Sensor and Simulation Notes

Note 372

November 1994

Impedances of Coplanar Conical Plates in a Uniform Dielectric Lens and Matching Conical Plates for Feeding a Paraboloidal Reflector

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Abstract

In this paper we investigate impedance characteristics of coplanar conical plate geometries which pass through a lens boundary. The plates are initially in a dielectric lens matching to exterior conical plates which serve as a paraboloidal reflector feed. The lens impedance Z_{in}, is a function of the "half-angle", α', of the interior conical plates, but is independent of the ratio, F/D, of focal length to reflector diameter. Various practical choices of this ratio are made and impedances, Z_{out}, of the exterior region are calculated. As $\alpha' \longrightarrow 0$, the ratio Z_{out}/Z_{in} approaches $\sqrt{\epsilon_r}$, where ϵ_r is the relative permittivity of the lens region. As α' increases, the impedance ratio also increases. Numerical and graphical impedance results are presented as a function of F/D, so that one can trade off the various performance characteristics.

1 Introduction

One form of an impulse radiating antenna (IRA) consists of a paraboloidal reflector fed by conical transmission lines that propagate a spherical TEM wave, which originates from the focal point of the reflector. A diagram which depicts such an IRA is shown schematically in Figure 1. The design considerations of a uniform dielectric lens useful in launching a spherical TEM wave onto such a paraboloidal reflector have been investigated in earlier work [1, 2, 3], and the geometrical parameters for the lens design are indicated in Figure 2. The lens design in essence consists of specifying the following parameters.

a) h, the cylindrical radius of the outermost ray which intercepts the reflector,

b) a dielectric medium for the lens, with relative permittivity ϵ_r,

c) the angle, $\theta_r^{(out)}$, from the apex of the conical transmission line (focal point) to the edge of the reflector. This angle is determined by the reflector geometry through the ratio F/D, where F is the focal distance from the center of the reflector and D is the reflector diameter,

d) the angle $\theta_r^{(in)}$ made by the line inside the lens with respect to the directions of launch.

There are several reasons for employing dielectric lenses in such an application, and these have been discussed in [3]. The reasons may be briefly summarized as follows. First, the ideal point source at the focal point is physically realized by a closing switch. The high voltage between the switch electrodes dictates the use of a dielectric medium surrounding the switch enclosure. Secondly, a dielectric medium can also be used to ensure a spherical TEM wave launch inside the lens. All rays emanating from the switch center should exit the lens boundary and end up on a spherical wavefront centered on the focal point at the same instant. The "equal transit-time" condition, which provides the basis for the lens design, generates an optimal spherical wave (not truly TEM at very early times due to different

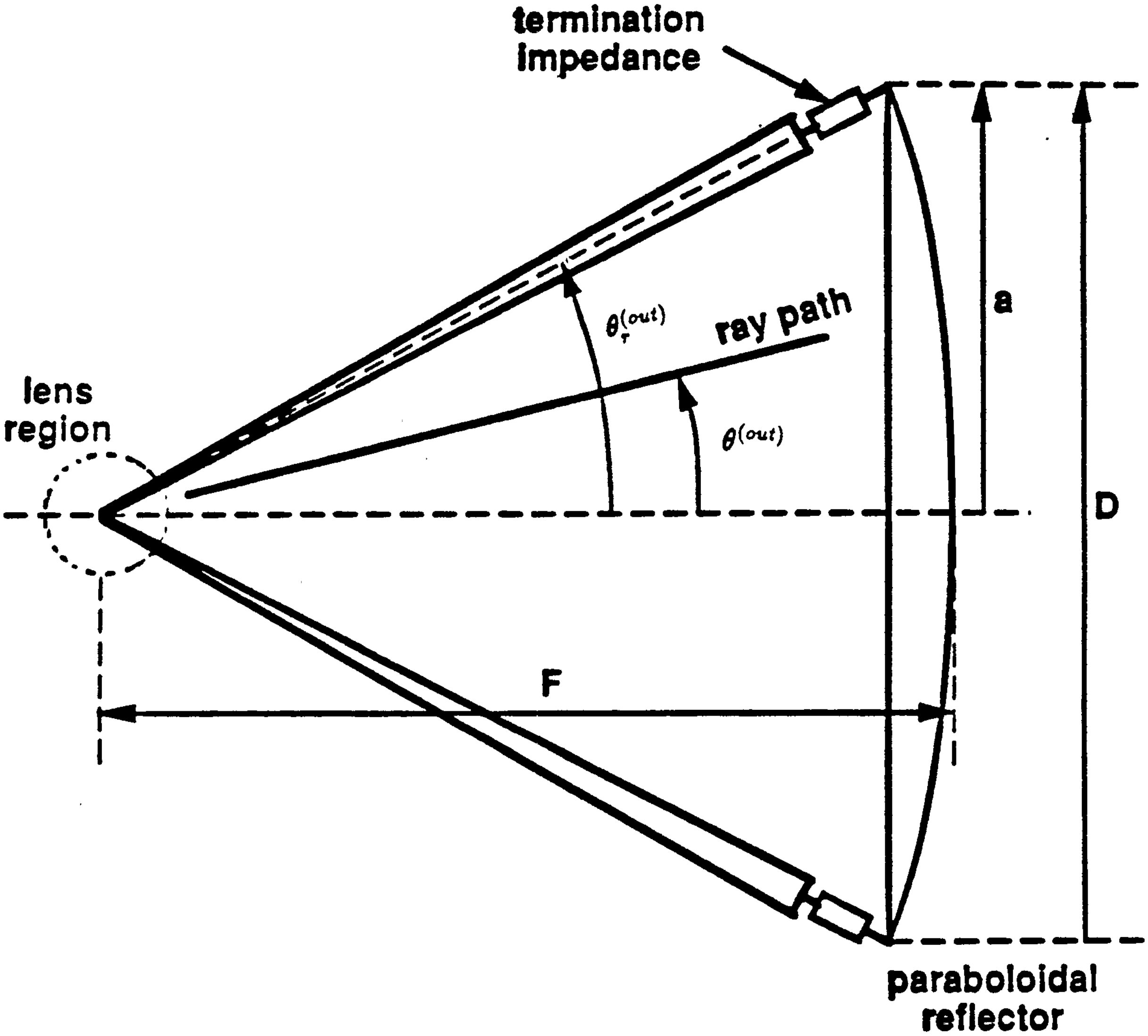

Figure 1: Schematic diagram of Prototype IRA-2

transmission coefficients at different points on the lens surface) launch onto the reflector. Note that $\theta_r^{(in)}$ in [1] is taken as 90° and defines the center (symmetry plane) of the conical plate, thereby maximizing the conical-transmission line impedance inside the lens, and the plate spacing for high voltage reasons. The frequency dependence of the dielectric constant of the lens medium is also an important consideration in the choice of the material. The dielectric constant chosen is approximately 2.26, which is the dielectric constant for materials such as transformer oil or polyethylene and is approximately frequency independent over the frequency range of interest.

