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A Symmetry Result for an Antenna on a Truncated Ground Plane

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Abstract

In some circumstances it is desired to mount an antenna on a finite-size (truncated) ground plane and drive it with a single ended source. For an observer on the reference plane (infinite) on which the ground plane lies, one can use symmetry considerations (partial symmetry) to analyze and simplify the results. Considering the radiation of a transient pulse, for a time window of width t_g (ground-plane-contour clear time) the fields are precisely 1/2 the fields for the antenna plus image, a symmetrical configuration. This and other considerations influence the choice of ground-plane size and shape for best antenna performance.

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1. Introduction

In some applications it is desirable to mount an antenna on a finite-size or truncated ground plane which is separated from other nearby scatterers (e.g., the earth surface). One class of such applications concerns the matching of single-ended sources producing transient waveforms to a single-ended antenna, the truncated ground plane being the "ground" or reference potential for the source. Examples of interest are the half reflector impulse radiating antenna (IRA) [5, 7] and the half lens IRA (TEM horn with lens) [4, 6, 8]. The alternate to this approach is a differential source driving a differential (symmetric) antenna, or a single ended source matched to a differential antenna via a balun, a difficult task for extremely high powers and extreme bandwidth pulses.

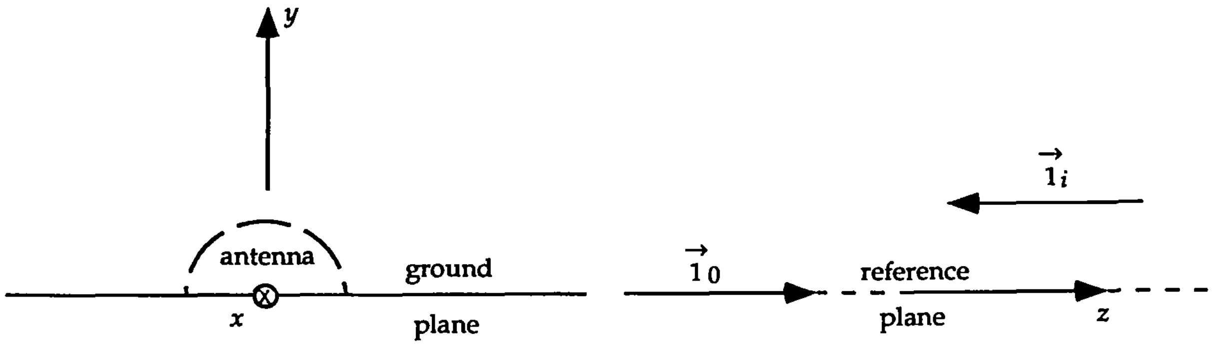
Figure 1.1 illustrates the truncated ground plane which is taken to lie on the (x, z) plane $(y = 0)$ which we term the reference plane. Note that the truncated ground plane is assumed to be larger than the linear dimensions of the antenna. For the antenna in transmission, the observer is taken to be on the reference plane at some distance away from the truncated ground plane. The direction to the observer is then taken as parallel to the reference plane, i.e.,

