eqn. 133

Sensor and Simulation Notes
Note XLVI
15 July 1967

The Single-Conductor, Planar, Uniform Surface Transmission Line, Driven from One End

Capt Carl E. Baum
Air Force Weapons Laboratory

Abstract

A transmission line consisting of a conducting sheet parallel to the ground surface is discussed as a possible simulator for placing fast-rising pulsed electromagnetic fields over a ground surface. The response characteristics of such a structure are considered for both a short circuit termination and a resistive termination which matches the transmission-line impedance at high frequencies. This type of simulator may be appropriate in the case of high ground conductivities.

PL/PA 10/27/94

PL 94-0916

Foreword

In this note the figures giving the graphical results for the pulse shapes are grouped together for convenience at the ends of their respective sections or subsections of the note. We would like to thank A1C Franklin Brewster, Jr., Mr. Robert Myers, A1C Antonio Regal, Mr. Robert Thompson, and Mr. John N. Wood for the numerical calculations and the resulting graphs.

Contents

I. Introduction

There are various problems associated with the simulation of the nuclear electromagnetic pulse from a surface burst over a ground based facility. One of these problems concerns the simulation of the fast-rising portions of the pulse, particularly if the rise times of interest are smaller than the transit times (in nonconducting air) across the surface extent of the facility. One approach to this problem is the Brewster angle wave matcher discussed in a previous note.[1] In this note we discuss another approach to the problem.

A general class of simulators, which we might consider for the fast-rising portions of the pulse, consists of some sort of transmission-line structure over the ground surface. The ground is actually a part of the transmission line. The electromagnetic characteristics of the ground influence the pulse shapes over the ground surface. There may be various types of such transmission lines with single or multiple conductors above the ground and single or multiple electrical sources at various positions along the transmission line. The transmission line might also be uniform or nonuniform along its length. Terminating impedances can also be added at the ends of the transmission line to try to improve the characteristics of the pulse shapes. One might calculate the electromagnetic characteristics of such transmission lines by considering longitudinal and transverse impedances and/or by considering the expansion of the electromagnetic fields on the structure.

In this note we consider one of the simpler types of transmission lines for placing fast-rising pulses over the ground. As illustrated in figure 1, the transmission line consists of a flat conducting plate (or perhaps wire grid with wires in the direction of current flow) of constant width, w, length, d, and height, h, above an assumed smooth ground surface. The permittivity, ε, permeability, μ, and conductivity, σ, of both the ground and the air (or other medium) above the ground are assumed to be scalar constants, independent of position. These parameters are also assumed to be real numbers and independent of frequency for the calculation of the time domain waveforms. A subscript, 1, on these parameters is used for the medium above the ground, and a subscript, 2, is used for the ground. We call the transmission line being considered a single-conductor, planar, uniform surface transmission line.

The characteristics of this transmission line are considered by solving the wave equation in the simple geometry. A cartesian (x,y,z) coordinate system is established as illustrated in figure 1. Variation with y is neglected by assuming $w \gg h$. Thus, a two-dimensional problem is left; x is the horizontal distance along the transmission line and z is the vertical distance above the ground surface. For convenience define

1. Capt Carl E. Baum, Sensor and Simulation Note XXXVII, The Brewster Angle Wave Matcher, March 1967.

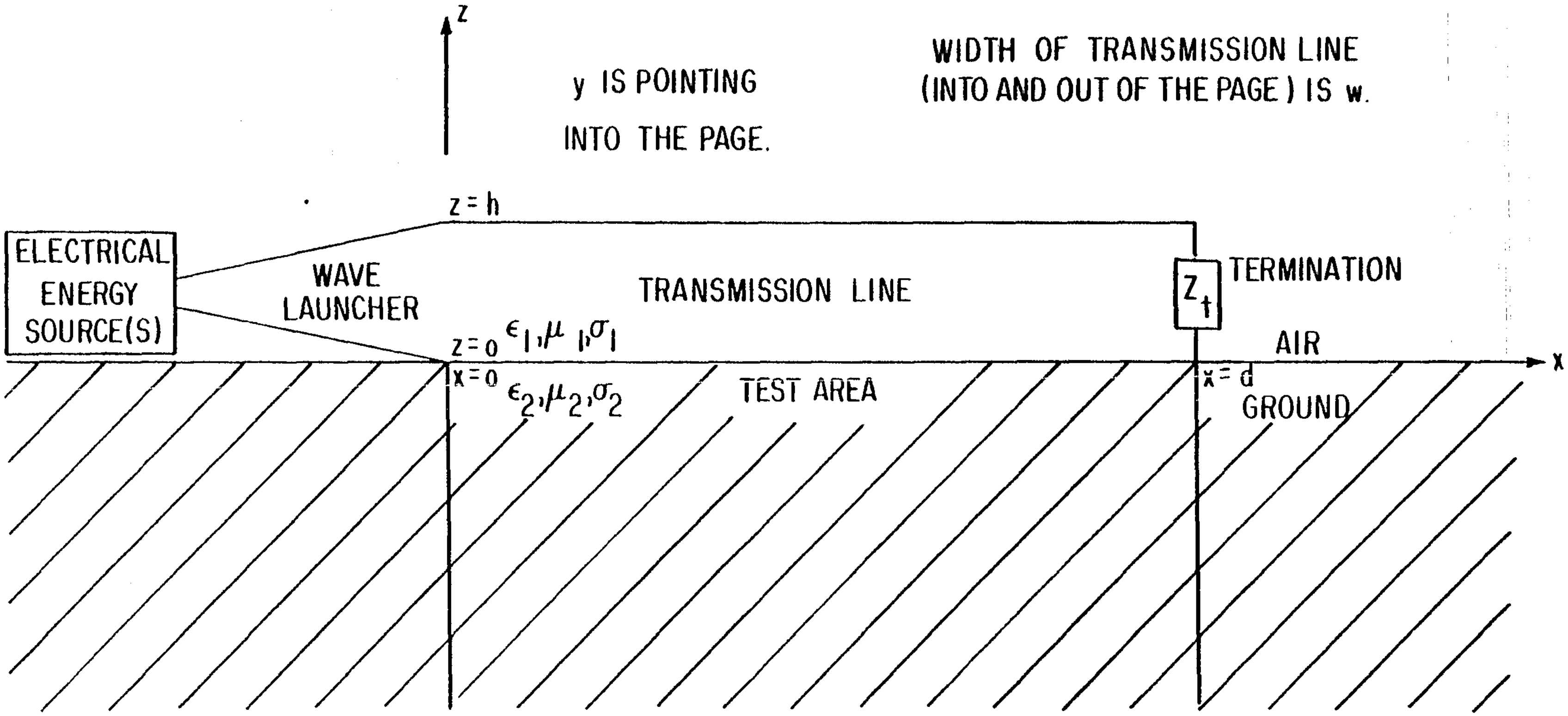


FIGURE 1. THE SINGLE-CONDUCTOR, PLANAR, UNIFORM SURFACE TRANSMISSION LINE, DRIVEN FROM ONE END

$$z' \equiv z-h \tag{1}$$

This may be more convenient to use than z in some cases. For this note we assume that a wave is launched onto the transmission line along the plane, $x = 0$, from some sort of electrical energy source(s) with appropriate wave launcher(s). A perfectly conducting sheet is assumed placed along the plane, $z' = 0$. (In a future note sources may also be distributed along the plane, $z' = 0$.) Some sort of termination for the wave is located along the plane, $x = d$. Note that conductors are used at each end of the transmission line to make electrical contact with the ground.

For the calculations in this note the parameters of the upper medium are assumed the same as those of free space. Certain approximations are made to simplify the results of the calculations, but these place some restrictions on the high-frequency response characteristics and on the allowable height, h, of the conducting sheet above the ground. One can vary h to vary the rate of attenuation of the high frequencies in the wave as the wave propagates over the ground surface. For a high conductivity ground h can be made rather small and so this type of surface transmission line may apply best for high σ_2. After solving for the electromagnetic fields in the frequency domain, the pulse shapes are determined for an assumed step function voltage or current at the input to the transmission line. This is accomplished for the most part by numerical inverse Fourier transforms, the resulting numerical errors being kept to less than about a percent in the computer calculations.[2] We first consider the case of a transmission line of effectively infinite length, followed by the case of a transmission line terminated in a short circuit or in a resistance which matches the transmission-line impedance for high frequencies.

2. Frank Sulkowski, Mathematics Note II, FORPLEX: A Program to Calculate Inverse Fourier Transforms, November 1966.

II. Boundary Value Problem

To calculate the response of this type of transmission line we expand the electromagnetic fields on the structure. Define a wave impedance as

$$Z \equiv \sqrt{\frac{s\mu}{\sigma+s\varepsilon}} \tag{2}$$

and a propagation constant as

$$\gamma \equiv \sqrt{s\mu(\sigma+s\varepsilon)} \tag{3}$$

where s is the Laplace transform variable. These two parameters, and other related impedances and propagation constants, are appropriately subscripted to apply to the two media. There are also the convenient relationships

$$s\mu = \gamma Z \tag{4}$$

and

$$\sigma+s\varepsilon = \frac{\gamma}{Z} \tag{5}$$

Since we assume a solution independent of y, then in Cartesian (x,y,z) coordinates the solution of the wave equation is of the form, $e^{\pm\gamma_x x \pm \gamma_z z}$, where γ_x and γ_z are propagation constants pertaining to the x and z directions, respectively. The propagation constants are related as

$$\gamma_x^2 + \gamma_z^2 = \gamma^2 \tag{6}$$

Assume a wave propagating in the +x direction along the transmission line. The electromagnetic fields then are of the form in medium 1 $(0 < z < h)$

$$\tilde{E}_{z_1} = \tilde{A}_1 e^{\gamma_{z_1} z' - \gamma_x x} + \tilde{B}_1 e^{-\gamma_{z_1} z' - \gamma_x x} \tag{7}$$

$$\tilde{E}_{x_1} = \tilde{A}_2 e^{\gamma_{z_1} z' - \gamma_x x} + \tilde{B}_2 e^{-\gamma_{z_1} z' - \gamma_x x} \tag{8}$$

and

$$\tilde{H}_{y_1} = \tilde{A}_3 e^{\gamma_{z_1} z' - \gamma_x x} + \tilde{B}_3 e^{-\gamma_{z_1} z' - \gamma_x x} \tag{9}$$

and in medium 2 $(z<0)$

$$\tilde{E}_{z_2} = \tilde{A}_4 e^{\gamma_{z_2} z - \gamma_x x} \tag{10}$$

$$\tilde{E}_{x_2} = \tilde{A}_5 e^{\gamma_{z_2} z - \gamma_x x} \tag{11}$$

and

$$\tilde{H}_{y_2} = \tilde{A}_6 e^{\gamma_{z_2} z - \gamma_x x} \tag{12}$$

where the $\tilde{A}$'s and $\tilde{B}$'s are coefficients to be determined. A tilde, $\sim$, is used over a quantity to indicate the Laplace transform of the quantity. The letter subscripts of the electromagnetic field symbols indicate which Cartesian component is symbolized. Note that the form of the solution in equations (7) through (12) is similar to that for a surface bound wave, except that two terms, with opposite signs on γ_{z_1}, are included for medium 1 because of the boundary at z = h. Also note that γ_x applies to the waves in both media and that no subscripts are used in this case.

To relate the coefficients expand

$$\nabla \times \vec{\tilde{H}} = (\sigma + s\varepsilon)\, \vec{\tilde{E}} \tag{13}$$

as

$$-\frac{\partial \tilde{H}_y}{\partial z} = (\sigma + s\varepsilon)\, \tilde{E}_x = \frac{\gamma}{Z}\, \tilde{E}_x \tag{14}$$

and

$$\frac{\partial \tilde{H}_y}{\partial x} = (\sigma + s\varepsilon)\, \tilde{E}_z = \frac{\gamma}{Z}\, \tilde{E}_z \tag{15}$$

Then the coefficients for the field components in both media can be related for medium 1 as

$$-\gamma_{z_1} \tilde{A}_3 = \frac{\gamma_1}{Z_1} \tilde{A}_2 \tag{16}$$

$$\gamma_{z_1} \tilde{B}_3 = \frac{\gamma_1}{Z_1} \tilde{B}_2 \tag{17}$$

$$-\gamma_x \tilde{A}_3 = \frac{\gamma_1}{Z_1} \tilde{A}_1 \tag{18}$$

and

$$-\gamma_x \tilde{B}_3 = \frac{\gamma_1}{Z_1} \tilde{B}_1 \tag{19}$$

and for medium 2 as

$$-\gamma_{z_2}\tilde{A}_6 = \frac{\gamma_2}{Z_2}\tilde{A}_5 \tag{20}$$

and

$$-\gamma_x\tilde{A}_6 = \frac{\gamma_2}{Z_2}\tilde{A}_4 \tag{21}$$

Now apply the boundary conditions that $\tilde{E}_x$ and $\tilde{H}_y$ are continuous at $z = 0$ giving

$$\tilde{A}_2 e^{-\gamma_{z_1}h} + \tilde{B}_2 e^{\gamma_{z_1}h} = \tilde{A}_5 \tag{22}$$

and

$$\tilde{A}_3 e^{-\gamma_{z_1}h} + \tilde{B}_3 e^{\gamma_{z_1}h} = \tilde{A}_6 \tag{23}$$

Substituting for $\tilde{A}_2$, $\tilde{B}_2$, and $\tilde{A}_5$ in equation (22) gives

$$\frac{\gamma_{z_1}}{\gamma_1} Z_1 \left[\tilde{A}_3 e^{-\gamma_{z_1}h} - \tilde{B}_3 e^{\gamma_{z_1}h}\right] = \frac{\gamma_{z_2}}{\gamma_2} Z_2\tilde{A}_6 \tag{24}$$

or

$$\tilde{A}_3 e^{-\gamma_{z_1}h} - \tilde{B}_3 e^{\gamma_{z_1}h} = \frac{\gamma_{z_2}}{\gamma_{z_1}} \frac{\gamma_1}{\gamma_2} \frac{Z_2}{Z_1}\tilde{A}_6 = \frac{\gamma_{z_2}}{\gamma_{z_1}}\left(\frac{\gamma_1}{\gamma_2}\right)^2 \frac{\mu_2}{\mu_1}\tilde{A}_6 \tag{25}$$

Combining equations (23) and (25) gives

$$\tilde{A}_3 = \frac{e^{\gamma_{z_1}h}}{2}\left[1 + \frac{\gamma_{z_2}}{\gamma_{z_1}}\left(\frac{\gamma_1}{\gamma_2}\right)^2 \frac{\mu_2}{\mu_1}\right]\tilde{A}_6 \tag{26}$$

and

$$\tilde{B}_3 = \frac{e^{-\gamma_{z_1}h}}{2}\left[1 - \frac{\gamma_{z_2}}{\gamma_{z_1}}\left(\frac{\gamma_1}{\gamma_2}\right)^2 \frac{\mu_2}{\mu_1}\right]\tilde{A}_6 \tag{27}$$

With equations (26) and (27) and equations (16) through (21) the field coefficients can be expressed in terms of one of the coefficients, $\tilde{A}_6$, and the various other parameters.

For convenience define

$$\tilde{A}_2 + \tilde{B}_2 \equiv \tilde{A} \tag{28}$$

and

$$\tilde{A}_3 + \tilde{B}_3 \equiv \tilde{B} \tag{29}$$

Then at z = h the field components are

$$\tilde{E}_{z_1} = -\frac{\gamma_x}{\gamma_1} Z_1 \tilde{B} e^{-\gamma_x x} \tag{30}$$

$$\tilde{E}_{x_1} = \tilde{A} e^{-\gamma_x x} \tag{31}$$

and

$$\tilde{H}_{y_1} = \tilde{B} e^{-\gamma_x x} \tag{32}$$

For the calculations in this note we assume a perfectly conducting sheet at z = h. This makes $\tilde{A} = 0$ implying, from equation (28) and (29) and equations (16) through (19),

$$\tilde{A}_2 = -\tilde{B}_2 = -\frac{\gamma_{z_1}}{\gamma_1} Z_1 \frac{\tilde{B}}{2} \tag{33}$$

$$\tilde{A}_3 = \tilde{B}_3 = \frac{\tilde{B}}{2} \tag{34}$$

and

$$\tilde{A}_1 = \tilde{B}_1 = -\frac{\gamma_x}{\gamma_1} Z_1 \frac{\tilde{B}}{2} \tag{35}$$

The fields in medium 1 then can be written as

$$\tilde{E}_{z_1} = -\frac{\gamma_x}{\gamma_1} Z_1 \tilde{B} \cosh (\gamma_{z_1} z') e^{-\gamma_x x} \tag{36}$$

$$\tilde{E}_{x_1} = -\frac{\gamma_{z_1}}{\gamma_1} Z_1 \tilde{B} \sinh (\gamma_{z_1} z') e^{-\gamma_x x} \tag{37}$$

and

$$\tilde{H}_{y_1} = \tilde{B} \cosh (\gamma_{z_1} z') e^{-\gamma_x x} \tag{38}$$

Including equations (20) through (23), the fields in medium 2 can then be written as

$$\tilde{E}_{z_2} = -\frac{\gamma_x}{\gamma_2} Z_2 \tilde{B} \cosh(\gamma_{z_1} h) e^{\gamma_{z_2} z - \gamma_x x} \tag{39}$$

$$\tilde{E}_{x_2} = \frac{\gamma_{z_1}}{\gamma_1} Z_1 \tilde{B} \sinh(\gamma_{z_1} h)\, e^{\gamma_{z_2} z - \gamma_x x} \tag{40}$$

and

$$\tilde{H}_{y_2} = \tilde{B} \cosh(\gamma_{z_1} h)\, e^{\gamma_{z_2} z - \gamma_x x} \tag{41}$$

All the field components now contain the same coefficient, B.

The relationship among the various propagation constants is not arbitrary. From equations (22) and (23) $\tilde{A}_5$ and $\tilde{A}_6$ can now be related as

$$\frac{\tilde{A}_5}{\tilde{A}_6} = \frac{\gamma_{z_1}}{\gamma_1} Z_1 \tanh(\gamma_{z_1} h) \tag{42}$$

These two coefficients are also related in equation (20) giving then

$$\frac{\gamma_{z_2}}{\gamma_2} Z_2 + \frac{\gamma_{z_1}}{\gamma_1} Z_1 \tanh(\gamma_{z_1} h) = 0 \tag{43}$$

or

$$1 + \frac{\mu_1}{\mu_2} \frac{\gamma_{z_1}}{\gamma_{z_2}} \left(\frac{\gamma_2}{\gamma_1}\right)^2 \tanh(\gamma_{z_1} h) = 0 \tag{44}$$

From equation (6) we have

$$\gamma_{z_1}^2 = \gamma_1^2 - \gamma_x^2 \tag{45}$$

and

$$\gamma_{z_2}^2 = \gamma_2^2 - \gamma_x^2 \tag{46}$$

so that substituting for γ_{z_1} and γ_{z_2} in equation (44), γ_x can be solved in terms of γ_1, γ_2, μ_1, μ_2, and h.

III. Propagation Constant of Transmission Line

Having a solution for the propagation constant in equations (44) through (46), we now simplify the results by making certain approximations. In doing this restrict the parameters of medium 1 to those of free space so that $\varepsilon_1 = \varepsilon_o$, $\mu_1 = \mu_o$, and $\sigma_1 = 0$. Also restrict the permeability of the lower medium as $\mu_2 = \mu_1$. Then define some convenient parameters. The relative dielectric constant of the ground is

$$\varepsilon_r \equiv \frac{\varepsilon_2}{\varepsilon_o} \tag{47}$$

the relaxation time of the ground is

$$t_r \equiv \frac{\varepsilon_2}{\sigma_2} \tag{48}$$

a modification of this relaxation time is

$$t_r' \equiv \frac{\varepsilon_o}{\sigma_2} \tag{49}$$

and the transit time from the ground to z = h is

$$t_h \equiv h\sqrt{\mu_o\varepsilon_o} = \frac{h}{c} \tag{50}$$

For convenience we include a table of the relaxation times for various conductivities and relative dielectric constants.

σ_2 (mhos/m) \ ε_r	10	20	40	80	
10^{-4}	885	1770	3540	7080	88.5
10^{-3}	88.5	177	354	708	8.85
10^{-2}	8.85	17.7	35.4	70.8	.885
10^{-1}	.885	1.77	3.54	7.08	.0885
4 (sea water)				.177	2.21×10^{-3}
	t_r (ns)				t_r' (ns)

Table I. Relaxation Times

Now assume that $|\gamma_{z_1} h| << 1$ so that $\tilde{E}_{z_1}$ and $\tilde{H}_{y_1}$ are approximately uniform with z. Then we have

$$\tanh(\gamma_{z_1} h) \approx \gamma_{z_1} h \tag{51}$$

so that equation (44) reduces to

$$1 + \frac{\mu_1}{\mu_2} \frac{\gamma_{z_1}^2 h}{\gamma_{z_2}} \left(\frac{\gamma_2}{\gamma_1}\right)^2 \approx 0 \tag{52}$$

and since we assume $\mu_1 = \mu_2$ this becomes

$$1 + \frac{\gamma_{z_1}^2 h}{\gamma_{z_2}} \left(\frac{\gamma_2}{\gamma_1}\right)^2 \approx 0 \tag{53}$$

This assumption of small $|\gamma_{z_1} h|$ restricts the field distribution in region 1 to a single dominant mode which is similar to the TEM mode on a lossless transmission line.