In this report we are concerned with impedance calculations for a lens geometry, which we take as a coplaner conical plate geometry as investigated in [2]. The lens impedance as a function of the half-angle, α', (Figure 3) of the interior conical plates, as well as the impedance outside of the lens, is calculated on the basis of this coplaner conical geometry. The reflector geometry, specified by the F/D ratio, determines the acceptable range for the angle α'. The notation in Figure 3 is the same as that which appears in [2]. For consistency, changes have been made in the notation which appears in [1]. For example, θ_1 becomes $\beta_1^{(in)}$ or $\beta_2^{(in)}$, θ_2 becomes $\beta_1^{(out)}$ or $\beta_2^{(out)}$, while θ_{1max} is replaced by $\theta_r^{(in)}$ and θ_{2max} is replaced by $\theta_r^{(out)}$. The lens characteristic impedance is denoted by Z_{in} while the characteristic impedance of the region outside of the lens is denoted by Z_{out}. These impedances are calculated and tabulated for various F/D ratios in the later sections of this report.

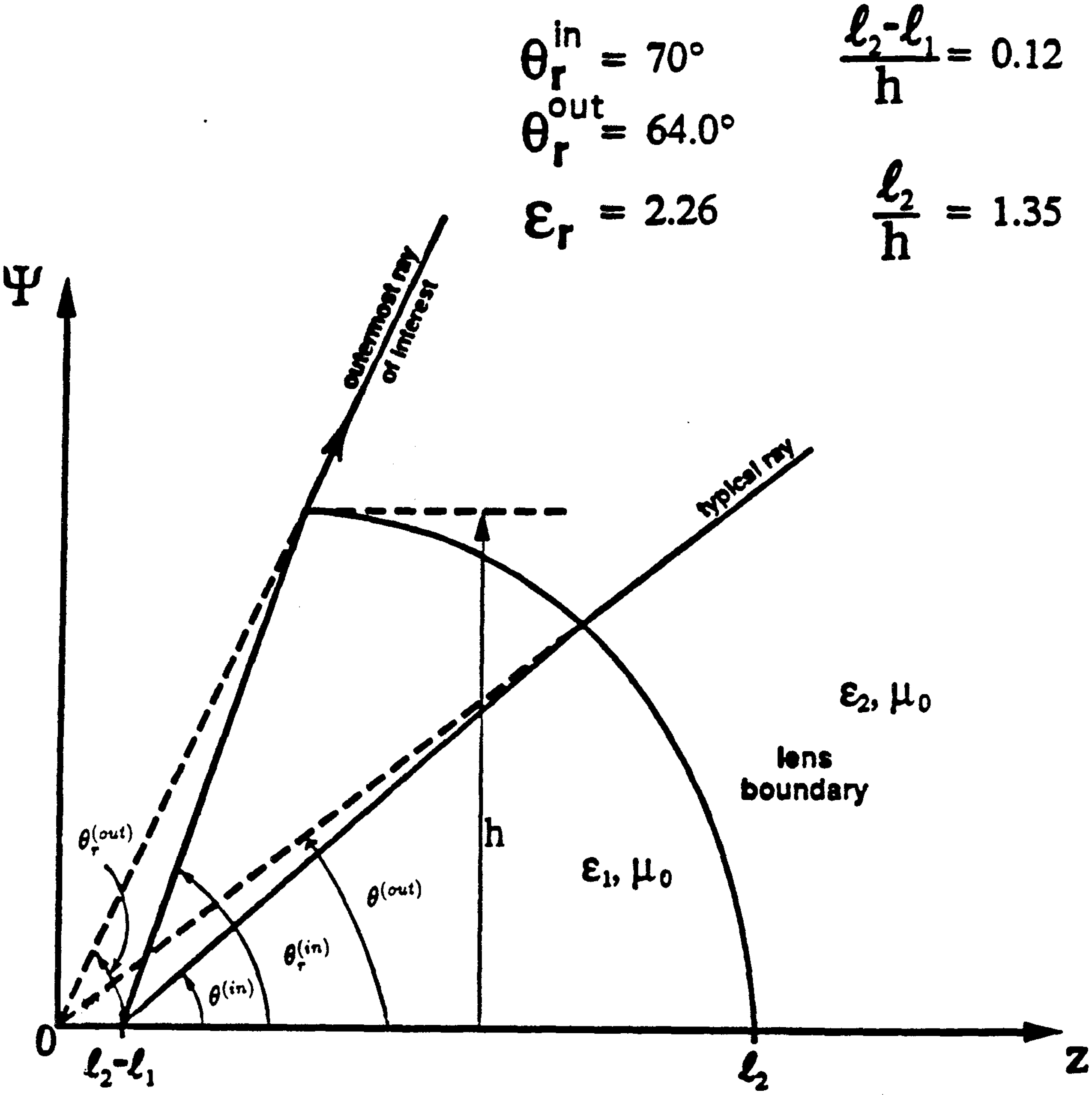

Figure 2: Lens Design Parameters

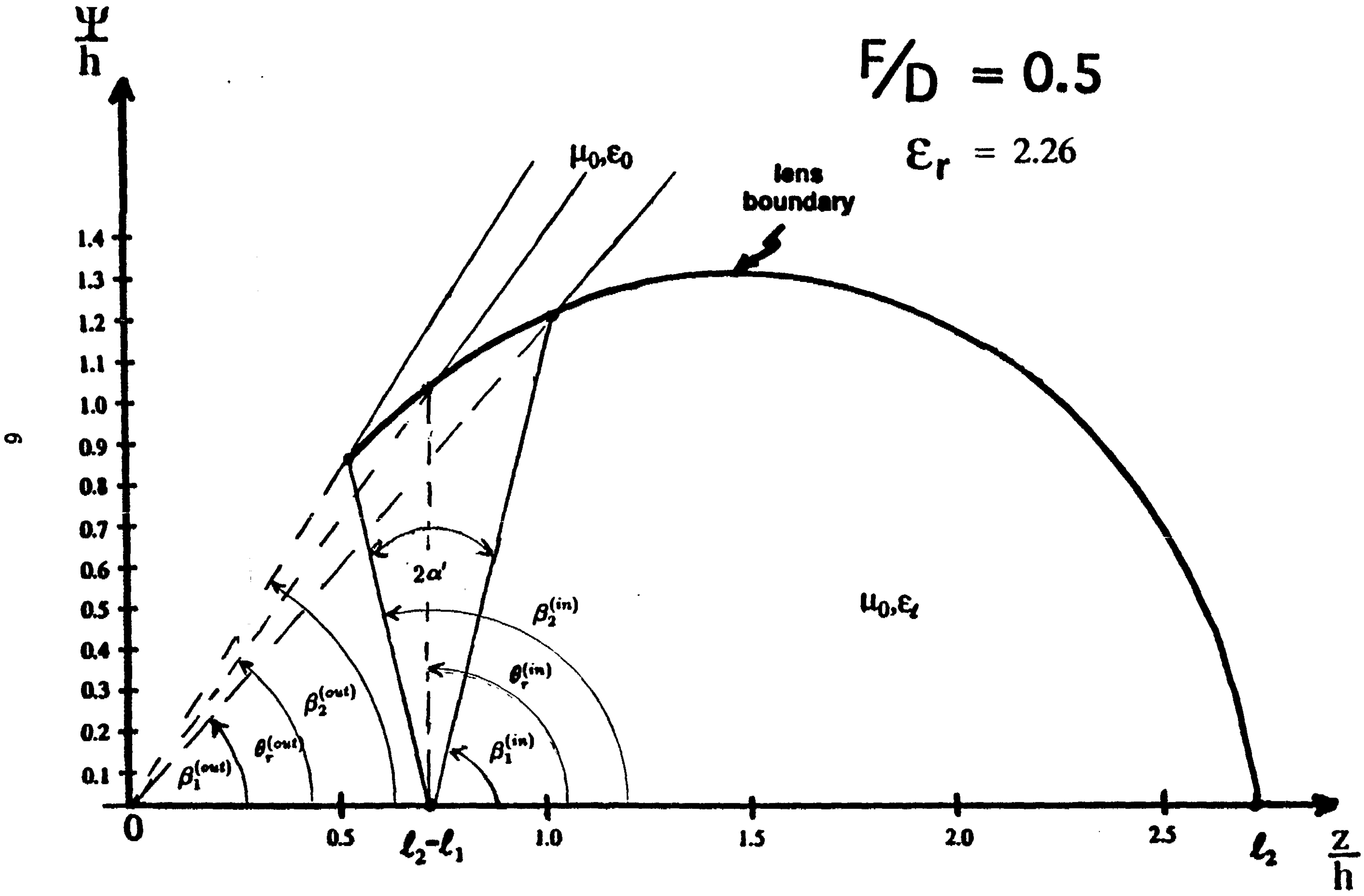

Figure 3: Coplanar Conical Plate Geometry

2 Theoretical Considerations in Lens Design

Let us recall the equal-time condition for a diverging spherical wave in a medium with permittivity ϵ_1 going into another diverging spherical wave in a second medium with permittivity ϵ_2. The equation which describes this equation is derived in [4] and it also appears in [3] in the form

$$\sqrt{\epsilon_1}\,\{[(z_b-\ell_2+\ell_1)^2+\Psi_b^2]^{1/2}-\ell_1\}=\sqrt{\epsilon_2}\,\{[z_b^2+\Psi_b^2]^{1/2}-\ell_2\} \tag{2.1}$$