$$\vec{1}_0 \cdot \vec{1}_y = 0 \tag{1.1}$$

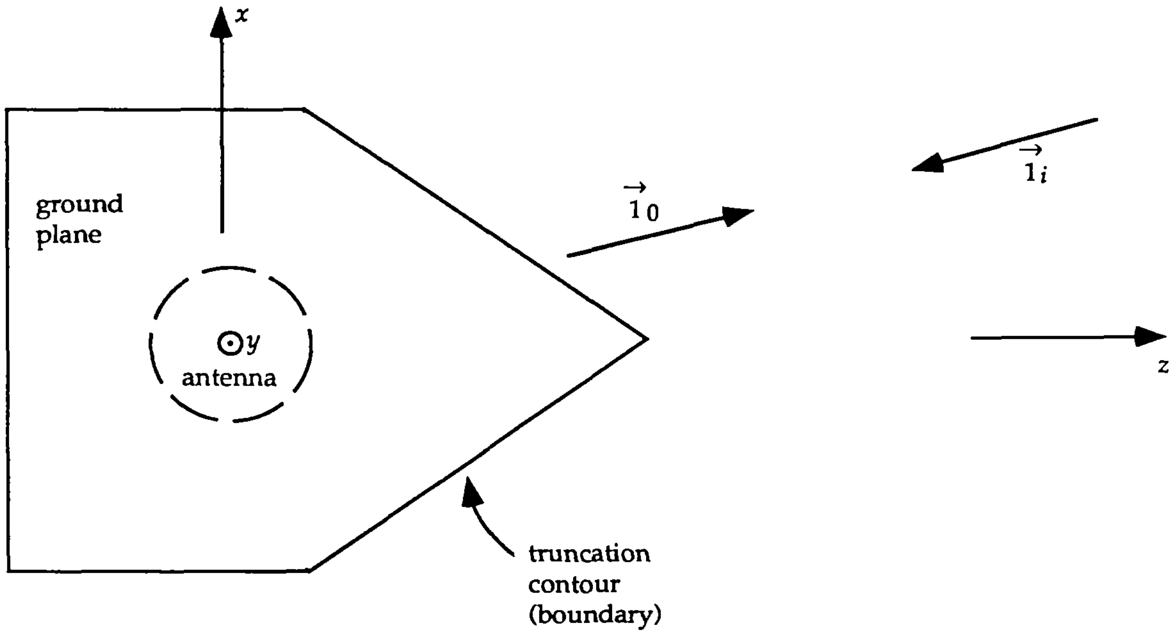
Similarly in reception the incoming plane wave is assumed to have the direction of incidence parallel to the reference plane (with the electric field polarized in the y direction) so that

$$\vec{1}_i \cdot \vec{1}_y = 0 \tag{1.2}$$

This special case will allow the introduction of symmetry considerations [11, 12]



A. Side view



B. Top view

Fig. 1.1 Antenna on Truncated Ground Plane

2. Antenna Viewed in Reception with Incidence Parallel to Reference Plane

Consider an incoming plane wave of the form

$$\begin{aligned}
\vec{E}^{(inc)}(\vec{r},t) &= E_0 \, \vec{1}_y \, f\left(t - \frac{\vec{1}_i \cdot \vec{r}}{c}\right) \\
\vec{H}^{(inc)}(\vec{r},t) &= \frac{1}{Z_0} \vec{1}_i \times \vec{E}^{(inc)}(\vec{r},t) \\
c &= [\mu_0 \, \varepsilon_0]^{-\frac{1}{2}} \equiv \text{speed of light} \\
Z_0 &= \left[\frac{\mu_0}{\varepsilon_0}\right]^{\frac{1}{2}} \equiv \text{wave impedance}
\end{aligned} \tag{2.1}$$

As a gedankenexperiment first remove the antenna so that the only thing to interact with this wave is the truncated ground plane, assumed perfectly conducting with zero thickness. In this case the above wave propagates over and past the truncated ground plane as though it were not there (no scattering).

Second, introduce the antenna on top of the ground plane. This interacts with the incident wave to produce scattered fields as well as a signal (voltage/current time function) at the antenna port(s). Initially the antenna behaves as though the ground plane were infinite (not truncated) because it takes some time for the scattered wave to propagate to the edge of the ground plane (the ground-plane contour) and return to the antenna as illustrated in fig. 2.1. Furthermore the return scattering from the contour is not in general to the nearest part of the antenna, but rather by the shortest path to the antenna port where the signal, including this scattering, is measured. In fig. 2.1, the two scattering paths are shown the same, each with length $d_g/2$ from which we define

$$t_g \equiv \frac{d_g}{c} \equiv \text{ground-plane clear time} \tag{2.2}$$

Note that this scattering path need not be parallel to (just above the surface of) the ground plane, but can include paths to/from elevated portions of the antenna.

