CAPACITIVE SOURCES

placing a short circuit across the load. For convenience we might consider a V_N as zero volts and a C_N as an infinitely large capacitance. In some cases it may be desirable to have the last switch closure at t_{N-1} in which case the final short circuit is not switched across the load. In this note we choose the V_k's and C_k's to make the current waveform roughly flat after the initial rise associated with the first capacitor. Eventually, of course, in a practical case the current waveform decays to zero. Other kinds of waveforms can also be produced by this technique but are not considered here.

For the present calculations we use the equivalent circuits of figure 1 with ideal capacitors and switches. The impedance of the buried transmission line is taken to be the same as in references 1 and 2. The ground conductivity, σ, and permeability, μ, are assumed to be frequency and depth independent; frequencies of interest are assumed low enough that the ground permittivity, ϵ, is unimportant. In calculating the waveforms only the voltages and currents, or electric and magnetic fields, at the ground surface are considered. These can be transported down into the ground as in references 1 and 2.

As in reference 2 we first consider the simpler case of an effectively infinite-length transmission line followed by the case of a finite-length, open circuited transmission line. Up to three separate capacitive sources plus a possible switched electrical short are considered. The calculation of the waveforms was accomplished by a numerical solution of the differential equations in the time domain. Accuracy was determined by halving the time step and recalculating the waveform. The indicated relative RMS errors were, for most of the waveforms, less than about 2% with errors of about 3% occurring in a few cases.

II. Effectively Infinite-Length Transmission Line

Let the buried transmission line be effectively infinitely long, i.e., let times of interest be much less than the diffusion time for the transmission line so that the bottom end of the transmission line has negligible effect on the impedance. Then the impedance of the buried transmission line is

$$Z_{L_\infty} = f_g \sqrt{\frac{s\mu}{\sigma}} \tag{4}$$

where s is the Laplace transform variable and f_g is a dimensionless factor related to the geometry of a cross section of the transmission line. For the moment consider just the first capacitor and switch. From reference 2 we have

$$\tilde{I}(s) = \frac{V_1 C_1}{(st_c)^{3/2} + 1} \tag{5}$$

where

$$t_c \equiv \left[C_1^2 f_g^{\,2} \frac{\mu}{\sigma}\right]^{1/2} \tag{6}$$

The tilde, ~, over a quantity indicates the Laplace transform of the quantity. Then define an appropriate normalized time as

$$\tau_c \equiv \frac{t}{t_c} \tag{7}$$

and a normalized Laplace transform variable as

$$s_c \equiv st_c \tag{8}$$

There is also a characteristic current

$$I_c \equiv \frac{V_1 C_1}{t_c} \tag{9}$$

from which we define a normalized current or magnetic field at the ground surface as

$$h_c(\tau_c) \equiv \frac{I}{I_c} \tag{10}$$

which has a normalized Laplace transform as

$$\tilde{h}_c(s_c) = \left[s_c^{\,3/2} + 1\right]^{-1} \tag{11}$$

The normalized voltage or electric field at the ground surface is defined as

$$e_c(\tau_c) \equiv \frac{V}{V_1} \tag{12}$$

which has a normalized Laplace transform as

$$\tilde{e}_c(s_c) = \sqrt{s_c}\left[s_c^{3/2} + 1\right]^{-1} \tag{13}$$

Now generalize the normalized voltage or electric field and current or magnetic field to the case of several capacitors switched in at different times. First note that the normalized Laplace transforms of these two quantities are related as

$$\tilde{h}_c(s_c) = \tilde{e}_c(s_c)\,\tilde{g}_c(s_c) \tag{14}$$

where

$$\tilde{g}_c(s_c) = \frac{1}{\sqrt{s_c}} \tag{15}$$

This $\tilde{g}_c(s_c)$ is the normalized Laplace transform of a normalized impulse response function. In the normalized time domain this is

$$g_c(\tau_c) = \frac{1}{\sqrt{\pi\tau_c}} \tag{16}$$

where τ_c is only taken positive. In the normalized time domain the normalized current and voltage are related by

$$h_c(\tau_c) = e_c(\tau_c) * g_c(\tau_c) = \int_0^{\tau_c} e_c(\tau_c')g_c(\tau_c-\tau_c')d\tau_c' \tag{17}$$

The asterisk, *, denotes the convolution of the two functions as explicitly written in the integral. The normalized impulse response function, $\tilde{g}_c(s_c)$, used here is simply a constant times the admittance of the transmission line. As long as the normalization factors are kept the same the normalized impulse response function relates h_c and e_c independent of the sources. We then use the parameters of the first capacitor in the normalization factors and use equation (17) as one of the equations relating h_c and e_c.

The voltage and current can also be related through the capacitors. For $t_k \le t \le t_{k+1}$ we have

$$V = V_k - \left[\sum_{n=1}^{k} C_n\right]^{-1} \int_{t_k}^{t} I\, dt' \tag{18}$$

For convenience normalize the initial voltage on the kth capacitor as

$$v_k \equiv \frac{V_k}{V_1} \tag{19}$$

define a normalized capacitance as

$$c_k \equiv \frac{C_k}{C_1} \tag{20}$$

and define a normalized time for the kth switch to close as

$$\tau_{c_k} \equiv \frac{t_k}{t_c} \tag{21}$$

In terms of the normalized parameters, then for $\tau_{c_k} \leq \tau_c \leq \tau_{c_{k+1}}$ we have

$$e_c(\tau_c) = v_k - \left[\sum_{n=1}^{k} c_n\right]^{-1} \int_{\tau_{c_k}}^{\tau_c} h_c(\tau_c')d\tau_c' \tag{22}$$

For convenience define

$$S_k \equiv \left[\sum_{n=1}^{k} c_n\right]^{-1} \tag{23}$$

Then for $\tau_{c_k} \leq \tau_c \leq \tau_{c_{k+1}}$ we have

$$e_c(\tau_c) = v_k - S_k \int_{\tau_{c_k}}^{\tau_c} h_c(\tau_c')d\tau_c' \tag{24}$$

This equation, together with equation (17), can be used to solve for both e_c and h_c. Note that some values of the normalized parameters have simple values, including

$$\left.\begin{array}{lll} v_1 = 1 & S_1 = 1 & h_c(0) = 0 \\ v_N = 0 & S_N = 0 & e_c(0) = 1 \\ \tau_{c_1} = 0 & & \end{array}\right] \tag{25}$$

There are also the relations

$$\left.\begin{aligned} \tau_{c_{k+1}} &> \tau_{c_k} \\ v_{k+1} &< v_k \\ S_{k+1} &< S_k \end{aligned}\right] \qquad (26)$$

In obtaining a numerical solution of equations (17) and (24) define a set of variables for discrete times based on a positive integer, I, as

$$T_I \equiv \left(I - 1\right) \Delta T \qquad (27)$$

where ΔT is a positive time increment. At the normalized time, T_I, there are the normalized voltage or electric field, E_I, and the normalized current or magnetic field, H_I. Equation (16) for the normalized impulse response function, for $I \geq 2$, becomes

$$G_I = \frac{1}{\sqrt{\pi T_I}} \qquad (28)$$

This function is not defined for I = 1. The initial values from equation (25) are

$$E_1 = 1 \qquad H_1 = 0 \qquad (29)$$

First consider equation (24) and write it as a recurrence relationship in the variables as

$$E_I \simeq E_{I-1} - S_k \frac{H_{I-1} + H_I}{2} \Delta T \qquad (30)$$

Then define

$$A_{I-1} \equiv E_{I-1} - \frac{1}{2} S_k \Delta T \, H_{I-1} \qquad (31)$$

so that

$$E_I \simeq A_{I-1} - \frac{1}{2} S_k \Delta T \, H_I \qquad (32)$$

Second consider equation (17) and write the convolution integral as

$$H_I \simeq \frac{\Delta T}{2}\sum_{J=1}^{I-2}\left[E_J G_{I-J+1} + E_{J+1} G_{I-J}\right] + \frac{1}{2}\left[E_{I-1}+E_I\right]\int_{T_{I-1}}^{T_I}\frac{d\tau_c'}{\sqrt{\pi(T_I-\tau_c')}} \tag{33}$$

Note that the integrals are approximated in a simple trapezoidal fashion except for $\tau_c-\tau_c'$ near zero in $g_c(\tau_c-\tau_c')$ where it is approximated as in the last term of equation (33). Equation (33) reduces to

$$H_I \simeq \frac{\Delta T}{2}\left[E_1 G_I + E_{I-1} G_2\right] + \Delta T\sum_{J=2}^{I-2} E_J G_{I-J+1} + \left[E_{I-1}+E_I\right]\sqrt{\frac{\Delta T}{\pi}} \tag{34}$$

Then define

$$B_{I-1} \equiv \frac{\Delta T}{2}\left[E_1 G_I + E_{I-1} G_2\right] + \Delta T\sum_{J=2}^{I-2} E_J G_{I-J+1} + \sqrt{\frac{\Delta T}{\pi}}\, E_{I-1} \tag{35}$$

so that

$$H_I \simeq B_{I-1} + \sqrt{\frac{\Delta T}{\pi}}\, E_I \tag{36}$$

Note that these expressions for H_I apply for $I \geq 4$ but can be used for $I = 2$ and $I = 3$ if the appropriate terms are removed from the expressions.

Equations (32) and (36) can be combined giving

$$H_I \simeq \frac{\sqrt{\frac{\Delta T}{\pi}}\, A_{I-1} + B_{I-1}}{1 + \frac{S_k \Delta T}{2}\sqrt{\frac{\Delta T}{\pi}}} \tag{37}$$

If E_{I-1} and H_{I-1} are known then A_{I-1} and B_{I-1} can be calculated and then H_I can be calculated. Having H_I then E_I can be calculated from equation (32). Starting from $I = 1$ in equation (29) then E_I and H_I can be calculated for any I. If ΔT is small E_I and H_I approximate $e_c(\tau_c)$ and $h_c(\tau_c)$. Note that k in equations (31), (32), (35), and (37) is chosen for each value of I depending on the value of E_{I-1} so that $v_k \geq E_{I-1} > v_{k+1}$.

In some cases a final short is switched across the buried transmission line at a normalized time, τ_{c_N}. For long times such that $\tau_c >> \tau_{c_N}$ there is little variation between $g_c(\tau_c)$ and $g_c(\tau_c-\tau_{c_N})$ because of the mathematical form of g_c in equation (16). Then for $\tau_c >> \tau_{c_N}$ we can approximate $h_c(\tau_c)$ from equation (17) as

$$h_c(\tau_c) \approx \frac{1}{\sqrt{\pi\tau_c}} \int_0^{\tau_{c_N}} e_c(\tau_c')d\tau_c' \qquad (38)$$

This is just a constant times $\tau_c^{-1/2}$, the constant being obtained from the integral of e_c in the numerical calculations.

Figures 2 through 5 give the results of the numerical calculations. In figure 2, with a single capacitor, note the change in h_c produced by switching in the short when e_c reaches zero. The magnetic-field waveform becomes nonoscillatory and it is significantly broadened; this is accomplished with no change in either the peak or time to peak of the waveform. Figures 3 and 4 consider the cases of two and three capacitors, respectively, with a final short included in all cases. In both figures the effect of varying the capacitance of the last capacitor is illustrated. The v_k's and S_k's are chosen with the idea of making the magnetic-field waveform approximate a constant after the initial rise and before the final decay. We define an "optimum" case as one in which the total variation or ripple in the flat top is held to approximately .1 times the initial peak and the flat top of the waveform is extended to the largest time. For the "optimum" cases the approximate forms of the long-time decay (from equation (38)) are included as dotted lines on the graphs. In figure 5 the "optimum" cases, plus the single-capacitor case, are summarized with a logarithmic time scale. For convenience some of the parameters for the single-capacitor case and "optimum" multiple-capacitor cases are summarized in the following table.

number of capacitors	v_2	v_3	S_2	S_3	τ_{c_2}	τ_{c_3}	τ_{c_4}	long-time h_c
1	--	--	--	--	1.65	--	--	$.481\ \tau_c^{-1/2}$
2	.3	--	.038	--	1.14	13.1	--	$1.41\ \tau_c^{-1/2}$
3	.3	.11	.038	.002	1.14	8.24	90.6	$2.52\ \tau_c^{-1/2}$

Table I. Summary of Parameters for Effectively Infinite-Length Transmission Line

Note that the initial peak of h_c is .726 in all cases.