where ℓ_1, ℓ_2, z_b, and Ψ_b are as they appear in Figure 2. The coordinates (z_b, Ψ_b) are coordinates of a point on the lens boundary. This equation may then be used to obtain an analytical expression for the lens boundary curve, which expresses the angle $\theta^{(out)}$ as a function of $\theta^{(in)}$. The resulting expression is

$$\cos(\theta^{(out)})=\frac{AB\sin^2(\theta^{(in)})+|B\cos(\theta^{(in)})-A\epsilon_r^{\frac{1}{2}}|\sqrt{\left[B^2-2AB\epsilon_r^{\frac{1}{2}}\cos(\theta^{(in)})+A^2\epsilon_r\right]-A^2\sin^2(\theta^{(in)})}}{B^2-2AB\epsilon_r^{\frac{1}{2}}\cos(\theta^{(in)})+A^2\epsilon_r} \tag{2.2}$$

$$\begin{aligned} A &= (\ell_2/\ell_1)-1 \\ B &= (\ell_2/\ell_1)-\epsilon_r^{\frac{1}{2}} \end{aligned}$$

The parameter ℓ_2/ℓ_1 is determined once $\theta_r^{(in)}, \theta_r^{(out)}$, are ϵ_r are chosen. If $\theta_r^{(in)} = 90°$, the formula for ℓ_2/ℓ_1 is, as in [1],

$$\frac{\ell_2}{\ell_1}=\epsilon_r^{\frac{1}{2}}\frac{[\cos(\theta_r^{(out)})+\sin(\theta_r^{(out)})]-1}{\cos(\theta_r^{(out)})+\epsilon_r^{\frac{1}{2}}\sin(\theta_r^{(out)})-1} \tag{2.3}$$

Thus when $\theta_r^{(in)} = 90°$, the parameters $\ell_2/\ell_1, A$, and B may be expressed in terms of ϵ_r and F/D. The results are:

$$A=\frac{(\epsilon_r^{\frac{1}{2}}-1)}{2}\left[\frac{16(F/D)^2-1}{4\epsilon_r^{\frac{1}{2}}(F/D)-1}\right] \tag{2.4}$$

$$B=\frac{(\epsilon_r^{\frac{1}{2}}-1)}{2}\left[\frac{16(F/D)^2-8\epsilon_r^{\frac{1}{2}}(F/D)+1}{4\epsilon_r^{\frac{1}{2}}(F/D)-1}\right] \tag{2.5}$$

$$\frac{\ell_2}{\ell_1} = \frac{\frac{1}{2}\left\{(\epsilon_r^{\frac{1}{2}} - 1)[16(F/D)^2 + 8(F/D) - 1] + 2[4(F/D) - 1]\right\}}{4\sqrt{\epsilon_r}(F/D) - 1} \tag{2.6}$$