In fig. 2.1, the direction of incidence has been chosen as $-\vec{1}_z$, a special case of interest in that the (y, z) plane $(x = 0)$ can be chosen as a symmetry plane for the antenna and truncated ground plane, about which the incident and scattered fields are purely symmetric as defined by [9, 11]

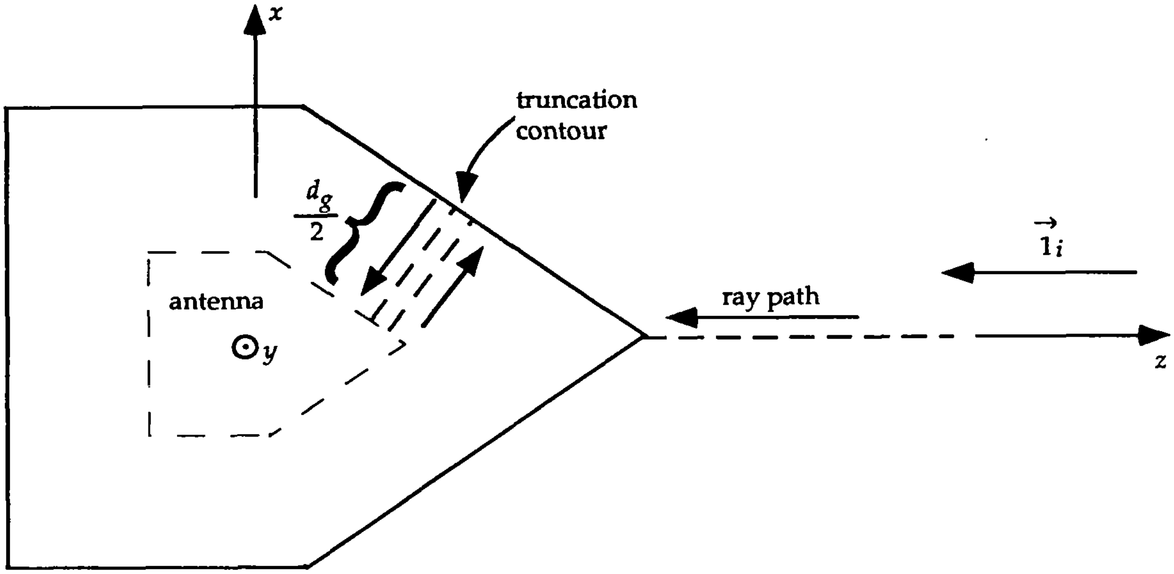


Fig. 2.1 Ground-Plane Clear Time in Reception (Simple Case).

$$R_x = \{(1),\ (R_x)\} \equiv \text{reflection symmetry group}$$

$$\overset{\leftrightarrow}{1} \equiv \overrightarrow{1}_x \overrightarrow{1}_x + \overrightarrow{1}_y \overrightarrow{1}_y + \overrightarrow{1}_z \overrightarrow{1}_z \equiv \text{identity (representation of (1))}$$

$$\overset{\leftrightarrow}{R}_x \equiv \overset{\leftrightarrow}{1} - 2\, \overrightarrow{1}_x \overrightarrow{1}_x \ \text{reflection dyadic (representation of } (R_x)) \qquad (2.3)$$

$$\begin{aligned} \overrightarrow{E}(\overset{\leftrightarrow}{R}_x \cdot \overrightarrow{r}, t) &= \overset{\leftrightarrow}{R}_x \cdot \overrightarrow{E}(\overrightarrow{r}, t) \\ \overrightarrow{H}(\overset{\leftrightarrow}{R}_x \cdot \overrightarrow{r}, t) &= -\overset{\leftrightarrow}{R}_x \cdot \overrightarrow{H}(\overrightarrow{r}, t) \end{aligned}$$

Such a symmetrical antenna then produces an electric field in transmission on the symmetry ($x = 0$) plane which is polarized parallel to this plane. In the far field for $\overrightarrow{1}_0 = \overrightarrow{1}_z$ the polarization is then $\overrightarrow{1}_y$.