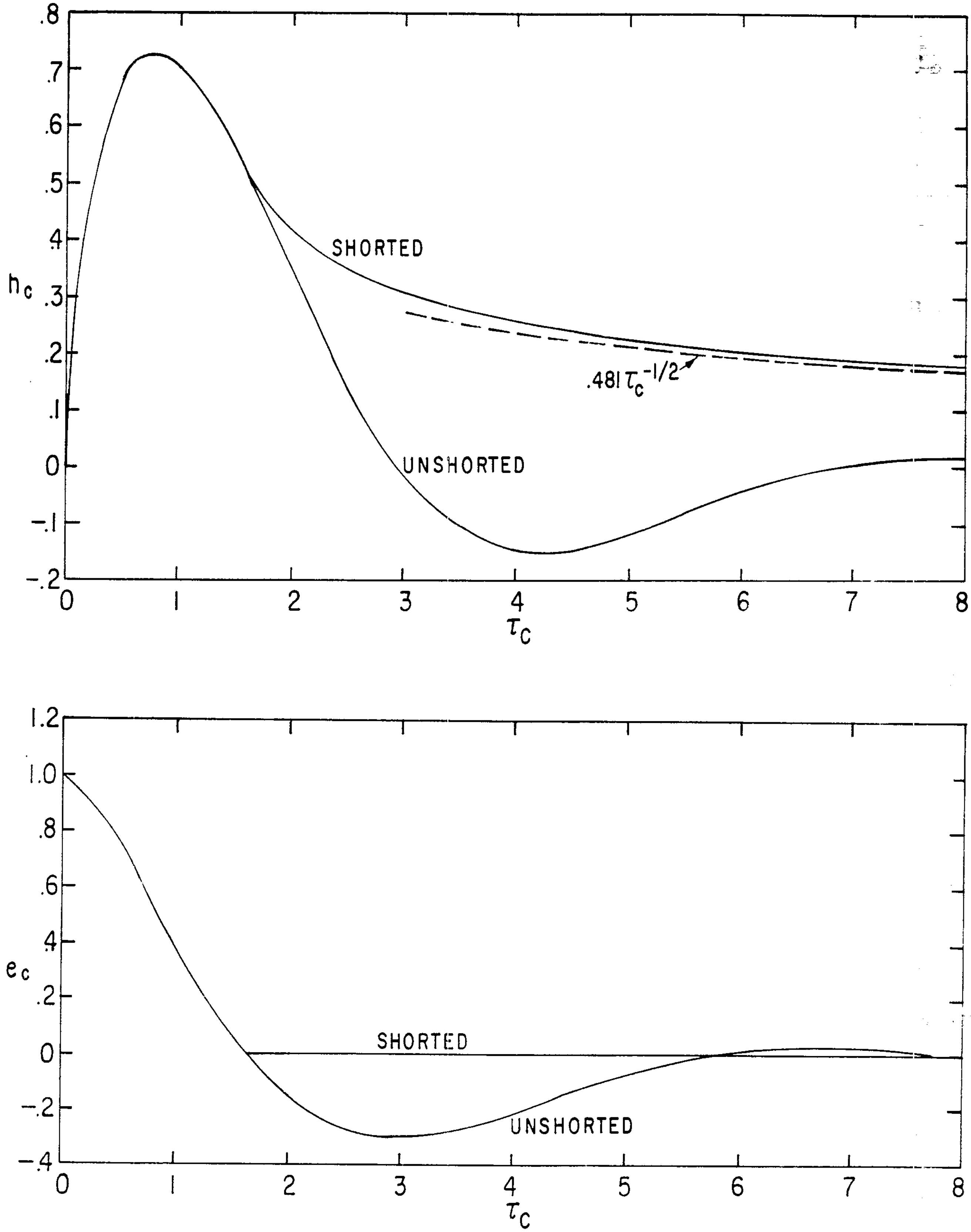


FIGURE 2. PULSE SHAPES FOR INFINITE-LENGTH TRANSMISSION LINE WITH ONE CAPACITOR

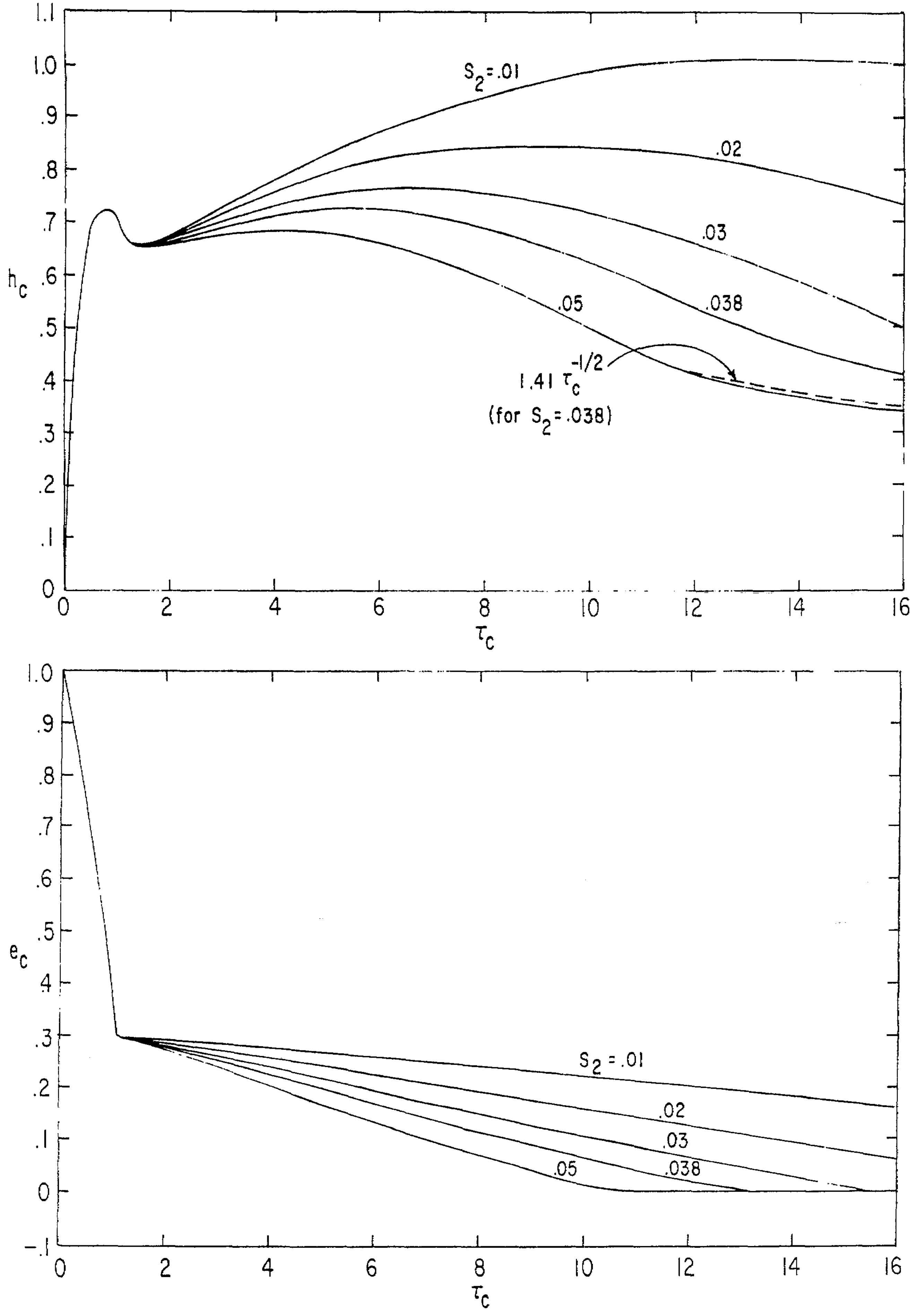


FIGURE 3. PULSE SHAPES FOR INFINITE-LENGTH TRANSMISSION LINE WITH TWO CAPACITORS: v_2 =.3

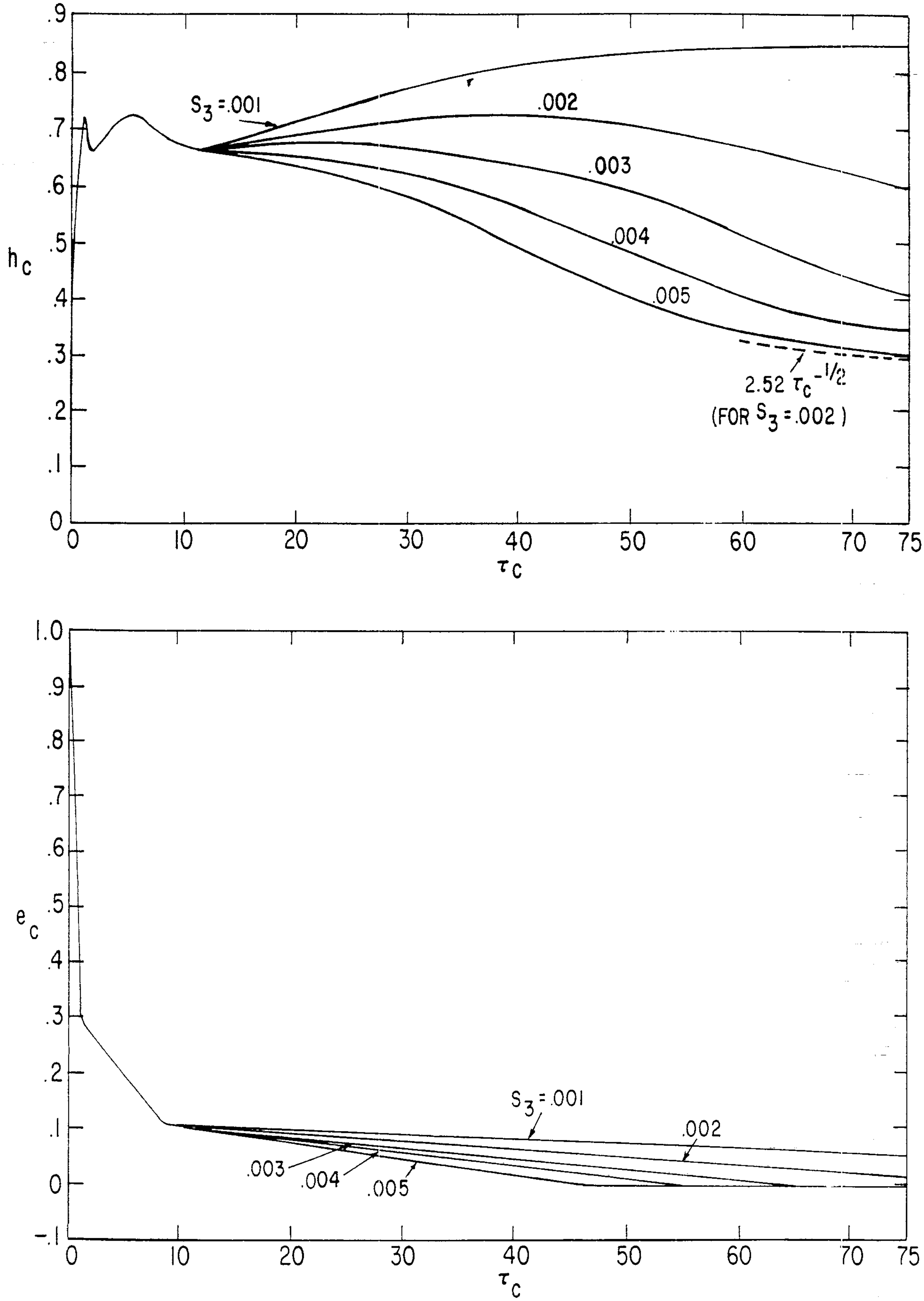


FIGURE 4. PULSE SHAPES FOR INFINITE-LENGTH TRANSMISSION LINE WITH THREE CAPACITORS: $v_2 = .3$, $S_2 = .038$, $v_3 = .11$