Rearranging equation (53) gives

$$\left(\frac{\gamma_{z_1}}{\gamma_1}\right)^4 \left(\gamma_2 h\right)^2 = \left(\frac{\gamma_{z_2}}{\gamma_2}\right)^2 \tag{54}$$

Substituting for γ_{z_1} and γ_{z_2} gives

$$\left[1 - \left(\frac{\gamma_x}{\gamma_1}\right)^2\right]^2 \left(\gamma_2 h\right)^2 \approx 1 - \left(\frac{\gamma_x}{\gamma_2}\right)^2 \tag{55}$$

or

$$\left(\frac{\gamma_x}{\gamma_1}\right)^4 + \left[\frac{1}{(\gamma_2 h)^2} \left(\frac{\gamma_1}{\gamma_2}\right)^2 - 2\right] \left(\frac{\gamma_x}{\gamma_1}\right)^2 + 1 - \frac{1}{(\gamma_2 h)^2} \approx 0 \tag{56}$$

Solving this as a quadratic equation gives

$$\left(\frac{\gamma_x}{\gamma_1}\right)^2 \approx 1 - \frac{1}{2(\gamma_2 h)^2} \left(\frac{\gamma_1}{\gamma_2}\right)^2 + \left\{\left[\frac{1}{2(\gamma_2 h)^2} \left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^2 + \frac{1}{(\gamma_2 h)^2} \left[1 - \left(\frac{\gamma_1}{\gamma_2}\right)^2\right]\right\}^{1/2} \tag{57}$$

or

$$\left(\frac{\gamma_x}{\gamma_1}\right)^2 \approx 1 - \frac{1}{2(\gamma_2 h)^2} \left(\frac{\gamma_1}{\gamma_2}\right)^2 + \frac{1}{\gamma_2 h} \left[1 - \left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^{1/2} \left\{1 + \frac{1}{4(\gamma_2 h)^2} \left(\frac{\gamma_1}{\gamma_2}\right)^4 \left[1 - \left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^{-1}\right\}^{1/2} \tag{58}$$

The plus sign has been used for the square root in the quadratic solution so that γ_x will apply to an attenuating wave.

To expand the square root consider the approximation that

$$\left| \frac{1}{4(\gamma_2 h)^2} \left(\frac{\gamma_1}{\gamma_2}\right)^4 \left[1 - \left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^{-1} \right| << 1 \tag{59}$$

for all frequencies of interest. We call this particular restriction on the solution as restriction 1. Note that

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2 = \epsilon_r \left[\frac{1}{st_r} + 1\right] \tag{60}$$

For $\epsilon_r >> 1$ (which we assume), then for all frequencies

$$\left| \left(\frac{\gamma_2}{\gamma_1}\right)^2 \right| >> 1 \tag{61}$$

and restriction 1 can be simplified to

$$\left| \frac{1}{4(\gamma_2 h)^2} \left(\frac{\gamma_1}{\gamma_2}\right)^4 \right| << 1 \tag{62}$$

Substituting for γ_1 and γ_2 gives

$$\left| \frac{st_r \left(\frac{t_r}{t_h}\right)^2}{4\epsilon_r^3 (st_r + 1)^3} \right| << 1 \tag{63}$$

Substituting $j\omega$ for s, the expression on the left of equation (63) has a maximum at $\omega t_r = 1/\sqrt{2}$. Substituting this value restriction 1 becomes

$$\frac{1}{10.4\epsilon_r^3} \left(\frac{t_r}{t_h}\right)^2 << 1 \tag{64}$$

so that if this restriction is met it applies for all frequencies. Large ϵ_r, σ_2, and h help to meet this restriction. Note, however, that if all frequencies of interest are much less than t_r^{-1}, then restriction 1 can be relaxed from its form in equation (64).

Now expand equation (58) as

$$\left(\frac{\gamma_x}{\gamma_1}\right)^2 \approx 1 - \frac{1}{2(\gamma_2 h)^2}\left(\frac{\gamma_1}{\gamma_2}\right)^2 + \frac{1}{\gamma_2 h}\left[1-\left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^{1/2}\left\{1+\frac{1}{8(\gamma_2 h)^2}\left(\frac{\gamma_1}{\gamma_2}\right)^4\left[1-\left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^{-1}\right\} \quad (65)$$

Simplify this further by ignoring the second term in the braces compared to 1. Then take the ratio of the second to the third term on the right side of the equation giving

$$\left\{\frac{1}{2(\gamma_2 h)^2}\left(\frac{\gamma_1}{\gamma_2}\right)^2\right\}\left\{\frac{1}{\gamma_2 h}\left[1-\left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^{1/2}\right\}^{-1} = \frac{1}{2\gamma_2 h}\left(\frac{\gamma_1}{\gamma_2}\right)^2\left[1-\left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^{-1/2} \quad (66)$$

However, the magnitude of the right side of equation (66) is just the square root of the left side of equation (59) and is therefore small compared to 1. Thus, applying restriction 1, equation (65) simplifies to

$$\left(\frac{\gamma_x}{\gamma_1}\right)^2 \approx 1 + \frac{1}{\gamma_2 h}\left[1-\left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^{1/2} \quad (67)$$

It is this form for γ_x that we use for the calculations.

Return now to the assumption that $|\gamma_{z_1} h| \ll 1$ which we call restriction 2. First solve for $\gamma_{z_1} h$ as

$$\left(\gamma_{z_1} h\right)^2 = \left(\frac{\gamma_{z_1}}{\gamma_1}\right)^2 (\gamma_1 h)^2 = \left[1-\left(\frac{\gamma_x}{\gamma_1}\right)^2\right](\gamma_1 h)^2 \quad (68)$$

Substituting from equation (67) gives

$$(\gamma_{z_1} h)^2 = -\gamma_1 h\left(\frac{\gamma_1}{\gamma_2}\right)\left[1-\left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^{1/2} \quad (69)$$

Using the result of equation (61), restriction 2 becomes

$$\left|\left(-\frac{\gamma_1^2 h}{\gamma_2}\right)^{1/2}\right| \ll 1 \quad (70)$$

or

$$\left|\frac{s^{3/4} t_h^{1/2} t_r^{1/4}}{[\varepsilon_r(1+st_r)]^{1/4}}\right| \ll 1 \quad (71)$$

Replacing $1+st_r$ by st_r increases the left side giving

$$\left| \left(\frac{st_h}{\sqrt{\varepsilon_r}} \right)^{1/2} \right| << 1 \tag{72}$$

In terms of ω restriction 2 is then

$$\omega << \frac{\sqrt{\varepsilon_r}}{t_h} \tag{73}$$

Thus, restriction 2 is a restriction on how large ω can be for valid results. Note that if frequencies of interest are much less than t_r^{-1}, then restriction 2 can be relaxed from its form in equation (73).

Having solved for γ_x in equation (67), the other propagation constants can be obtained. These are

$$\left(\frac{\gamma_{z_1}}{\gamma_1} \right)^2 = 1 - \left(\frac{\gamma_x}{\gamma_1} \right)^2 \simeq - \frac{1}{\gamma_2 h} \left[1 - \left(\frac{\gamma_1}{\gamma_2} \right)^2 \right]^{1/2} \tag{74}$$

and

$$\left(\frac{\gamma_{z_2}}{\gamma_2} \right)^2 = 1 - \left(\frac{\gamma_x}{\gamma_2} \right)^2 \simeq 1 - \left(\frac{\gamma_1}{\gamma_2} \right)^2 \left\{ 1 + \frac{1}{\gamma_2 h} \left[1 - \left(\frac{\gamma_1}{\gamma_2} \right)^2 \right]^{1/2} \right\} \tag{75}$$

The field components in equations (36) through (41) can then be written for small $|\gamma_{z_1} h|$ as

$$\tilde{E}_{z_1} \simeq - \frac{\gamma_x}{\gamma_1} Z_1 \tilde{B} e^{-\gamma_x x} \tag{76}$$

$$\tilde{E}_{x_1} \simeq - \frac{\gamma_{z_1}^2 z'}{\gamma_1} Z_1 \tilde{B} e^{-\gamma_x x} \tag{77}$$

$$\tilde{H}_{y_1} \simeq \tilde{B} e^{-\gamma_x x} \tag{78}$$

$$\tilde{E}_{z_2} \simeq - \frac{\gamma_x}{\gamma_2} Z_2 \tilde{B} e^{\gamma_{z_2} z - \gamma_x x} \tag{79}$$

$$\tilde{E}_{x_2} \approx \frac{\gamma_{z_1}^2 h}{\gamma_1} Z_1 \tilde{B}\, e^{\gamma_{z_2} z - \gamma_x x} \tag{80}$$

and

$$\tilde{H}_{y_2} \approx \tilde{B}\, e^{\gamma_{z_2} z - \gamma_x x} \tag{81}$$

Substituting for γ_{z_1}, γ_{z_2}, and γ_x all the field components are solved in terms of ideally known parameters and one arbitrary coefficient, $\tilde{B}$.

IV. Impedance of Transmission Line

Now consider the impedance of this surface transmission line for the case that it is assumed infinitely long. At x = 0 the current into the conducting plate (at z=h) is

$$\tilde{I} = w\, \tilde{H}_{y_1}\Big|_{\substack{x=0\\z=h}} = w\, \tilde{B} \tag{82}$$

For the voltage driving the transmission line we have for positive z

$$\tilde{V}_1 = -\int_{-h}^{0} \tilde{E}_{z_1}\Big|_{x=0} dz' = \frac{\gamma_x}{\gamma_1} Z_1 \tilde{B} \int_{-h}^{0} \cosh\,(\gamma_{z_1} z')\, dz'$$

$$= \frac{\gamma_x}{\gamma_1 \gamma_{z_1}} Z_1 \tilde{B} \sinh\,(\gamma_{z_1} h) \tag{83}$$

For $|\gamma_{z_1} h| << 1$ this becomes

$$\tilde{V}_1 \simeq \frac{\gamma_x}{\gamma_1} Z_1 h \tilde{B} \tag{84}$$

For negative z calculate a similar parameter as

$$\tilde{V}_2 = -\int_{-\infty}^{0} \tilde{E}_{z_2}\Big|_{x=0} dz = \frac{\gamma_x}{\gamma_2} Z_2 \tilde{B} \cosh\,(\gamma_{z_1} h) \int_{-\infty}^{0} e^{\gamma_{z_2} z}\, dz$$

$$= \frac{\gamma_x}{\gamma_2 \gamma_{z_2}} Z_2 \tilde{B} \cosh\,(\gamma_{z_1} h) \tag{85}$$

which for $|\gamma_{z_1} h| << 1$ is

$$\tilde{V}_2 \simeq \frac{\gamma_x}{\gamma_2 \gamma_{z_2}} Z_2 \tilde{B} \tag{86}$$

The ratio of the two parameters in equations (85) and (83) is

$$\frac{\tilde{V}_2}{\tilde{V}_1} = \frac{\gamma_1 \gamma_{z_1}}{\gamma_2 \gamma_{z_2}} \frac{Z_2}{Z_1} \coth (\gamma_{z_1} h) \tag{87}$$

Substituting from equation (44) this becomes

$$\frac{\tilde{V}_2}{\tilde{V}_1} = - \frac{\mu_1}{\mu_2} \left(\frac{\gamma_{z_1}}{\gamma_{z_2}}\right)^2 \frac{\gamma_2}{\gamma_1} \frac{Z_2}{Z_1} = - \left(\frac{\gamma_{z_1}}{\gamma_{z_2}}\right)^2 \tag{88}$$

Substituting from equations (74) and (75) then

$$\frac{\tilde{V}_2}{\tilde{V}_1} \simeq \left(\frac{\gamma_1}{\gamma_2}\right)^2 \frac{1}{\gamma_2 h} \left[1-\left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^{1/2} \left\{1 - \left(\frac{\gamma_1}{\gamma_2}\right)^2 \left\{1 + \frac{1}{\gamma_2 h} \left[1-\left(\frac{\gamma_1}{\gamma_2}\right)^2\right]^{1/2}\right\}\right\}^{-1} \tag{89}$$

and using the approximation of equation (61) this is

$$\frac{\tilde{V}_2}{\tilde{V}_1} \simeq \frac{1}{\gamma_2 h} \left(\frac{\gamma_1}{\gamma_2}\right)^2 \left\{1 - \frac{1}{\gamma_2 h} \left(\frac{\gamma_1}{\gamma_2}\right)^2\right\}^{-1} \tag{90}$$

Using restriction 1 as expressed in equation (62) we then have

$$\left|\frac{\tilde{V}_2}{\tilde{V}_1}\right| << 1 \tag{91}$$

We then neglect $\tilde{V}_2$. The presence of the vertical conductors in the ground, shorting out $\tilde{E}_{z_2}$ at x=0, should have no significant effect on the impedance of the transmission line.

The impedance of the surface transmission line, with the length assumed infinite is then

$$Z_{L_\infty} \simeq \frac{\tilde{V}_1}{\tilde{I}} \simeq \frac{h}{w} \frac{\gamma_x}{\gamma_1} Z_1 \tag{92}$$

Note also that

$$Z_{L_\infty} \simeq - \frac{h}{w} \frac{\tilde{E}_{z_1}}{\tilde{H}_{y_1}} \tag{93}$$

Substituting from equation (67) we have

$$Z_{L_\infty} \approx \left\{ 1 + \frac{1}{\gamma_2 h} \left[1 - \left(\frac{\gamma_1}{\gamma_2} \right)^2 \right]^{1/2} \right\}^{1/2} \frac{h}{w} Z_1 \tag{94}$$

which we use for the calculations in this note.

V. Response of Infinite-Length Transmission Line

As the first case for the response of this surface transmission line let the transmission line be infinitely long. Define some additional parameters. The transit time to the position of interest is

$$t_x \equiv \frac{x}{c} \tag{95}$$

Another useful time is

$$t_o \equiv t_r' \left(\frac{t_x}{t_h}\right)^2 = t_r' \left(\frac{x}{h}\right)^2 \tag{96}$$

from which we define a normalized Laplace transform variable as

$$s_o \equiv st_o \tag{97}$$

and a normalized time as

$$\tau_o \equiv \frac{t-t_x}{t_o} \tag{98}$$

For convenience collect together some of the parameters as

$$p_1 \equiv \frac{t_r' t_x}{t_h^2} \tag{99}$$

and

$$p_2 \equiv \varepsilon_r \left(\frac{t_h}{t_x}\right)^2 = \varepsilon_r \left(\frac{h}{x}\right)^2 \tag{100}$$

Now apply a step-function voltage at $x=0$ and $t=0$. For this case then define an appropriate normalized voltage or vertical electric field as

$$\tilde{e}_{z_o}(s_o) \equiv \frac{e^{-\gamma_x x + \gamma_1 x}}{s_o} \tag{101}$$

Note the term, $\gamma_1 x$, in the exponential. This is included to shift the time scale so that the pulse starts at $\tau_o = 0$, the arrival time at x. Next define an appropriate normalized current or magnetic field as

$$\tilde{h}_o(s_o) \equiv \frac{h}{w} \frac{Z_1}{Z_{L_\infty}} \tilde{e}_{z_o}(s_o) = \frac{\gamma_1}{\gamma_x} \frac{e^{-\gamma_x x+\gamma_1 x}}{s_o} \tag{102}$$

Note that since we are assuming that $\varepsilon_1 = \varepsilon_o$, $\mu_1 = \mu_o$, and $\sigma_1 = 0$, then Z_1 is independent of s and is just the wave impedance of free space. For the calculations of this section rewrite equation (67) substituting for γ_1 and γ_2 from equation (3) giving

$$\left(\frac{\gamma_x}{\gamma_1}\right)^2 \approx 1 + \frac{1}{\sqrt{s\mu_o(\sigma_2+s\varepsilon_2)h}} \left[1 - \frac{s\varepsilon_o}{\sigma_2+s\varepsilon_2}\right]^{1/2}$$

$$= 1 + \left(\frac{t_r'}{t_h^2 s}\right)^{1/2} \frac{1}{1+st_r} \left[1+ \left(1-\frac{1}{\varepsilon_r}\right) st_r\right]^{1/2}$$

$$= 1 = \frac{t_r' t_x}{t_h^2} s_o^{-1/2} \frac{1}{1+s_o\left(\frac{h}{x}\right)^2 \varepsilon_r} \left[1+ \left(1-\frac{1}{\varepsilon_r}\right) s_o \left(\frac{h}{x}\right)^2 \varepsilon_r\right]^{1/2}$$

$$= 1 + \frac{p_1 s_o^{-1/2}}{1+s_o p_2} \left[1+ \left(1-\frac{1}{\varepsilon_r}\right) s_o p_2\right]^{1/2} \qquad (103)$$

For the terms in the exponential we have

$$-\gamma_x x+\gamma_1 x = -st_x \left(\frac{\gamma_x}{\gamma_1} -1\right) = -\frac{s_o}{p_1}\left(\frac{\gamma_x}{\gamma_1} - 1\right) \qquad (104)$$

Using the initial value theorem of the Laplace transform we can obtain an analytic expression for the initial rise of the time-domain waveforms as

$$e_{z_o}(0+) = \lim_{s_o \to \infty} s_o \tilde{e}_{z_o}(s_o) = e^{-\frac{1}{2}\sqrt{\frac{1-\frac{1}{\varepsilon_r}}{p_2}}} \qquad (105)$$

and

$$h_o(0+) = \lim_{s_o \to \infty} s_o \tilde{h}_o(s_o) = e^{-\frac{1}{2}\sqrt{\frac{1-\frac{1}{\varepsilon_r}}{p_2}}} \qquad (106)$$

Note that the initial rise is the same for both waveforms.

There is a special limiting case for $p_1 = 0$, in which

$$\left(\frac{\gamma_x}{\gamma_1}\right)^2\Bigg|_{p_1=0} = 1 \tag{107}$$

but

$$-\gamma_x x+\gamma_1 x\Bigg|_{p_1=0} = -\frac{1}{2}\frac{s_o^{1/2}}{1+s_o p_2}\left[1+\left(1-\frac{1}{\varepsilon_r}\right)s_o p_2\right]^{1/2} \tag{108}$$

For p_2 also zero then

$$\tilde{h}_o(s_o)\Bigg|_{\substack{p_1=0\\p_2=0}} = \tilde{e}_o(s_o)\Bigg|_{\substack{p_1=0\\p_2=0}} = \frac{e^{-\frac{\sqrt{s_o}}{2}}}{s_o} \tag{109}$$

which has the solution

$$h_o(\tau_o)\Bigg|_{\substack{p_1=0\\p_2=0}} = e_o(\tau_o)\Bigg|_{\substack{p_1=0\\p_2=0}} = \text{erfc}\left(\frac{1}{4\sqrt{\tau_o}}\right) \tag{110}$$

This special simple solution applies for frequencies of interest, ω, such that $\omega >> t_r^{-1}$ so that $|s_o p_2| << 1$, and $\omega >> t_r'/t_h^2$ so that $|s_o^{1/2}| >> p_1$. For such a frequency band to exist it is necessary that $t_h >> t_r' \sqrt{\varepsilon_r}$.