The time-domain reciprocity theorem [3] relates the transmission and reception properties of the antenna (including truncated ground plane), the reception (effective height convolved with incident electric field) being the time integral of the transmitted far field times some constants (for driving waveform into the antenna the same as the incident-field waveform). This means in transmission that *for a clear time t_g the far field is not influenced by the ground-plane contour* (truncation). This remarkable result of course only applies to observers on the reference plane with $\overrightarrow{1}_0$ as restricted in (1.1). Observers at significant elevations above the reference plane will see a diffraction from the truncation contour at a retarded time very soon after the first signal from the antenna. Below the ground plane this diffraction will be simultaneous with the first signal

The above result can also be applied, by superposition, to the various constituent parts of a waveform. For example, a reflector IRA has a prepulse before the main impulsive part seen in the far field on boresight [10]. The comparatively low-level, long-pulse-width prepulse may exceed t_g in pulse width and thereby be influenced by ground-plane-contour diffraction (on the reference plane). However, the width of the impulsive part can easily be much less than t_g in the far field, and thereby not be altered in shape by the ground plane contour, again on the reference plane.

3. Antenna Viewed in Transmission with Observer on Reference Plane: Partial Symmetry

Now let us consider the antenna in transmission and apply symmetry concepts. As in fig. 3.1, add the image of the antenna above the ground plane as an identical one except for reflection $(\overleftrightarrow{R}_y)$ through the reference ($y = 0$) plane. Here the example antenna is a half IRA, but the argument applies much more generally. The antenna with truncated ground plane need not even be symmetrical $(\overleftrightarrow{R}_x)$ with respect to the $y = 0$ plane.

The image antenna is driven like the original antenna, except that we have two choices of sign. We can drive with the same sign (+ for common mode in the figure) producing a symmetric field $\vec{E}_{sy}$ with respect to the reference plane (using $(\overleftrightarrow{R}_y)$ in (2.3)). Driving with the opposite sign (– for differential mode in the figure) produces an antisymmetric field $\vec{E}_{as}$ with respect to the reference plane (using $-(\overleftrightarrow{R}_y)$ in (2.3)). The resulting field is

$$\vec{E}(\vec{r},t) = \frac{1}{2}\left[\vec{E}_{sy}(\vec{r},t) + \vec{E}_{as}(\vec{r},t)\right] \tag{3.1}$$

On the reference plane away from the truncated ground plane the symmetric electric field has no y component giving

$$\vec{1}_y \cdot \vec{E}(\vec{r},t) = E_y(\vec{r},t) = \frac{1}{2}\,\vec{1}_y \cdot \vec{E}_{as}(\vec{r},t) \tag{3.2}$$

noting that $\vec{E}_{as}$ includes both the antenna and its image. The factor of $1/2$ accounts for the fact that in the two configurations there is a voltage V each time on the antenna, giving $2V$ when the results are summed, the sum of the voltages on the image antenna being zero. If in addition the antenna and truncated ground plane are symmetric $(\overleftrightarrow{R}_x)$ with respect to the $y = 0$ plane there is no x component of the electric field at an observer on this plane. The two planes intersect on the z axis where (3.2) is augmented by

$$E_x(\vec{r},t) = 0 \tag{3.3}$$

so that the only transverse (to z) field on the z axis is polarized in the y direction. Any z component falls off rapidly, not appearing in the far field on the z axis.