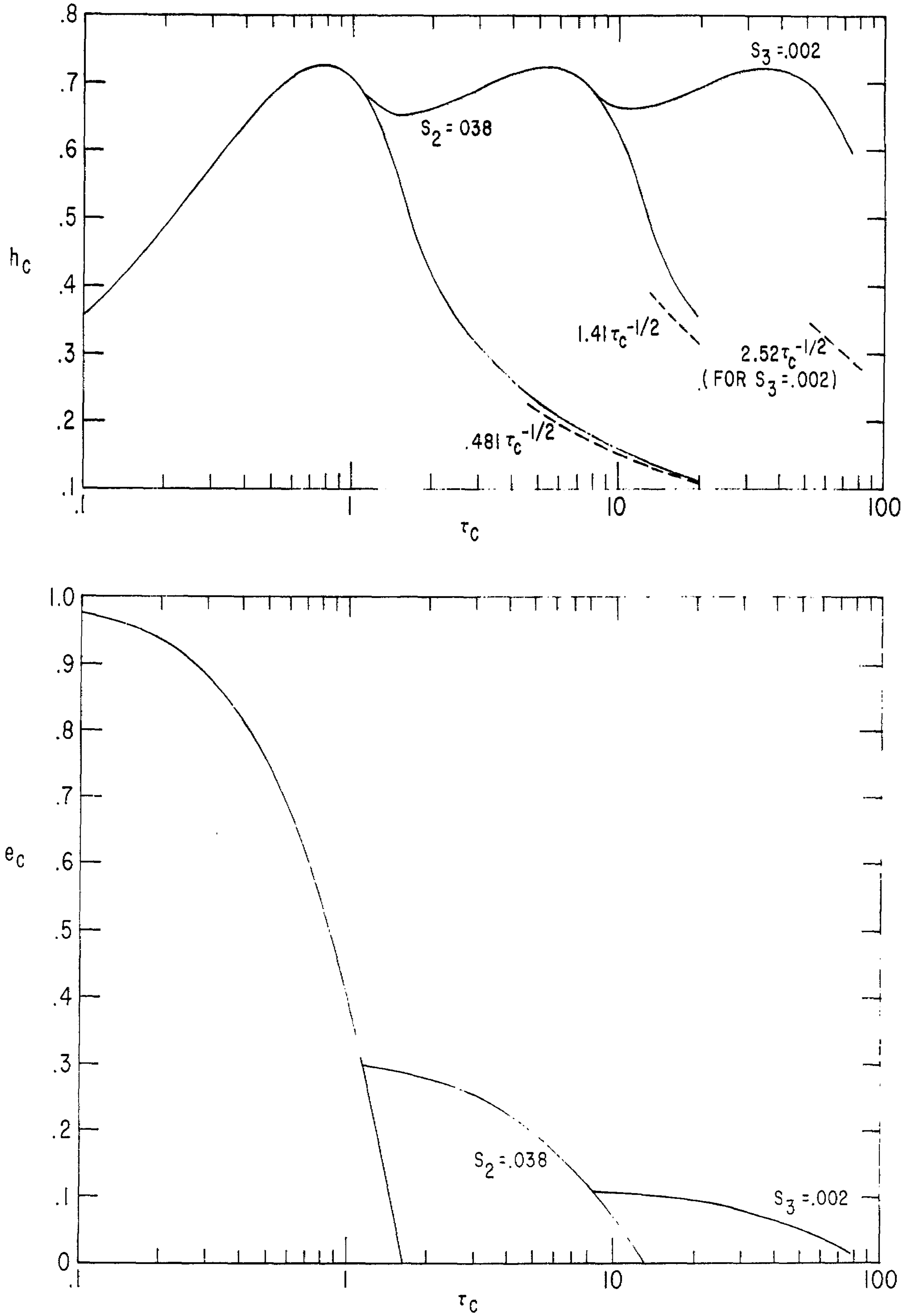


FIGURE 5. PULSE SHAPES FOR INFINITE-LENGTH TRANSMISSION LINE WITH VARYING NUMBER OF CAPACITORS

III. Finite-Length, Open Circuited Transmission Line

Now include an open-circuit termination at the bottom of the buried transmission line. As in reference 2 there is a characteristic time

$$t_\ell \equiv \frac{\mu\sigma\ell^2}{4} \tag{39}$$

where ℓ is the length of the transmission line. From this we define a normalized time as

$$\tau_\ell \equiv \frac{t}{t_\ell} \tag{40}$$

and a normalized Laplace transform variable as

$$s_\ell \equiv st_\ell \tag{41}$$

Define a characteristic current as

$$I_o \equiv \frac{V_1}{R_o} \tag{42}$$

where

$$R_o \equiv \frac{f_g}{\ell\sigma} \tag{43}$$

and define another characteristic time as

$$t_o \equiv R_o C_1 \tag{44}$$

From reference 2 there is a normalized current or magnetic field at the ground surface for a single capacitor and switch with a normalized Laplace transform given by

$$\tilde{h}_\ell(s_\ell) = \left\{2\, s_\ell^{3/2}\, \frac{1+e^{-4\sqrt{s_\ell}}}{1-e^{-4\sqrt{s_\ell}}} + \frac{t_\ell}{t_o}\right\}^{-1} \tag{45}$$

and a corresponding normalized voltage or electric field at the ground surface with a normalized Laplace transform given by

$$\tilde{e}_\ell(s_\ell) \equiv \ell\tilde{j}_\ell(s_\ell) = 2\sqrt{s_\ell}\, \frac{1+e^{-4\sqrt{s_\ell}}}{1-e^{-4\sqrt{s_\ell}}} \left\{2\, s_\ell^{3/2}\, \frac{1+e^{-4\sqrt{s_\ell}}}{1-e^{-4\sqrt{s_\ell}}} + \frac{t_\ell}{t_o}\right\}^{-1} \tag{46}$$

In the time domain these are related to the current and voltage as

$$h_\ell(\tau_\ell) = \frac{I}{I_o} \tag{47}$$

and

$$e_\ell(\tau_\ell) = \frac{V}{V_1} \tag{48}$$

Generalize these currents and voltages to the case of several capacitors switched in at different times. Relate the normalized Laplace transforms of the normalized current and voltage as

$$\tilde{h}_\ell(s_\ell) = \tilde{e}_\ell(s_\ell)\ \tilde{g}_\ell(s_\ell) \tag{49}$$

where

$$\tilde{g}_\ell(s_\ell) = \frac{1}{2\sqrt{s_\ell}}\ \frac{1-e^{-4\sqrt{s_\ell}}}{1+e^{-4\sqrt{s_\ell}}} \tag{50}$$

which can be written as

$$\tilde{g}_\ell(s_\ell) = \frac{1}{2\sqrt{s_\ell}}\left\{1 + 2\sum_{n=1}^{\infty} (-1)^n\ e^{-4n\sqrt{s_\ell}}\right\} \tag{51}$$

Then in the normalized time domain there is a normalized impulse response function given by

$$g_\ell(\tau_\ell) = \frac{1}{2\sqrt{\pi\tau_\ell}}\left\{1 + 2\sum_{n=1}^{\infty} (-1)^n\ e^{-\frac{4n^2}{\tau_\ell}}\right\} \tag{52}$$

Note that for $\tau_\ell << 1$

$$g_\ell(\tau_\ell) \simeq \frac{1}{2\sqrt{\pi\tau_\ell}} \tag{53}$$

In the normalized time domain the normalized current and voltage are related by

$$h_\ell(\tau_\ell) = e_\ell(\tau_\ell) * g_\ell(\tau_\ell) = \int_0^{\tau_\ell} e_\ell(\tau_\ell')\ g_\ell(\tau_\ell-\tau_\ell')d\tau_\ell' \tag{54}$$

We include a table of $g_\ell(\tau_\ell)$ calculated from equation (52). Alternatively $g_\ell(\tau_\ell)$ can be calculated from equation (50) using a numerical inverse Fourier transform. Note that for $\tau_\ell < .5$ the first term (with no exponential) in equation (52) gives quite accurate results. Note that g_ℓ decays quite rapidly for $\tau_\ell >> 1$.

τ_ℓ	g_ℓ
.5	3.987×10^{-1}
.6	3.633×10^{-1}
.7	3.349×10^{-1}
.8	3.111×10^{-1}
.9	2.904×10^{-1}
1.0	2.718×10^{-1}
1.5	1.983×10^{-1}
2.0	1.456×10^{-1}
2.5	1.070×10^{-1}
3.0	7.858×10^{-2}
3.5	5.772×10^{-2}
4.0	4.240×10^{-2}
4.5	3.115×10^{-2}
5.0	2.288×10^{-2}

τ_ℓ	g_ℓ
6	1.235×10^{-2}
7	6.664×10^{-3}
8	3.596×10^{-3}
9	1.941×10^{-3}
10	1.047×10^{-3}
12	3.050×10^{-4}
14	8.881×10^{-5}
16	2.586×10^{-5}
18	7.531×10^{-6}
20	2.193×10^{-6}
25	1.004×10^{-7}
30	4.593×10^{-9}
35	2.102×10^{-10}

Table II. Impulse Response Function of Finite-Length, Open Circuited Transmission Line

The voltage and current are related through the capacitors in equation (18). Define a normalized time for the kth switch to close as

$$\tau_{\ell_k} \equiv \frac{t_k}{t_\ell} \tag{55}$$

In terms of the normalized parameters, then for $\tau_{\ell_k} \leq \tau_\ell \leq \tau_{\ell_{k+1}}$ we have

$$e_\ell(\tau_\ell) = v_k - S_k \frac{t_\ell}{t_o} \int_{\tau_{\ell_k}}^{\tau_\ell} h_\ell(\tau_\ell')d\tau_\ell' \tag{56}$$

Equations (54) and (56) can be used to solve for e_ℓ and h_ℓ. Some of the new parameters have simple values, including

$$h_\ell(0) = 0 \qquad e_\ell(0) = 1 \qquad \tau_{\ell_1} = 0 \tag{57}$$

There is also the relation

$$\tau_{\ell_{k+1}} > \tau_{\ell_k} \tag{58}$$

For this case of the finite-length transmission line define a discrete time to replace τ_ℓ for numerical calculations based on a positive integer, I, as

$$T'_I \equiv (I-1)\ \Delta T' \tag{59}$$

where $\Delta T'$ is a positive time increment. Define

$$S'_k \equiv S_k \frac{t_\ell}{t_o} \tag{60}$$

The normalized impulse response function, for $I \geq 2$, becomes

$$G'_I = g_\ell(T'_I) \tag{61}$$

The normalized voltage or electric field is E'_I and the normalized current or magnetic field is H'_I. The initial values are

$$E'_1 = 1 \qquad\qquad H'_1 = 0 \tag{62}$$

Similar to equation (32) we have a recurrence relation for E'_I as

$$E'_I \simeq A'_{I-1} - \frac{1}{2} S'_k\ \Delta T'\ H'_I \tag{63}$$

where we define

$$A'_{I-1} \equiv E'_{I-1} - \frac{1}{2} S'_k\ \Delta T'\ H'_{I-1} \tag{64}$$

From equation (54) H'_I is given by

$$H'_I \simeq \frac{\Delta T'}{2} \sum_{J=1}^{I-2} \left[E'_J G'_{I-J+1} + E'_{J+1} G'_{I-J}\right] + \frac{1}{2}\left[E'_{I-1} + E'_I\right] \int_{T'_{I-1}}^{T'_I} \frac{d\tau'_\ell}{2\sqrt{\pi(T'_I - \tau'_\ell)}} \tag{65}$$