We may also express $\theta_r^{(out)}$ in terms of F/D by the formulas

$$\sin(\theta_r^{(out)}) = \frac{8(F/D)}{16(F/D)^2 + 1} \tag{2.7}$$

$$\cos(\theta_r^{(out)}) = \frac{16(F/D)^2 - 1}{16(F/D)^2 + 1} \tag{2.8}$$

The results obtained above then may be used to obtain the table below:

F/D	$\theta_r^{(out)}$	A	B	ℓ_2/ℓ_1
.25	90°	0	-.503	1.0
.30	79.6°	.138	-.365	1.14
.33	74.3°	.190	-.314	1.19
.4	64°	.279	-.224	1.28
.5	53°	.378	-.127	1.34
.657	41.7°	.503	0	1.503

Thus for natural choices of F/D, the parameter B will be negative. Closer examination of equation (2.2) and the relevant equations in [1] indicates that for all values of $\theta^{(in)}$ we have

$$\cos(\theta^{(out)}) = \frac{AB\sin^2(\theta^{(in)}) + (A\epsilon_r^{\frac{1}{2}} - B\cos\theta^{(in)})\sqrt{\left[B^2 - 2AB\epsilon_r^{\frac{1}{2}}\cos(\theta^{(in)}) + A^2\epsilon_r\right] - A^2\sin^2(\theta^{(in)})}}{B^2 - 2AB\epsilon_r^{\frac{1}{2}}\cos(\theta^{(in)}) + A^2\epsilon_r} \tag{2.9}$$

Further calculations also show that when $(F/D) > 0.337$ that $\theta^{(out)}$ attains a maximum value when $\theta^{(in)}$ exceeds 90°, since $(A\epsilon_r^{\frac{1}{2}} - B\cos(\theta^{(in)}))$ is positive. When $(F/D) < 0.337$, $\theta^{(out)}$ does not attain a maximum value for any $\theta^{(in)}$ in the range $0 \leq \theta^{(in)} \leq 180°$.

The lens boundary curve is now specified by first choosing an appropriate F/D ratio which determines the angle $\theta_r^{(out)}$ of Figure 1. The lens parameters h, ϵ_r, and $\theta_r^{(in)}$ are then chosen

The lens boundary curve is now specified by first choosing an appropriate F/D ratio which determines the angle $\theta_r^{(out)}$ of Figure 1. The lens parameters h, ϵ_r, and $\theta_r^{(in)}$ are then chosen (consistent with certain constraints which are discussed in [1]) so that the lens parameters ℓ_1/h and ℓ_2/h are obtained and a boundary curve is generated with $0 \leq \theta^{(in)}$ and $0 \leq \theta^{(out)}$. Note that $\theta_r^{(out)}$ is not the largest $\theta^{(out)}$ on the lens boundary, but that $\theta^{(out)}$ which corresponds to the edge of the reflector and similarly for $\theta_r^{(in)}$. In the cases which we investigate the angle $\theta_r^{(in)}$ will be 90°. For the chosen value of F/D, a family of lens designs may then be generated by various choices of $\theta_r^{(in)}$, subject to the constraints mentioned in [1]. In Figure 4, lens boundary curves for $\theta_r^{(in)} = 90°$ and F/D values of 0.25, 0.3, 0.33, 0.4, and 0.5 are displayed. Note that larger values of F/D lead to larger lenses, while the value of $\theta_r^{(out)}$ decreases with increasing F/D. The limiting or trivial case occurs when $F/D = 0.25$. In this case, unlike the cases where $F/D > 0.25$, reflections from the lens surface (spherical) do occur, but the TEM wave is not distorted. There are also some limitations on how small $\theta^{(out)}$ can be for a given $\theta_r^{(in)}$. These are based on the slope of the lens boundary matching the ray direction.