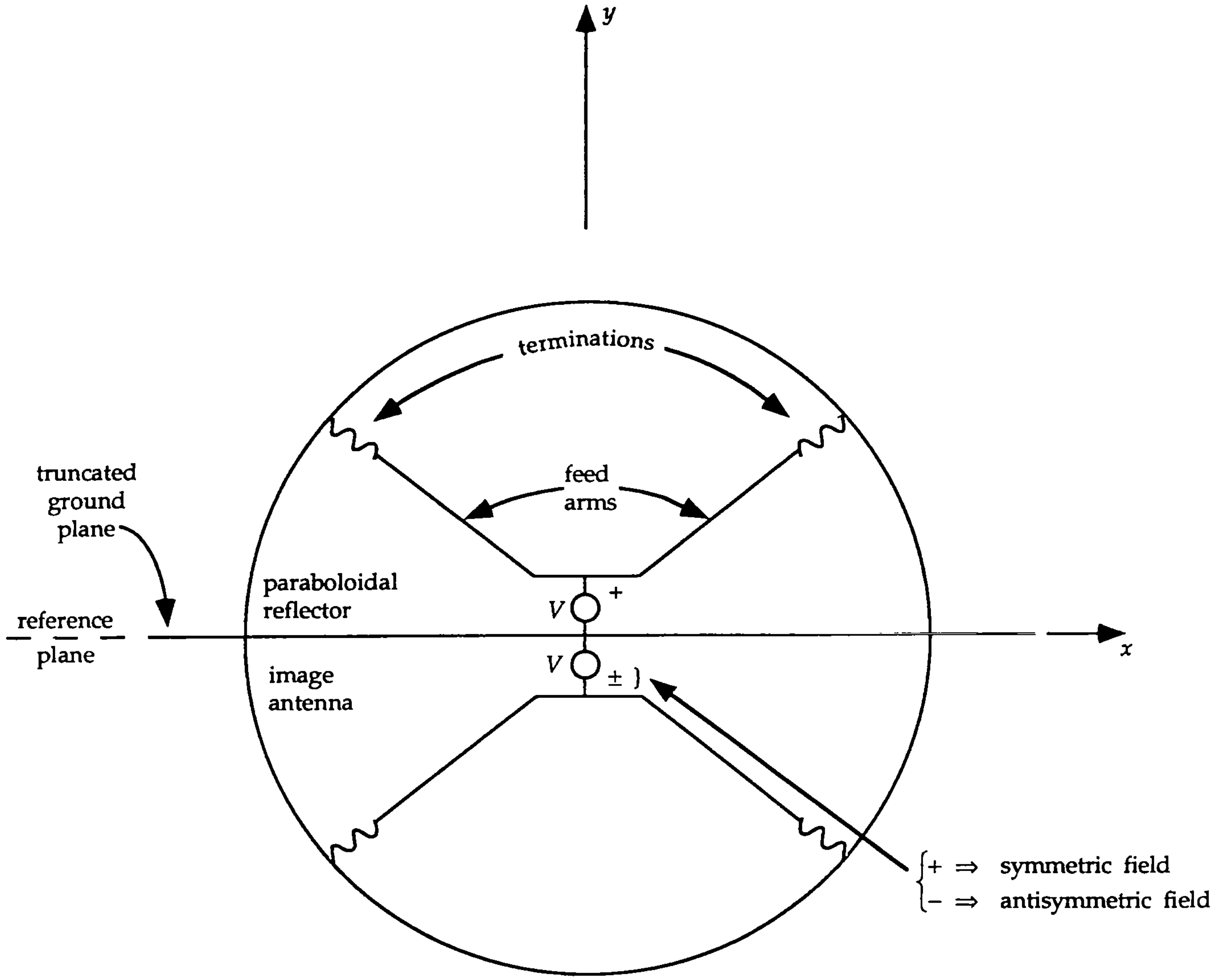


Fig. 3.1 Antenna and Image (Example as Half IRA)

So symmetry considerations (reflection) involving the antenna image are an accurate procedure for a time window of width t_g. However, strictly speaking, this symmetry does not exist due to the absence of the image antenna. This is an example of "partial symmetry" discussed in [12].

This simple factor of 1/2 has limitations. After the field from the antenna propagates around the truncated ground plane and reaches some point on the image antenna, the field does not scatter from the image antenna which is in fact not present. Similarly there is no field from the image antenna to propagate around the truncated ground plane and scatter from the antenna. However, the time for this to occur is t_g (as in fig. 2.1) since propagation paths above and below the truncated ground plane involving the image antenna are the same (symmetry). Hence the above results are valid (exact) up to a time t_g after the initial signal reaches the observer (on the reference plane). This also applies to various constituent parts of the waveform (e.g., prepulse, impulse, etc.)

At this point let us consider what else might be below the truncated ground plane (pulsers, power supplies, support structures, etc.). Such things can also scatter fields from the antenna above the ground plane. However, the effect of this at an observer on the reference plane is only seen at some time t_g after the initial signal, where t_g is now generalized to include this additional scattering. If the below-ground-plane equipment is sufficiently removed from the ground plane contour, then t_g will be governed by the image as before. However, as this equipment approaches the ground-plane contour, t_g will eventually be reduced.