Note that the form is slightly different from equation (33) due to the factor of 1/2 in g_ℓ in equation (53) for small τ_ℓ. Then for H'_I we have

$$H'_I \simeq B'_{I-1} + \frac{1}{2}\sqrt{\frac{\Delta T'}{\pi}}\ E'_I \tag{66}$$

where we define

$$B'_{I-1} \equiv \frac{\Delta T'}{2}\left[E'_1\ G'_I + E'_{I-1}\ G'_2\right] + \Delta T' \sum_{J=2}^{I-2} E'_J G'_{I-J+1} + \frac{1}{2}\sqrt{\frac{\Delta T'}{\pi}}\ E'_{I-1} \tag{67}$$

Again equations (65) and (67) apply for $I \geq 4$ and with slight modification for $I = 2$ and $I = 3$. Solving for H'_I we have

$$H'_I \approx \frac{\frac{1}{2}\sqrt{\frac{\Delta T'}{\pi}}\, A'_{I-1} + B'_{I-1}}{1 + \frac{S'_k \Delta T'}{4}\sqrt{\frac{\Delta T'}{\pi}}} \tag{68}$$

Having H'_I then E'_I can be calculated from equation (63). For small $\Delta T'$ then E'_I and H'_I approximate $e_\ell(\tau_\ell)$ and $h_\ell(\tau_\ell)$. Again k is chosen in these equations for each I depending on E'_{I-1} so that $v_k \geq E'_{I-1} > v_{k+1}$.

Figures 6 through 16 give the results of the numerical calculations. Four values of t_o/t_ℓ are considered: 1, .1, .01, and .001, in that order. For each value of t_o/t_ℓ the number of capacitors is varied up to a maximum of three capacitors, except for $t_o/t_\ell = 1$ for which the maximum number of capacitors included is two. The first figure for each value of t_o/t_ℓ illustrates the change produced in h_ℓ by switching in a short when e_ℓ reaches zero. As in the previous section (for the infinite-length transmission line) additional capacitors are added with the v_k's and S_k's chosen to make the magnetic-field waveform approximate a constant (after the initial rise) with a ripple of about 10% and with the flat top of the waveform extended to the largest time. There is a significant difference between this case of a finite-length transmission line and the previous case of an infinite-length transmission line in that, for characteristic times in the waveforms much larger than t_ℓ, the impedance of the finite-length transmission line approximates a constant resistance. If the last capacitor is switched in at sufficiently large τ_ℓ, then the associated S_k can be set to zero and the associated v_k can be chosen so as to make the roughly flat top of the magnetic field waveform extend to arbitrarily large times. Of course, since $S_k = 0$ corresponds to $C_k = \infty$, such a case does not correspond to a real capacitor but to an ideal voltage source. Also, in practice, one would have the waveform eventually decay to zero. For $t_o/t_\ell = 1$ one can make $S_2 = 0$ and for a t_o/t_ℓ of .1 and .01 one can make $S_3 = 0$ and achieve an arbitrarily long, flat-topped waveform. However, for $t_o/t_\ell = .001$, one needs more than three capacitors to be able to set the last S_k to zero and maintain the same kind of flat-topped magnetic-field waveform. Note that when the last S_k is set to zero in this manner there is no final short put on the transmission line. For convenience some of the parameters for the single-capacitor cases and "optimum" multiple-capacitor cases are summarized in the following table.

$\frac{t_o}{t_\ell}$	number of capacitors	v_2	v_3	S_2	S_3	τ_{ℓ_2}	τ_{ℓ_3}	τ_{ℓ_4}	initial peak of h_c
1	1	--	--	--	--	2.72	--	--	.457
	2	.45	--	0	--	1.40	--	--	"
.1	1	--	--	--	--	.552	--	--	.216
	2	.3	--	.03	--	.380	7.32	--	"
	3	.3	.21	.03	0	.380	1.84	--	"
.01	1	--	--	--	--	.116	--	--	.103
	2	.3	--	.03	--	.080	1.15	--	"
	3	.3	.1	.03	0	.080	.74	--	"
.001	1	--	--	--	--	.024	--	--	.0469
	2	.3	--	.034	--	.018	.226	--	"
	3	.3	.11	.034	.002	.018	.14	1.41	"

Table III. Summary of Parameters for Finite-Length, Open Circuited Transmission Line

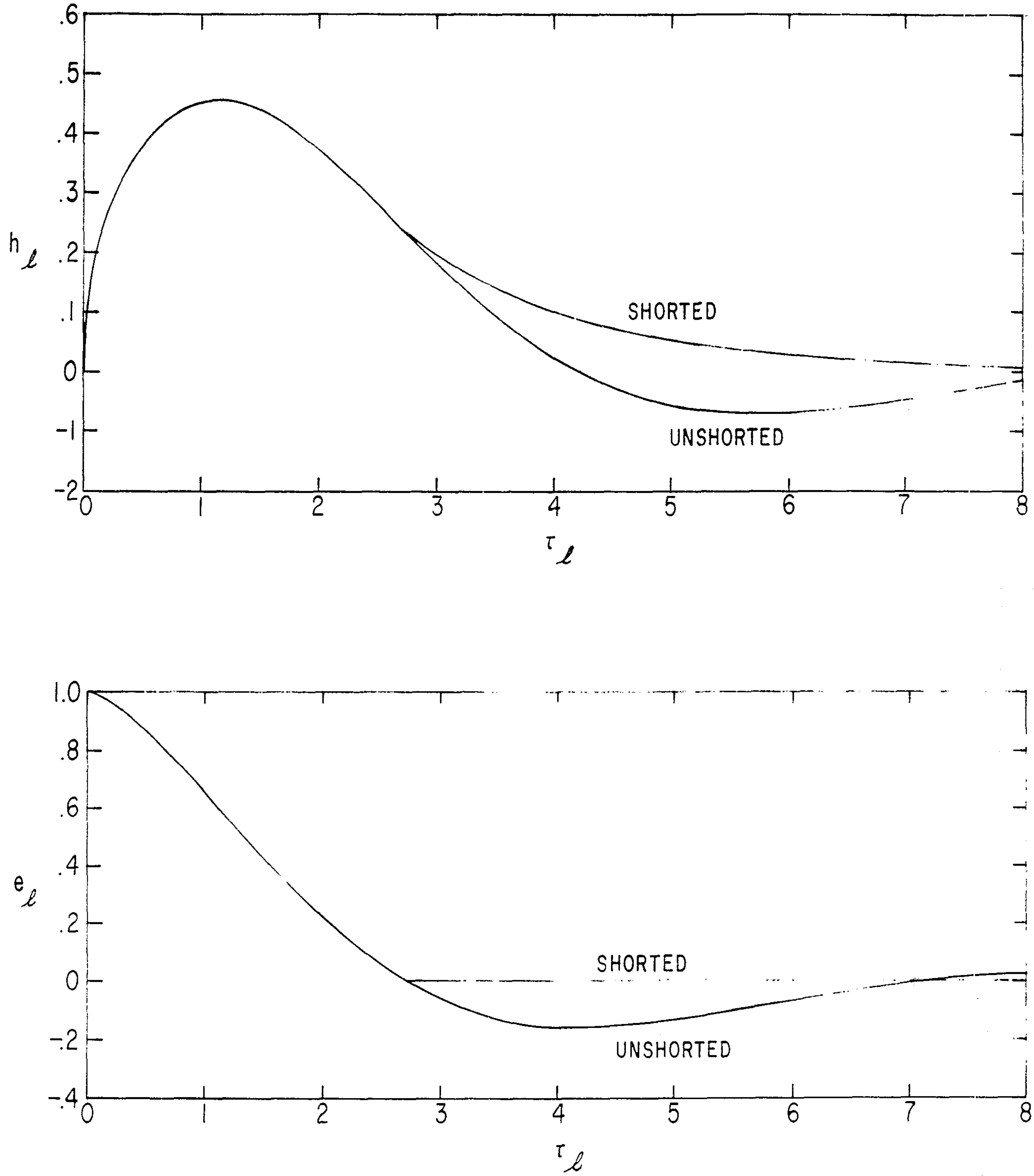


FIGURE 6. PULSE SHAPES FOR FINITE-LENGTH TRANSMISSION LINE WITH ONE CAPACITOR : $\frac{t_0}{t_\ell} = 1$.