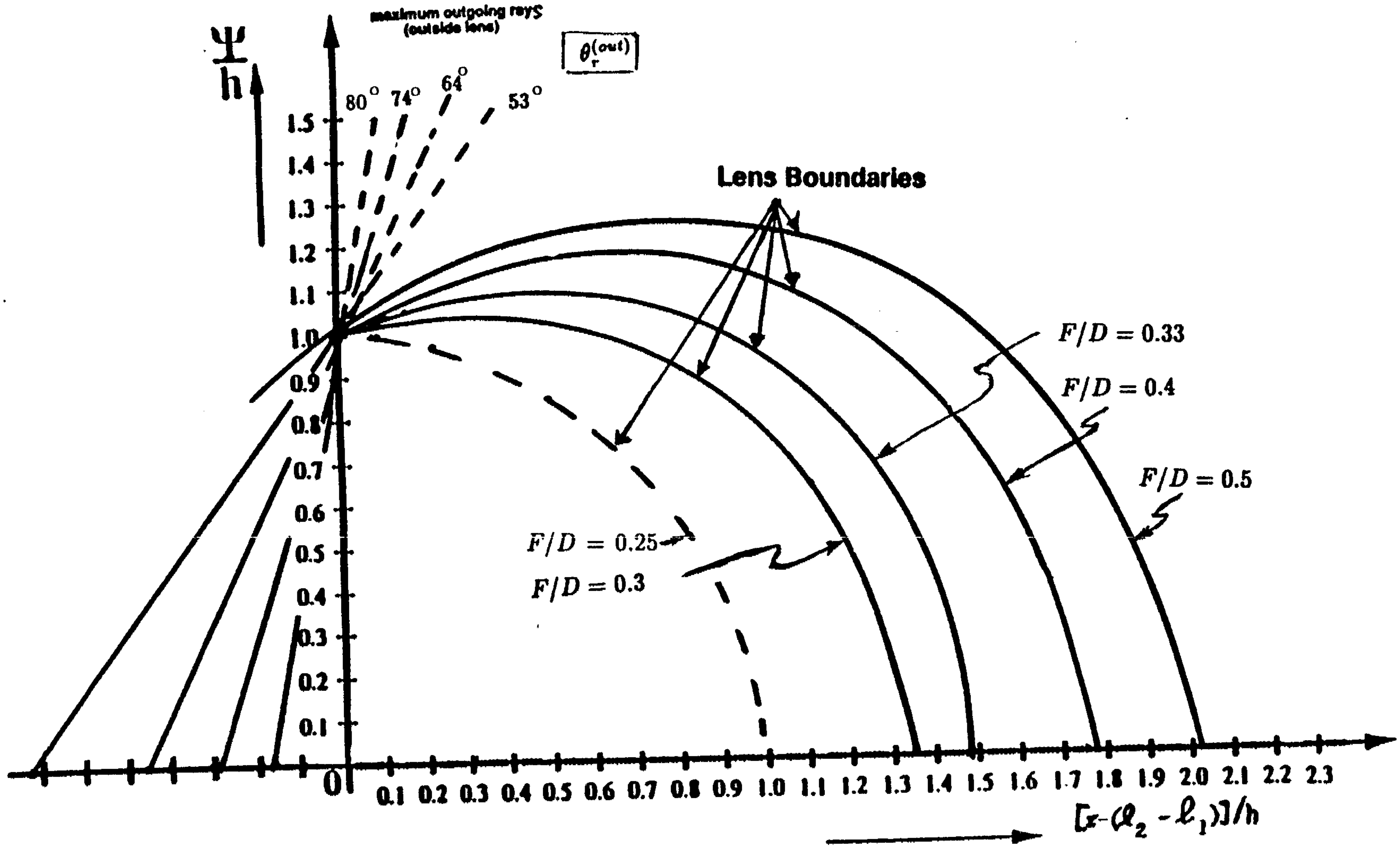

Figure 4: Lens Shapes with $\theta_r^{(in)} = 90°$, $\epsilon_r = 2.26$ (Cylindrical Coordinates)

3 Impedance Calculations

We begin by fixing the F/D ratio which determines the angle $\theta_r^{(out)}$ of Figure 1. Since the angle $\theta_r^{(in)}$ is 90° for all chosen values of F/D, and $\epsilon_r = 2.26$, the range of the angle α' is determined. Thus for all values of F/D considered, the value of Z_{in} depends only on α'. The lens characteristic impedance Z_{in} can then be calculated by the procedure described in this section. Note that $2\alpha' = \beta_2^{(in)} - \beta_1^{(in)}$ and so the value of α' determines the angles $\beta_i^{(in)}, i = 1, 2$. The angles $\beta_i^{(out)}$ are then found from equation (2.2) and thus the lens shape will be determined.

The problem of calculating the characteristic impedance of the lens region and the region exterior to the lens can now be considered. The geometry, that of coplanar conical plates, is described in Figure 3. A formula for the characteristic impedance of such a geometry appears in [2] and is given by

$$Z_{in} = \frac{Z_0}{\sqrt{\epsilon_r}} \quad \frac{K(m_{in})}{K'(m_{in})} \tag{3.1}$$

where ϵ_r is the relative permittivity for the lens material and the parameter m_{in} is given by

$$m_{in} = \frac{\tan^2(\beta_1^{(in)}/2)}{\tan^2(\beta_2^{(in)}/2)} \tag{3.2}$$

The impedance Z_0 is 376.73 ohms. The quantities $K(m_{in})$ and $K'(m_{in})$ are complete elliptic integrals of the first kind. Formulas for these integrals appear in many places (for example, in [5]) and are given by

$$K(m_{in}) = \int_0^{\pi/2} (1 - m_{in}\sin^2(\theta))^{-\frac{1}{2}} d\theta \tag{3.3}$$

$$K'(m_{in}) = K(1 - m_{in}) \equiv K(m_1^{(in)}) \tag{3.4}$$

where $0 \leq m_{in} < 1$ and $m_1^{(in)} = 1 - m_{in}$. We may also introduce a geometric impedance factor $f_g^{(in)}$ as

$$f_g^{(in)} = \frac{K(m_{in})}{K'(m_{in})} \tag{3.5}$$

and thus rewrite (3.1) in the form

$$Z_{in} = f_g^{(in)} Z_0/\sqrt{\epsilon_r} \tag{3.6}$$

Similar expressions hold for the region exterior to the lens. Thus we take for the characteristic impedance of this region

$$Z_{out} = Z_0 \frac{K(m_{out})}{K'(m_{out})} \tag{3.7}$$

where

$$m_{out} = \frac{\tan^2(\beta_1^{(out)}/2)}{\tan^2(\beta_2^{(out)}/2)} \tag{3.8}$$

and

$$K'(m_{out}) = K(1 - m_{out}) \tag{3.9}$$

Likewise a geometric factor, $f_g^{(out)}$, is given by

$$f_g^{(out)} = \frac{K(m_{out})}{K'(m_{out})}. \tag{3.10}$$

The elliptic integrals which depend on the parameters m_{in} and m_{out} may be evaluated by the tables in [5] or by use of the Mathematica program. The integrals can also be evaluated numerically by expanding the integral and performing a term-wise integration which yields, for $|m_{in}| < 1$,

$$K(m_{in}) = \frac{\pi}{2}\left\{1 + \left(\frac{1}{2}\right)^2 m_{in} + \left(\frac{1.3}{2.4}\right)^2 m_{in}^2 + \ldots\right\} \tag{3.11}$$

$$= \frac{\pi}{2} \sum_{n=0}^{\infty} \frac{1}{4}n \binom{2n}{n}^2 m_{in}^n \tag{3.12}$$

where

$$\binom{2n}{n} = \frac{(2n)!}{(n!)^2} .$$

When m_{in}, or m_{out}, is near 0 or 1, there are expressions for $K(m)/K'(m)$ which may be derived. We consider approximations for m_{in} first.

The formula (3.2) for m_{in} may be reexpressed in the form

$$m_{in} = \frac{(1-\cos(\beta_1^{(in)}))\,(1+\cos(\beta_2^{(in)}))}{(1+\cos(\beta_1^{(in)}))\,(1-\cos(\beta_2^{(in)}))} \tag{3.13}$$

and since

$$\begin{aligned} \beta_1^{(in)} &= \frac{\pi}{2} - \alpha' \\ \beta_2^{(in)} &= \frac{\pi}{2} + \alpha' \end{aligned} \tag{3.14}$$

we then obtain, for small α',

$$m_{in} \sim \frac{(1-\sin(\alpha))^2}{(1+\sin(\alpha))^2} \sim \left(\frac{1-\alpha}{1+\alpha}\right)^2 . \tag{3.15}$$

