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The Brewster Angle Wave Matcher

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Abstract

A technique, utilizing the high-frequency Brewster angle, is discussed as a possible simulator for placing fast-rising pulsed electromagnetic fields over a ground surface. The reflection of a rectangular pulse at the high-frequency Brewster angle is calculated, showing that for pulse widths less than the relaxation time of the ground, the reflection amplitude is small compared to the incident amplitude. This type of simulator may be appropriate for simulation tests involving low ground conductivities.

I. Introduction

The problem of the simulation of the nuclear electromagnetic pulse from a surface burst over an extensive ground-based facility is a complex one. Conceptually it may be possible to divide the problem into several different (but not entirely separable) problems. One such division might be based on the transit time (in non-conducting air) over the surface extent of the system. For pulses (or that part of a pulse) with characteristic times greater than this transit time the above-surface structure can be considered as one or more electrical lumped elements. Something like the buried transmission line can then be used to transport these longer-time fields deep into the ground.[1] Suppose that it is desired to produce a clean, fast-rising pulse over the ground surface, with a rise time smaller than the transit time across the surface extent of the system. Then the above-surface simulator structure may have to be analyzed as a wave-guiding structure and/or a transmission line.

Another problem concerns the air conductivity and Compton current density in the source region for the nuclear electromagnetic pulse. A simulation technique using electrical energy sources is quite deficient in this regard. In some cases, particularly for parts of a system (under test) at and above the surface, the interaction of the simulated electromagnetic pulse with the system may be somewhat different from the real case. For this note we do not consider the problems associated with the Compton current density or the air conductivity. We approach this simulation problem from the point of view of producing a desired magnetic field pulse shape over the ground surface. The associated vertical electric field may not be very similar to the vertical electric field, E_z, in the nuclear electromagnetic pulse which is being simulated since the properties of the air above the ground differ from those in a nuclear environment. Since the air conductivity is not considered, however, the difference in the vertical electric field may not be a significant point, because the vertical conduction current density, σE_z, in the air is entirely different. Neglecting the air conductivity and Compton current density in regions where they significantly influence the electromagnetic fields (i.e., the source region) is then a major assumption, at least with regards to the fields above the ground surface. There may also be problems regarding the ground conductivity in the presence of the nuclear radiation, but this is also neglected. Then we only consider the simulation problem of producing a fast-rising magnetic field pulse (with other associated field components) over a ground surface, in the absence of other characteristic features of the nuclear source region. These other features may turn out to be significant in some cases, but perhaps difficult to adequately simulate.

There may be several approaches to putting a fast-rising magnetic field pulse over the ground surface. Malik has proposed a simulator consisting of several conical sheets (or wire grids), from a common energy source, connected to rows of stakes in the ground. Appropriate pairs of the conical sheets form conical transmission lines which are driven in an electrical series configuration. The conical transmission lines then transmit the pulse to the

1. Lt Carl E. Baum, Sensor and Simulation Note XXII, A Transmission Line EMP Simulation Technique for Buried Structures, June 1966.

ground, each transmission line covering a part of the total ground area. One conical sheet or grid can be part of two of the conical transmission lines.[2] Some experimental work has been done in the case of two such conical grids forming a single conical transmission line.[3]

In the above approach one can, to some limited extent, think of the problem of matching the pulse into the ground by considering the matching of a plane wave into the ground. For the polarization of an incident plane wave with the magnetic field parallel to the ground surface, and for frequencies much greater than the relaxation frequency (σ/ε) of the ground, there is an angle for the incident wave at which there is very little reflection of the incident wave from the ground surface. We call this angle the high-frequency Brewster angle. At this particular angle a pulsed incident wave, with pulse width much less than the relaxation time (ε/σ) of the ground, produces a reflection with much smaller amplitude than the incident wave. In some cases, one might be able to use this high-frequency Brewster angle in designing a simulator to produce a fast-rising pulsed magnetic field over the ground surface.

Consider then the structure in figure 1. A conducting sheet or wire grid is used as the top plate of a parallel plate transmission line with the plate width, w, much greater than the plate spacing, h. Over most of the width of the transmission line the magnetic field is parallel to the plate and to the ground surface. This is the orientation for a plane wave for which there is a high-frequency Brewster angle. Choose the angle, ψ_B, for the orientation of the plate with respect to the ground surface so as to match the wave into the ground at the high-frequency Brewster angle. In such a form we call this technique the Brewster angle wave matcher.