In solving for the propagation constant in section III certain restrictions were placed on various parameters. In terms of the parameters used in this section restriction 1 is

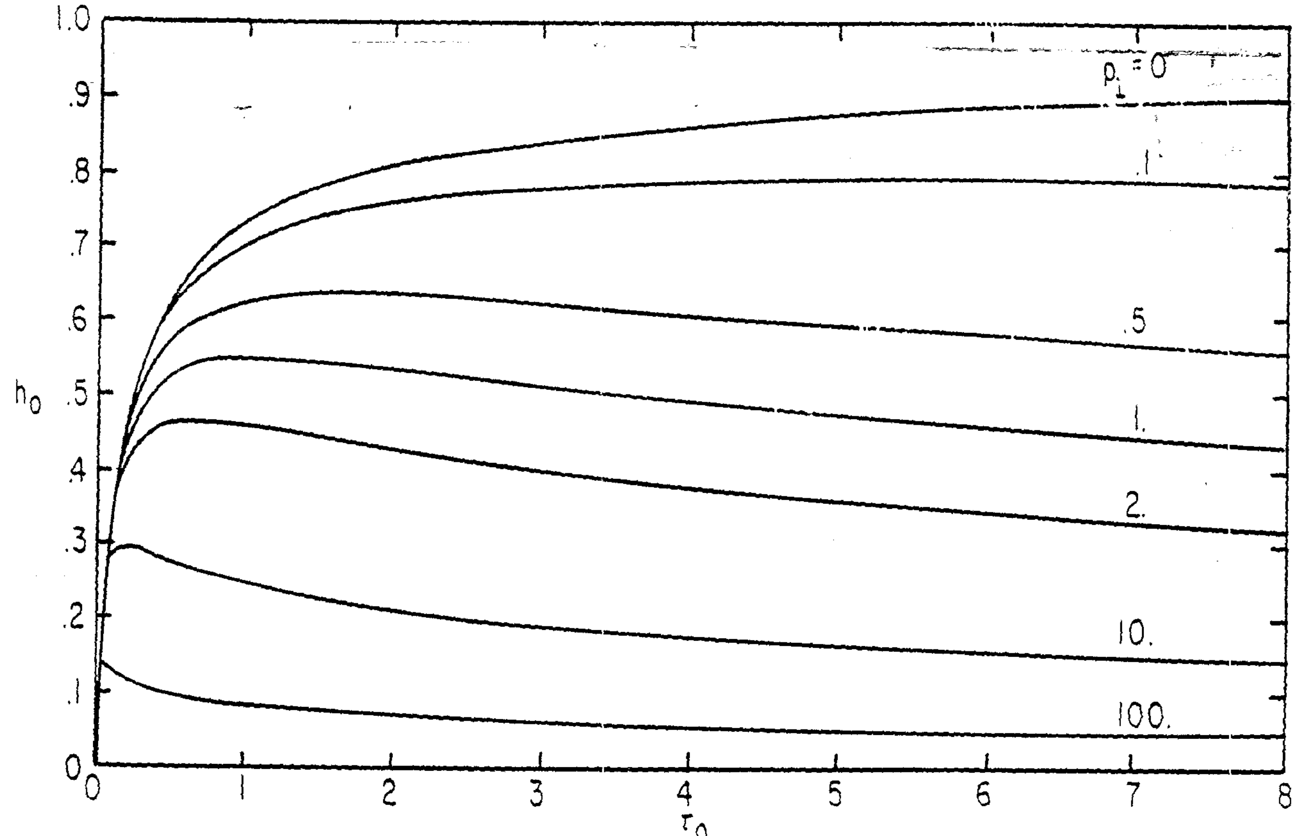
$$\frac{1}{10.4\varepsilon_r^2} p_1^2 p_2 << 1 \tag{111}$$

and restriction 2 restricts normalized frequencies of interest to

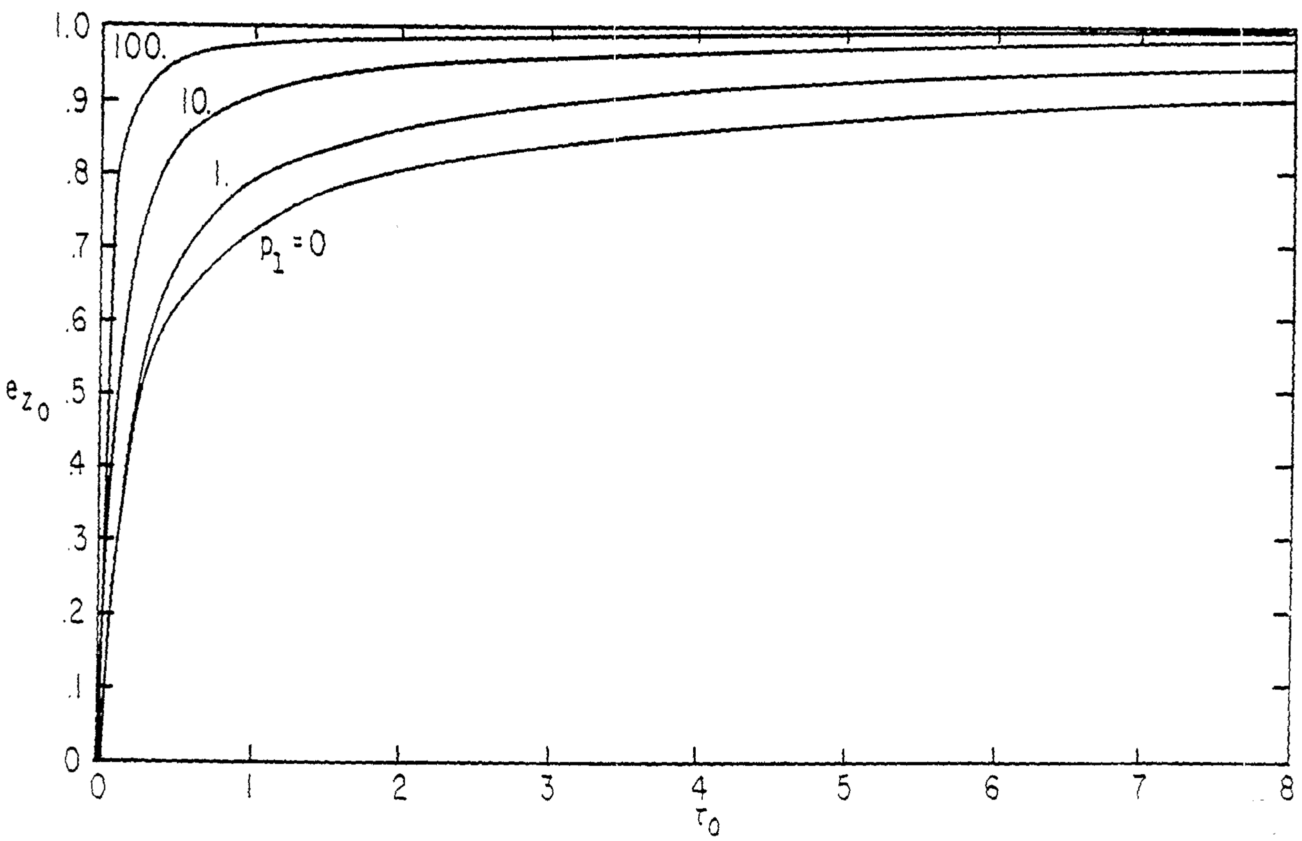
$$|s_o| = |\omega t_o| << \varepsilon_r \frac{p_1}{\sqrt{p_2}} \tag{112}$$

For the graphs the expressions in equations (101) through (104) are used as though they were exact, but the reader should take the restrictions into account when using the graphs. Note that if restriction 1, as in equation (111), is exceeded one can confine the frequencies of interest to much less than t_r^{-1}, relaxing the restriction to some desired degree.

The time-domain response characteristics of the infinite-length transmission line are plotted in figures 2 through 8. In figure 2 the pulse shapes are plotted for the limiting case of $p_2=0$ for several values of p_1. In figures 3 through 6 the pulse shapes are plotted with both p_1 and p_2 varied and for two values of ϵ_r. Define $h_{o_{max}}$ as the maximum value of the normalized current or magnetic field and t_{max} as the time of this maximum. These two parameters are plotted in figures 7 and 8 versus p_1 for several values of p_2 and two values of ϵ_r. Note that there is little difference between the pulse shapes for $\epsilon_r=10$ and $\epsilon_r=80$.

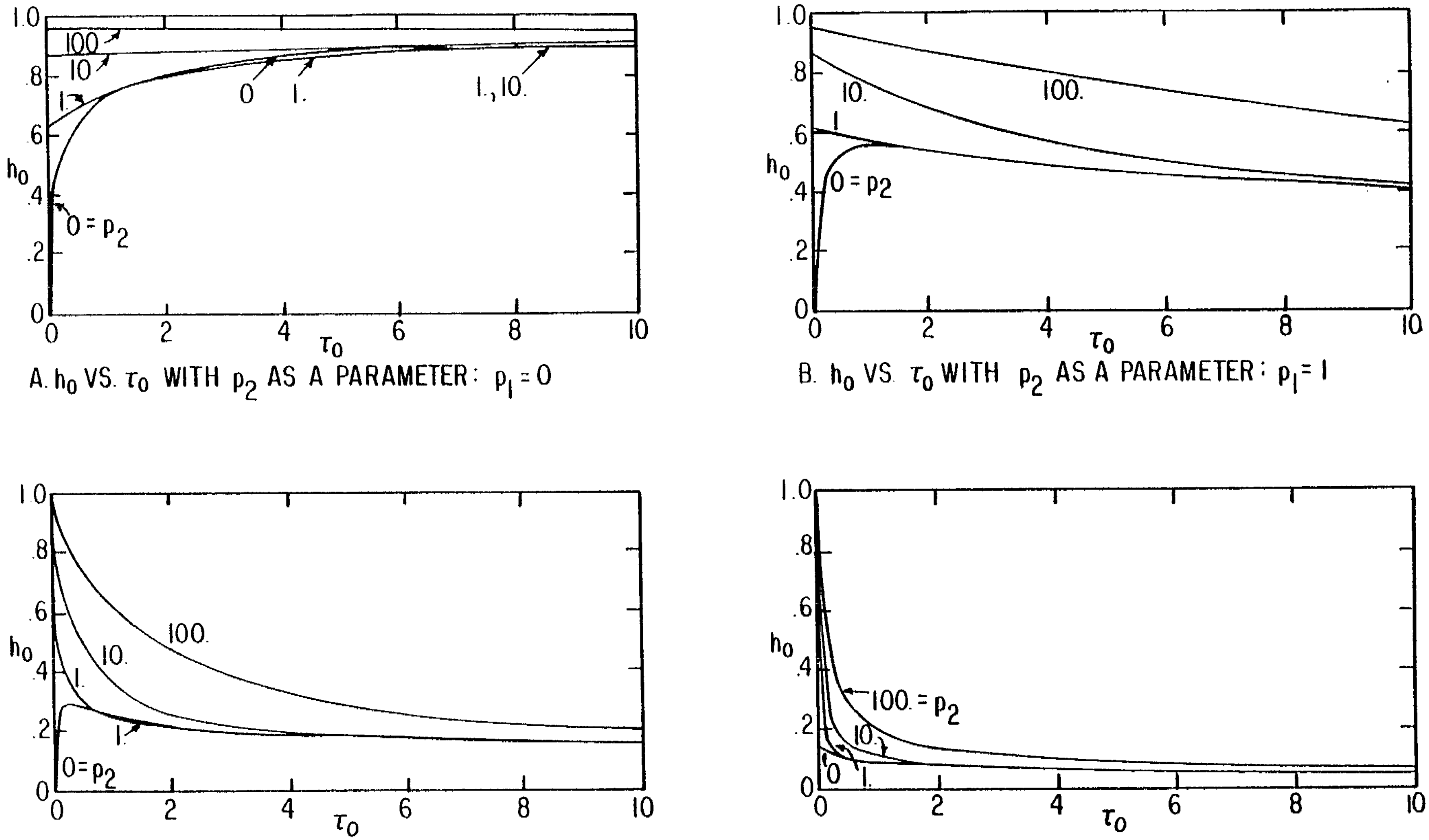


A. h_0 VS τ_0 WITH p_1 AS A PARAMETER



B. e_{z_0} VS τ_0 WITH p_1 AS A PARAMETER

FIGURE 2. PULSE SHAPES FOR STEP VOLTAGE ON INFINITE LENGTH TRANSMISSION LINE: $p_2 = 0$



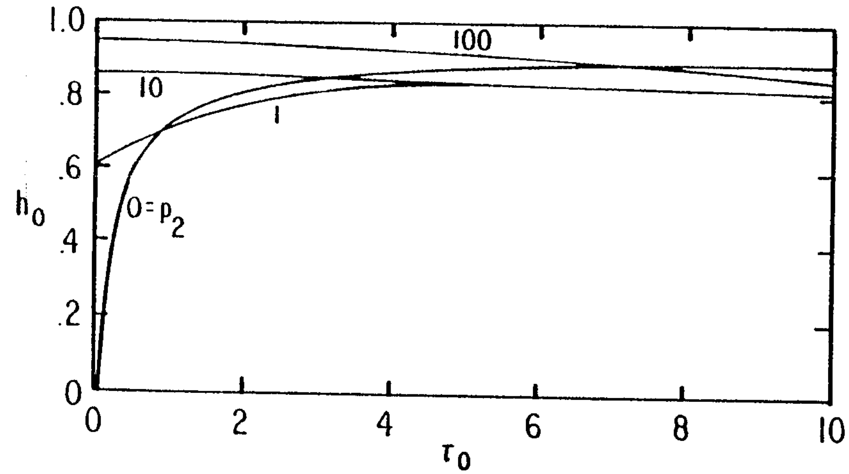
A. h_0 VS. τ_0 WITH p_2 AS A PARAMETER: $p_1 = 0$

B. h_0 VS. τ_0 WITH p_2 AS A PARAMETER: $p_1 = 1$

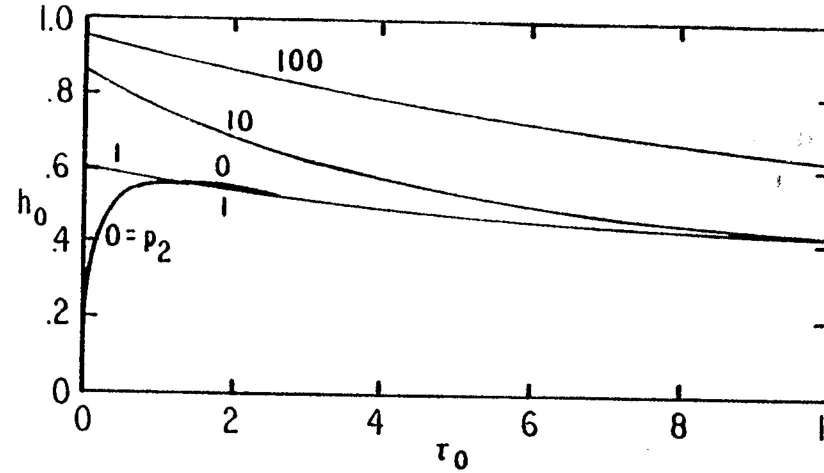
C. h_0 VS. τ_0 WITH p_2 AS A PARAMETER: $p_1 = 10$

D. h_0 VS. τ_0 WITH p_2 AS A PARAMETER: $p_1 = 100$

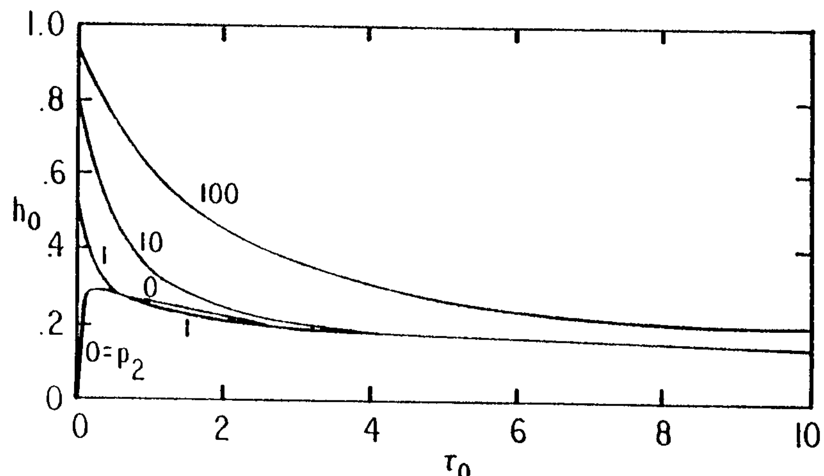
FIGURE 3. CURRENT OR MAGNETIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON INFINITE LENGTH TRANSMISSION LINE: $\epsilon_r = 10$



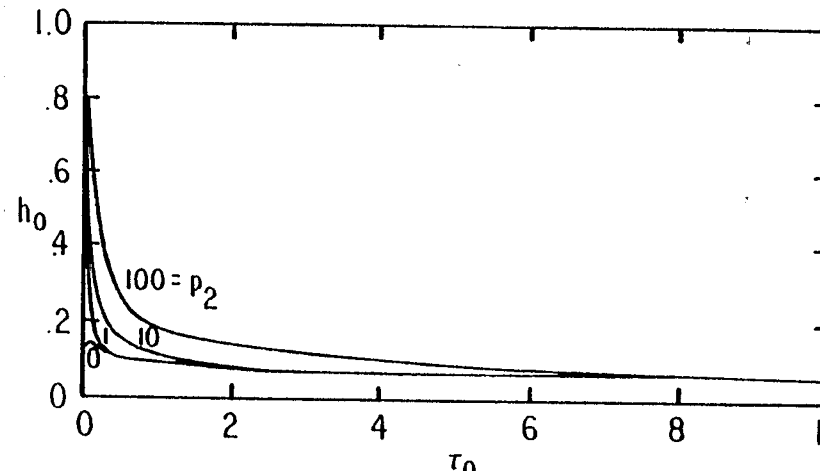
A. h_0 VS τ_0 WITH p_2 AS A PARAMETER: $p_1 = 0$



B. h_0 VS τ_0 WITH p_2 AS A PARAMETER: $p_1 = 1$

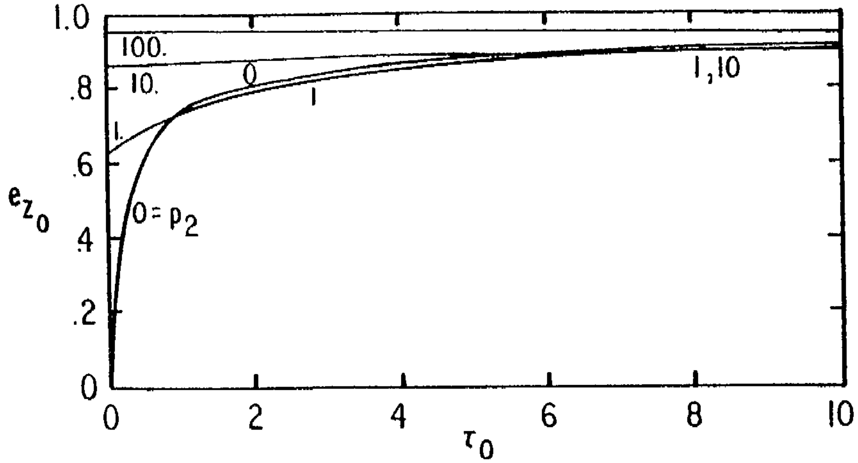


C. h_0 VS τ_0 WITH p_2 AS A PARAMETER: $p_1 = 10$

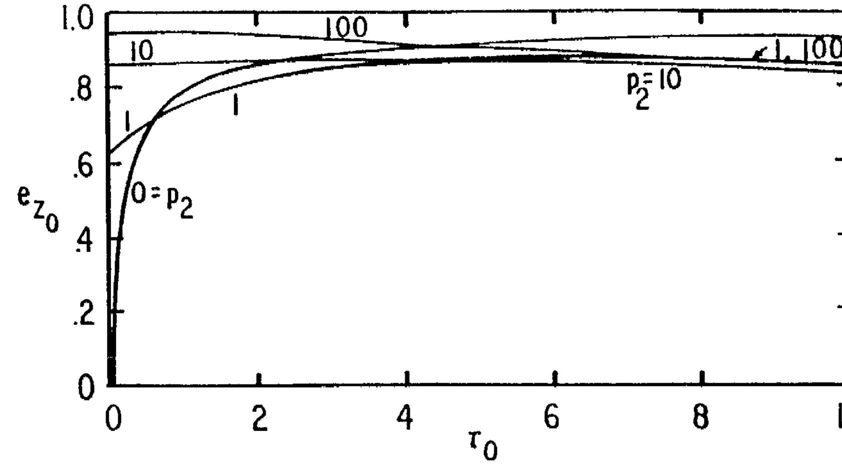


D. h_0 VS τ_0 WITH p_2 AS A PARAMETER: $p_1 = 100$

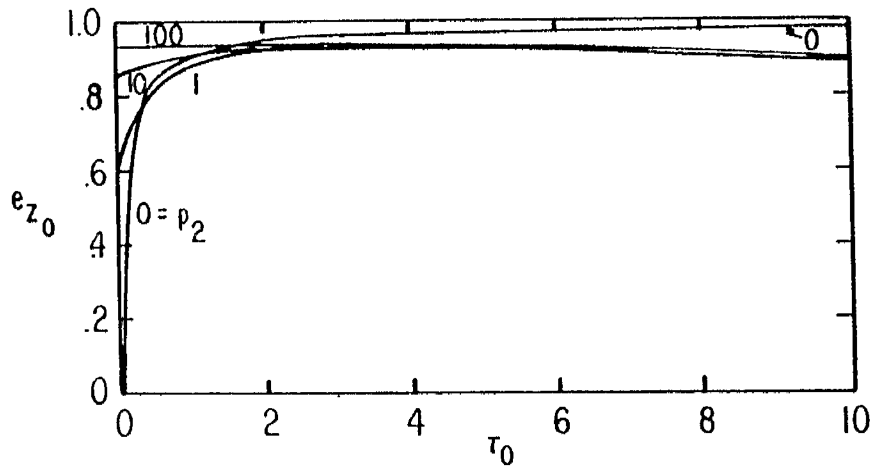
FIGURE 4. CURRENT OR MAGNETIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON INFINITE LENGTH TRANSMISSION LINE: $\epsilon_r = 80$



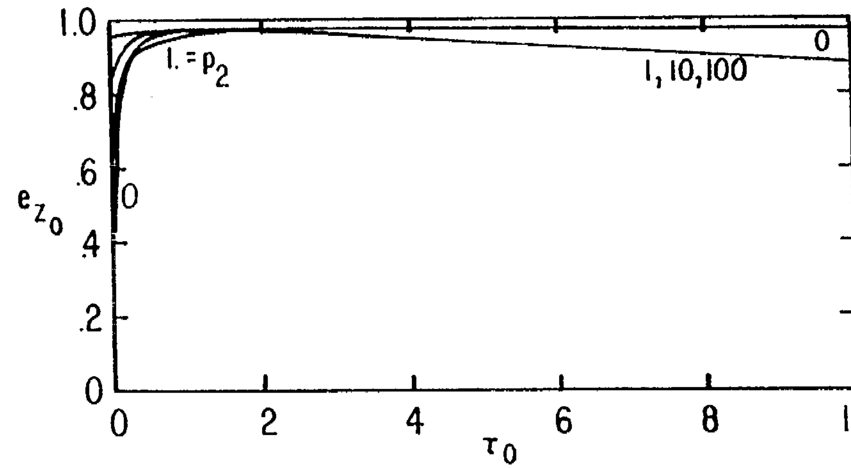
A. e_{z_0} VS τ_0 WITH p_2 AS A PARAMETER: $p_1 = 0$



B. e_{z_0} VS τ_0 WITH p_2 AS A PARAMETER: $p_1 = 1$



C. e_{z_0} VS τ_0 WITH p_2 AS A PARAMETER: $p_1 = 10$



D. e_{z_0} VS τ_0 WITH p_2 AS A PARAMETER: $p_1 = 100$

FIGURE 5. VOLTAGE OR VERTICAL ELECTRIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON INFINITE LENGTH TRANSMISSION LINE: $\epsilon_r = 10$