4. Considerations Concerning Size and Shape of Truncated Ground Plane

As we have seen, for observers on the reference plane there is no immediate effect of the ground plane truncation (other than the factor of 1/2 in the field) for a clear time t_g. As this is based on a transit time from the antenna to its image or other scatterers below the ground plane, one consideration is to increase t_g by extending the ground-plane contour (i.e., making the ground plane larger). However, one cannot, in general, continue this to fill the reference plane. There is some limit on practical ground-plane sizes.

Previous considerations have been limited to the magnitude of t_g as a clear time. One can also consider the magnitude of the effect of the scattering after the time t_g. As one extends the ground plane contour away from the antenna and equipment below the ground plane, the scattering returning to these is reduced by the distance. Furthermore, by judicious shaping of the ground-plane contour one can minimize the high-frequency diffraction to important positions on the antenna. In particular one can avoid focusing this diffraction at certain positions on the antenna.

From a low-frequency point of view one can consider the transmitting antenna as characterized by electric and/or magnetic dipole moments. (The reflector IRA, for example, can have both [2].) A relevant question concerns how close these moments are to 1/2 those for the antenna plus image (differential mode for addition with $2V$ excitation). One can also make a similar comparison of the input impedances. With no (or negligible) structures attached below the ground plane, as one makes the ground plane larger in all directions on the reference plane this ideal factor of 1/2 is asymptotically approached. How large one should make the ground plane can be estimated based on calculations for appropriate canonical geometries. From an impedance point of view, some calculations have been performed concerning ground-plane width for a parallel-plate transmission line which is also appropriate for a lens IRA [1, 6, 8].

5. Concluding Remarks

This is another example of how symmetry concepts can be used to simplify a problem at least in some respects. For a limited amount of time (partial symmetry) the unsymmetrical electromagnetic geometry (antenna on ground plane) behaves as though it were symmetrical, at least for observers on the reference plane.

This leaves the ground-plane size and shape for future optimization.

References

1. G. W. Carlisle, Impedance and Fields of Two parallel Plates of Unequal Breadths, Sensor and Simulation Note 90, July 1969.

2. C. E. Baum, Radiation of Impulse-Like Transient Fields, Sensor and Simulation Note 321, November 1989.

3. C. E. Baum, General Properties of Antennas, Sensor and Simulation Note 330, July 1991.

4. C. E. Baum, Low-Frequency-Compensated TEM Horn, Sensor and Simulation Note 377, January 1995.

5. C. E. Baum, Variations on the Impulse-Radiating-Antenna Theme, Sensor and Simulation Note 378, February 1995.

6. C. E. Baum, Brewster-Angle Interface Between Flat-Plate Conical Transmission Lines, Sensor and Simulation Note 389, November 1995.

7. E. G. Farr and G. D. Sower, Design Principles of Half Impulse Radiating Antennas, Sensor and Simulation Note 390, December 1995.

8. M. H. Vogel, Design of the Low-Frequency Compensation of an Extreme-Bandwidth TEM Horn and Lens IRA, Sensor and Simulation Note 391, April 1996.

9. C. E. Baum, Interaction of Electromagnetic Fields with an Object Which Has an Electromagnetic Symmetry Plane, Interaction Note 63, March 1971.

10. C. E. Baum and E. G. Farr, Impulse Radiating Antennas, pp. 139–147, in H. Bertoni et al (eds.), *Ultra-Wideband, Short-Pulse Electromagnetics*, Plenum Press, 1993.

11. C. E. Baum and H. N. Kritikos, Symmetry in Electromagnetics, ch. 1, pp. 1–90, in C. E. Baum and H. N. Kritikos (eds.), *Electromagnetic Symmetry*, Taylor & Francis, 1995.

12. C. E. Baum, Symmetry and Transforms of Waveforms and Waveform Spectra in Target Identification, ch. 7, pp. 309–343, in C. E. Baum and H. N. Kritikos (eds.), *Electromagnetic Symmetry*, Taylor & Francis, 1995.