The idea is then to launch a plane wave on the wide cylindrical transmission line, which is oriented with respect to the ground surface so as to match the wave into the ground at the high-frequency Brewster angle. To launch a fast-rising plane wave on the parallel plate transmission line one might use one or more conical transmission lines, matched to the parallel plate line, together with one or more fast-rising pulse generators.[4] Note that the bottom plate of the parallel plate transmission line is not really there since the ground surface begins right at what would be the beginning of the bottom plate.

There are various limitations in the Brewster angle wave matcher. Near the sides of the parallel plate line the magnetic field is not, in general, parallel to the ground surface so that the wave may not match into the ground surface as well near these sides. At each end of the test area, where there are two rows of stakes making contact with the ground, there may also be problems with the wave matching into the ground due to the distortion of the field orientation in the wave. Since the wave in the ground is not propagating in quite a vertical direction, then vertical conductors in the ground may distort the wave near these conductors. These same conductors could be slanted to more nearly match the direction of wave propagation in the ground, but this

2. J. Malik, letter of 21 May 1964: Generation of Surface Electric Fields.
3. M. M. Newman, et al, L&T Report 427, Experimental Study of the Generation of Surface and Subsurface Electromagnetic Fields, March 1965.
4. Capt Carl E. Baum, Sensor and Simulation Note XXXI, The Conical Transmission Line as a Wave Launcher and Terminator for a Cylindrical Transmission Line, January 1967.

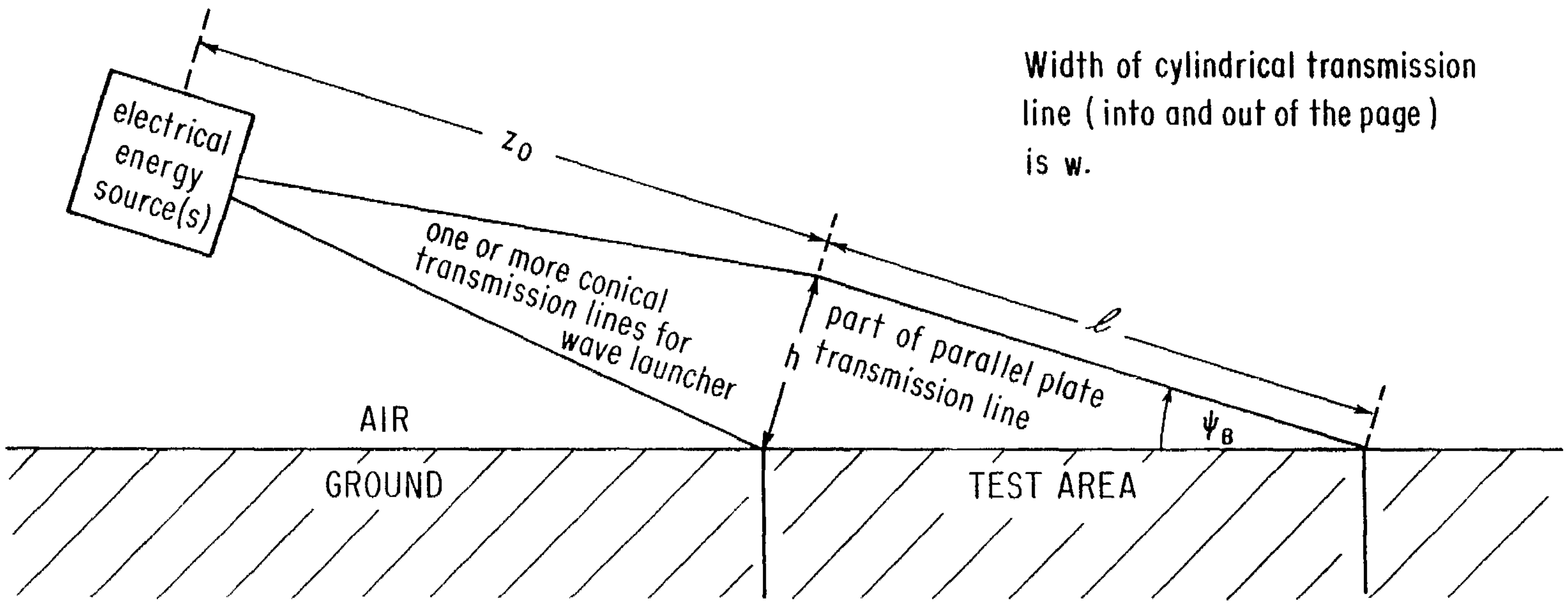

FIGURE 1. BREWSTER ANGLE WAVE MATCHER

may not be a significant problem due to the rapid attenuation of the high-frequency fields in the ground.