Hence

$$m_{in} \sim 1 - 4\alpha' \text{ , as } \alpha \longrightarrow 0 \tag{3.16}$$

and so

$$m_1^{(in)} = 1 - m_{in} \sim 4\alpha' \text{ , as } \alpha \longrightarrow 0 \text{ .} \tag{3.17}$$

We also have, from [5],

$$\frac{K'(m_{in})}{K(m_{in})} \sim \frac{1}{\pi} ln\left(\frac{16}{m_{in}}\right) \text{ , as } m_{in} \longrightarrow 0 \tag{3.18}$$

$$\frac{K(m_{in})}{K'(m_{in})} \sim \frac{1}{\pi} ln\left(\frac{16}{1-m_{in}}\right) \text{ , as } m_{in} \longrightarrow 1 \tag{3.19}$$

Similar expressions for m_{out} may also be derived from

$$m_{out} = \frac{(1-\cos(\beta_1^{(out)}))\,(1+\cos(\beta_2^{(out)}))}{(1+\cos(\beta_1^{(out)}))\,(1-\cos(\beta_2^{(out)}))} . \tag{3.20}$$

If we use formula (2.2) and let $\alpha' \longrightarrow 0$ we find

$$m_{out} \sim \frac{B^2 + 2AB\epsilon_r^{\frac{1}{2}}\alpha' + A^2\epsilon_r}{B^2 - 2AB\epsilon_r^{\frac{1}{2}}\alpha' + A^2\epsilon_r} . \tag{3.21}$$

Hence, if $m_{in} \longrightarrow 1$, $\alpha' \longrightarrow (1-m_{in})/4$ as $\alpha' \longrightarrow 0$. Thus

$$m_{out} \sim \frac{1 + a(1-m_{in})}{1 - a(1-m_{in})} \text{ , as } m_{in} \longrightarrow 1 \text{ ,} \tag{3.22}$$

where

$$a = \frac{AB\epsilon_r^{\frac{1}{2}}}{2(B^2 + A^2\epsilon_r)}\ .$$

Thus if $m_{in} \longrightarrow 1$, the impedance ratio

$$\frac{Z_{out}}{Z_{in}} = \sqrt{\epsilon_r}\frac{K(m_{out})}{K'(m_{out})} \cdot \frac{K'(m_{in})}{K(m_{in})} \sim \sqrt{\epsilon_r}\,\frac{\ell n\left[\frac{16}{1-(1+a(1-m_{in}))^2}\right]}{\ell n\left[\frac{16}{1-m_{in}}\right]} \quad \longrightarrow \epsilon_r^{\frac{1}{2}} \tag{3.23}$$

as $m_{in} \longrightarrow 1$.

Armed with the above formulas we may now calculate the impedances Z_{in} and Z_{out}, and also the ratio Z_{out}/Z_{in} for various practical values of F/D. In our impedance calculations we have chosen F/D ratios of 2.5, 3.0, 3.3, 4.0 and 5.0. For each of these F/D ratios, tabulations of α' , $\beta_i^{(in)}$, m_{in} , $f_g^{(in)}$, Z_{in} , $\beta_i^{(out)}$, m_{out} , $f_g^{(out)}$, Z_{out} and Z_{out}/Z_{in} are given. The data appears in Tables 1 through 5, while graphical results are presented in Figure 5, which gives plots of Z_{out}/Z_{in} versus α' , for each of the chosen values of F/D. The special case where $F/D = 0.25$ was mentioned in Section 2. Note that for our choice of $\epsilon_r = 2.26$ and for the limiting value of $F/D = 0.25$, we have $Z_{out}/Z_{in} = \sqrt{\epsilon_r} = 1.503$. Also, on each of the Tables 2 through 5 the calculation for $Z_{out} = 400$ ohms is listed separately.

4 Conclusions

In Tables 2 through 5 we observe that as α' increases, the ratio Z_{out}/Z_{in} increases. Small values of α' correspond to larger values of Z_{in}, and so as $\alpha' \longrightarrow 0$, Z_{in} increases and smaller values of Z_{out}/Z_{in} imply a better match at the lower frequencies (radian wavelengths of the order of h or larger) for which a transmission-line approximation is appropriate. As $\alpha' \longrightarrow 0$, $Z_{out}/Z_{in} \longrightarrow \epsilon_r^{\frac{1}{2}} = 1.503$, which is the ratio of the wave impedances of the media. For a ray propagating along the z axis (in the positive z direction) this represents a transmission coefficient of

$$T_\epsilon = \frac{2\epsilon_r^{\frac{1}{2}}}{\epsilon_r^{\frac{1}{2}} + 1} = 1.201 \tag{4.24}$$

while the smaller values of α' make the transmission-line transmission coefficient

$$T_Z = \frac{2\frac{Z_{out}}{Z_{in}}}{\frac{Z_{out}}{Z_{in}} + 1} \tag{4.25}$$

approach the same value. Note that for $F/D = 0.25$ the two transmission coefficients are the same for all α'.

The special case of $F/D = 0.25$ (a spherical lens) has the property that waves transmitted and reflected via the lens boundary are spherical TEM waves. However the reflected wave then is focused on the apex of the interior conical transmission line where it reflects back toward and through the lens boundary (with a fast rise time) unless there is a matched load at this apex. Larger F/D values make this wave reflected from the lens boundary dispersed in the lens and exiting the lens giving a smoother transition from the initially-transmitted fast-rising wave to the late-time behavior. It would then appear that F/D should be larger than 0.25, but not excessively so, since Z_{out}/Z_{in} increases significantly above $\sqrt{\epsilon_r}$.

We also note that for F/D values larger than 0.337, the values of α' cut off at an angle determined by the choice of F/D. When $F/D = 0.4$, this value of α' is approximately 33°, while for $F/D = 0.5$, the maximum value of α' is approximately 12°.