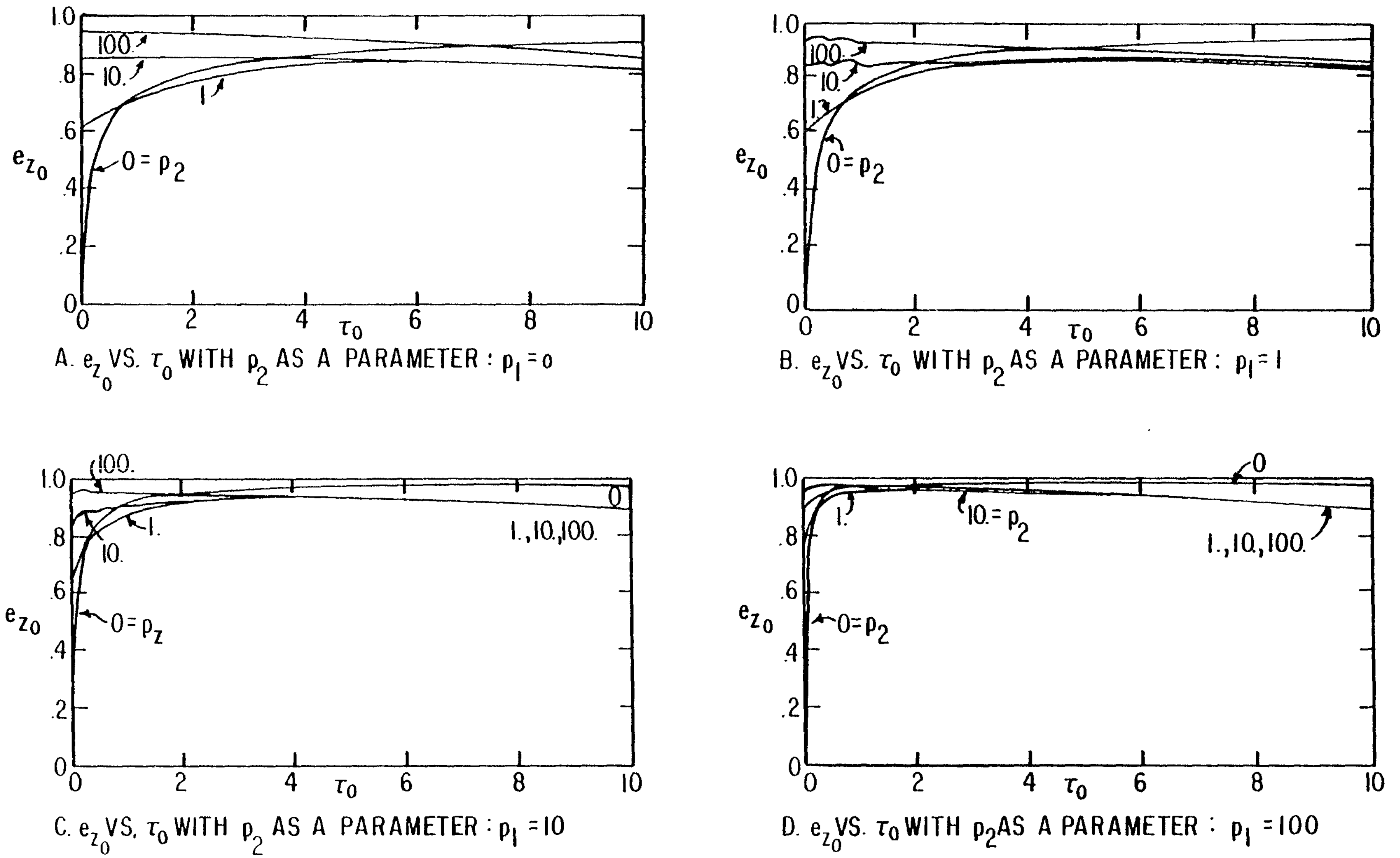
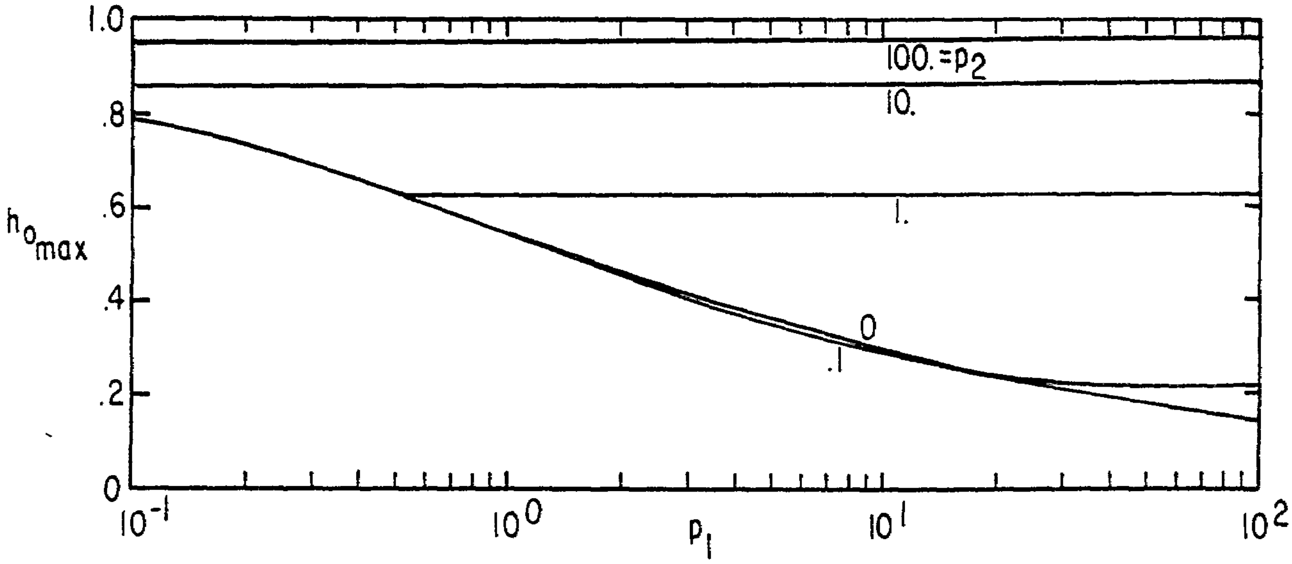
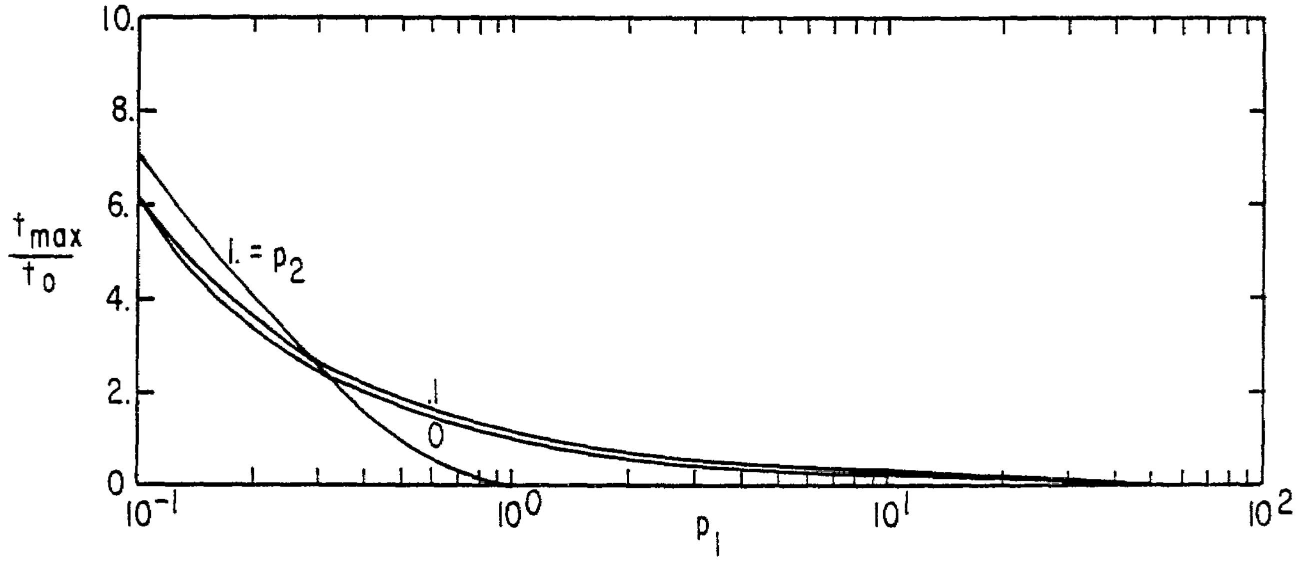


FIGURE 6. VOLTAGE OR VERTICAL ELECTRIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON INFINITE LENGTH TRANSMISSION LINE: $\epsilon_r = 80$

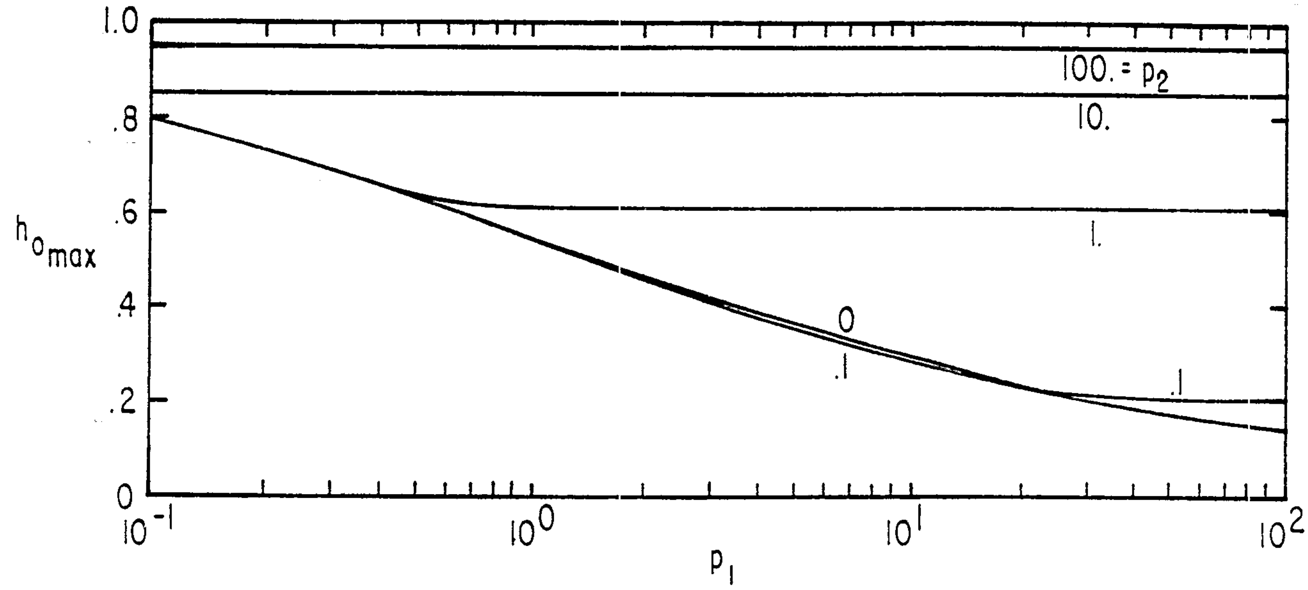


A. VALUE OF MAXIMUM VS. p_1 WITH p_2 AS A PARAMETER

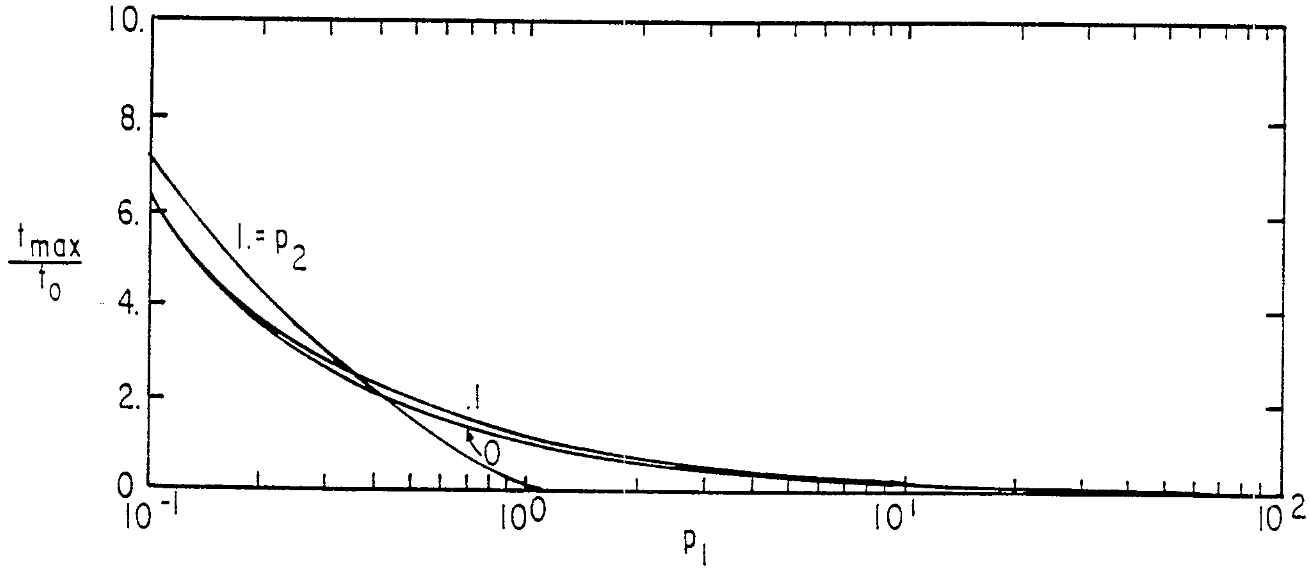


B. TIME OF MAXIMUM VS p_1 WITH p_2 AS A PARAMETER

FIGURE 7. PARAMETERS OF CURRENT OR MAGNETIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON INFINITE LENGTH TRANSMISSION LINE: $\epsilon_r = 10$



A. VALUE OF MAXIMUM VS p_1 WITH p_2 AS A PARAMETER



B. TIME OF MAXIMUM VS p_1 WITH p_2 AS A PARAMETER

FIGURE 8. PARAMETERS OF CURRENT OR MAGNETIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON INFINITE LENGTH TRANSMISSION LINE: $\epsilon_r = 80$

VI. Response of Finite-Length Transmission Line

Now include the end of the transmission line at $x=d$. Two cases are considered: a short-circuit termination and a resistive termination. Again define some convenient parameters. The transit time to the end of the transmission line is

$$t_d \equiv \frac{d}{c} \tag{113}$$

from which we define a normalized Laplace transform variable as

$$s_d \equiv s t_d \tag{114}$$

and a normalized time as

$$\tau_d \equiv \frac{t-t_x}{t_d} \tag{115}$$

Define a normalized distance as

$$x' \equiv \frac{x}{d} \tag{116}$$

Another convenient parameter is

$$p_3 \equiv \frac{t_r' t_d}{t_h^2} \tag{117}$$

Modify equation (103) in the last section to use the new parameters giving

$$\left(\frac{\gamma_x}{\gamma_1}\right)^2 = 1 + \left(\frac{t_r' t_d}{t_h^2 s_d}\right)^{1/2} \frac{1}{1+s_d \frac{t_r}{t_d}} \left[1 + \left(1-\frac{1}{\varepsilon_r}\right) s_d \frac{t_r}{t_d}\right]^{1/2}$$

$$= 1 + \left(\frac{p_3}{s_d}\right)^{1/2} \frac{1}{1+s_d \frac{t_r}{t_d}} \left[1+ \left(1-\frac{1}{\varepsilon_r}\right) s_d \frac{t_r}{t_d}\right]^{1/2} \tag{118}$$

We also have

$$-\gamma_x x+\gamma_1 x = - s_d x' \left(\frac{\gamma_x}{\gamma_1} - 1\right) \tag{119}$$

and

$$\gamma_x d = s_d \frac{\gamma_x}{\gamma_1} \tag{120}$$

These are the expressions used for the calculations in this section.

There are various forms for the normalized voltage or vertical electric field, $\tilde{e}_{z_d}(s_d)$, and normalized current or magnetic field, $\tilde{h}_d(s_d)$, used in this section but they all have the same initial rise characteristics, except at x'=1 where the first reflection is coincident with the initial rise. Using the same procedure as in equations (105) and (106) and restricting $0 \leq x' < 1$ we obtain

$$h_d(0+) = e_{z_d}(0+) = e^{-\frac{x'}{2}\sqrt{p_3 \frac{t_d}{t_r}\left(1-\frac{1}{\epsilon_r}\right)}} \tag{121}$$

The actual form of the normalized waveforms is considered later.

In terms of the parameters used in this section restriction 1 is

$$\frac{1}{10.4\epsilon_r^2} p_3 \frac{t_r}{t_d} << 1 \tag{122}$$

and restriction 2 restricts normalized frequencies of interest to

$$|s_d| = |\omega t_d| << \epsilon_r \left(p_3 \frac{t_d}{t_r}\right)^{1/2} \tag{123}$$

Again, for the graphs the expressions for the waveforms are considered as exact; so the reader should keep these restrictions in mind.

A. Short-Circuit Termination

Let the termination impedance at x=d be zero, a short circuit, distributed over the whole width, w, of the transmission line. First apply a step-function voltage at x=0 and t=0. For this case the appropriate normalized voltage or vertical electric field is

$$\tilde{e}_{z_d}(s_d) \equiv \frac{1}{s_d} \frac{1-e^{-2\gamma_x d(1-x')}}{1-e^{-2\gamma_x d}} e^{-\gamma_x x+\gamma_1 x} \tag{124}$$

Note the -1 reflection for the vertical electric field at x=d. The impedance of this short-circuited transmission line is

$$Z_L = Z_{L_\infty} \frac{1-e^{-2\gamma_x d}}{1+e^{-2\gamma_x d}} \tag{125}$$

The appropriate normalized current or magnetic field for this case of a step function voltage is then

$$\tilde{h}_d(s_d) \equiv \tilde{e}_{z_d}(s_d)\bigg|_{x'=0} \frac{h}{w}\frac{Z_1}{Z_L} \frac{1+e^{-2\gamma_x d(1-x')}}{1+e^{-2\gamma_x d}} e^{-\gamma_x d+\gamma_1 x}$$

$$= \frac{1}{s_d}\frac{\gamma_1}{\gamma_x} \frac{1+e^{-2\gamma_x d(1-x')}}{1-e^{-2\gamma_x d}} e^{-\gamma_x x+\gamma_1 x} \tag{126}$$

Note the +1 reflection for the magnetic field at both $x=d$ and $x=0$. If, in equation (121), x' is replaced by $x'+\tau_d$, where τ_d is taken as the normalized time of arrival of the various reflections at x', then equation (121) gives the magnitudes of the step discontinuities associated with various reflections. The sign of these step discontinuities depends on the +1 or −1 reflection coefficients at each end of the surface transmission line, as appropriate for the particular field component.

Figures 9 through 12 have the pulse shapes for a step function voltage driving the short-circuited transmission line for two values of t_r/t_d. Each graph is for a particular value of p_3 and each curve for a particular x'. All the curves are for the case of $\epsilon_r=10$. Note the step discontinuities in the waveforms for the case of $t_r/t_d=1$. As p_3 is increased these discontinuities become less noticeable. Note that h_d continually increases at long times because a step voltage is driving the short-circuited transmission line.

As a second case apply a step-function current to the short-circuited transmission line at $x=0$ and $t=0$. The normalized current or magnetic field is

$$\tilde{h}_d(s_d) \equiv \frac{1}{s_d} \frac{1+e^{-2\gamma_x d(1-x')}}{1+e^{-2\gamma_x d}} e^{-\gamma_x x+\gamma_1 x} \tag{127}$$

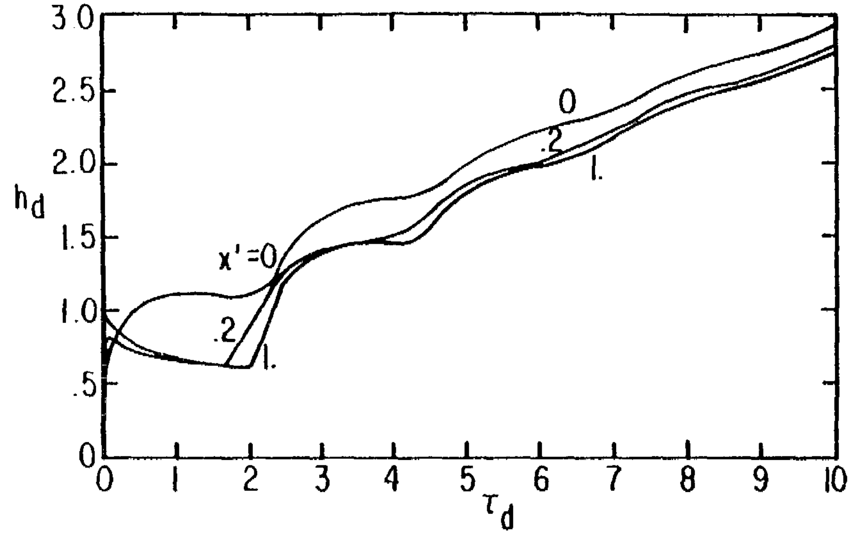
The associated normalized voltage or vertical electric field is

$$\tilde{e}_{z_d}(s_d) \equiv \tilde{h}_d(s_d)\Bigg|_{z''=0} \frac{w}{h}\frac{Z_L}{Z_1}\,\frac{1-e^{-2\gamma_x d(1-x')}}{1-e^{-2\gamma_x d}}\,e^{-\gamma_x x+\gamma_1 x}$$

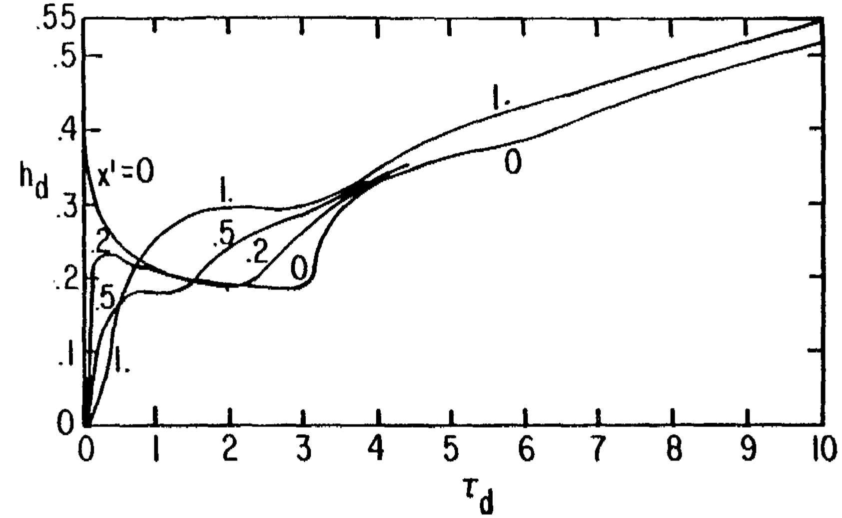
$$\simeq \frac{1}{s_d}\,\frac{\gamma_x}{\gamma_1}\,\frac{1-e^{-2\gamma_x d(1-x')}}{1+e^{-2\gamma_x d}}\,e^{-\gamma_x x+\gamma_1 x} \tag{128}$$

Note that the magnetic field has a + 1 reflection coefficient at $x=d$, but has a−1 reflection coefficient at $x=0$. Again equation (121) with x' replaced by $x' + \tau_d$ gives the magnitudes of the step discontinuities in the waveform associated with the reflections.