For sufficiently high frequencies, then, the Brewster angle wave matcher may place a plane wave over the ground surface with little significant reflections. As the frequency is lowered, however, the plane wave produces significant reflections which can in turn interact with the top plate of the transmission line and reflect back toward the ground and the energy source. These reflections may restrict the range of usefulness of this technique. Since we are interested in placing a fast pulse over the ground we should consider various times of interest. These include the pulse width, the relaxation time of the ground, and the transit times on the transmission line structure (including conical transition(s)), e.g., $(\ell+z_0)/c$. If the pulse width is less than the relaxation time then the amplitude of the reflection from the ground is small compared to the amplitude of the incident wave, a desirable situation. If the transit times on the structure are less than the relaxation time, then, for frequencies low enough for the reflections to be large, the above-ground structure can be treated as one or more electrical lumped elements (because wavelengths for such low frequencies are then much larger than the above-ground structure). For this latter case multiple reflections are then not a problem; if a pulse width is longer than the relaxation time the later parts of it may be distorted, but perhaps one can compensate for such distortion. Then for the pulse width and/or the transit times on the structure smaller than the relaxation time the Brewster angle wave matcher may be an appropriate simulator for a fast-rising magnetic field pulse over a ground surface.

The problem of multiple reflections in the structure (for relaxation time less than transit times) may not be as serious as one might think. For small ψ_B (associated with a large high-frequency permittivity of the ground) the reflections may not be distinct, so as to give a "ringing" effect, but may blend into the wave. Such phenomena are not considered in this note but can perhaps be included by considering the structure, including the ground as a transmission line. Another thing to notice about this simulator is that, for the incident wave striking the ground surface, the speed of the wave over the surface is greater than c, the speed of light in vacuum, because the wave is propagating at an angle, ψ_B, with respect to the surface. For small ψ_B this discrepancy is small since the speed over the surface is $c/\cos(\psi_B)$. Ideally one would like the speed over the surface to be c since in a nuclear source region the source currents and conductivity appear over the surface at this speed, as do the resulting fields. Thus, there are various imperfections in the Brewster angle wave matcher. Some of these, however, may not be significant in some cases of interest.

II. Pulse Matching into the Ground

Consider, now, the matching of the pulsed wave into the ground. Take the case of a plane wave incident on the ground with a polarization such that the magnetic field is parallel to the ground surface, as illustrated in figure 2. The ground surface (assumed flat) is taken to define the (x,y) plane. The incident wave propagates at an angle, θ_1, with respect to the positive z axis, or at an angle, ψ_1, with respect to the ground surface. The reflected wave has the same angles as the incident wave. The transmitted wave propagates at an angle, θ_2, with respect to the negative z axis. Medium