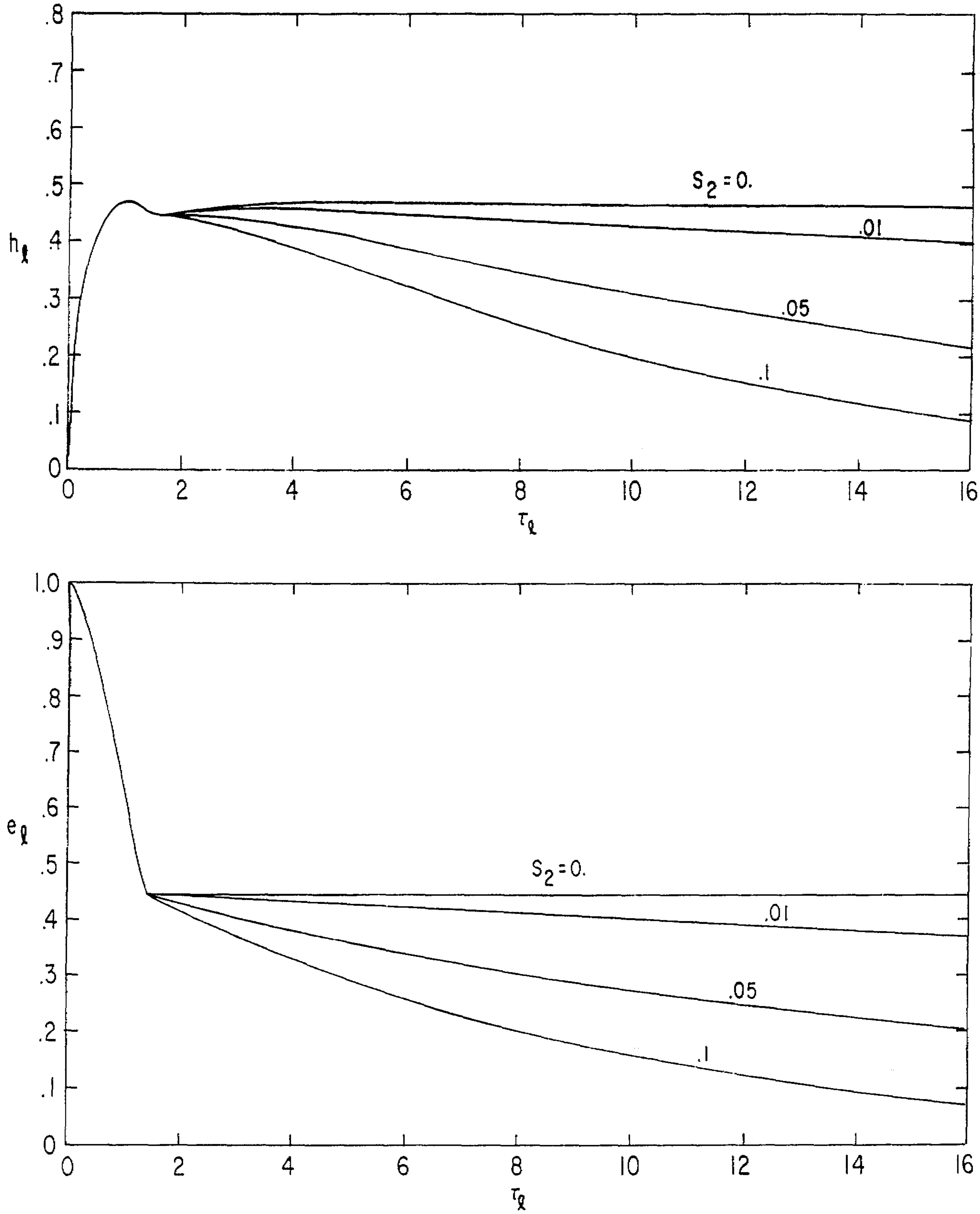
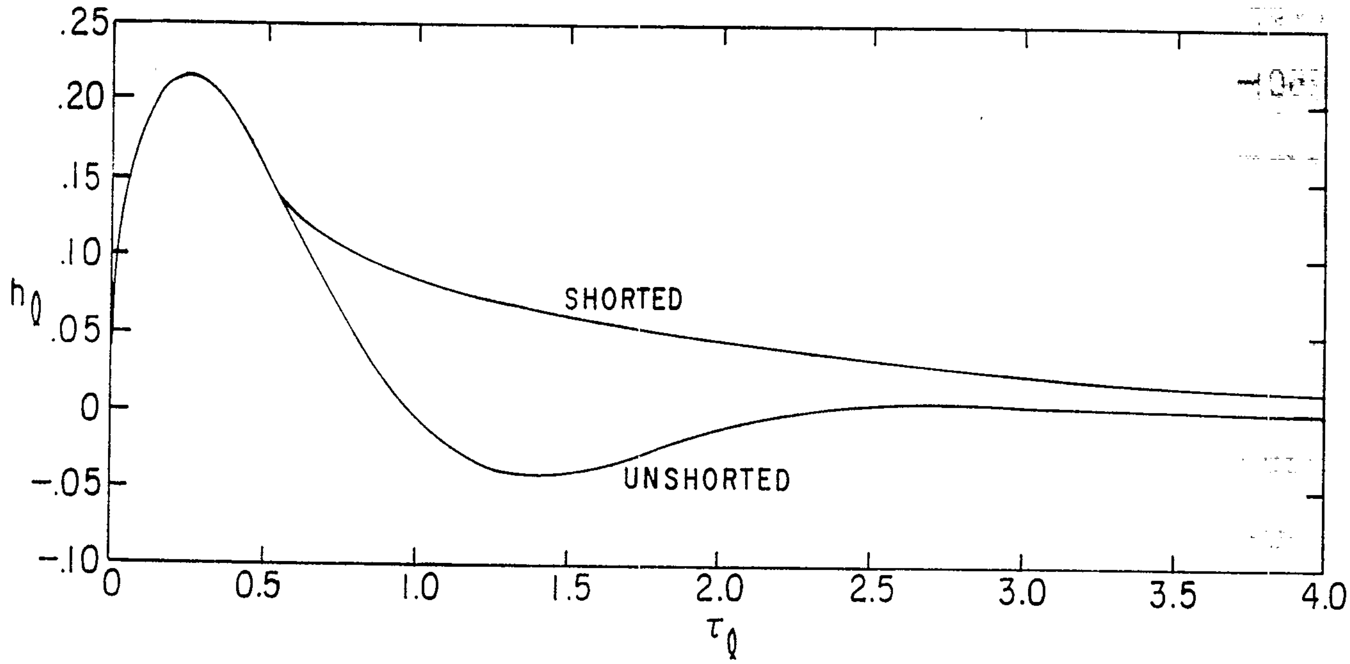


FIGURE 7. PULSE SHAPES FOR FINITE-LENGTH TRANSMISSION LINE WITH TWO CAPACITORS: $\frac{t_0}{t_\ell} = 1.$, $v_2 = .45$



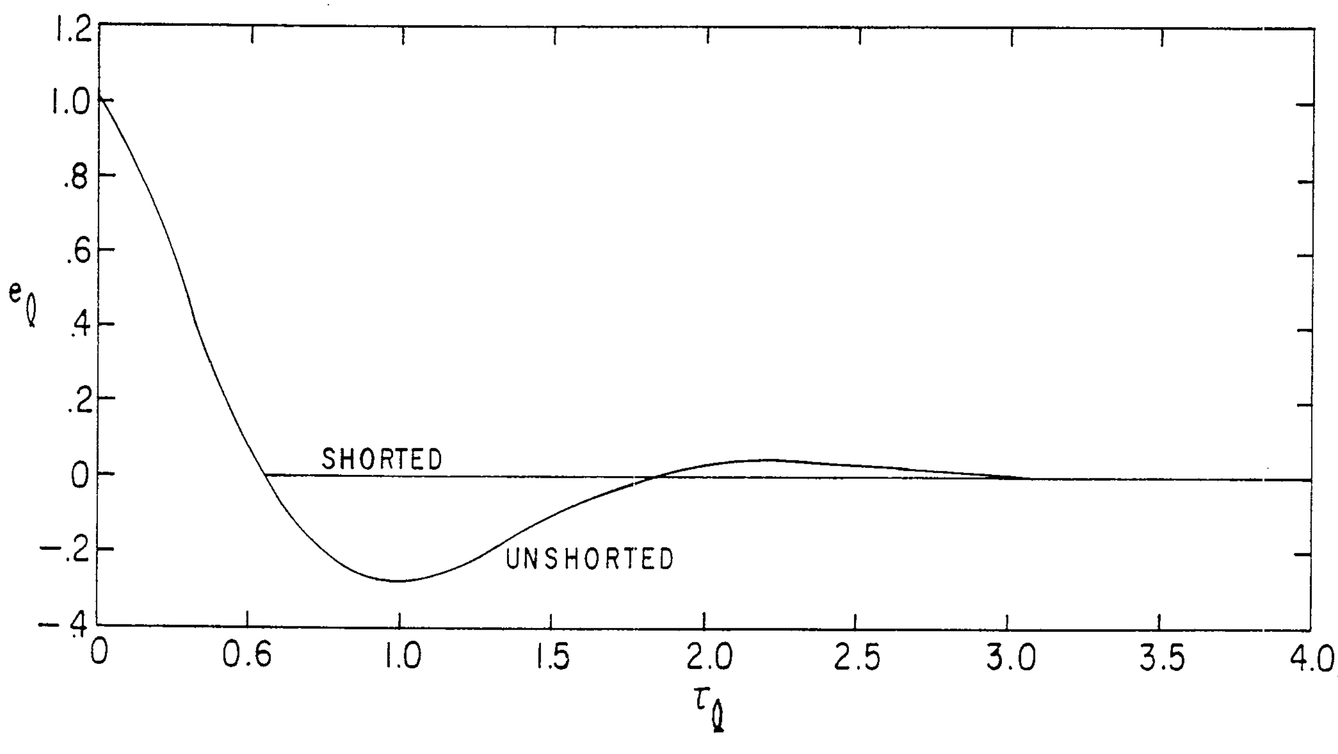


FIGURE 8. PULSE SHAPES FOR FINITE-LENGTH TRANSMISSION LINE WITH ONE CAPACITOR: $\frac{t_0}{t_\ell} = .1$

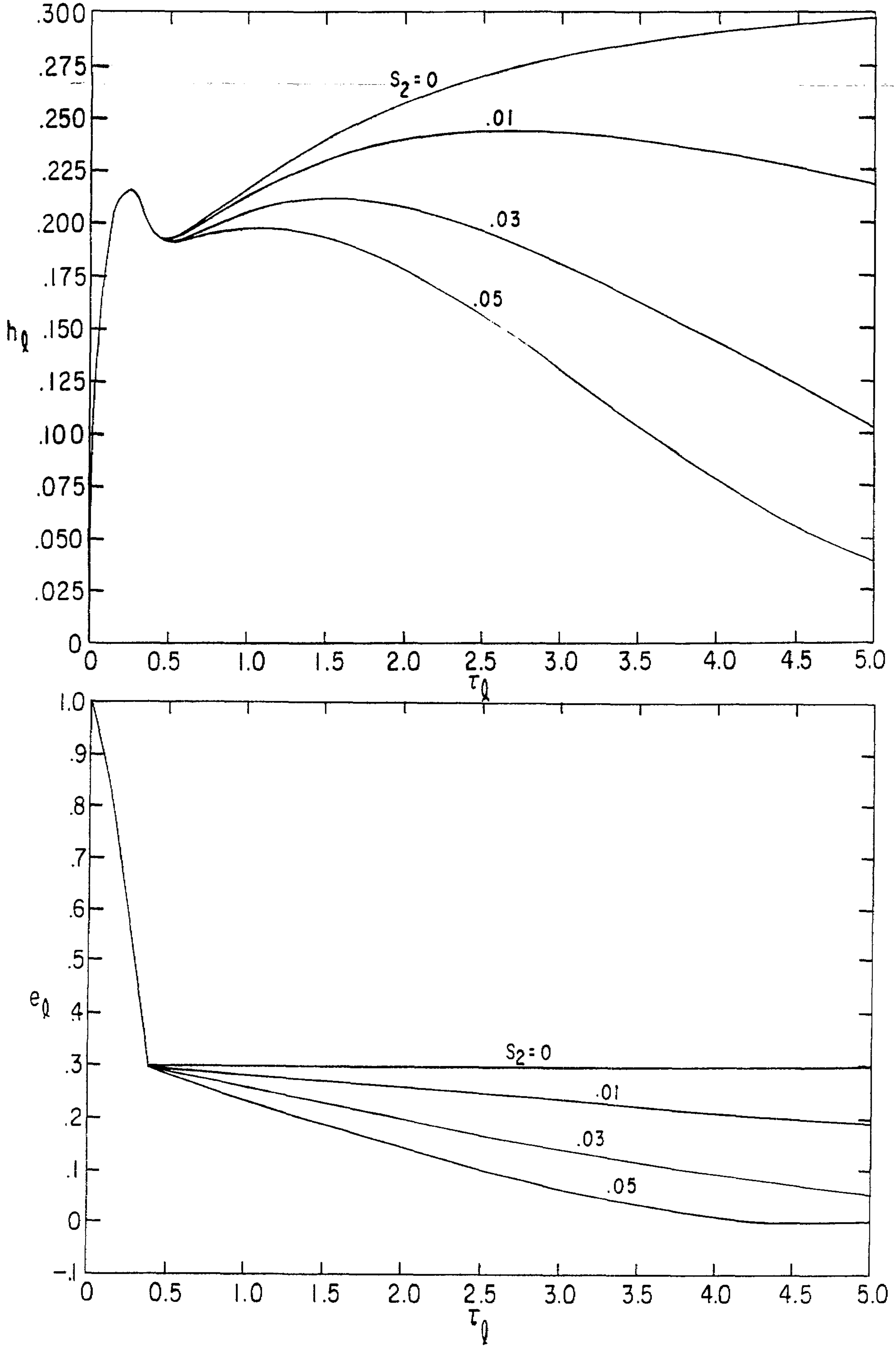
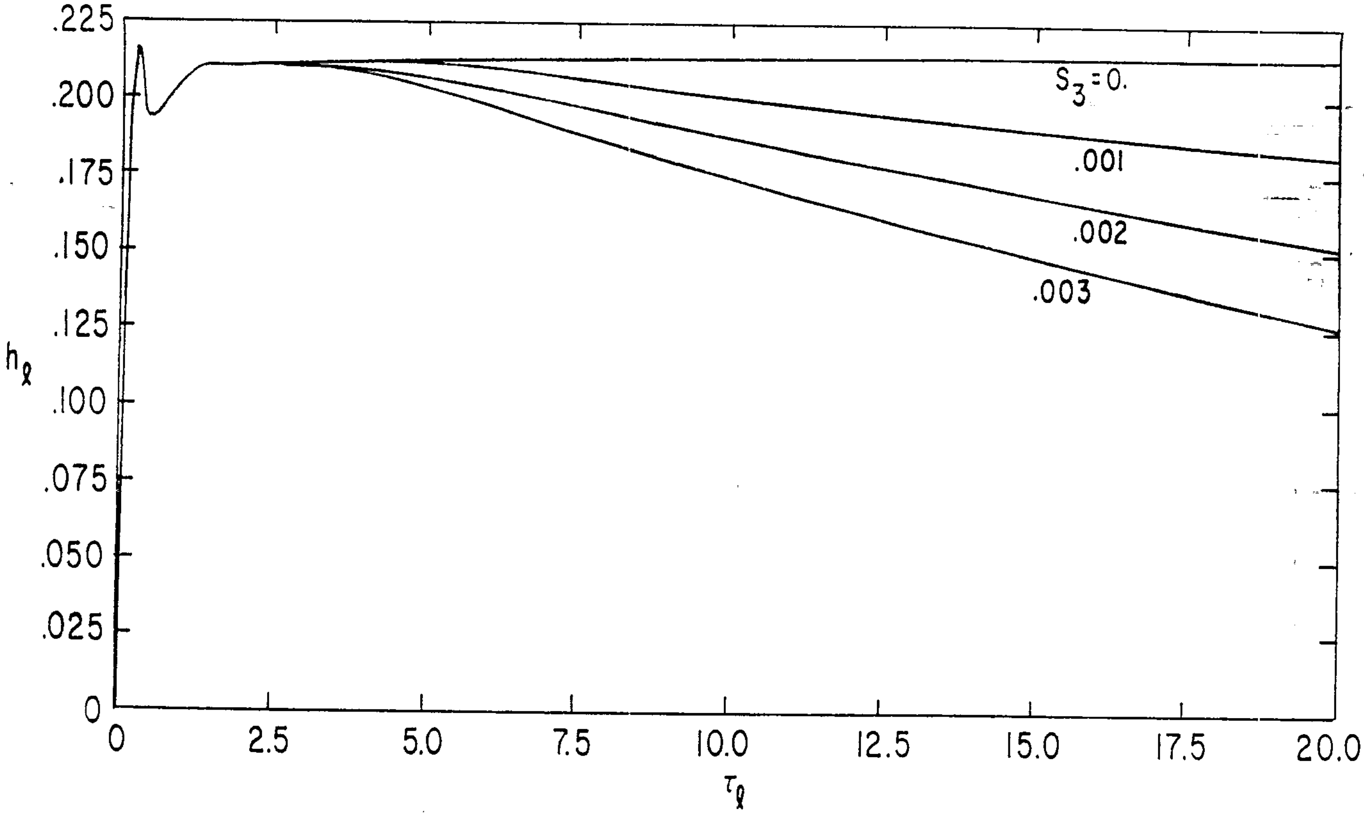