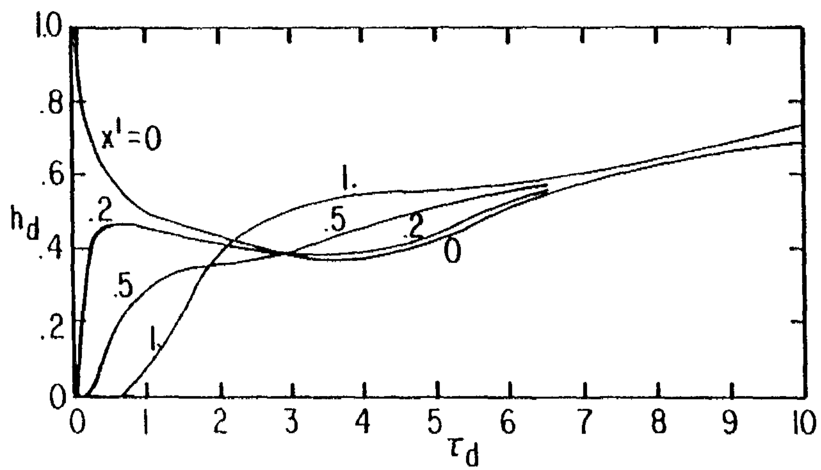
Figures 13 through 16 have the pulse shapes for a step function current driving the short-circuited transmission line. The choice of the various parameters is the same as for the previous case of a step function voltage. All the curves are for the case of $\epsilon_r = 10$. Note that with the use of a step function current h_d does not increase without bound at long times. Also note that, while large values of p_3 reduce the prominence of the recurring step discontinuities, large values of p_3 also increase the rise time of h_d.



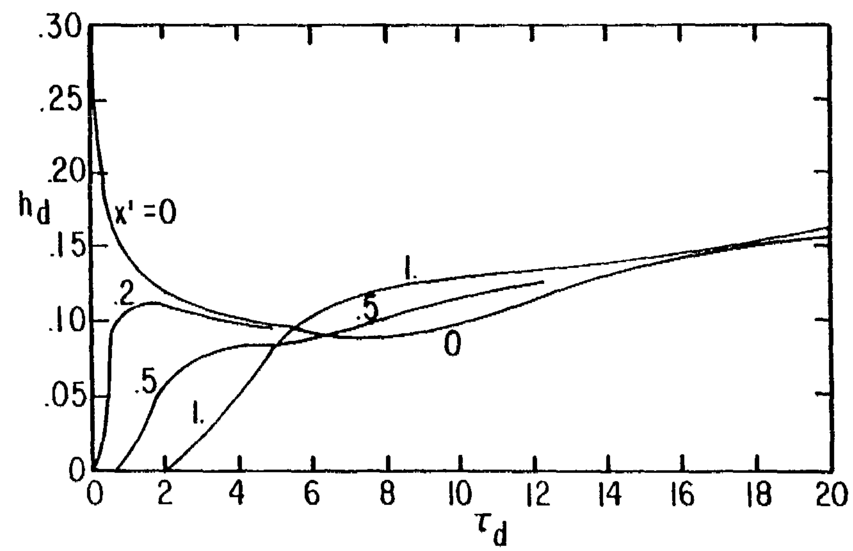
A. h_d VS. τ_d WITH x' AS A PARAMETER : $p_3 = 1$



B. h_d VS. τ_d WITH x' AS A PARAMETER : $p_3 = 10$

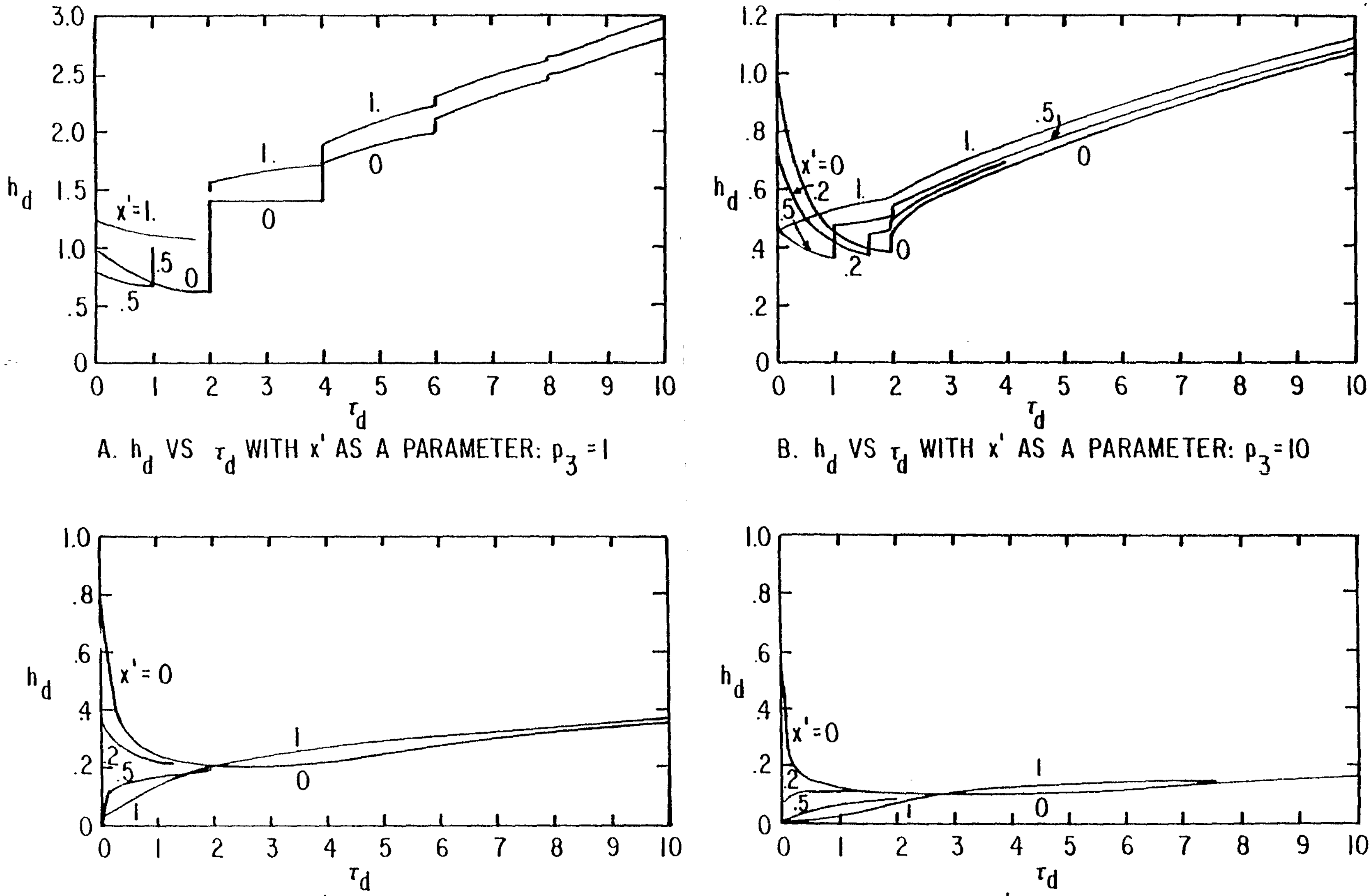


C h_d VS. τ_d WITH x' AS A PARAMETER : $p_3 = 100$



D. h_d VS. τ_d WITH x' AS A PARAMETER : $p_3 = 1000$

FIGURE 9. CURRENT OR MAGNETIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON SHORT CIRCUITED TRANSMISSION LINE : $t_r/t_d = 0$



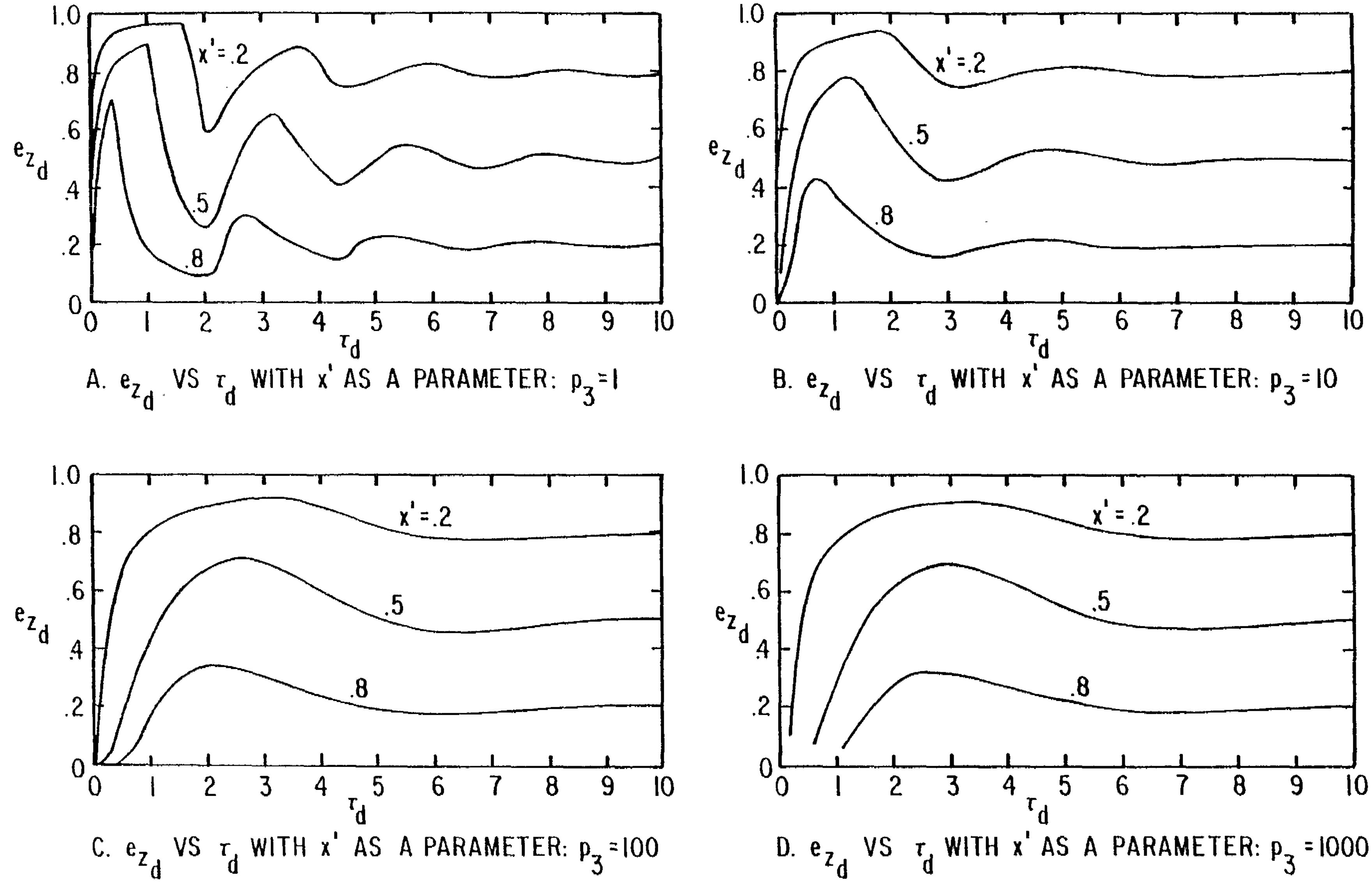
A. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 1$

B. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 10$

C. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 100$

D. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 1000$

FIGURE 10. CURRENT OF MAGNETIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON SHORT CIRCUITED TRANSMISSION LINE: $t_r/t_d = 1$.



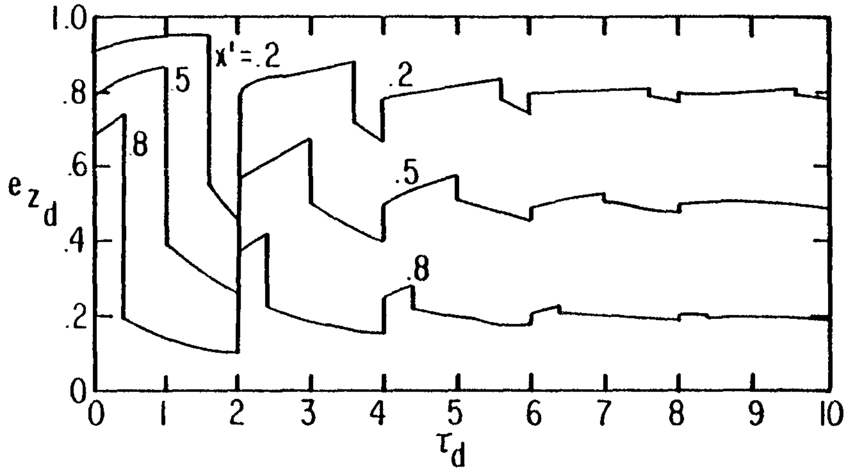
A. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 1$

B. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 10$

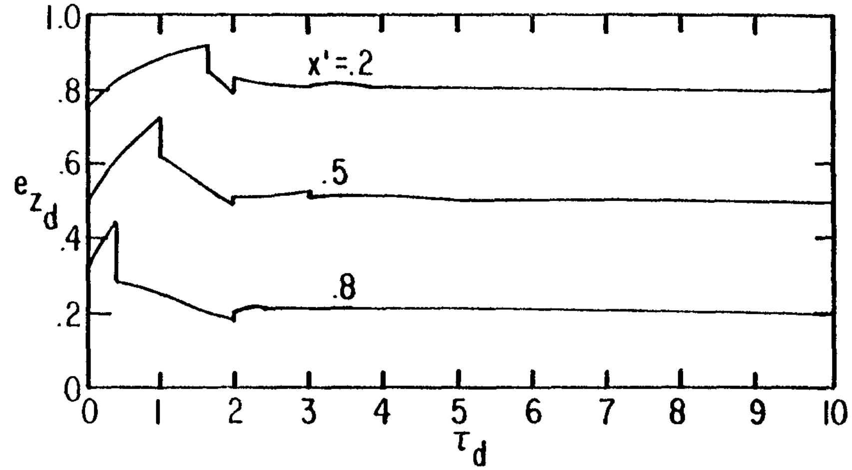
C. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 100$

D. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 1000$

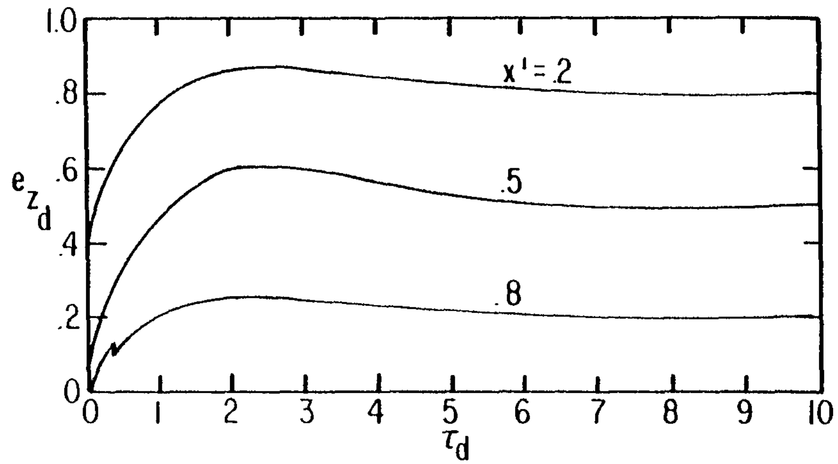
FIGURE 11. VOLTAGE OR VERTICAL ELECTRIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON SHORT CIRCUITED TRANSMISSION LINE: $t_r/t_d = 0$.



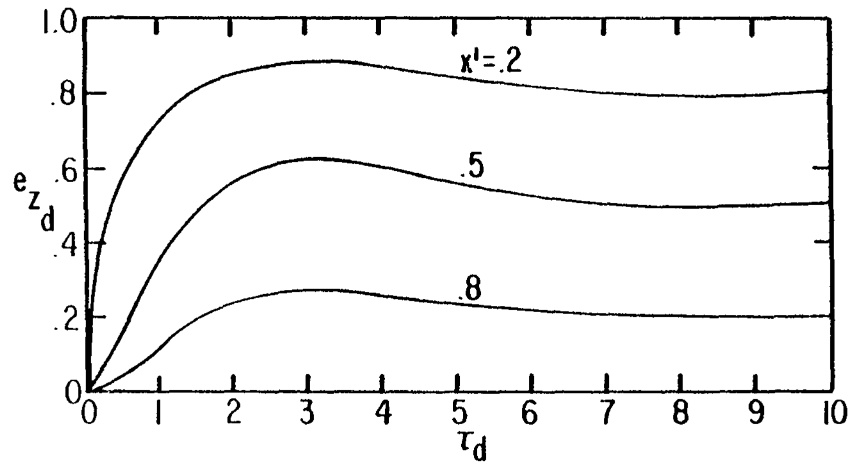
A. e_{z_d} VS. τ_d WITH x' AS A PARAMETER: $p_3 = 1$



B. e_{z_d} VS. τ_d WITH x' AS A PARAMETER: $p_3 = 10$

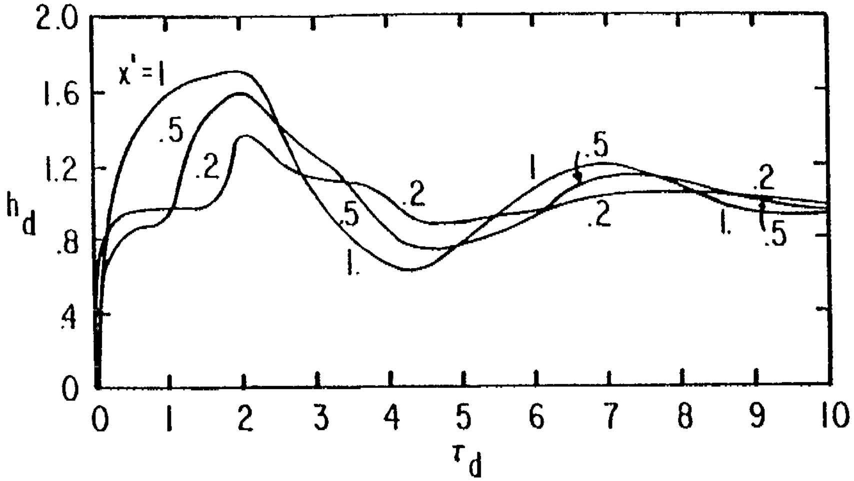


C. e_{z_d} VS. τ_d WITH x' AS A PARAMETER: $p_3 = 100$

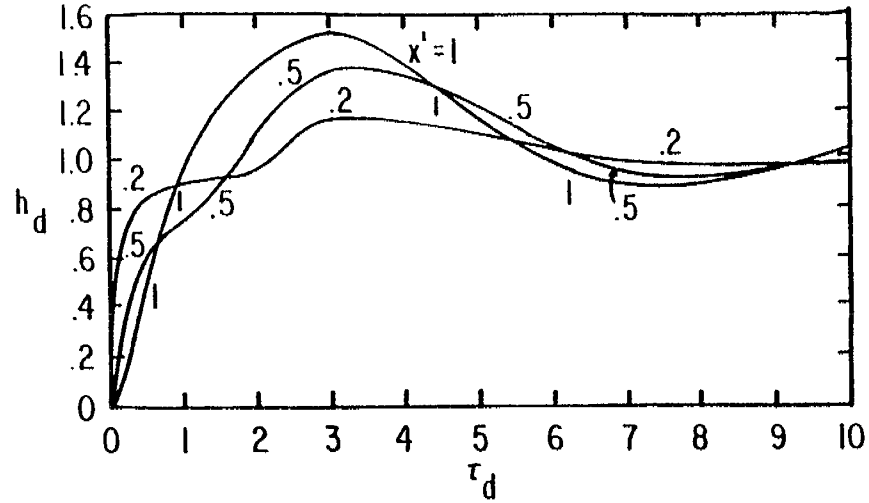


D. e_{z_d} VS. τ_d WITH x' AS A PARAMETER: $p_3 = 1000$

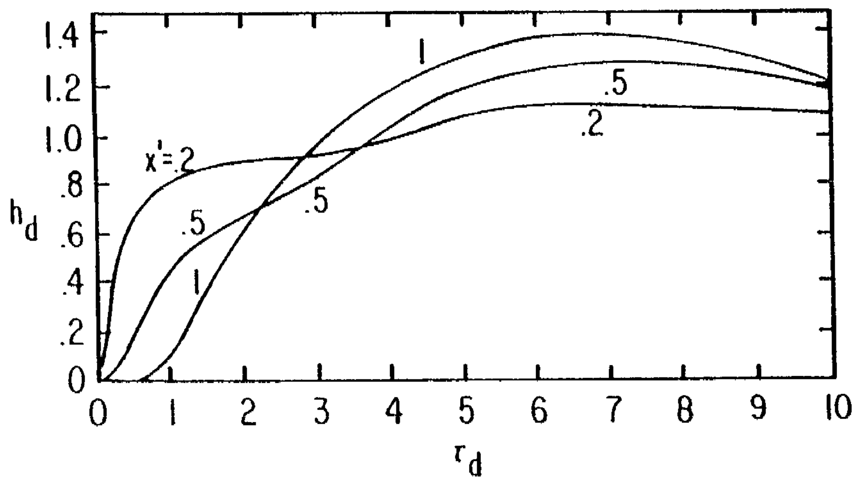
FIGURE 12. VOLTAGE OR VERTICAL ELECTRIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON SHORT CIRCUITED TRANSMISSION LINE: $t_r/t_d = 1$