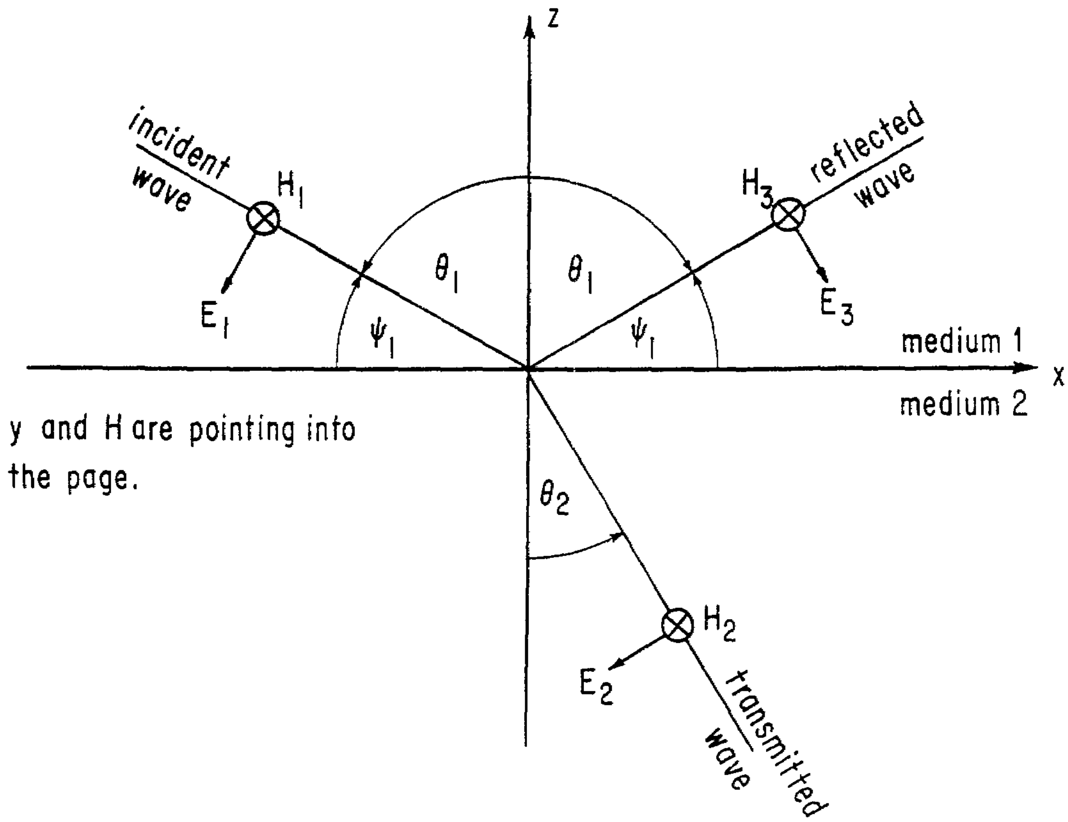

FIGURE 2. REFLECTION OF A PLANE WAVE AT THE SURFACE OF A CONDUCTING DIELECTRIC WITH THE MAGNETIC FIELD PARALLEL TO THE SURFACE

one (air) has permittivity, ε_o, permeability, μ_o, and zero conductivity. Medium two (ground) has permittivity, ε_2, permeability, μ_o, and conductivity, σ_2. The ground conductivity is not very frequency dependent for frequencies of interest; the ground permittivity is quite frequency dependent but seems to level off for high frequencies.[5] In any case we assume that both σ_2 and ε_2 are frequency independent for this analysis.

For convenience define a relaxation time for the ground as

$$t_r = \frac{\varepsilon_2}{\sigma_2} \tag{1}$$

Then define a normalized Laplace transform variable

$$s_r = st_r \tag{2}$$

In the time domain define a normalized time

$$\tau_r = \frac{t}{t_r} \tag{3}$$

where t is time. The relative dielectric constant of the ground is

$$\varepsilon_r = \frac{\varepsilon_2}{\varepsilon_o} \tag{4}$$

Substituting $j\omega$ for s_r in the Laplace transformed quantities (to be developed) we perform a numerical inverse Fourier transform to obtain the desired pulse shapes as a function of τ_r, the normalized time.[6]

In another note we develop the pulse reflection from such a ground surface for various wave polarizations and angles of incidence.[7] For the case of the magnetic field parallel to the surface the reflected wave for an incident step function wave (applying to both E and H) is

$$\tilde{R}_h = \frac{1}{s_r} \frac{\cos(\theta_1) - \sqrt{\frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}\left[1 - \frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}(\sin(\theta_1))^2\right]}}{\cos(\theta_1) + \sqrt{\frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}\left[1 - \frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}(\sin(\theta_1))^2\right]}} \tag{5}$$

where the tilde, $\sim$, over a quantity indicates the Laplace transform of the quantity. The positive directions for the fields in each of the waves are as indicated in figure 2. There are various pulse shapes one might use for the incident wave, but we consider the step function, and later a rectangular pulse, for convenience. This approach should show some of the significant features of a reflected pulse.