FIGURE 9. PULSE SHAPES FOR FINITE-LENGTH TRANSMISSION LINE WITH TWO CAPACITORS: $t_0/t_\ell = .1$, $v_2 = .3$



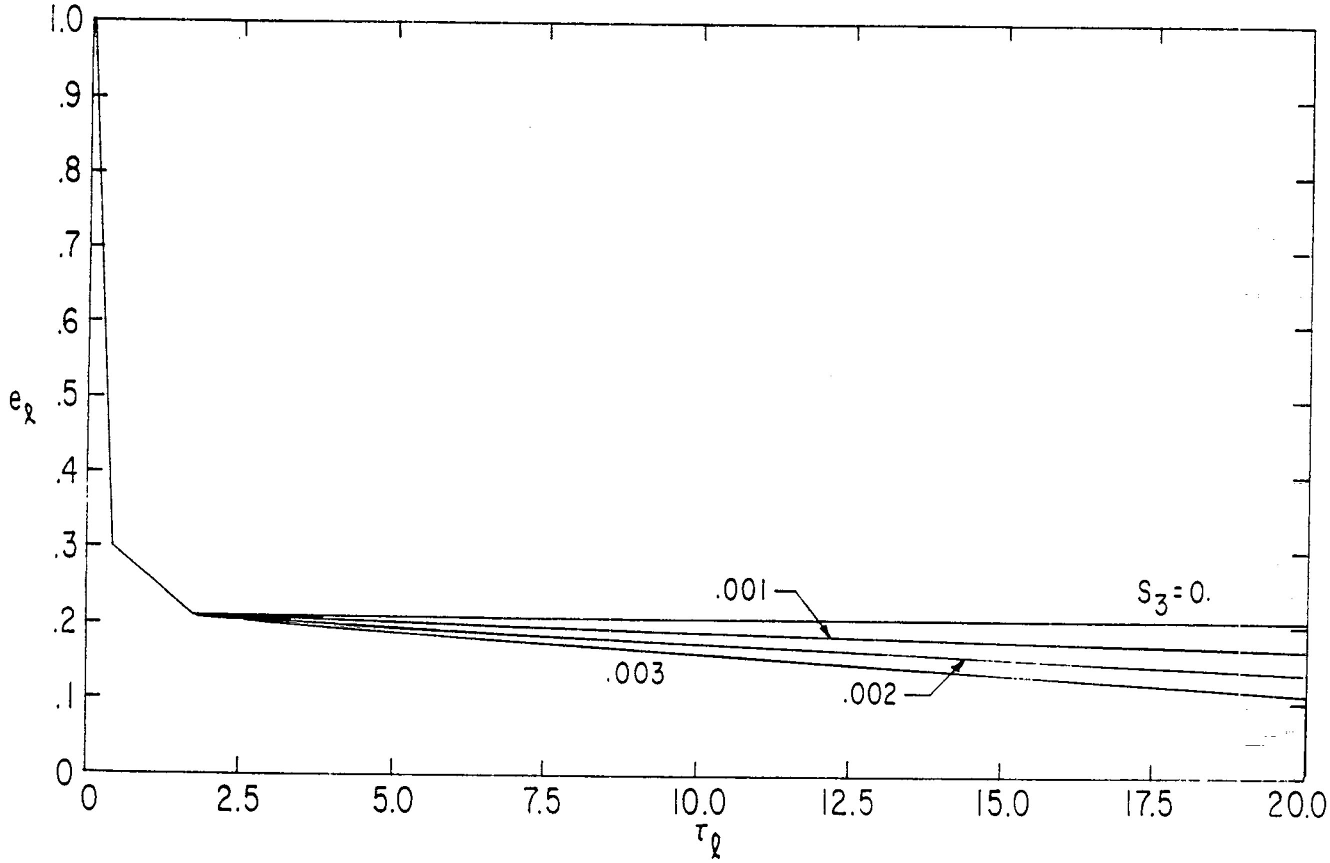


FIGURE 10. PULSE SHAPES FOR FINITE-LENGTH TRANSMISSION LINE WITH THREE CAPACITORS: $\frac{t_0}{t_\ell}=.1$, $v_2=.3$, $S_2=.03$, $v_3=.21$