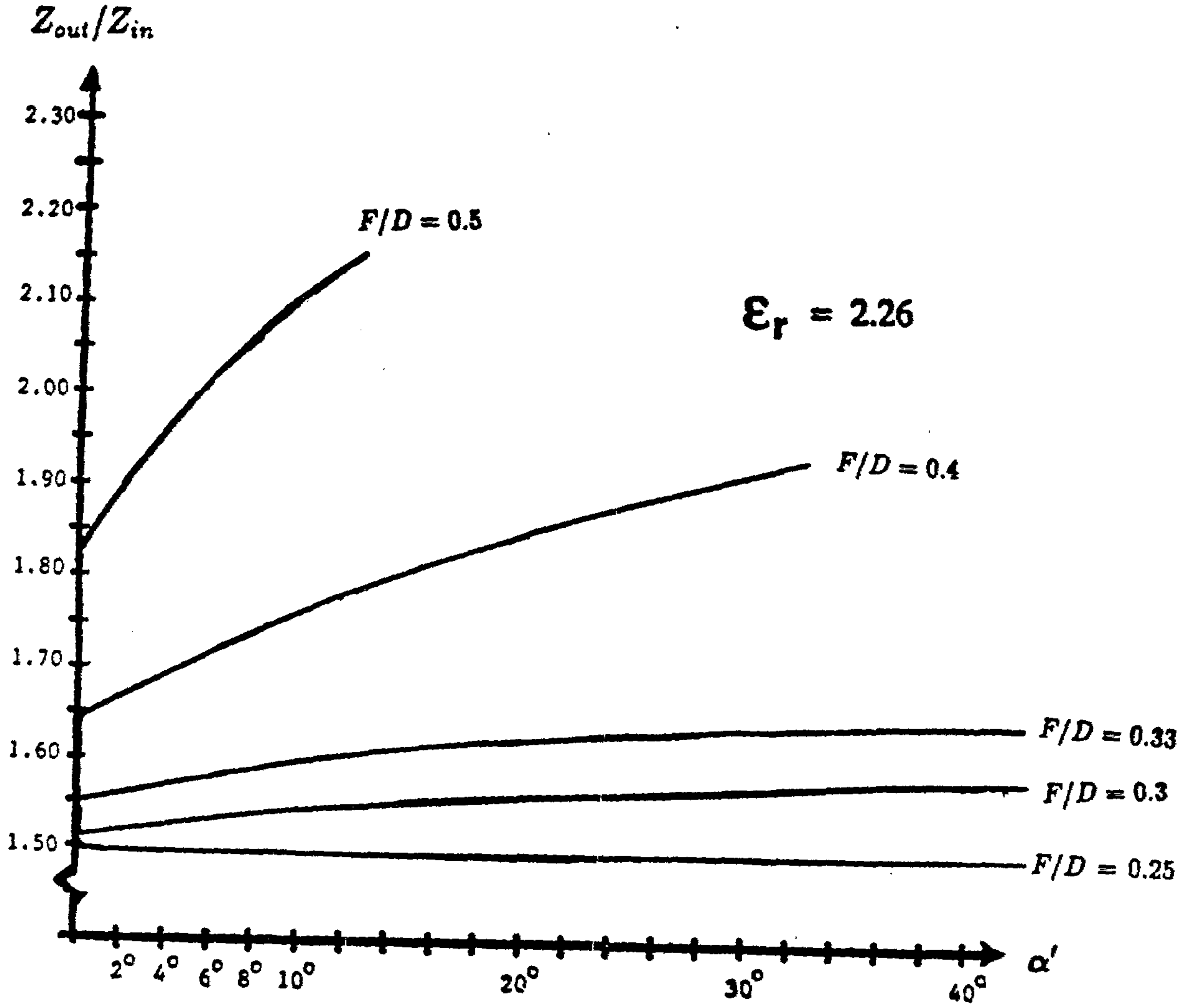

Figure 5: Impedance Ratio, Z_{out}/Z_{in}, versus half-angle, α'

α' (degrees)	$\beta_1^{(in)}$ (degrees)	$\beta_2^{(in)}$ (degrees)	m_{in}	$f_g^{(in)}$	Z_{in} (ohms)
0.5	89.5	90.5	.9657	1.951	488.8
1.0	89	91	.9326	1.730	433.5
2.0	88	92	.8697	1.509	378.2
3.0	87	93	.8110	1.380	345.8
4.0	86	94	.7562	1.288	322.9
5.0	85	95	.7050	1.217	305.1
6.0	84	96	.6573	1.159	290.5
7.0	83	97	.6127	1.110	278.2
8.0	82	98	.5710	1.067	267.5
9.0	81	99	.5321	1.030	258.1
10.0	80	100	.4957	0.9961	249.6
11.0	79	101	.4618	0.9656	242.0
12.0	78	102	.4300	0.9377	235.0
15.0	75	105	.3467	0.8660	217.0
18.0	72	108	.2786	0.8072	202.3
21.0	69	111	.2231	0.7571	189.7
24	66	114	.1779	0.7135	178.8
27	63	117	.1410	0.6747	169.1
30	60	120	.1111	0.6396	160.3
33	57	123	.0869	0.6076	152.3
36	54	126	.0674	0.5780	144.9
42	48	132	.0393	0.5245	131.4
48	42	138	.0217	0.4766	119.4
54	36	144	.0111	0.4325	108.4
60	30	150	.0052	0.3909	97.9
66	24	156	.0020	0.3504	87.8

Table 1: Coplanar Conical Plate Parameters Inside Lens (independent of F/D)

α' (degrees)	$\beta_1^{(out)}$ (degrees)	$\beta_2^{(out)}$ (degrees)	m_{out}	$f_g^{(out)}$	Z_{out} (ohms)	$\frac{Z_{out}}{Z_{in}}$
0.5	79.12	80.03	.9684	1.977	744.7	1.524
1.0	78.67	80.48	.9378	1.756	661.6	1.526
2.0	77.77	81.39	.8794	1.536	578.5	1.530
3.0	76.87	82.30	.8246	1.406	529.8	1.532
4.0	75.96	83.21	.7732	1.315	495.3	1.534
5.0	75.06	84.12	.7249	1.244	468.5	1.536
6.0	74.17	85.03	.6796	1.186	446.6	1.537
7.0	73.27	85.95	.6370	1.136	428.1	1.539
8.0	72.37	86.86	.5970	1.094	412.0	1.540
9.0	71.47	87.78	.5594	1.056	397.8	1.542
10.0	70.58	88.71	.5241	1.022	385.1	1.543
11.0	69.68	89.62	.4909	0.9918	373.6	1.544
12.0	68.79	90.55	.4598	0.9638	363.1	1.545
15.0	66.11	93.33	.3770	0.8920	336.0	1.548
18.0	63.44	96.14	.3082	0.8330	313.8	1.551
21.0	60.77	98.97	.2511	0.7827	294.9	1.554
24.0	58.11	101.8	.2037	0.7388	278.3	1.557
27.0	55.46	104.7	.1644	0.6997	263.6	1.559
30.0	52.80	107.6	.1319	0.6644	250.3	1.561
33.0	50.15	110.6	.1050	0.6320	238.1	1.564
36.0	47.50	113.6	.0829	0.6018	226.7	1.565
42.0	42.21	119.7	.0502	0.5474	206.2	1.569
48.0	36.92	126.1	.0288	0.4981	187.6	1.571
54.0	31.64	132.8	.0208	0.4733	178.3	1.645
60.0	26.37	139.7	.0074	0.4091	154.1	1.574
66.0	21.09	147.1	.0030	0.3657	137.8	1.569