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5. James H. Scott, EMP Theoretical Note XVIII, Electrical and Magnetic Properties of Rock and Soil, May 1966.
6. Frank Sulkowski, Mathematics Note II, FORPLEX: A Program to Calculate Inverse Fourier Transforms, November 1966.
7. Capt Carl E. Baum, EMP Theoretical Note XXV, The Reflection of Pulsed Waves from the Surface of a Conducting Dielectric, February 1967.

The initial rise of R_h is zero at one particular angle, θ_1 or ψ_1, which we denote as θ_B or ψ_B; the corresponding reflected pulse is denoted by R_B. The particular angle we have called the high-frequency Brewster angle. The initial rise of R_h is

$$R_h(0+) = \frac{\cos(\theta_1) - \frac{1}{\varepsilon_r}\sqrt{\varepsilon_r - (\sin(\theta_1))^2}}{\cos(\theta_1) + \frac{1}{\varepsilon_r}\sqrt{\varepsilon_r - (\sin(\theta_1))^2}} \tag{6}$$

Setting this to zero defines the high-frequency Brewster angle as

$$\cos(\theta_B) = \frac{1}{\sqrt{1+\varepsilon_r}} \tag{7}$$

or

$$\sin(\theta_B) = \sqrt{\frac{\varepsilon_r}{1+\varepsilon_r}} \tag{8}$$

There is also the convenient relationship

$$\tan(\psi_B) = \cot(\theta_B) = \frac{1}{\sqrt{\varepsilon_r}} \tag{9}$$

showing that large ε_r gives small ψ_B, making the size of the simulator structure smaller to cover a given test area.

Restrict the angle of incidence to be the high-frequency Brewster angle. Denote the reflection due to a step function incident wave as R_B for this special case, giving

$$\tilde{R}_B = \frac{1}{s_r}\,\frac{\cos(\theta_B) - \sqrt{\frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}\left[1 - \frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}(\sin(\theta_B))^2\right]}}{\cos(\theta_B) \mp \sqrt{\frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}\left[1 - \frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}(\sin(\theta_B))^2\right]}} \tag{10}$$

Multiply numerator and denominator of this expression by the denominator, giving

$$\tilde{R}_B = \frac{1}{s_r}\,\frac{(\cos(\theta_B))^2 - \frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}\left[1 - \frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}(\sin(\theta_B))^2\right]}{\left\{\cos(\theta_B) \mp \sqrt{\frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}\left[1 - \frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}(\sin(\theta_B))^2\right]}\right\}^2} \tag{11}$$

Substituting for $\cos(\theta_B)$ and $\sin(\theta_B)$ from equations (7) and (8) gives

$$\tilde{R}_B = \frac{1}{s_r} \frac{\frac{1}{1+\varepsilon_r} - \frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}\left[1-\frac{1}{1+\varepsilon_r}\frac{s_r}{1+s_r}\right]}{\left\{\sqrt{\frac{1}{1+\varepsilon_r}}+\sqrt{\frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}\left[1-\frac{1}{1+\varepsilon_r}\frac{s_r}{1+s_r}\right]}\right\}^2}$$

$$= \frac{1}{s_r} \frac{1-\frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}\left[1+\varepsilon_r-\frac{s_r}{1+s_r}\right]}{\left\{1+\sqrt{\frac{1}{\varepsilon_r}\frac{s_r}{1+s_r}\left[1+\varepsilon_r-\frac{s_r}{1+s_r}\right]}\right\}^2}$$

$$= \frac{1}{s_r} \frac{(1+s_r)^2-\frac{1+\varepsilon_r}{\varepsilon_r}s_r(1+s_r)+\frac{1}{\varepsilon_r}s_r^2}{\left\{1+s_r+\sqrt{\frac{1+\varepsilon_r}{\varepsilon_r}s_r(1+s_r)-\frac{1}{\varepsilon_r}s_r^2}\right\}^2}$$

$$= \frac{1}{s_r} \frac{1+\left[1-\frac{1}{\varepsilon_r}\right]s_r}{\left\{1+s_r+\sqrt{\left[1+\frac{1}{\varepsilon_r}\right]s_r+s_r^2}\right\}^2} \qquad (12)$$

Look at the initial behavior of R_B by taking the limiting form of $\tilde{R}_B$ for large $|s_r|$ which is

$$\tilde{R}_B \simeq \frac{1-\frac{1}{\varepsilon_r}}{4\,s_r^2} \qquad (13)$$

Then for $\tau_r \ll 1$ the initial form of the reflected wave is

$$R_B \simeq \frac{1-\frac{1}{\varepsilon_r}}{4}\,\tau_r \qquad (14)$$

which is a linear ramp starting at zero amplitude. The form of this result and that of equation (12) indicates that, for $\varepsilon_r >> 1$, R_B is not very dependent on ε_r when R_B is expressed in terms of the normalized time, τ_r. Remember also that the angle of incidence (and reflection) is changed as ε_r is varied.