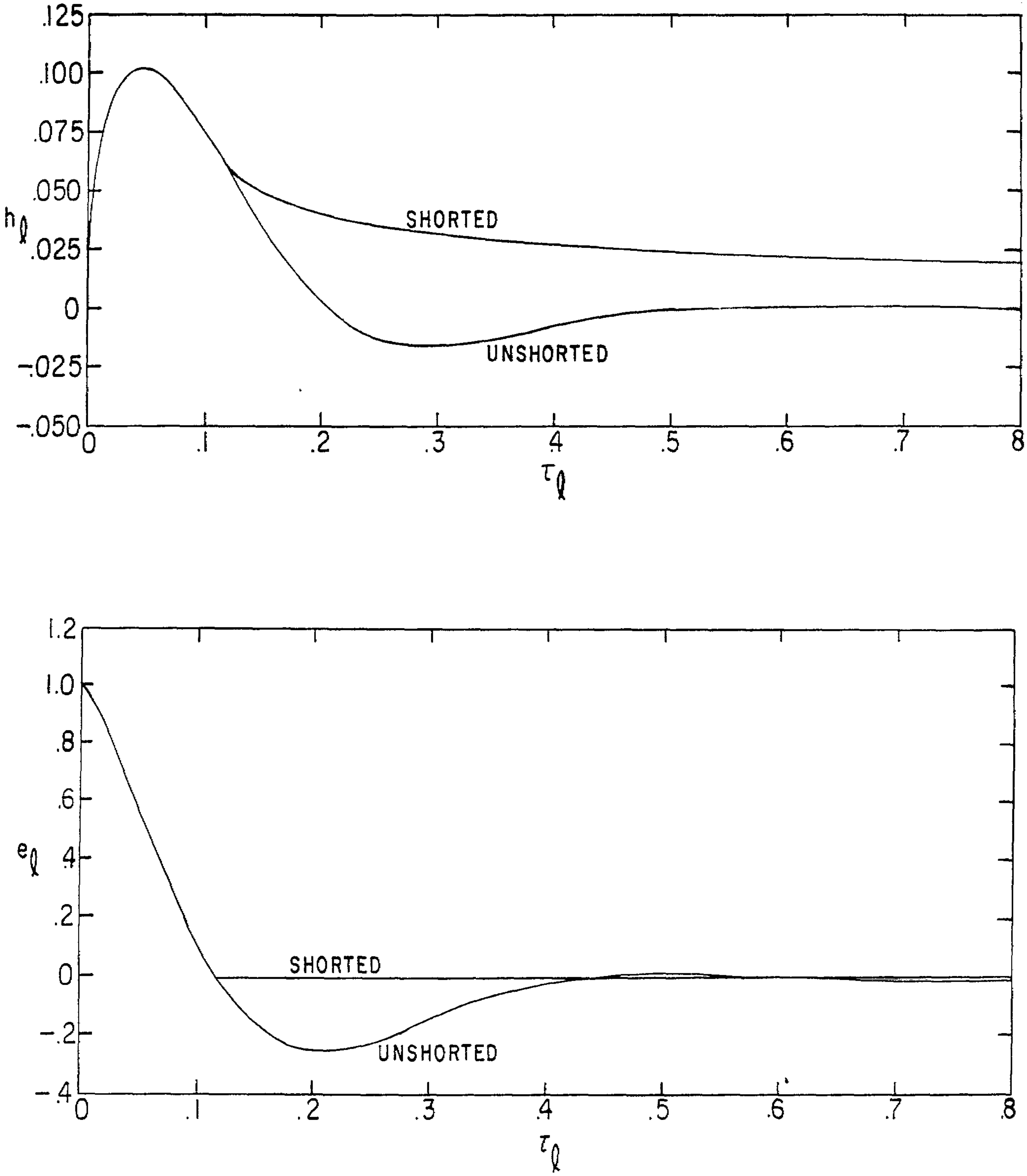


FIGURE 11. PULSE SHAPES FOR FINITE-LENGTH TRANSMISSION LINE WITH ONE CAPACITOR: t_o/t_ℓ = .01

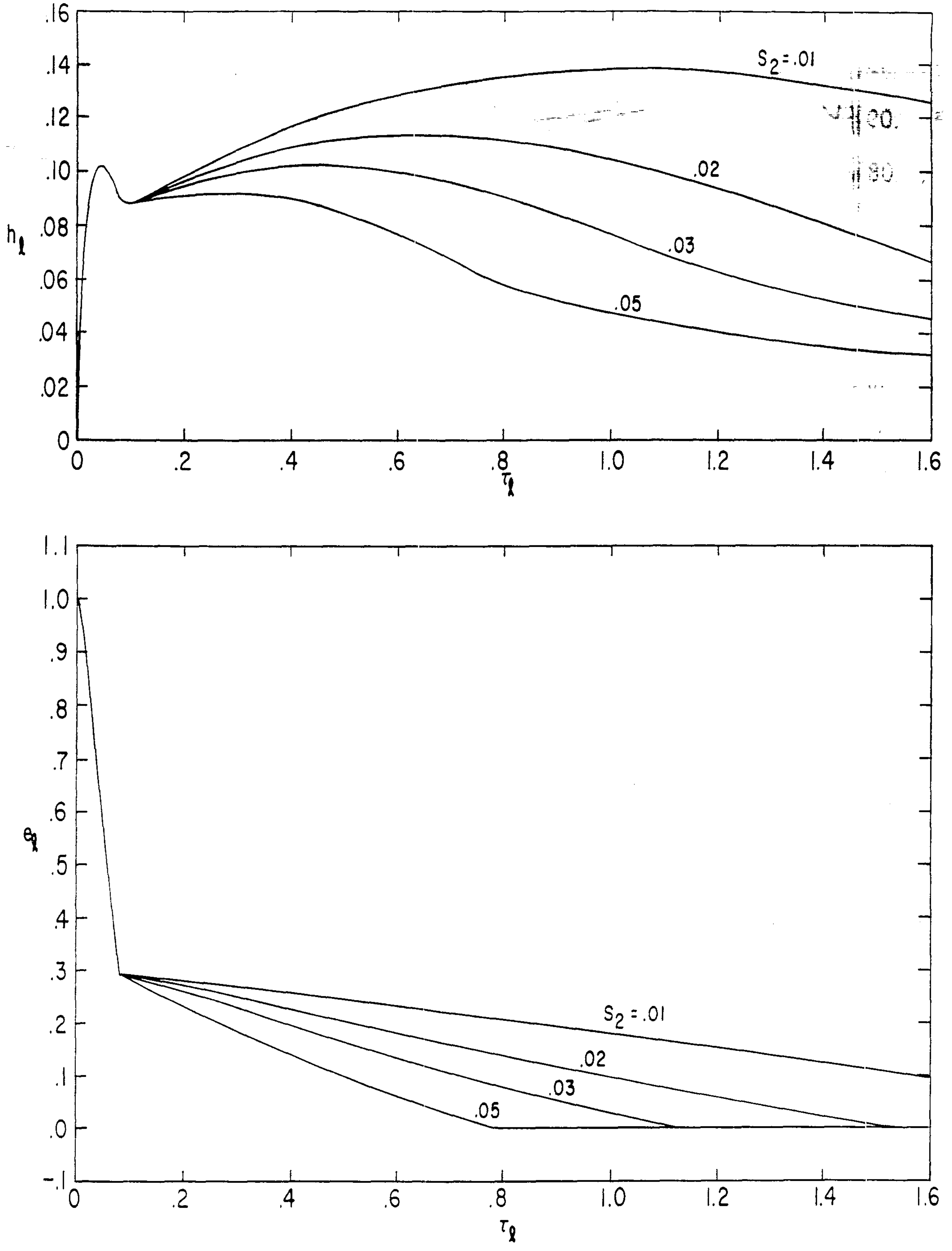


FIGURE 12. PULSE SHAPES FOR FINITE-LENGTH TRANSMISSION LINE WITH TWO CAPACITORS: $\frac{t_0}{t_\ell} = .01$, $v_2 = .3$

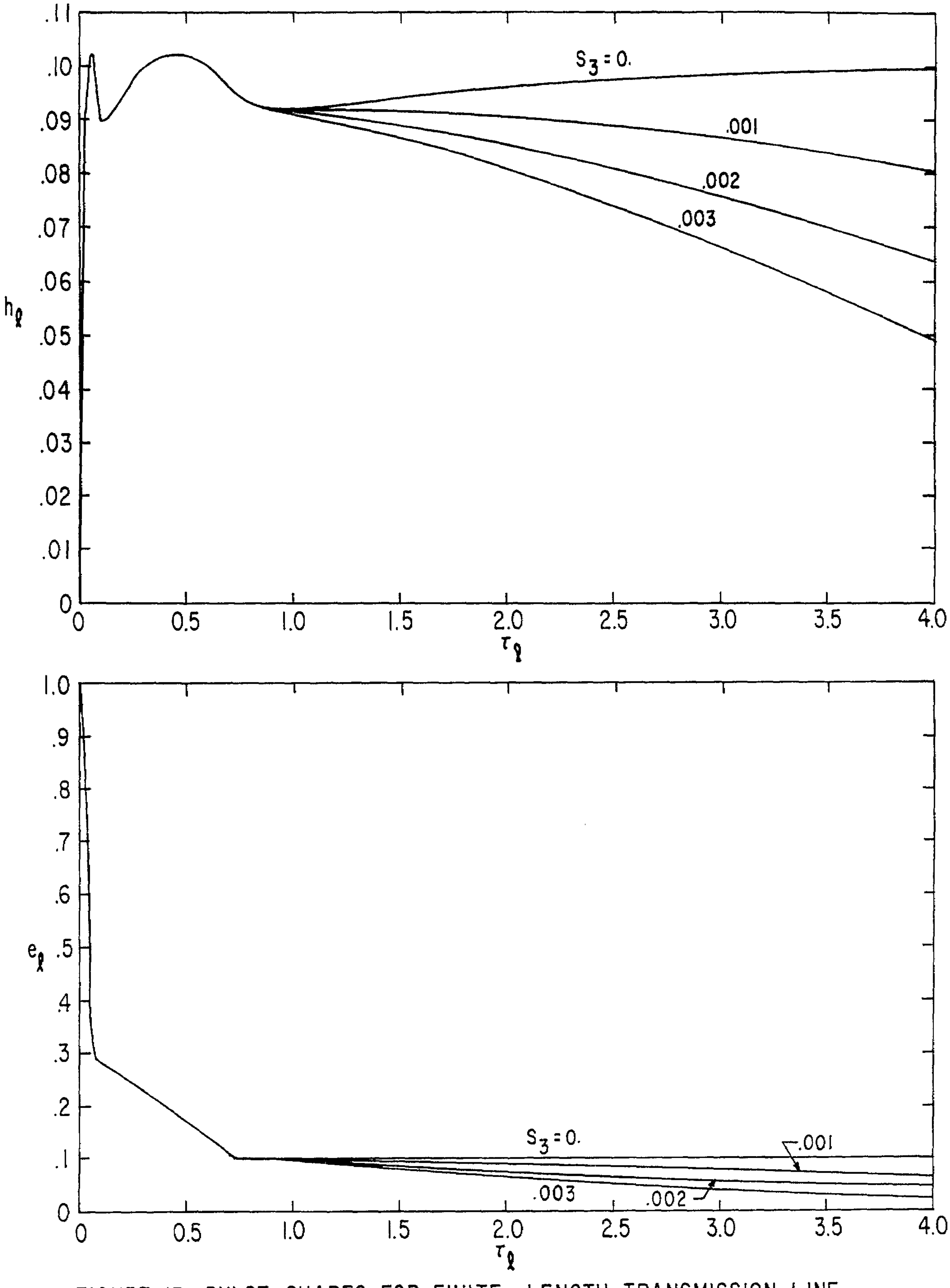


FIGURE 13. PULSE SHAPES FOR FINITE - LENGTH TRANSMISSION LINE WITH THREE CAPACITORS: $\frac{t_0}{t_\ell} = .01$, $v_2 = .3$, $S_2 = .03$, $v_3 = .1$