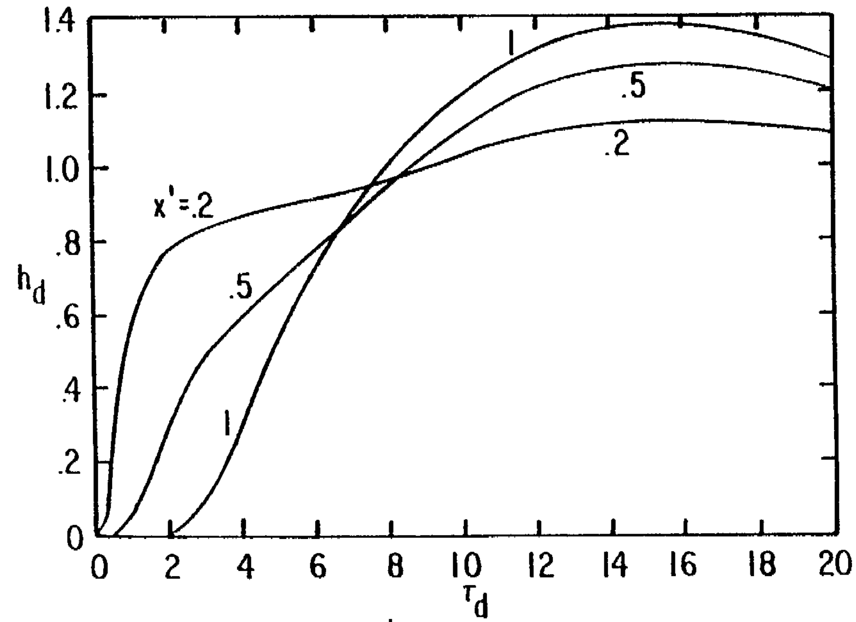
A. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 1$



B. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 10$

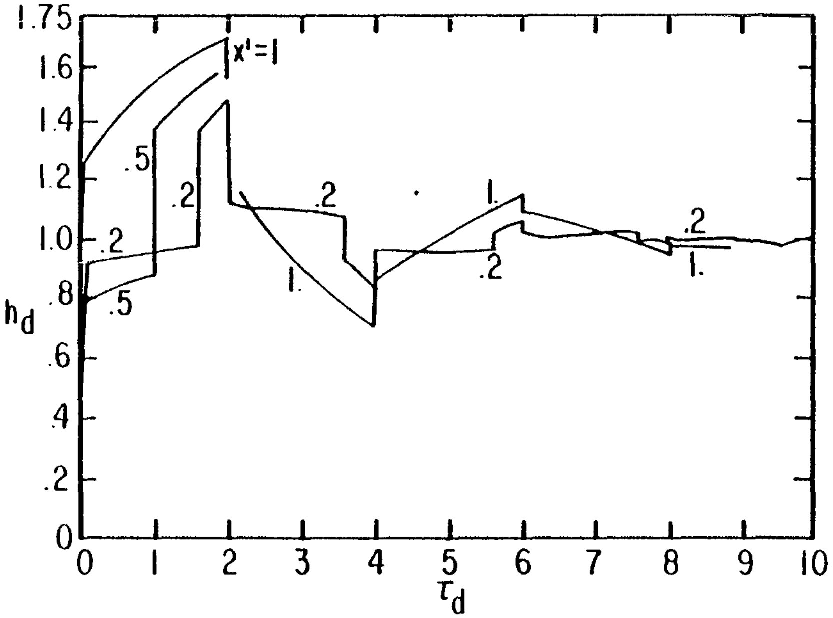


C. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 100$

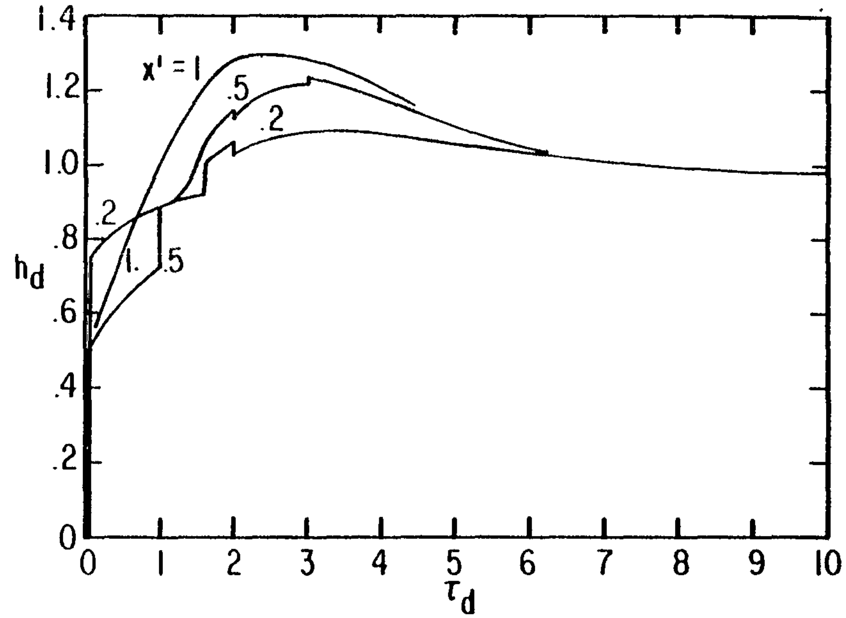


D. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 1000$

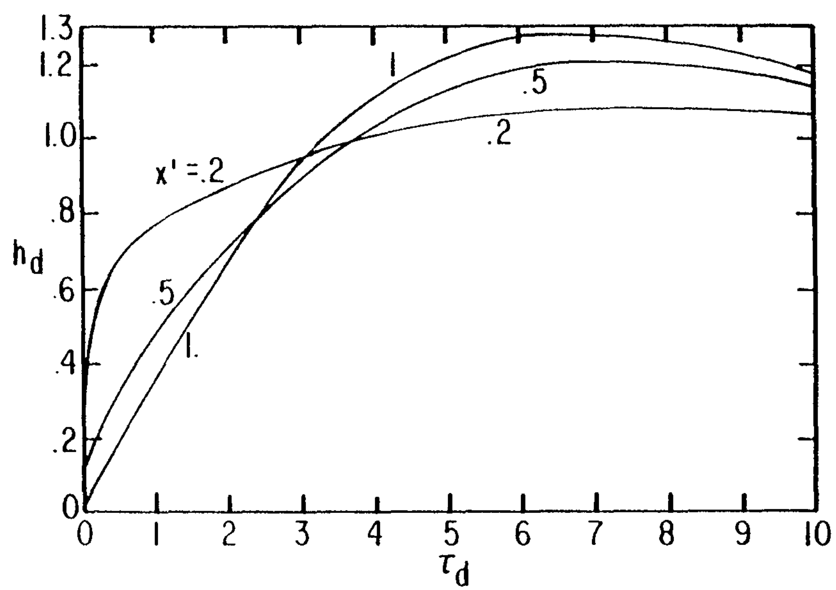
FIGURE 13. CURRENT OR MAGNETIC FIELD PULSE SHAPE FOR STEP CURRENT ON SHORT CIRCUITED TRANSMISSION LINE: $t_r/t_d = 0$.



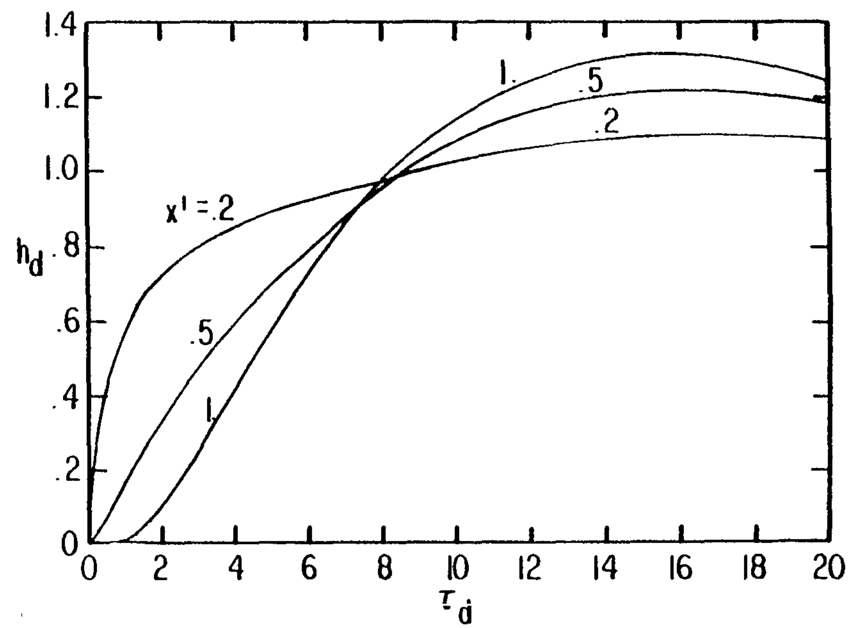
A. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 1$



B. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 10$

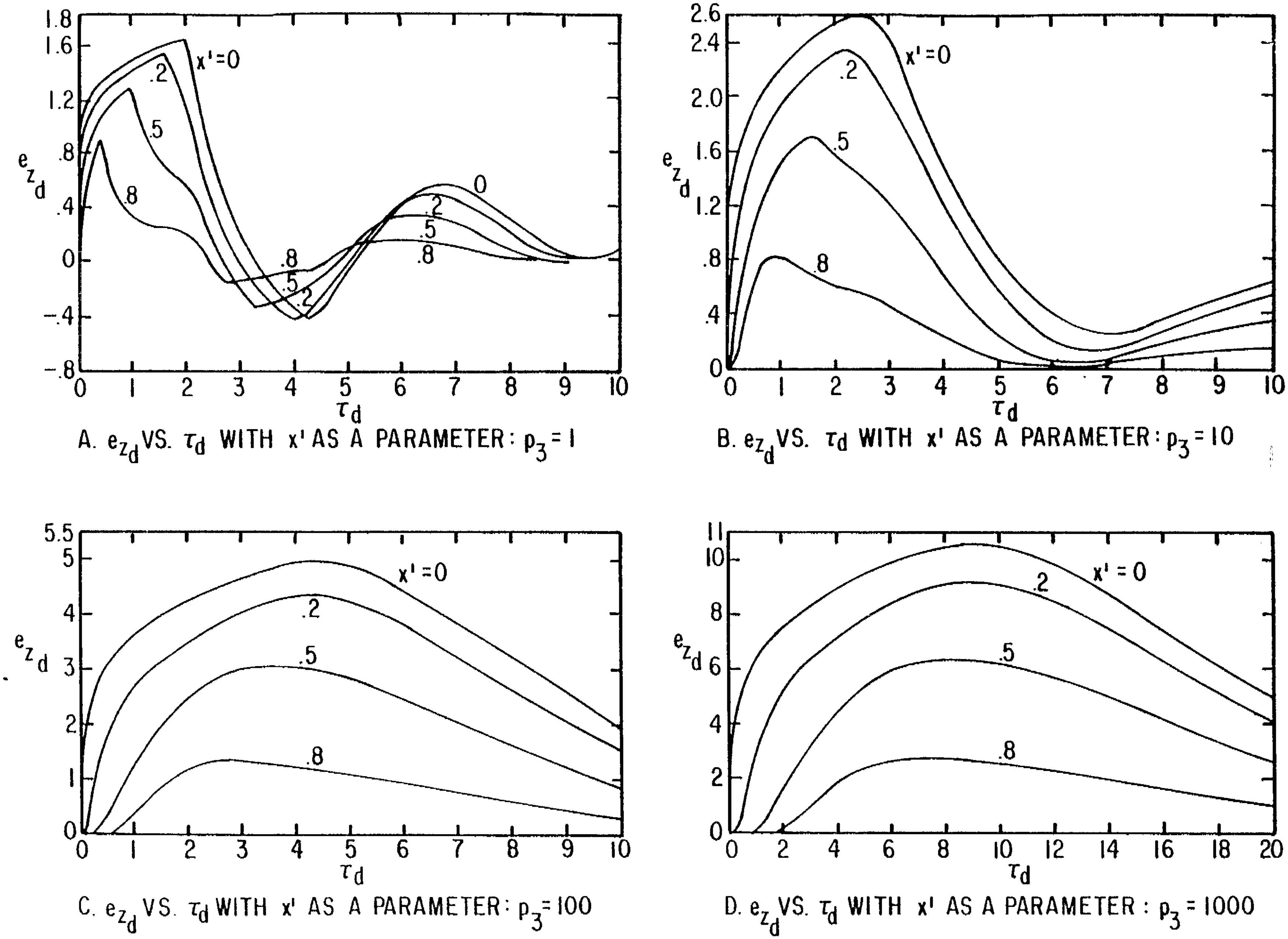


C. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 100$



D. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 1000$

FIGURE 14. CURRENT OR MAGNETIC FIELD PULSE SHAPE FOR STEP CURRENT ON SHORT CIRCUITED TRANSMISSION LINE: $t_{r/+} = 1$



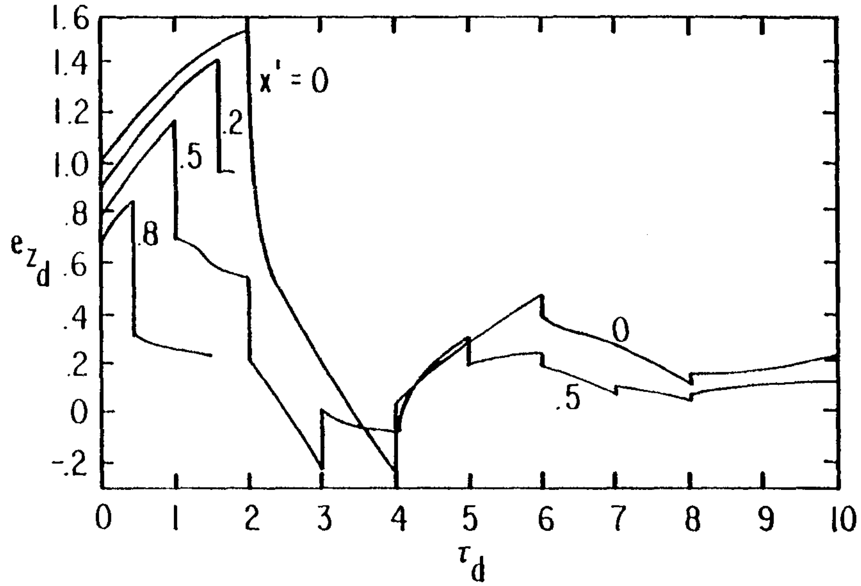
A. e_{z_d} VS. τ_d WITH x' AS A PARAMETER: $p_3 = 1$

B. e_{z_d} VS. τ_d WITH x' AS A PARAMETER: $p_3 = 10$

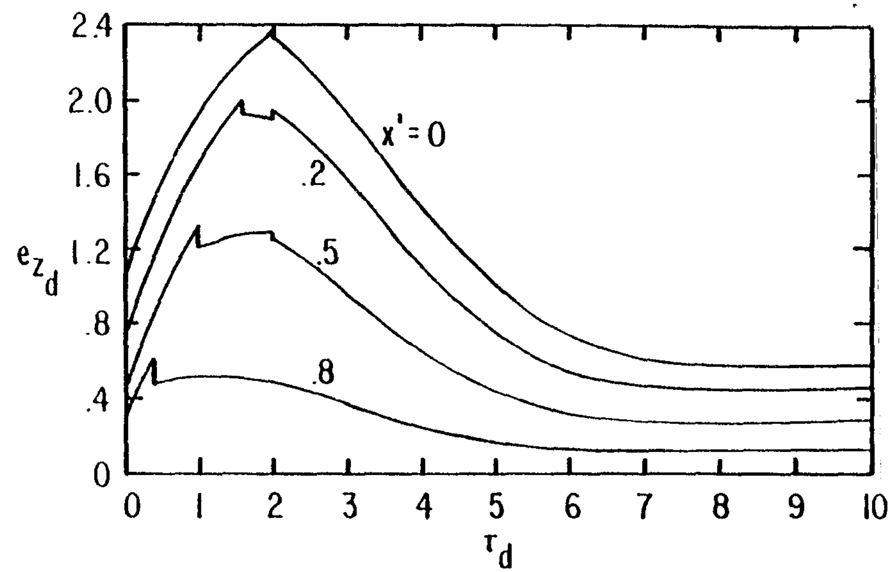
C. e_{z_d} VS. τ_d WITH x' AS A PARAMETER: $p_3 = 100$

D. e_{z_d} VS. τ_d WITH x' AS A PARAMETER: $p_3 = 1000$

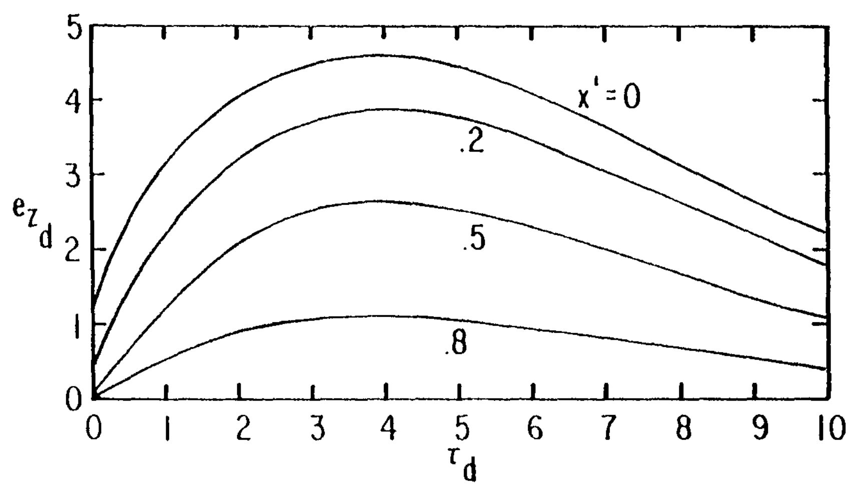
FIGURE 15. VOLTAGE OR VERTICAL ELECTRIC FIELD PULSE SHAPE FOR STEP CURRENT ON SHORT CIRCUITED TRANSMISSION LINE: $t_r/t_d = 0$



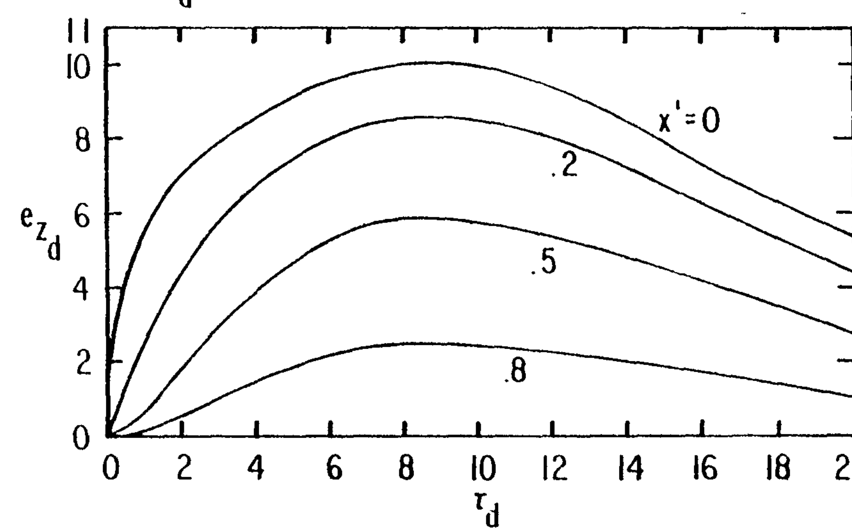
A. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 1$



B. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 10$



C. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 100$



D. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 1000$

FIGURE 16. VOLTAGE OR VERTICAL ELECTRIC FIELD PULSE SHAPE FOR STEP CURRENT ON SHORT CIRCUITED TRANSMISSION LINE: $t_r / t_d = 1$.