Having the reflection due to an incident step function wave one can easily construct the reflection due to an incident rectangular wave. The incident rectangular wave, of unity amplitude and time width, Δt, is of the form $u(\tau_r) - u(\tau_r - \frac{\Delta t}{t_r})$ where $u(\tau_r)$ is a unit step function which rises at $\tau_r = 0$. The reflection, R_B', in response to such a rectangular pulse (in the normalized time) is just

$$R_B' = R_B(\tau_r) - R_B(\tau_r - \frac{\Delta t}{t_r}) \qquad (15)$$

Performing a numerical inverse Fourier transform on equation (12) gives the reflection due to a step function incident wave; this is plotted in figures 3 and 4 for the cases of $\varepsilon_r = 10$ and $\varepsilon_r = 80$, respectively. Also included on these graphs are the reflections due to incident rectangular waves for various values of $\Delta t/t_r$, computed from equation (15). The indicated errors in the calculation of R_B (from successive calculations of the inverse Fourier transform computer code[8]) are about 3×10^{-3}.

Note in figures 3 and 4 that there is little difference between the two cases for ε_r, illustrating the relative independence of the result on ε_r for large ε_r. Since the upper medium (air) is assumed to be nonconducting the reflected pulse is not distorted as it propagates away from the surface. There is, however, a time delay introduced for positions away from the surface because $\tau_r = 0$ is defined by the arrival of the incident wave at a particular position on the ground surface, such as the coordinate origin in figure 2. The amplitude of the reflection is small for small $\Delta t/t_r$; for $\Delta t/t_r = 1$ it is only up to about .2. For convenience we include a table of the relaxation time and high-frequency Brewster angle for various conductivities and relative dielectric constants for both ground and sea water. These conductivities roughly cover the typical range of rock and soil conductivities.[9]