Note: When $\alpha' = 8.8°$, $Z_{in} = 259.7\Omega$, $Z_{out} = 400\Omega$, and $Z_{out}/Z_{in} = 1.542$

Table 2: Coplanar Conical Plate Parameters Outside Lens ($F/D = 0.3$)

α' (degrees)	$\beta_1^{(out)}$ (degrees)	$\beta_2^{(out)}$ (degrees)	m_{out}	$f_g^{(out)}$	Z_{out} (ohms)	$\frac{Z_{out}}{Z_{in}}$
0.5	73.90	74.70	.9715	2.011	757.6	1.550
1.0	73.50	75.09	.9439	1.790	674.4	1.556
2.0	72.70	75.89	.8909	1.570	591.3	1.564
3.0	71.91	76.68	.8409	1.440	542.7	1.569
4.0	71.11	77.48	.7936	1.349	508.2	1.574
5.0	70.31	78.27	.7490	1.278	481.4	1.578
6.0	69.51	79.06	.7068	1.220	459.5	1.581
7.0	68.70	79.85	.6670	1.170	441.0	1.585
8.0	67.90	80.64	.6293	1.128	424.9	1.588
9.0	67.10	81.43	.5937	1.090	410.7	1.592
10.0	66.29	82.22	.5600	1.057	398.1	1.595
11.0	65.49	83.01	.5282	1.026	386.6	1.597
12.0	64.68	83.79	.4981	0.9982	376.1	1.600
15.0	62.26	86.15	.4172	0.9266	349.1	1.608
18.0	59.82	88.50	.3488	0.8679	326.9	1.616
21.0	57.39	90.84	.2909	0.8179	308.1	1.624
24.0	54.94	93.18	.2419	0.7744	291.7	1.632
27.0	52.49	95.51	.2005	0.7357	277.2	1.639
30.0	50.03	97.84	.1654	0.7008	264.0	1.647
33.0	47.57	100.2	.1359	0.6689	252.0	1.655
36.0	45.10	102.5	.1110	0.6395	240.9	1.663
42.0	40.14	107.21	.0725	0.5862	220.8	1.686
48.0	35.16	111.98	.0457	0.5384	202.8	1.698
54.0	30.17	116.90	.0274	0.4943	186.2	1.718
60.0	25.17	122.08	.0153	0.4522	170.3	1.739
66.0	20.15	126.69	.0080	0.4132	155.7	1.773

Note: When $\alpha' = 9.8°$, $Z_{in} = 250.9\Omega$, $Z_{out} = 400\Omega$, and $Z_{out}/Z_{in} = 1.594$

Table 3: Coplanar Conical Plate Parameters Outside Lens ($F/D = 0.33$)

α' (degrees)	$\beta_1^{(out)}$ (degrees)	$\beta_2^{(out)}$ (degrees)	m_{out}	$f_g^{(out)}$	Z_{out} (ohms)	$\frac{Z_{out}}{Z_{in}}$
0.5	63.77	64.29	.9799	2.123	799.8	1.636
1.0	63.50	64.55	.9602	1.902	716.7	1.653
2.0	62.97	65.06	.9220	1.682	633.6	1.675
3.0	62.42	65.56	.8853	1.553	585.0	1.691
4.0	61.87	66.05	.8501	1.461	551.0	1.705
5.0	61.31	66.53	.8164	1.390	523.8	1.717
6.0	60.74	67.00	.7840	1.332	502.0	1.728
7.0	60.17	67.46	.7529	1.284	483.6	1.738
8.0	59.58	67.90	.7231	1.241	467.6	1.748
9.0	58.99	68.34	.6946	1.204	453.6	1.758
10.0	58.40	68.76	.6672	1.171	441.1	1.767
11.0	57.79	69.17	.6409	1.141	429.7	1.776
12.0	57.19	69.57	.6158	1.113	419.4	1.785
15.0	55.32	70.68	.5465	1.043	393.1	1.811
18.0	53.41	71.65	.4855	0.9869	371.8	1.838
21.0	51.45	72.48	.4321	0.9395	353.9	1.865
24.0	49.45	73.14	.3852	0.8990	338.7	1.894
27.0	47.41	73.63	.3441	0.8639	325.4	1.925
30.0	45.34	73.91	.3082	0.8329	313.8	1.958
33.0	43.23	73.97	.2768	0.8056	303.5	1.993

Note: When $\alpha' = 14.2°$, $Z_{in} = 221.7\Omega$, $Z_{out} = 400\Omega$, and $Z_{out}/Z_{in} = 1.804$

Table 4: Coplanar Conical Plate Parameters Outside Lens ($F/D = 0.4$)

α' (degrees)	$\beta_1^{(out)}$ (degrees)	$\beta_2^{(out)}$ (degrees)	m_{out}	$f_g^{(out)}$	Z_{out} (ohms)	$\frac{Z_{out}}{Z_{in}}$
0.5	53.01	53.24	.9899	2.347	884.0	1.809
1.0	52.89	53.35	.9800	2.126	800.9	1.848
2.0	52.64	53.56	.9606	1.905	717.8	1.898
3.0	52.37	53.75	.9415	1.777	669.3	1.935
4.0	52.08	53.92	.9228	1.685	634.9	1.966
5.0	51.79	54.08	.9046	1.615	608.3	1.994
6.0	51.48	54.22	.8868	1.557	586.6	2.019
7.0	51.15	54.34	.8694	1.509	568.3	2.043
8.0	50.81	54.44	.8525	1.467	552.6	2.066
9.0	50.46	54.53	.8360	1.430	538.7	2.088
10.0	50.09	54.59	.8199	1.397	526.4	2.109
11.0	49.72	54.64	.8043	1.368	515.3	2.129
12.0	49.33	54.67	.7891	1.341	505.2	2.150

Table 5: Coplanar Conical Plate Parameters Outside Lens ($F/D = 0.5$)

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