B. Resistive Termination

Let the termination impedance, Z_t, be resistive and match Z_{L_∞} in the high-frequency limit. Thus we let

$$Z_t = \frac{h}{w} Z_1 \tag{129}$$

and this is assumed uniformly distributed over the width, w, and height, h, of the transmission line above the ground at x=d. There are various ways this might be done, including the use of a structure similar to the wave launcher at the input of the transmission line. How this termination is accomplished is influenced by what high-frequency characteristics are needed. The reflection coefficient for the voltage or vertical electric field is then

$$r_e = \frac{Z_t - Z_{L_\infty}}{Z_t + Z_{L_\infty}} \approx \frac{1 - \frac{\gamma_x}{\gamma_1}}{1 + \frac{\gamma_x}{\gamma_1}} \tag{130}$$

For the present case of a resistive termination we assume a step function voltage at x=0 and t=0. The normalized voltage or vertical electric field is

$$\tilde{e}_{z_d}(s_d) \equiv \frac{1}{s_d} \frac{1 + r_e\, e^{-2\gamma_x d(1-x')}}{1 + r_e\, e^{-2\gamma_x d}}\, e^{-\gamma_x x + \gamma_1 x} \tag{131}$$

The impedance of the resistance - terminated transmission line is

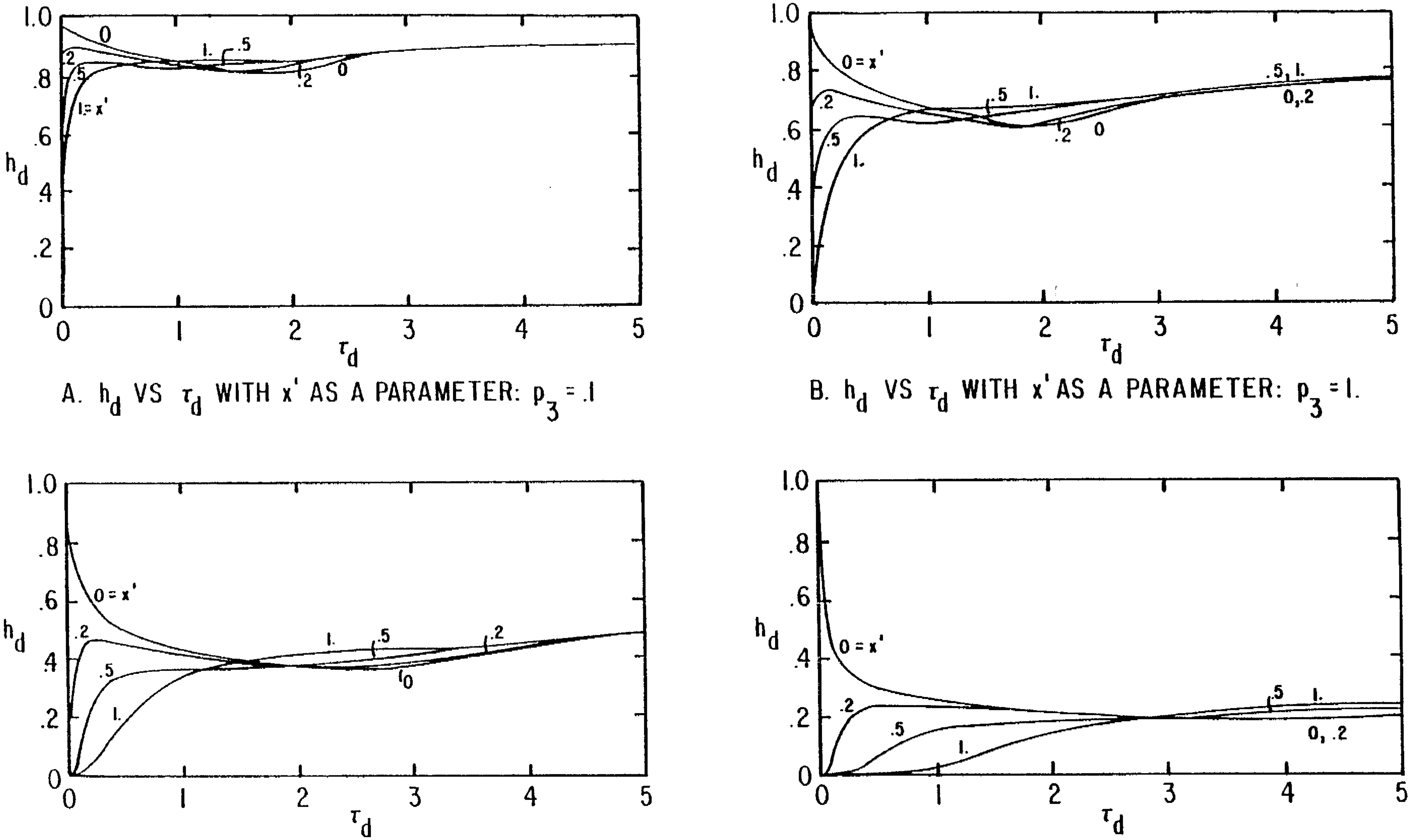
$$Z_L = Z_{L_\infty} \frac{1 + r_e\, e^{-2\gamma_x d}}{1 - r_e\, e^{-2\gamma_x d}} \tag{132}$$

The normalized current or magnetic field is then

$$\tilde{h}_d(s_d) \equiv \tilde{e}_{z_d}(s_d)\Big|_{x'=0} \frac{h}{w} \frac{Z_1}{Z_L} \frac{1 - r_e\, e^{-2\gamma_x d(1-x')}}{1 - r_e\, e^{-2\gamma_x d}}\, e^{-\gamma_x x + \gamma_1 x}$$

$$\approx \frac{1}{s_d} \frac{\gamma_1}{\gamma_x} \frac{1 - r_e\, e^{-2\gamma_x d(1-x')}}{1 + r_e\, e^{-2\gamma_x d}}\, e^{-\gamma_x x + \gamma_1 x} \tag{133}$$

Figures 17 through 24 have the pulse shapes for this case of a step function voltage driving the terminated transmission line. Each figure is for a particular value of t_r/t_d, each graph is for a particular value of p_3, and each curve is for a particular value of x'. All the curves are for the case of $\varepsilon_r=10$. Note that with the use of this particular resistive termination there are no step discontinuities in the pulse shapes after the initial rise. In the limit of large τ_d both h_d and e_{z_d} approach one. For large values of p_3, however, h_d rises to one comparatively slowly.



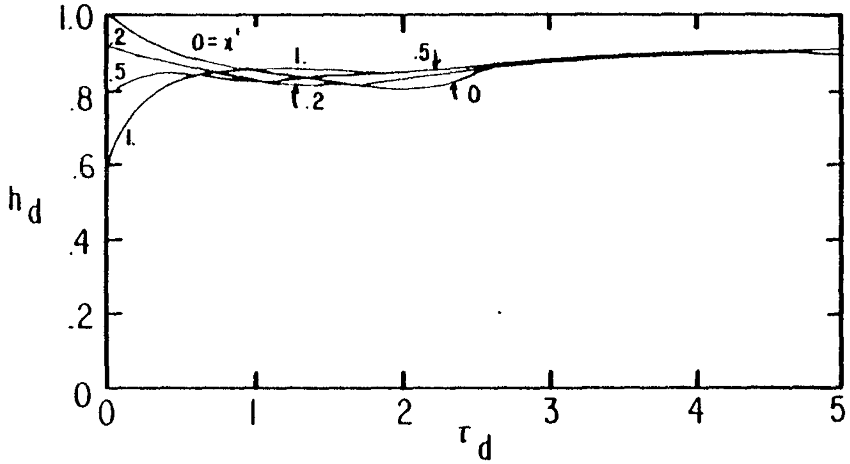
A. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = .1

B. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = 1.

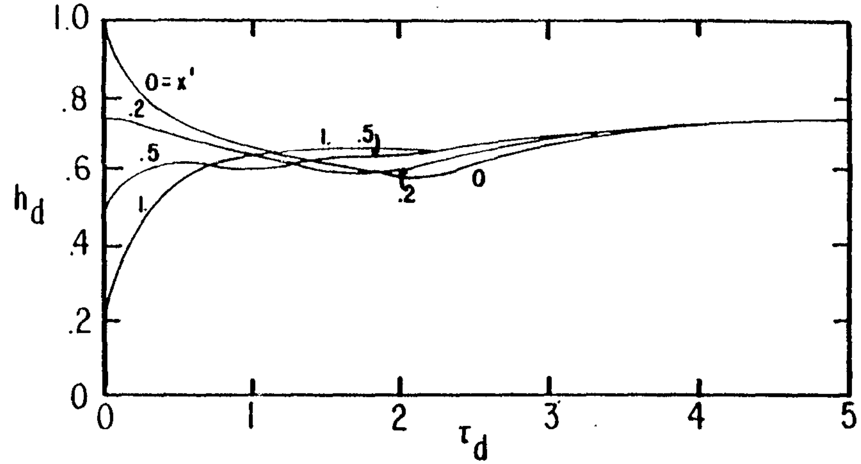
C. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = 10

D. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = 100

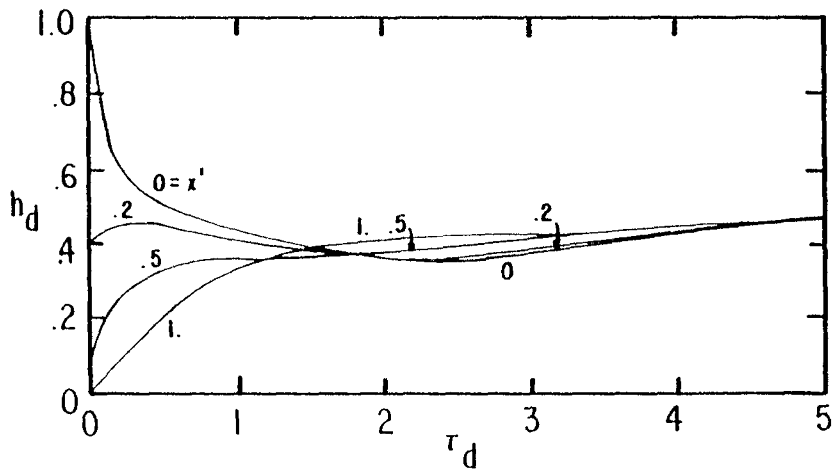
FIGURE 17. CURRENT OR MAGNETIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON TERMINATED TRANSMISSION LINE: t_r/t_d = 0.



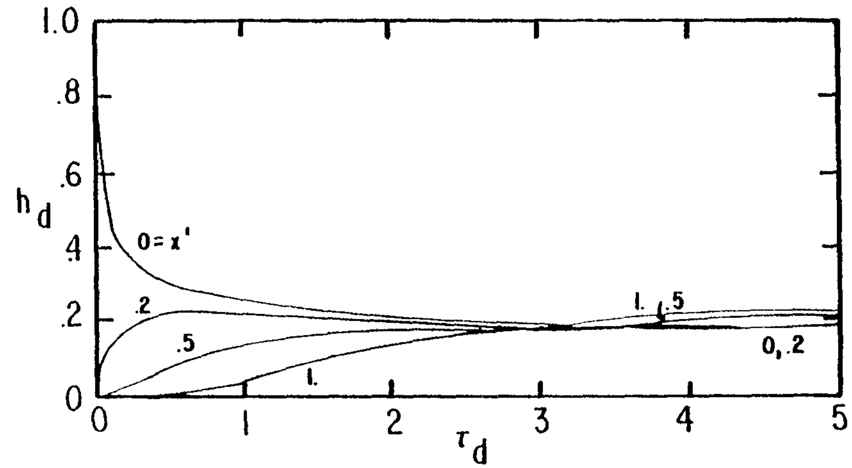
A. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = .1



B. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = 1

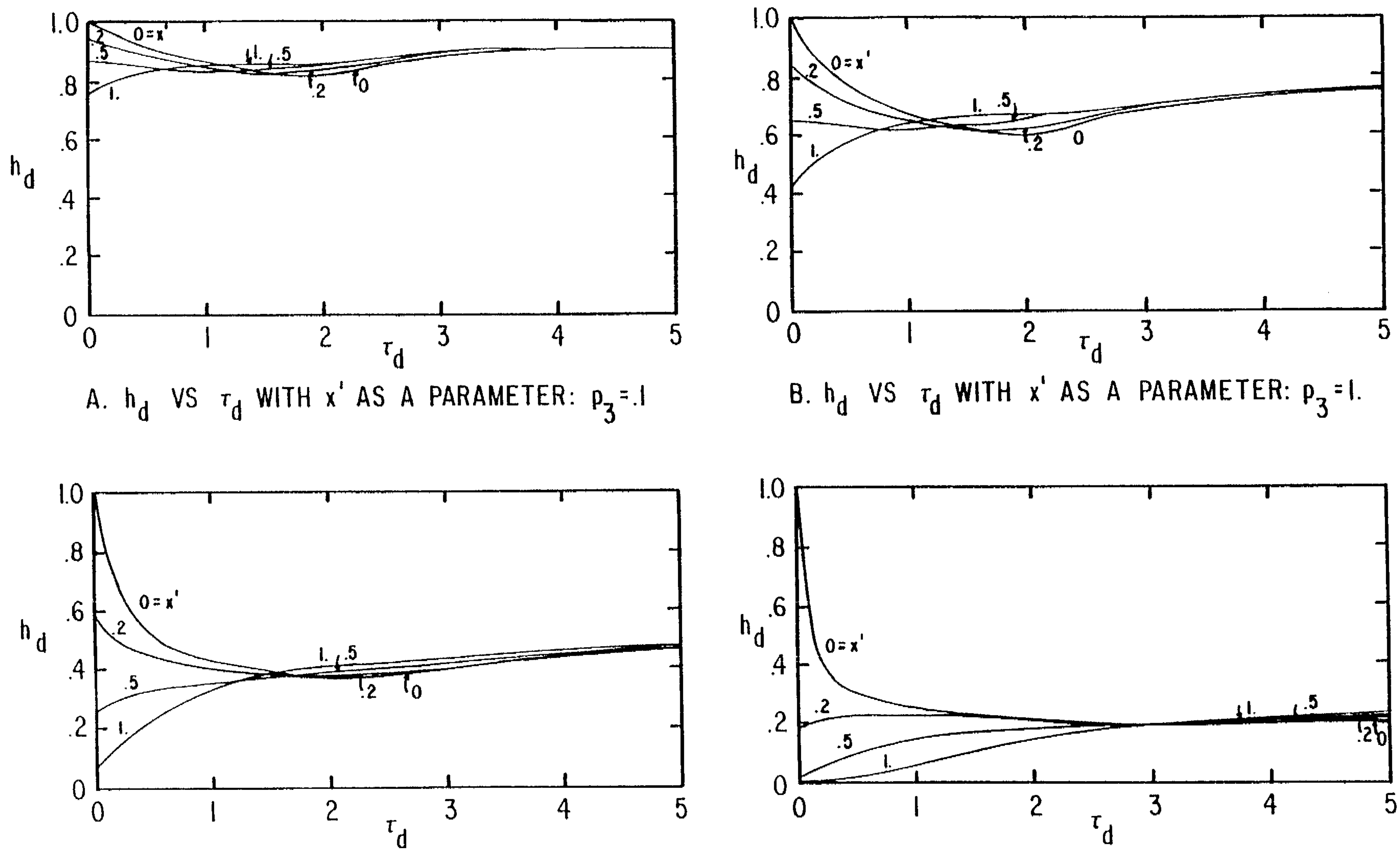


C. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = 10.



D. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = 100.

FIGURE 18. CURRENT OR MAGNETIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON TERMINATED TRANSMISSION LINE: $t_r/\tau_d = .1$



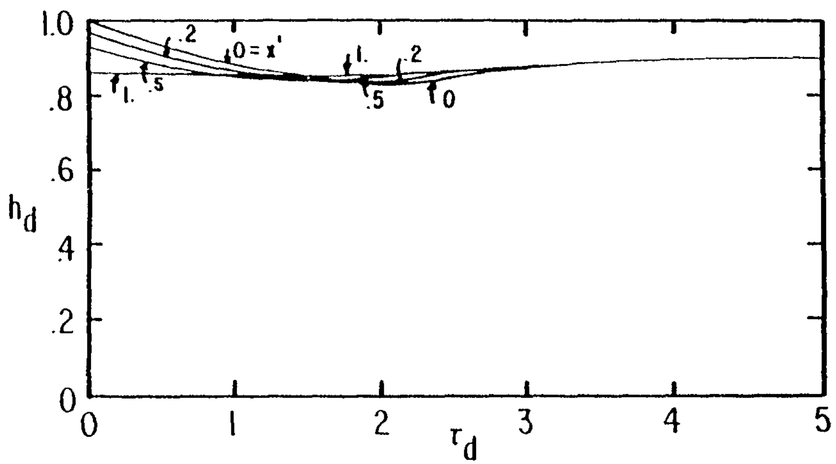
A. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = .1$

B. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 1.$

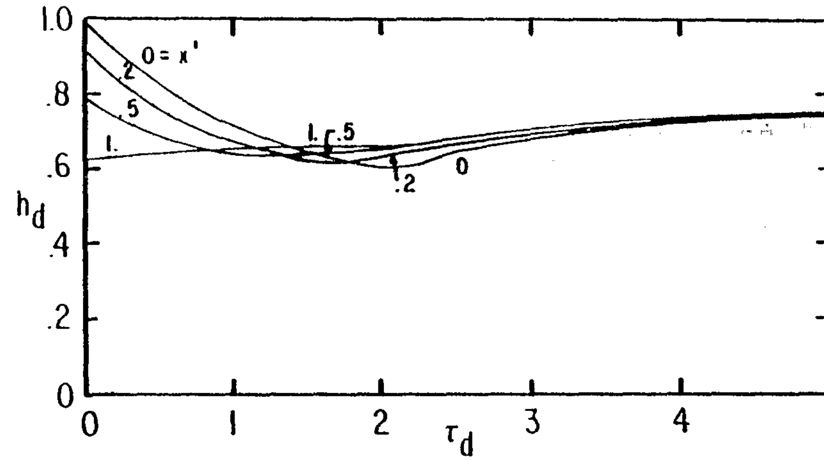
C. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 10.$

D. h_d VS τ_d WITH x' AS A PARAMETER: $p_3 = 100.$

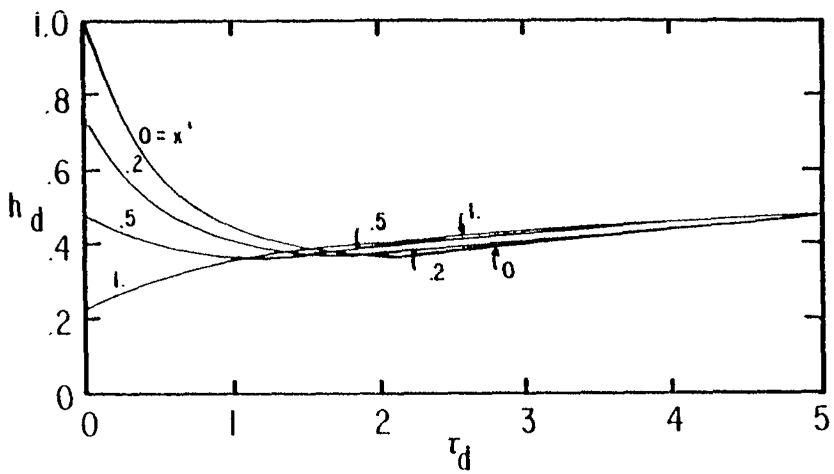
FIGURE 19. CURRENT OR MAGNETIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON SHORT CIRCUITED TRANSMISSION LINE: $t_r / t_d = .3$



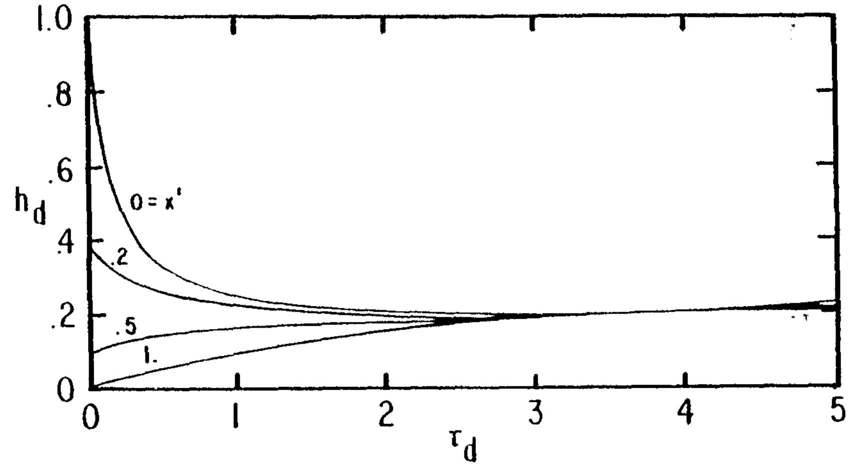
A. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = .1



B. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = 1.