8. See reference 6.
9. See reference 5.

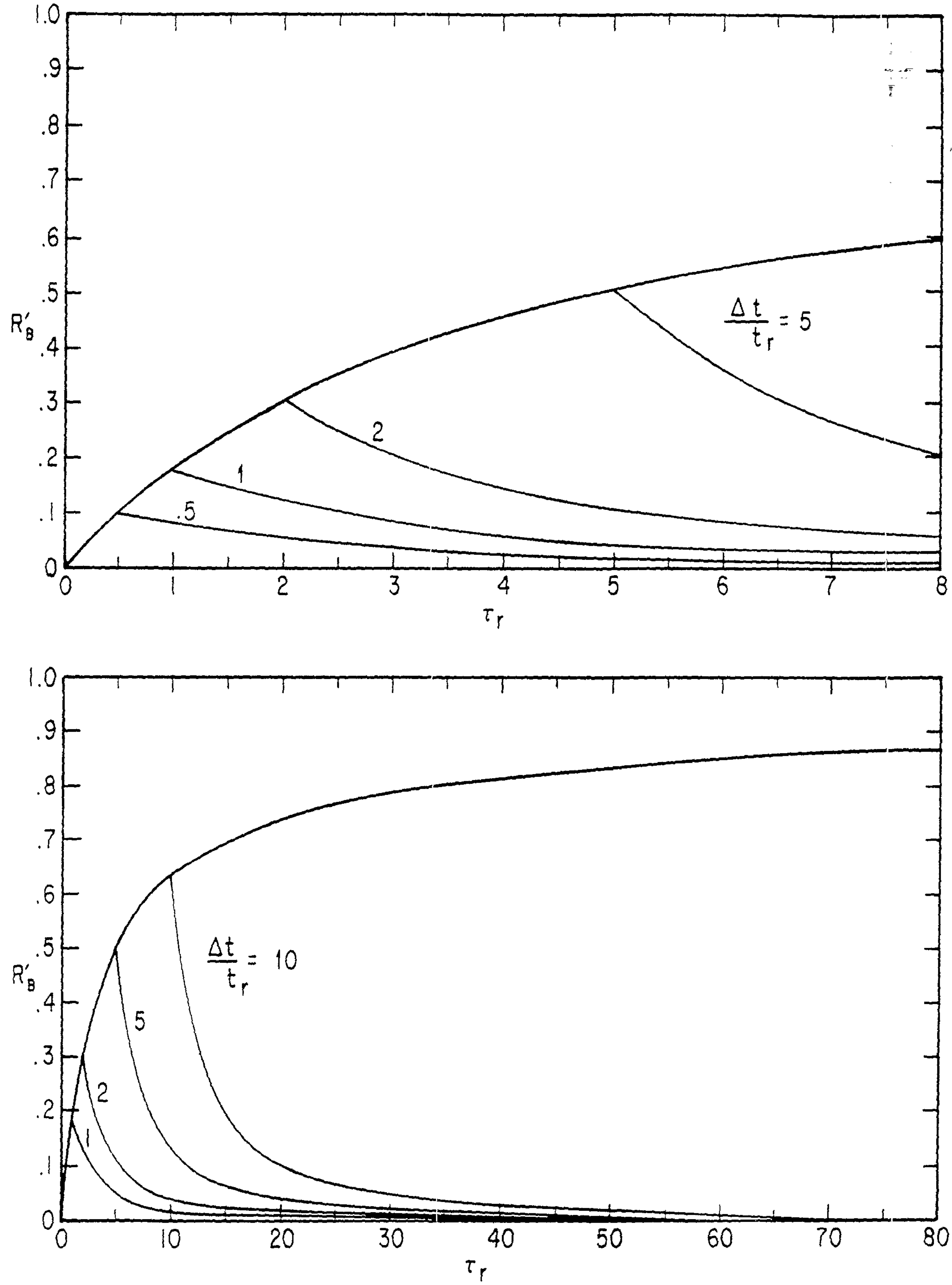

FIGURE 3. REFLECTION OF A RECTANGULAR PULSE AT THE HIGH-FREQUENCY BREWSTER ANGLE: $\epsilon_r = 10$

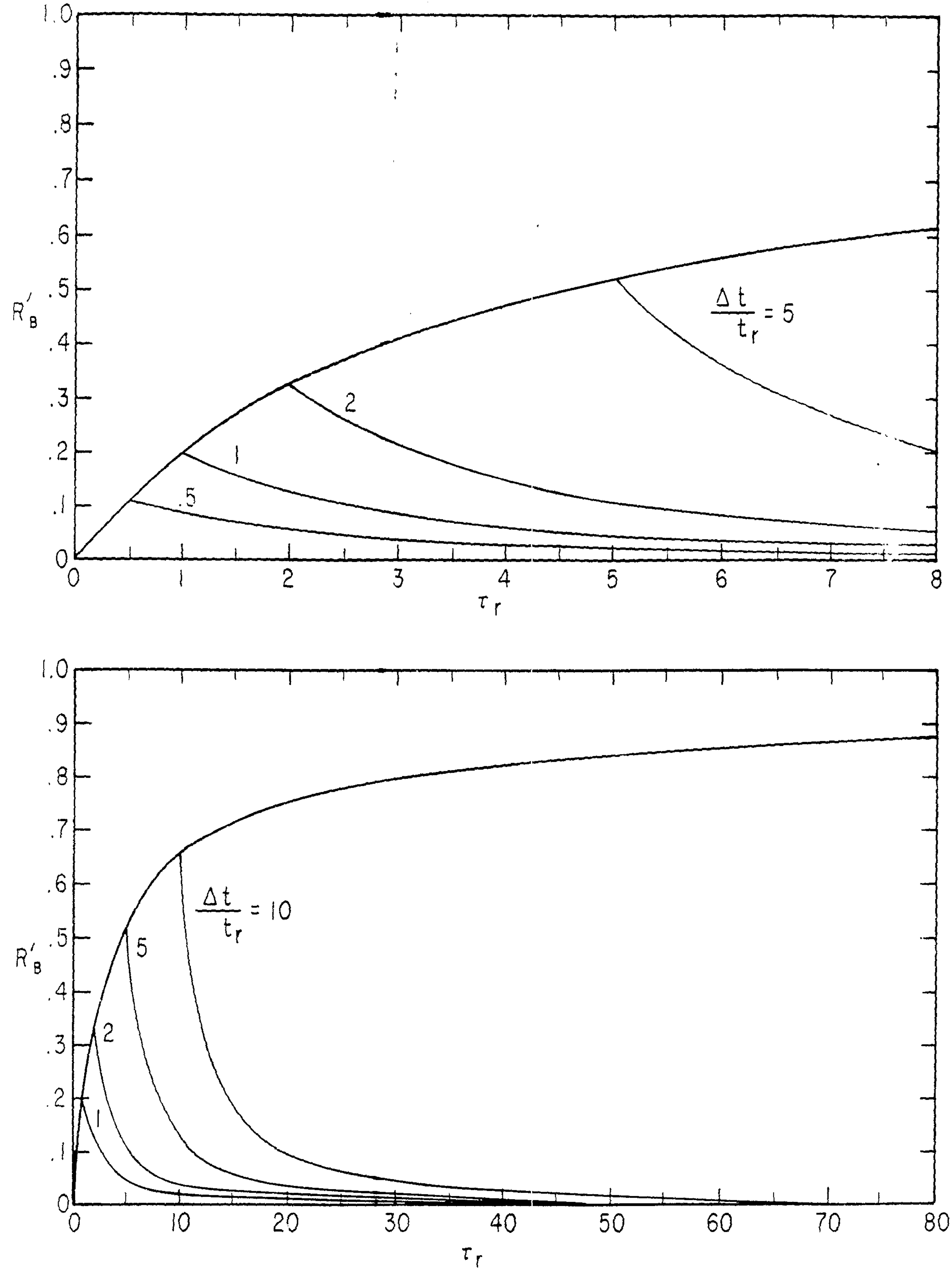

FIGURE 4. REFLECTION OF A RECTANGULAR PULSE AT THE HIGH-FREQUENCY BREWSTER ANGLE : $\epsilon_r = 80$

σ (mhos/m) \ ϵ_r	10	20	40	80	
10^{-4}	885	1770	3540	7080	t_r (ns)
10^{-3}	88.5	177	354	708	t_r (ns)
10^{-2}	8.85	17.7	35.4	70.8	t_r (ns)
10^{-1}	.885	1.77	3.54	7.08	t_r (ns)
4 (sea water)				.177	t_r (ns)
$\frac{2}{\pi}\psi_B$	.195	.1401	.0998	.0709	

Table I. Relaxation Time and Brewster Angle

The larger values of ϵ_r minimize ψ_B, possibly making the simulator structure more convenient for covering a large test area. However, one may not be able to choose this parameter. Note the range of the relaxation time for the permittivities and conductivities in table I. For the highest conductivities t_r is rather small compared to the transit times on simulator structures of about a hundred meters in extent; for the lowest conductivities t_r is rather large compared to such transit times. The Brewster angle wave matcher may then be an appropriate high-frequency simulator for the lower ground conductivities. In the case of the higher conductivities there may be significant reflections and the Brewster angle wave matcher might not be a good simulator. As an extreme case consider the very short relaxation time for sea water, much shorter than typical pulse widths of interest.

III. Summary

The Brewster angle wave matcher is a possible simulator for placing a fast-rising electromagnetic pulse over a ground surface. There are certain limitations on this technique in that the pulse width should be smaller than the relaxation time of the ground, and/or the transit times on the above-ground simulator structure should be smaller than this relaxation time. Otherwise, there may be a "ringing" effect due to multiple reflections of the incident pulse.

There are various imperfections in this approach, but they may not be overly significant in many cases. The high-frequency ground characteristics are not exactly the same as the idealizations in this note. There are fringing fields (on the sides of the cylindrical transmission line) which are not properly oriented to match into the ground at the high-frequency Brewster angle. There may be perturbations in the ground surface due to the presence of a system under test. It may be advisable to determine the effects of these various imperfections by actual measurements using the Brewster angle wave matcher. Fast, low-amplitude pulses could perhaps be used for measurements of the transfer function of such a simulator to determine some of the practical limitations of this simulation technique.

We would like to thank Mr. John N. Wood for the numerical calculations and the resulting graphs.