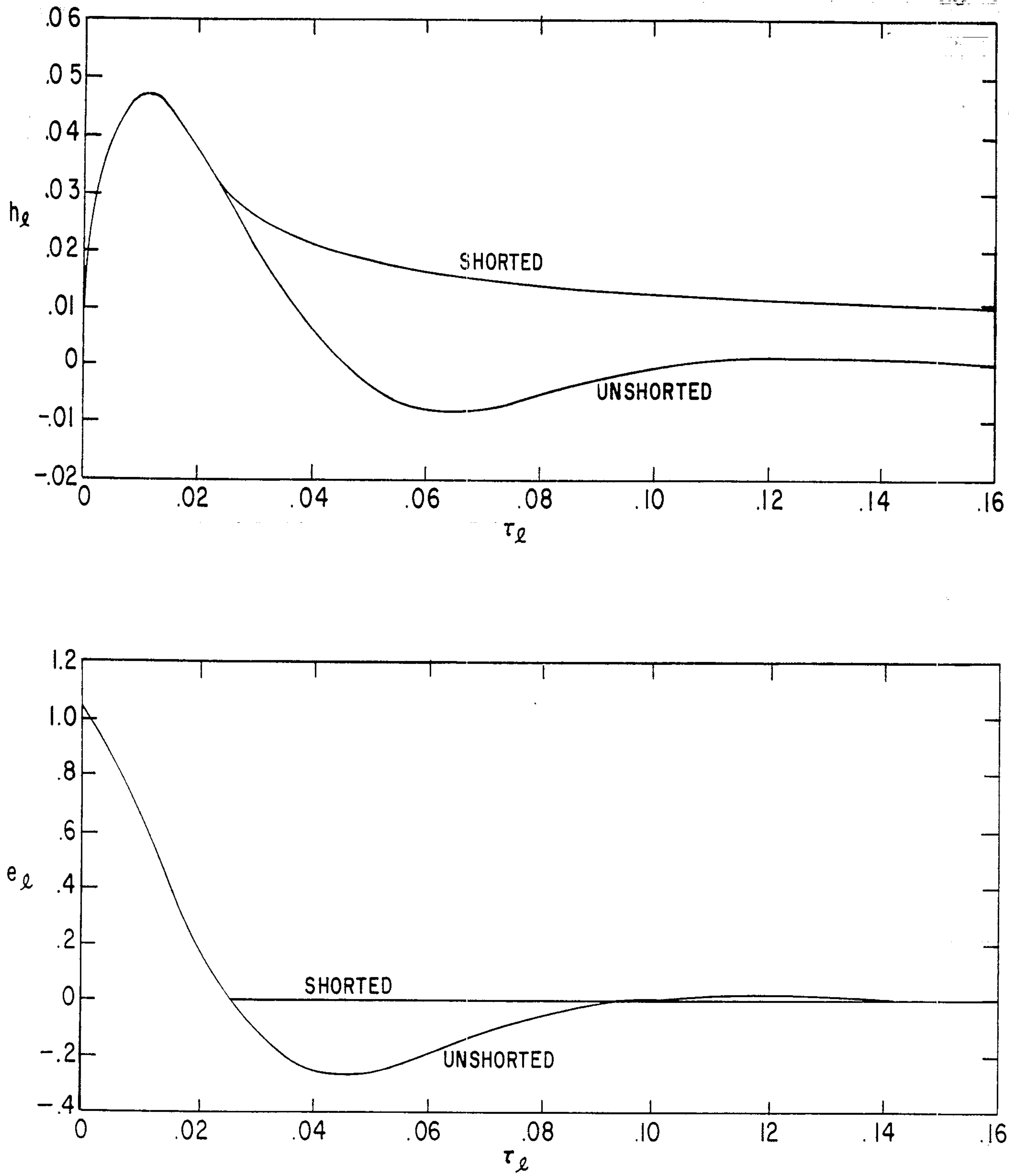


FIGURE 14. PULSE SHAPES FOR FINITE-LENGTH TRANSMISSION LINE WITH ONE CAPACITOR : $\frac{t_0}{t_\ell} = .001$

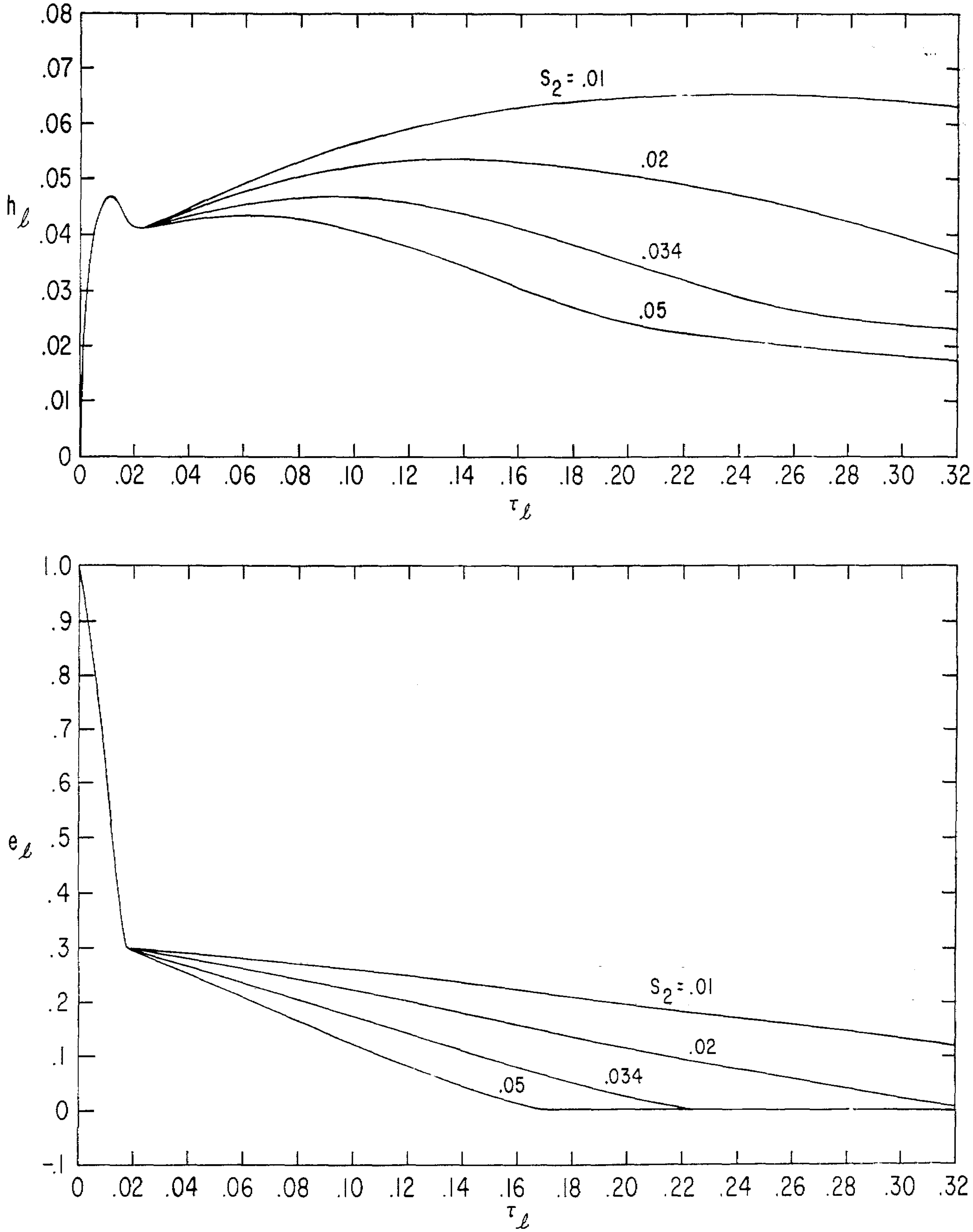
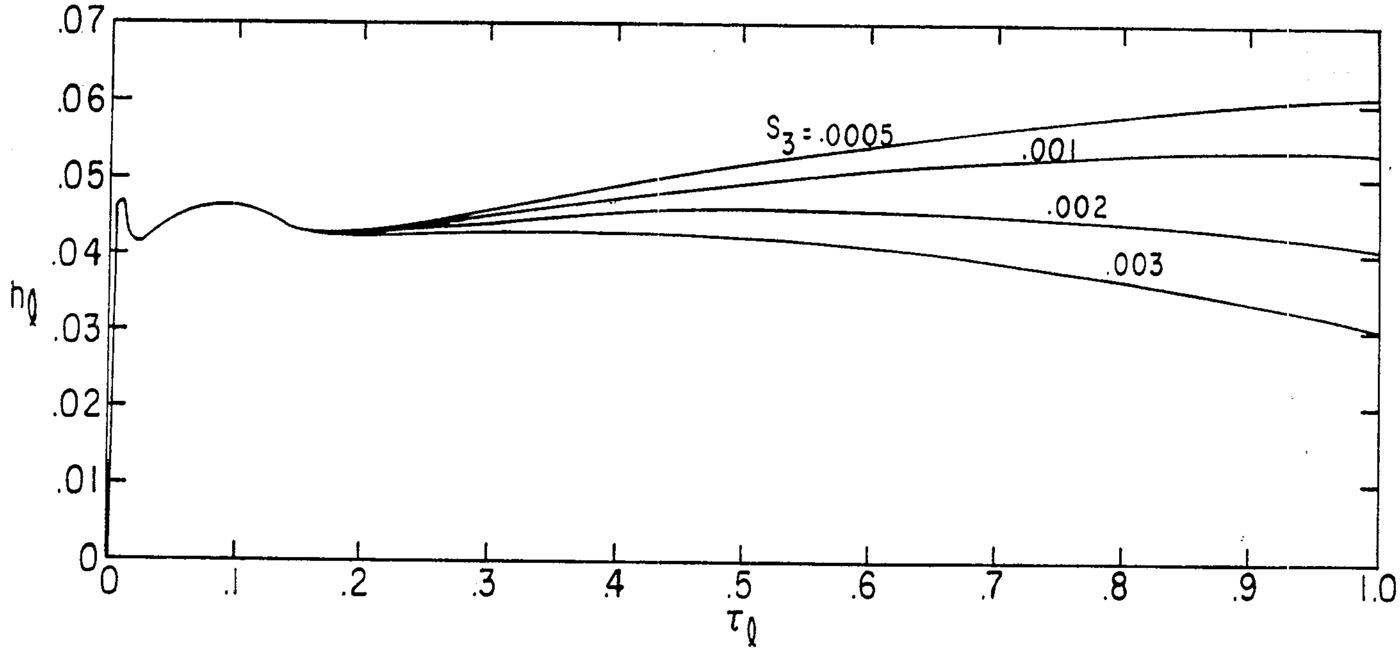


FIGURE 15. PULSE SHAPES FOR FINITE-LENGTH TRANSMISSION LINE WITH TWO CAPACITORS: $\frac{t_0}{t_\ell} = .001$, $v_2 = .3$



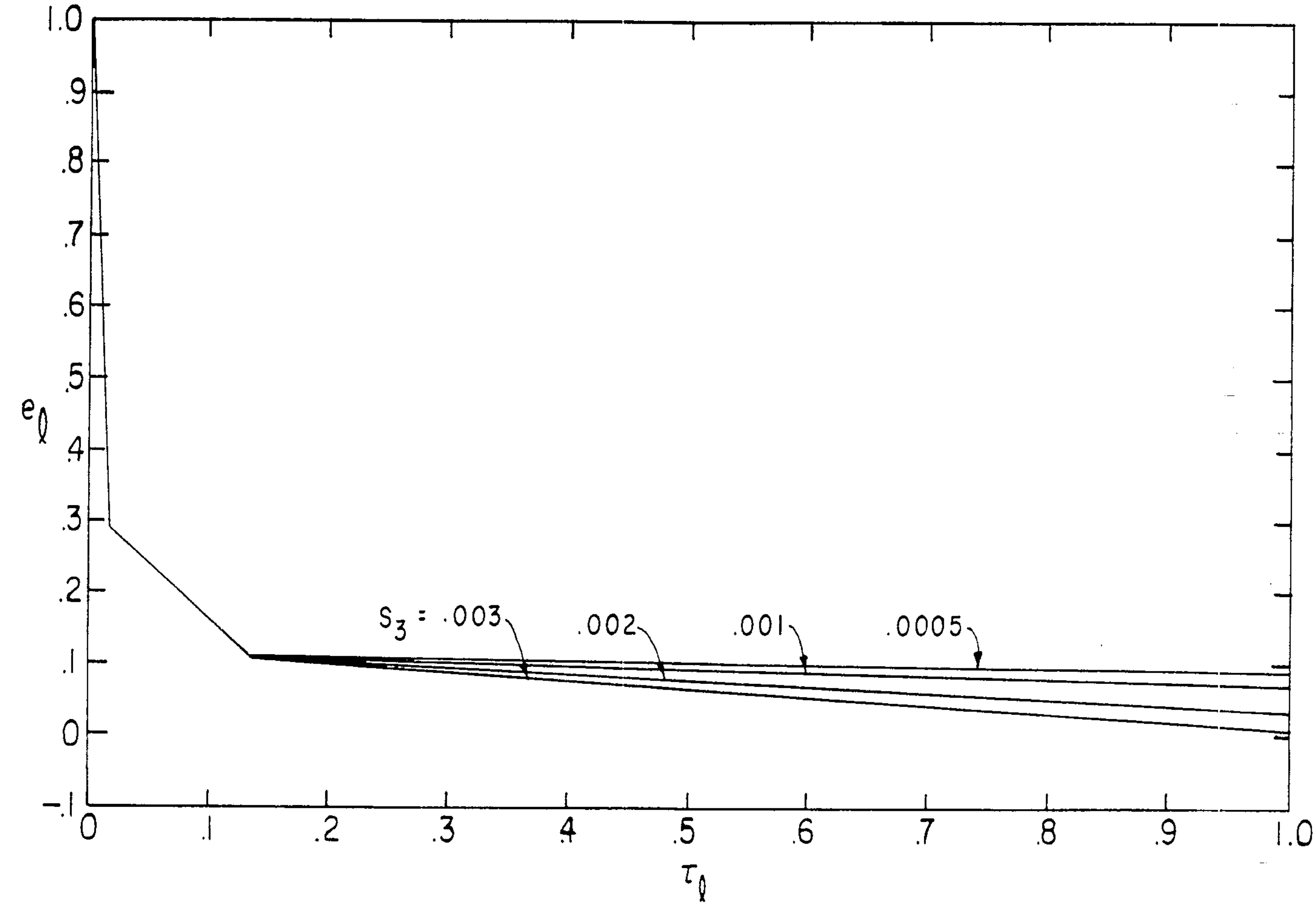


FIGURE 16. PULSE SHAPES FOR FINITE-LENGTH TRANSMISSION LINE WITH THREE CAPACITORS: $t_0/t_\ell = .001$, $v_2 = .3$, $S_2 = .034$, $v_3 = .11$

IV. Summary

By using multiple capacitive energy sources (perhaps plus an electrical short) which are switched onto a buried transmission line at appropriate times, one can achieve a considerable flexibility in shaping the waveforms for the electromagnetic fields. This can be accomplished without having to insert series resistors which would absorb energy. The present calculations have been pointed at achieving a roughly flat-topped waveform for the magnetic field, after the initial rise associated with the first capacitor. For a given number of capacitors the capacitances and initial voltages are chosen to give maximum length to this flat top with no more than about 10% ripple in the flat top. This choice of waveform is somewhat arbitrary and is used for illustration. Using the same calculational techniques one can extend these results to include other desirable waveforms.

Another possibility is that using this kind of multiple switching approach other kinds of lumped-element or distributed-element generators might be used, perhaps in combination with capacitors. Perhaps similar calculational techniques can be used for such generators by characterizing the generator in terms of an impulse response function and using a convolution integral, or in terms of some other mathematical operation. Note that for sufficiently small rise times in the waveforms the impulse response function for the simulator (buried transmission line plus materials above the ground surface) must be modified. Also some of the early-time behavior of the simulator might not be easily characterized by an impulse response function due to the distributed nature of the simulator.