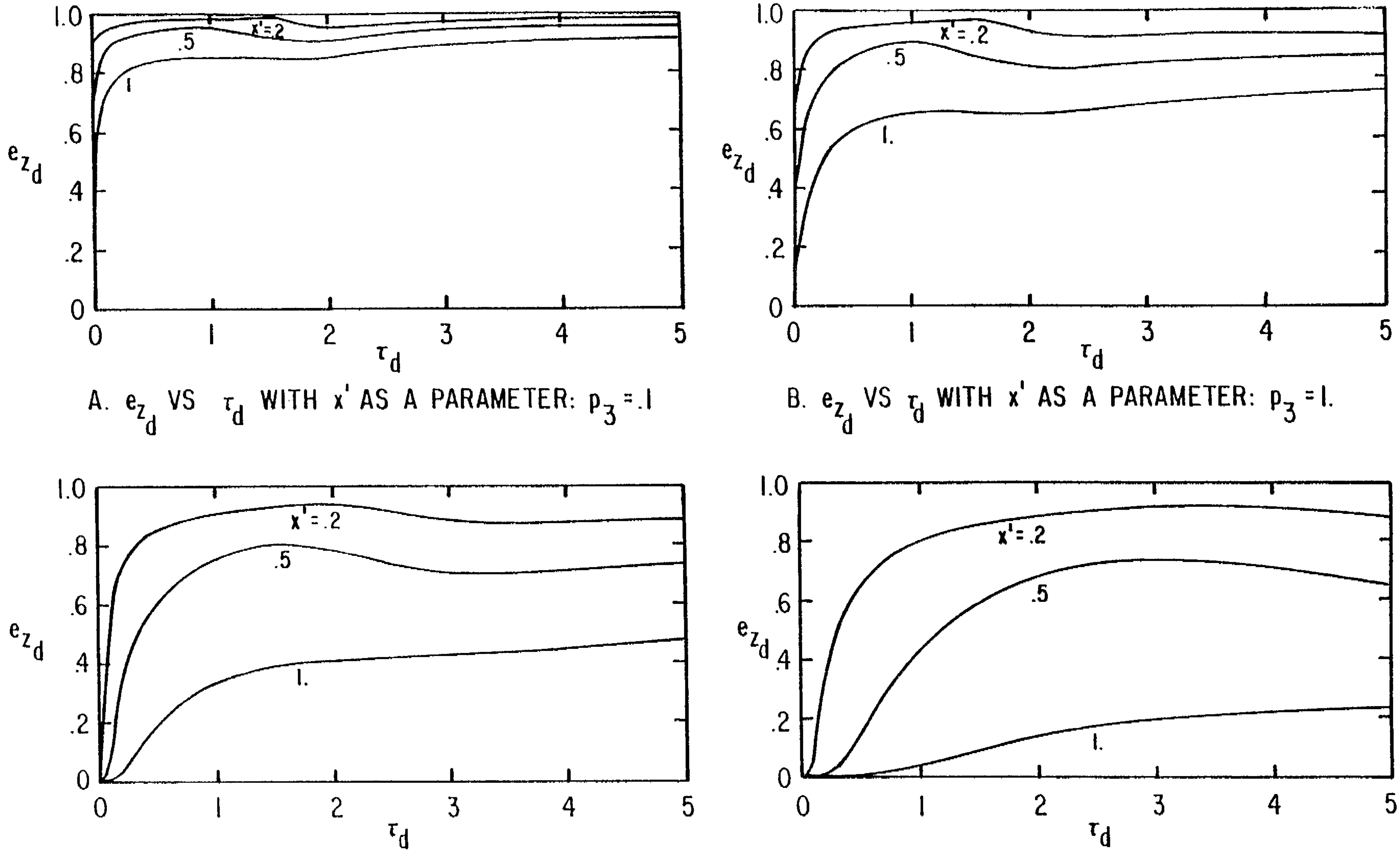


C. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = 10.



D. h_d VS τ_d WITH x' AS A PARAMETER: p_3 = 100.

FIGURE 20. CURRENT OR MAGNETIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON TERMINATED TRANSMISSION LINE: t_r/t_d = 1.



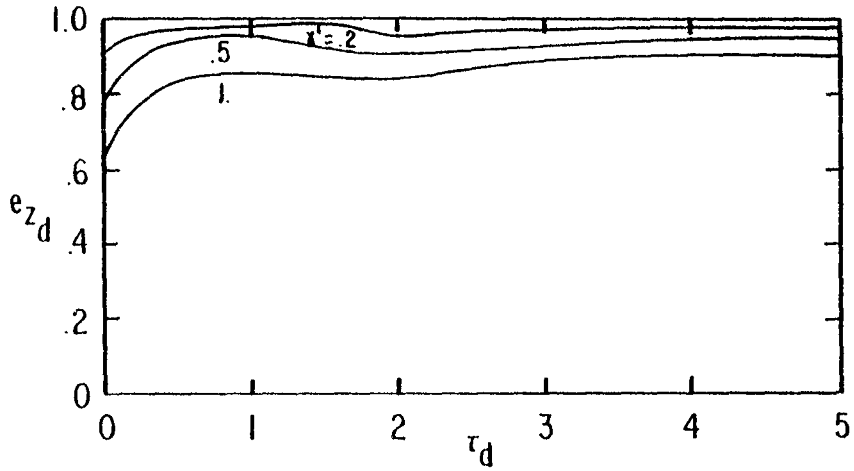
A. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = .1$

B. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 1.$

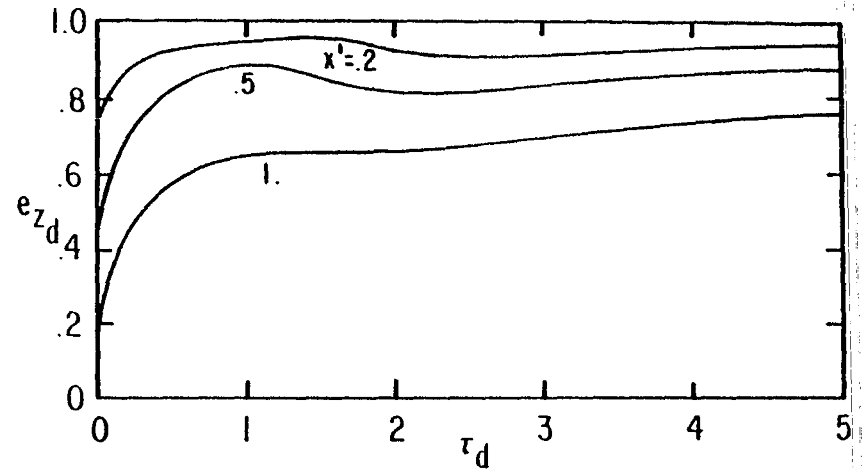
C. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 10.$

D. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 100.$

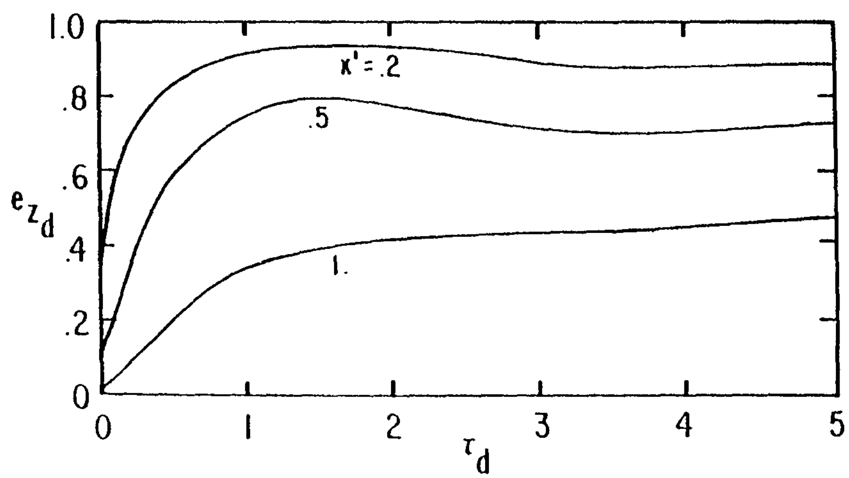
FIGURE 21. VOLTAGE OR VERTICAL ELECTRIC FIELD PULSE SHAPE FOR STEP VOLTAGE OR TERMINATED TRANSMISSION LINE: $t_r/t_d = 0.$



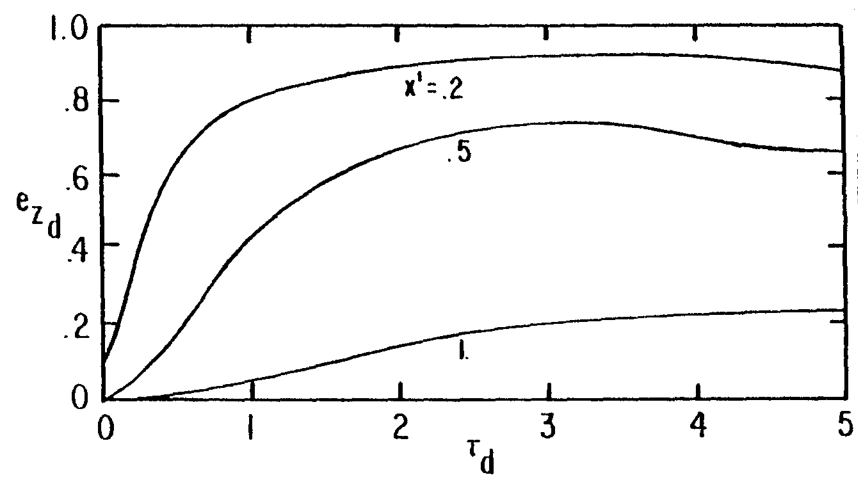
A. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = .1$



B. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 1.$

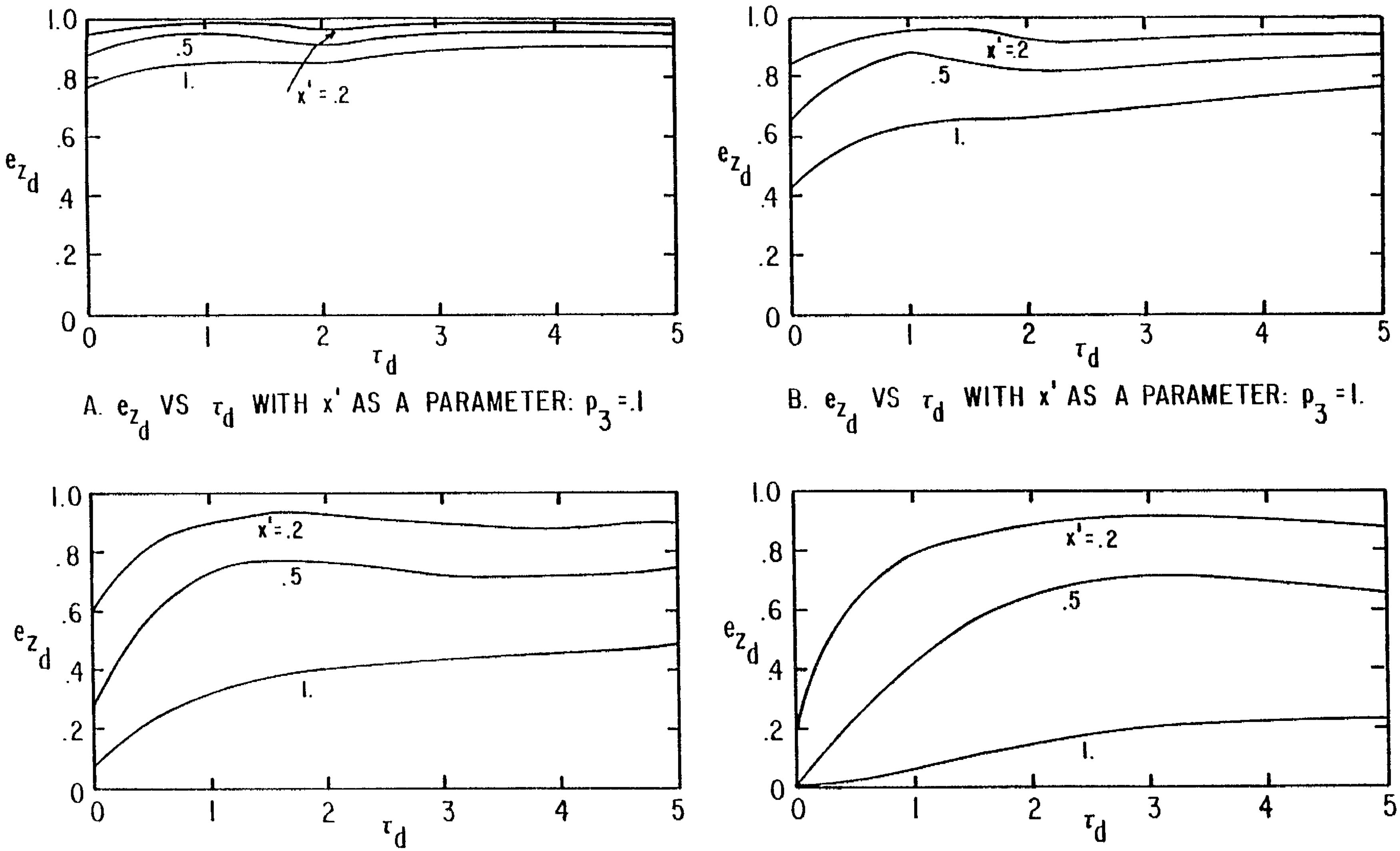


C. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 10.$



D. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 100$

FIGURE 22. VOLTAGE OR VERTICAL ELECTRIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON TERMINATED TRANSMISSION LINE: $t_r/t_d = .1$



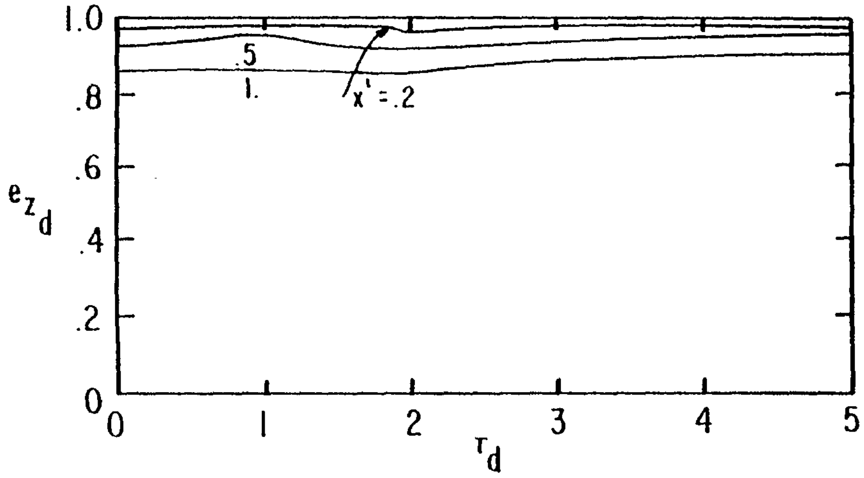
A. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = .1$

B. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 1.$

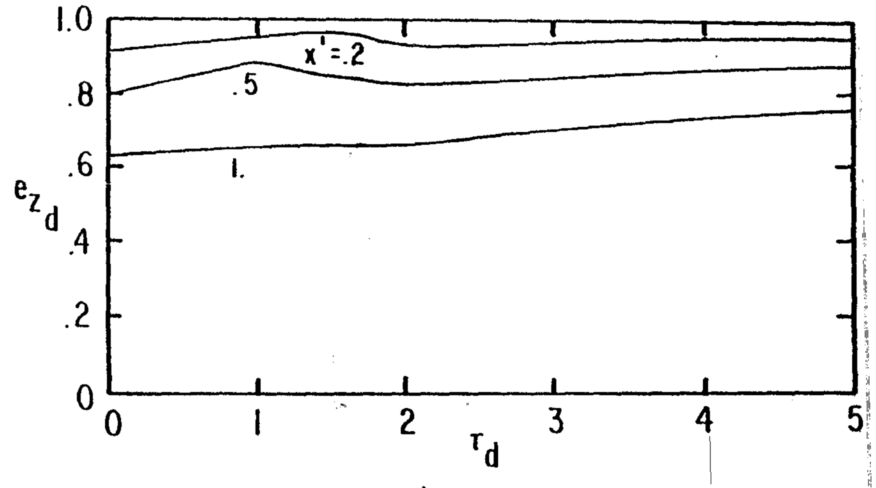
C. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 10.$

D. e_{z_d} VS τ_d WITH x' AS A PARAMETER: $p_3 = 100.$

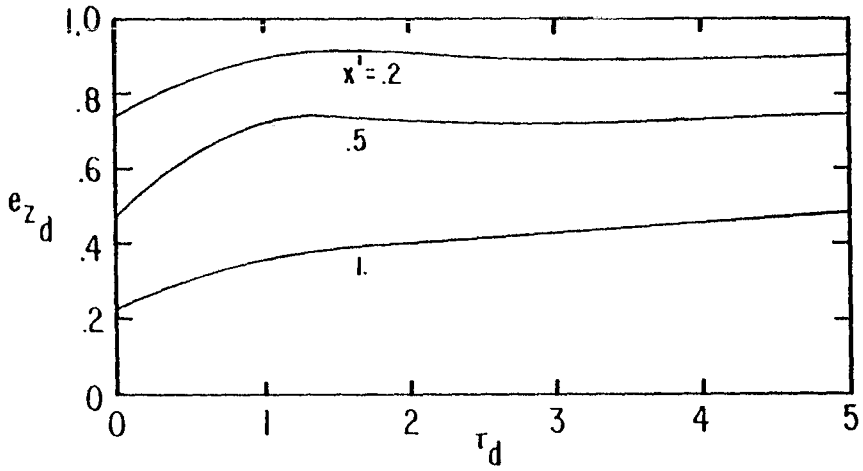
FIGURE 23. VOLTAGE OR VERTICAL ELECTRIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON TERMINATED TRANSMISSION LINE: $t_r/t_d = .3$



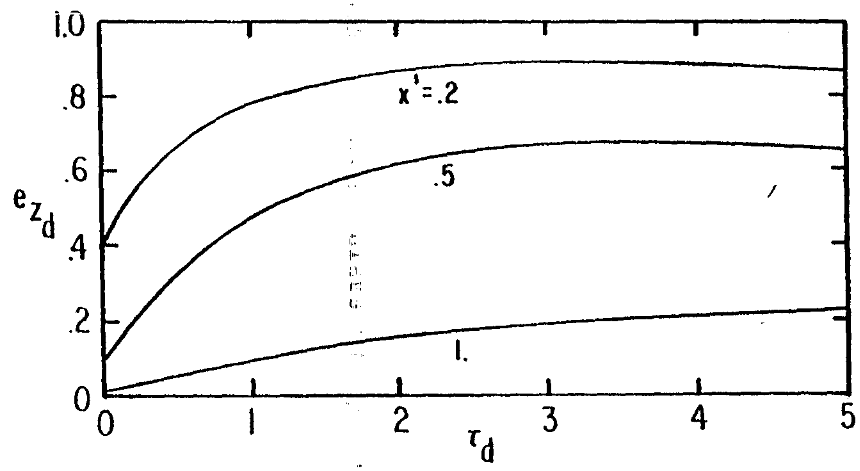
A. e_{z_d} VS τ_d WITH x' AS A PARAMETER: p_3 = .1



B. e_{z_d} VS τ_d WITH x' AS A PARAMETER: p_3 = 1.



C. e_{z_d} VS τ_d WITH x' AS A PARAMETER: p_3 = 10.



D. e_{z_d} VS τ_d WITH x' AS A PARAMETER: p_3 = 100.

FIGURE 24. VOLTAGE OR VERTICAL ELECTRIC FIELD PULSE SHAPE FOR STEP VOLTAGE ON TERMINATED TRANSMISSION LINE: t_r/t_d = 1.

VII. Summary

In this note we have considered some of the response characteristics of a possible simulator design for placing a fast-rising electromagnetic pulse over a ground surface. This type of simulator consists of a wide conducting sheet above and parallel to the ground surface. It is driven by electrical energy sources at one end and terminated in some fashion at the other end. The medium above the ground is assumed to have electromagnetic parameters the same as free space so that the wave propagates with approximately the speed of light in vacuum. As the wave propagates along this type of surface transmission line the high frequencies are preferentially attenuated. This is reflected in an increase in the rise time along the transmission line. However, the rate of increase of the rise time along the transmission line can be decreased by increasing h which decreases both p_1 and p_3. Two types of termination are considered for this type of surface transmission line. With a short-circuit termination the multiple reflections are quite pronounced unless there is sufficient loss of high frequencies in the wave to significantly degrade the rise time. With the transmission line terminated resistively so as to avoid the reflection of the highest frequency components at the termination, the rise time can be made small without introducing significant reflections into the wave. However, this is accomplished at the expense of efficiency since energy is lost in the resistive termination. Perhaps other kinds of terminations with frequency dependent impedances can be used for greater efficiency while still terminating the high frequencies. Note that there are restrictions on the ranges of various parameters for the derivation of the results in this note.

The performance of this type of surface transmission line is improved in the case of large ground conductivities. A large σ_2 makes t_r' small and reduces the loss of the high frequencies in the wave propagating over the ground. On the other hand the Brewster angle wave matcher (Ref. 1) works best with small σ_2. Thus these two types of simulation techniques are somewhat complementary. Note that for both of these techniques the ratio of the electric and magnetic fields is of the order of 377 ohms, the wave impedance of free space. If very large magnetic fields are desired, in order to simulate the electromagnetic pulse close to a nuclear surface burst, then the corresponding electric fields are quite large, perhaps leading to problems with electrical breakdown. The surface transmission line discussed in this note and the Brewster angle wave matcher may then be more appropriate for simulating the fast-rising portions of the nuclear electromagnetic pulse at somewhat less than full levels. Note that the influence of the air conductivity is not included in either of these simulation techniques.